Homework 3, Due: Friday, 9/25

This assignment is due on **Friday, September 25**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, please remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

1 (10 points) Prove that if $f(p) = f'(p) = 0$ and $f''(p) \neq 0$, then Newton's method converges linearly to p with asymptotic error constant $k = \dfrac{1}{2}$.

2 As discussed in lecture, the success of Newton's method depends upon the initial approximation p_0. In cases where p_0 is not sufficient, a global method (such as the bisection method) can be used to find a better starting point for Newton's method. A hybrid algorithm of this type is called a quasi-Newton algorithm.

(a) (10 points) Write a MATLAB code implementing a hybrid quasi-Newton algorithm that first takes a few iterations of the bisection method (global step) before applying Newton's method (local step). You can make use of the `bisection.m` code on Canvas (or your own version) for the global step and write your own Newton's method code for the local step. Apply your hybrid scheme to find the root of

$$f(x) = \sin(x) - \cos(x) \quad \text{for} \quad -1 \leq x \leq 3$$

stopping either when the relative error is less than 10^{-6} or when a maximum number of iterations is reached. Compare your results from the hybrid scheme to those using only Newton's method with $p_0 = 2$ and those using only the bisection method with the same stopping criteria. Describe the convergence (and the rate of convergence) observed in each case.

(b) (10 points) Write a MATLAB code that implements your hybrid scheme using the secant method for the local step (in place of Newton's method). You will need to write code that implements the secant method, and modify your hybrid algorithm from part (a) accordingly. Apply your hybrid scheme to find the root of

$$f(x) = x^3 - 7x^2 + 11x - 5 \quad \text{for} \quad 2 \leq x \leq 9$$

stopping either when the relative error is less than 10^{-6} or when a maximum number of iterations is reached. Compare your results from the hybrid scheme to those using only the secant method with $p_0 = 2$ and $p_1 = 7$ and those using only the bisection method with the same stopping criteria. Describe the convergence (and the rate of convergence) observed in each case.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Any code that you submit should be your own. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.