Homework 4, Due: Friday, 10/2

This assignment is due on **Friday, October 2**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, please remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

1 Consider the 4×4 linear system:

$$\begin{aligned} 4x_1 + x_2 - x_3 + x_4 &= -2 \\ x_1 + 4x_2 - x_3 - x_4 &= -1 \\ -x_1 - x_2 + 5x_3 + x_4 &= 0 \\ x_1 - x_2 + x_3 + 3x_4 &= 1 \end{aligned}$$

(a) (10 points) Will Jacobi's method converge for solving this linear system? Will the Gauss-Seidel method? Prove for each method using the convergence theorem given in lecture.

(b) (10 points) Write MATLAB code using Jacobi's method and the Gauss-Seidel method to solve this linear system. Start with $\mathbf{x}^{(0)} = \mathbf{0}$ and stop when $||\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}||_\infty < 10^{-5}$. Note that your results from part (a) should tell you whether or not each method will converge to the true solution. For each method, report your solution and the number of iterations it took the algorithm to converge (if it converges). Discuss your results.

2 (10 points) The nonlinear system

$$\begin{aligned} 3x_1 - \cos(x_2 x_3) - \frac{1}{2} &= 0 \\ x_1^2 - 625x_2^2 - \frac{1}{4} &= 0 \\ e^{-x_1 x_2} + 20x_3 + \frac{10\pi - 3}{3} &= 0 \end{aligned}$$

has a singular Jacobian matrix at the solution. Write a MATLAB code applying Newton's method with $\mathbf{x}^{(0)} = (1, 1, -1)^\mathsf{T}$ to approximate the solution to within 10^{-5} in the ℓ_∞ norm, and report your results. Does Newton's method converge? What if you use $\mathbf{x}^{(0)} = (1, 0, -1)^\mathsf{T}$ instead? Report your findings in each case.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Any code that you submit should be your own. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.