Homework 2, Due: Friday, 9/18

This assignment is due on **Friday, September 18**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, please remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

1 (10 points) The function defined by $f(x) = \sin(\pi x)$ has a root at every integer. Show that when $-1 < a < 0$ and $2 < b < 3$, the Bisection method converges to:

(a) 0, if $a + b < 2$

(b) 2, if $a + b > 2$

(c) 1, if $a + b = 2$

2 (10 points) If g is a function satisfying the conditions of the Fixed-Point Theorem, prove that the bounds for the error involved in using p_n (generated by fixed-point iteration) to approximate the fixed point p are given by

$$| p_n - p | \leq k^n \max\{p_0 - a, b - p_0\}$$

and

$$| p_n - p | \leq \frac{k^n}{1-k} | p_1 - p_0 | \quad \text{for all } n \geq 1.$$

This is a corollary to the Fixed-Point Theorem (see Slide 9 of the "Root-Finding Methods (Part 2)" lecture).

3 (10 points) Use the Existence and Uniqueness Theorem for Fixed Points (see Slide 3 of the "Root-Finding Methods (Part 2)" lecture) to show that $g(x) = \pi + 0.5 \sin(x/2)$ has a unique fixed point on $[0, 2\pi]$. Implement and use fixed-point iteration in MATLAB to find an approximation to the fixed point that is accurate to within 10^{-2}. Use the corollary of the Fixed-Point Theorem from Problem 2 to estimate the number of iterations required to achieve 10^{-2} accuracy, and compare this theoretical estimate to the number actually needed.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Any code that you submit should be your own. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.