Homework 6, Due: Friday, 10/16

This assignment is due on **Friday, October 16**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, please remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

$\boxed{1}$ Let f be defined on $[a, b]$ and let the nodes $a = x_0 < x_1 < x_2 = b$ be given. A quadratic spline interpolating function S consists of the quadratic polynomial

$$S_0(x) = a_0 + b_0(x - x_0) + c_0(x - x_0)^2 \quad \text{for} \quad x_0 \leq x \leq x_1$$

and the quadratic polynomial

$$S_1(x) = a_1 + b_1(x - x_1) + c_1(x - x_1)^2 \quad \text{for} \quad x_1 \leq x \leq x_2$$

such that

(i) $S(x_0) = f(x_0), \quad S(x_1) = f(x_1), \quad \text{and} \quad S(x_2) = f(x_2)$

(ii) $S \in C^1[x_0, x_2]$

(a) (5 points) Show that conditions (i) and (ii) lead to five equations in the six unknowns a_0, b_0, c_0, a_1, b_1, and c_1.

(b) (5 points) It remains to decide what additional condition to impose to make the solution unique. Does the condition $S \in C^2[x_0, x_2]$ lead to a meaningful solution? Explain why or why not.

$\boxed{2}$ Consider the following three data points:

$$(0, 0), \qquad (1, 1), \qquad (2, 2)$$

(a) (5 points) Determine a quadratic spline $S(x)$ that interpolates the given data and satisfies $S'(0) = 2$.

(b) (5 points) Plot and compare your quadratic spline from part (a) to the linear spline and natural cubic spline interpolants through this same set of data. What do you notice about each spline?

$\boxed{3}$ (10 points) Write a MATLAB code that generates a natural cubic spline $S(x)$ through the "ruddy duck" data discussed in class:

x_i	0.9	1.3	1.9	2.1	2.6	3.0	3.9	4.4	4.7	5.0	6.0	7.0	8.0	9.2	⋯
y_i	1.3	1.5	1.85	2.1	2.6	2.7	2.4	2.15	2.05	2.1	2.25	2.3	2.25	1.95	

⋯	10.5	11.3	11.6	12.0	12.6	13.0	13.3
	1.4	0.9	0.7	0.6	0.5	0.4	0.25

The data is contained in `ruddyduck_data_script.m`, which is posted on Canvas.

Your code should build the matrix A and vector $\mathbf{b}$ for the $(n+1) \times (n+1)$ linear system $\mathsf{A}\mathbf{x} = \mathsf{b}$, whose solution vector $\mathbf{x}$ contains the unknown c_j coefficients for the natural cubic spline. You can solve this system using the backslash command \ in MATLAB, then use the c_j coefficients to solve for the remaining unknown coefficients.

Once you have obtained all $4n$ coefficients, you can plug them into the appropriate pieces of the cubic spline and interpolate at a dense set of evaluation points (as in the example `Arnold_MA510_naturalspline_ex.m` shown in class and posted on Canvas). Plot your resulting cubic spline $S(x)$ at your evaluation points, along with markers showing the data points (x_i, y_i) for each $i = 0, \ldots, n$.

(Note that while you may follow the steps outlined in `Arnold_MA510_naturalspline_ex.m`, you'll want to modify your code for this problem to account for the fact that there are many more data points!)

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Any code that you submit should be your own. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.