COMP90038
Algorithms and Complexity

Lecture 2: Review of Basic Concepts
(with thanks to Harald Søndergaard)

Toby Murray

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DMD 8.17 (Level 8, Doug McDonell Bldg)

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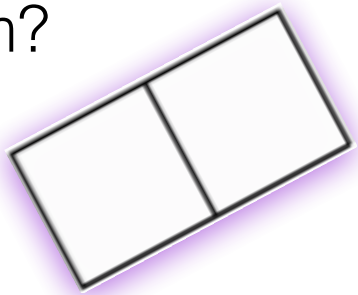
@tobycmurray

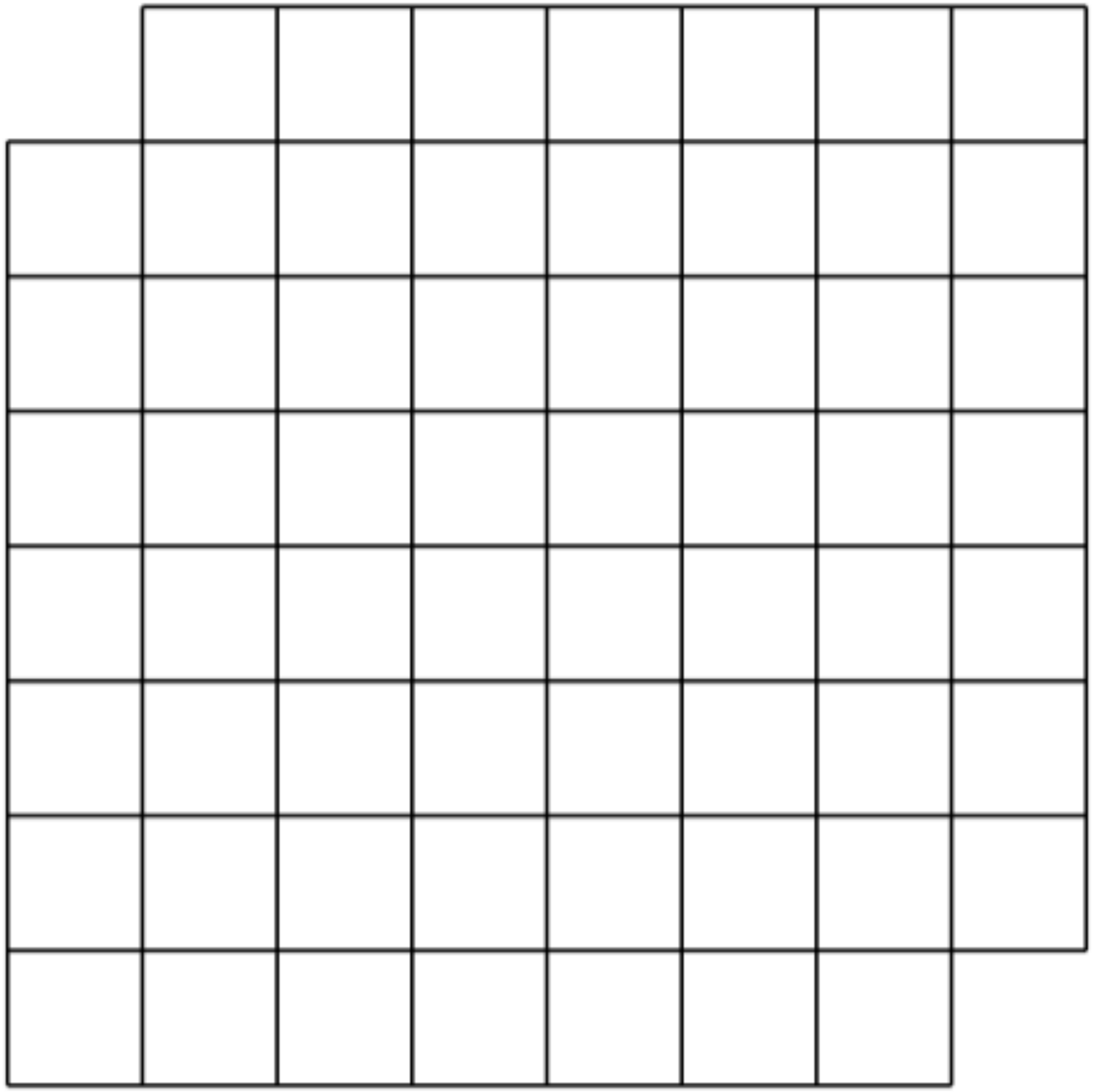
Approaching a problem

- Can we cover this board with 31 tiles of the following form?



- This is the **mutilated checkerboard problem**.

- There are only finitely many ways we can arrange the 31 tiles, so there is a brute-force (and very inefficient) way of solving the problem.



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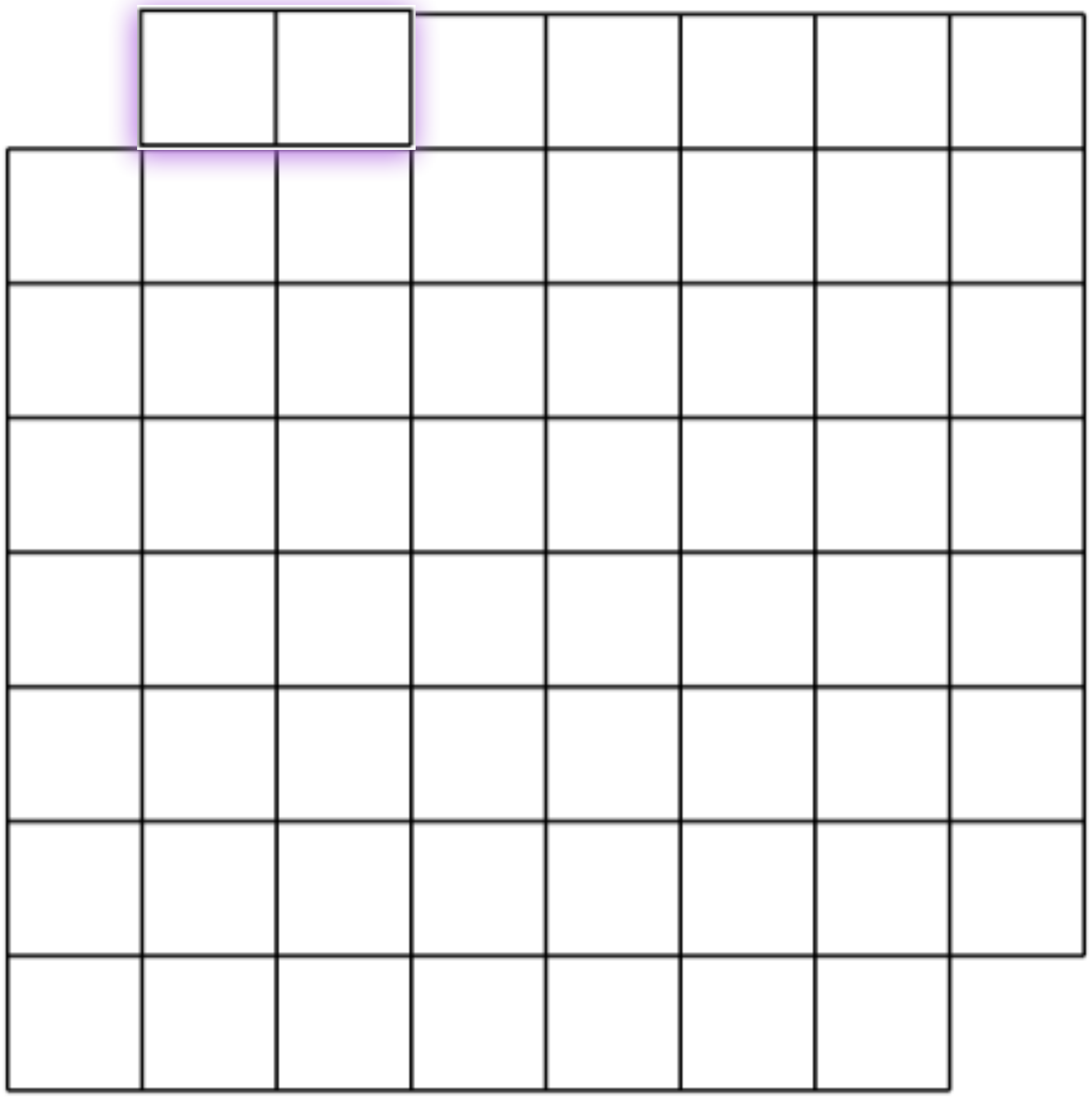

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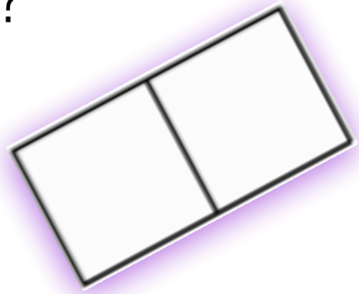

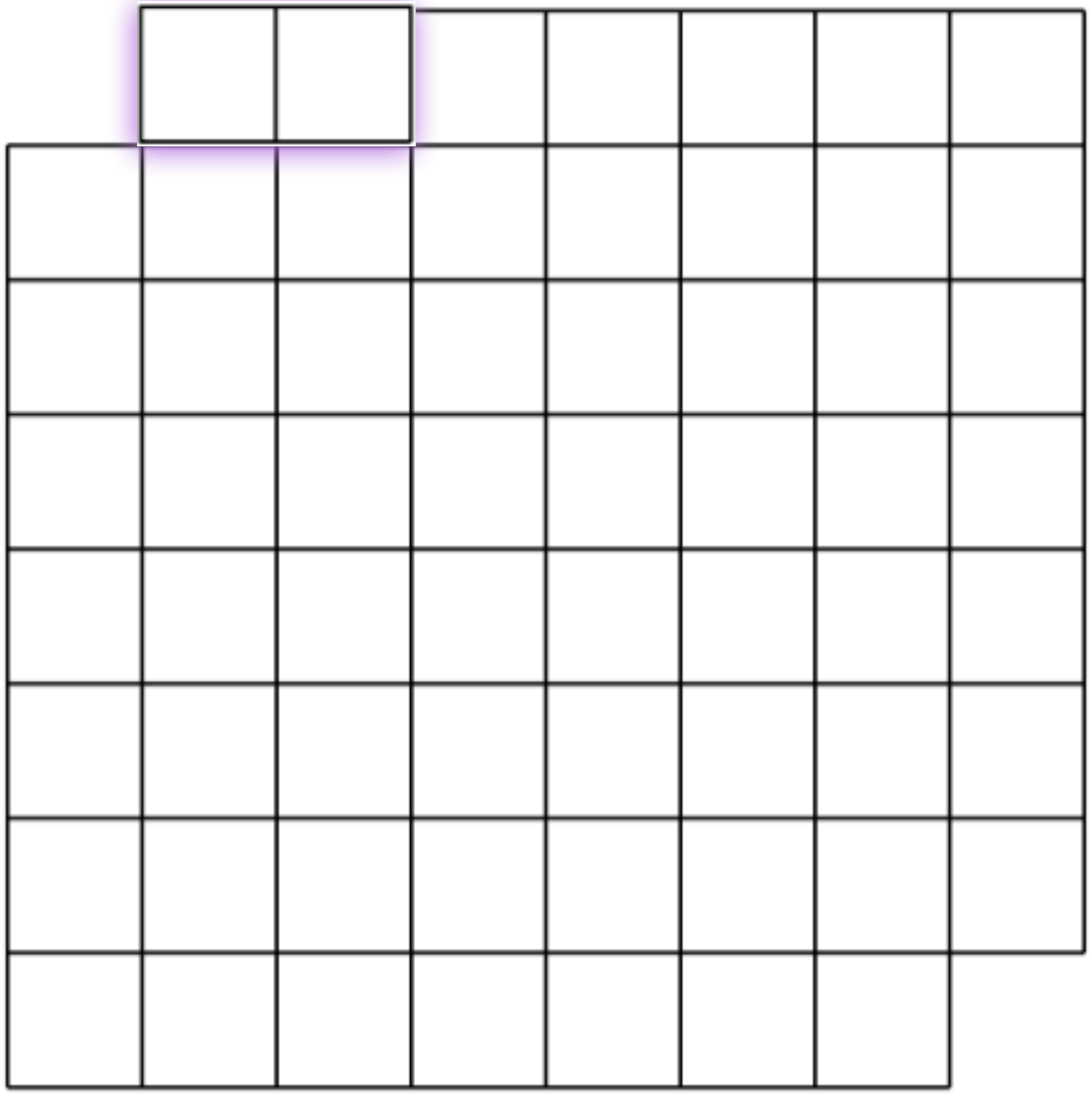

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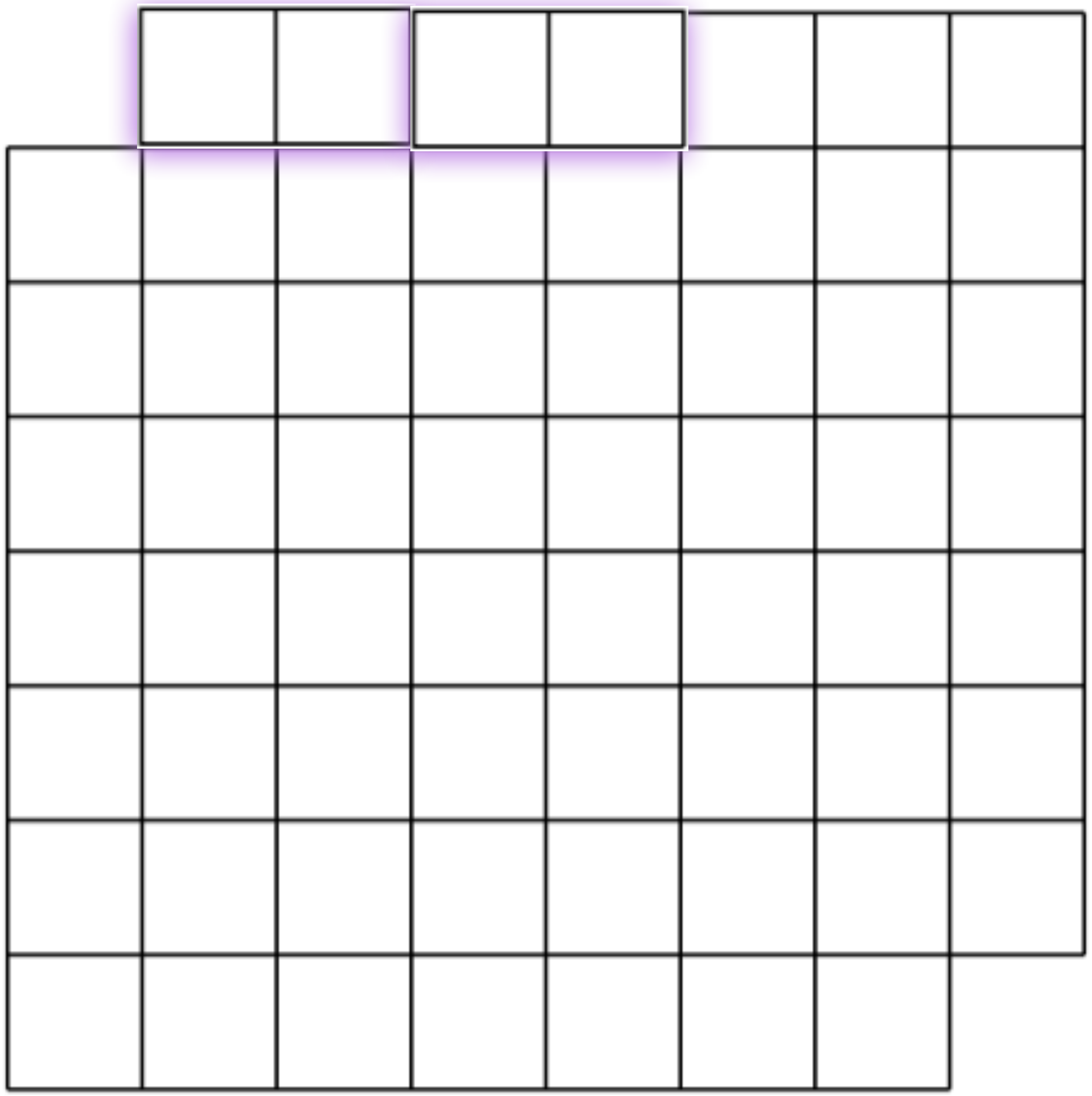

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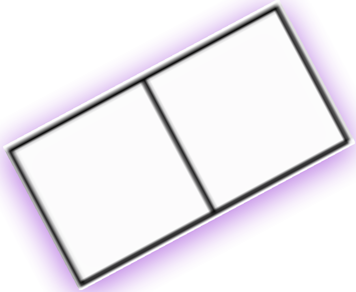

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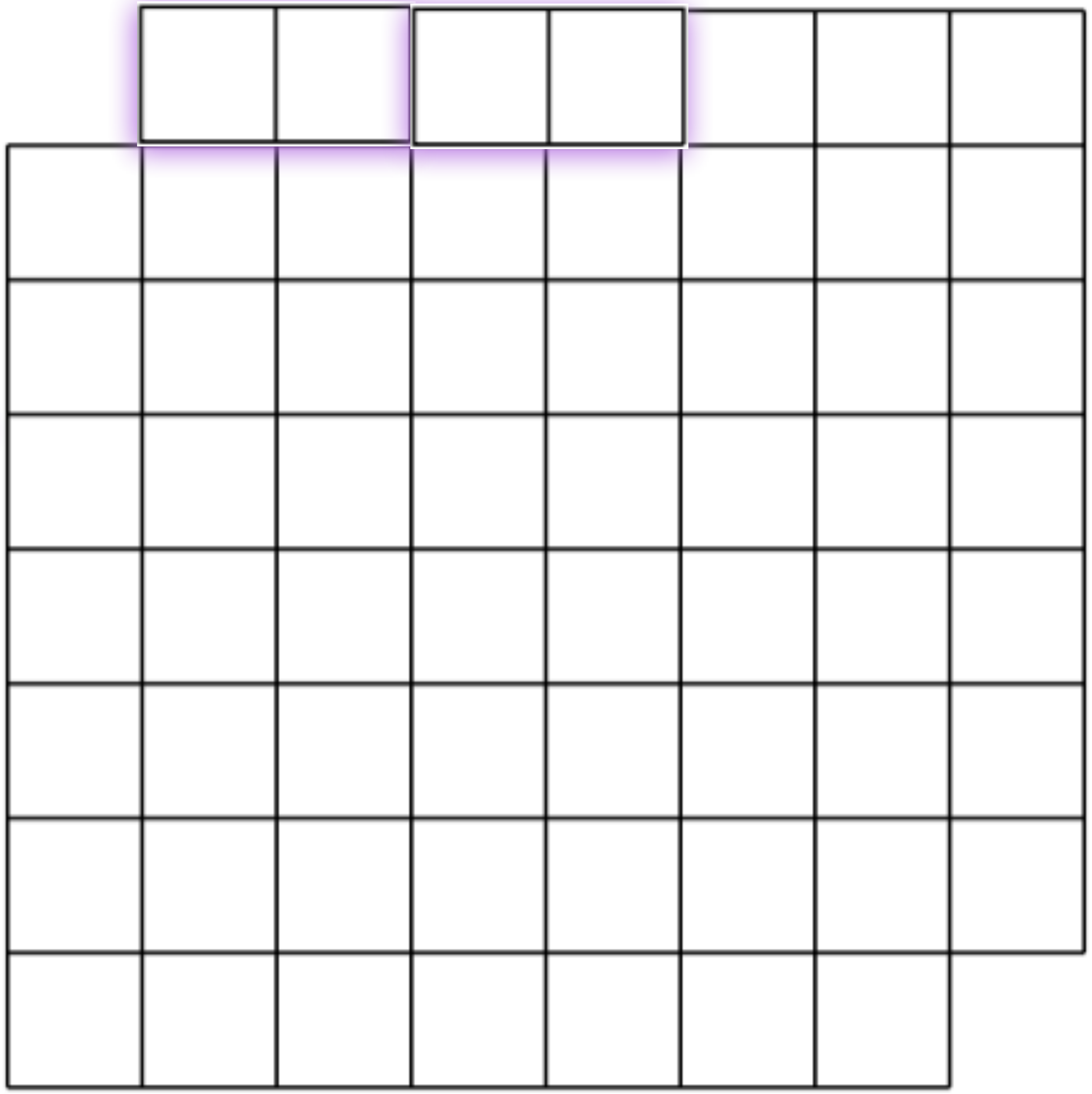

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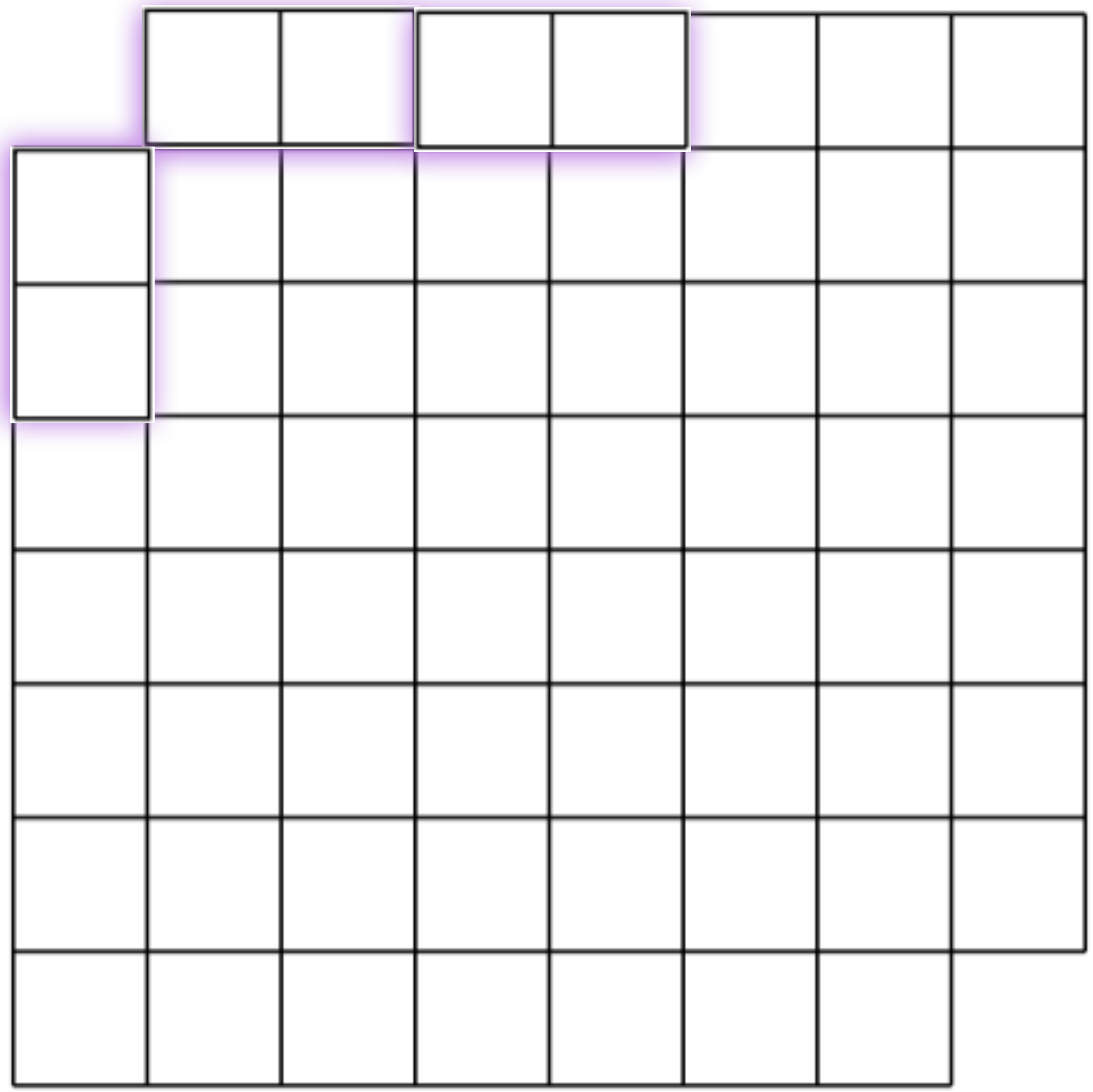

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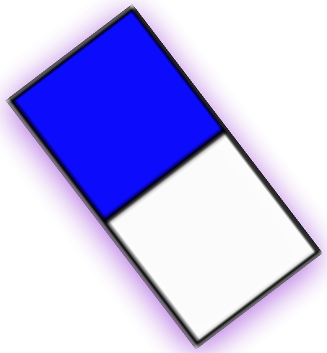

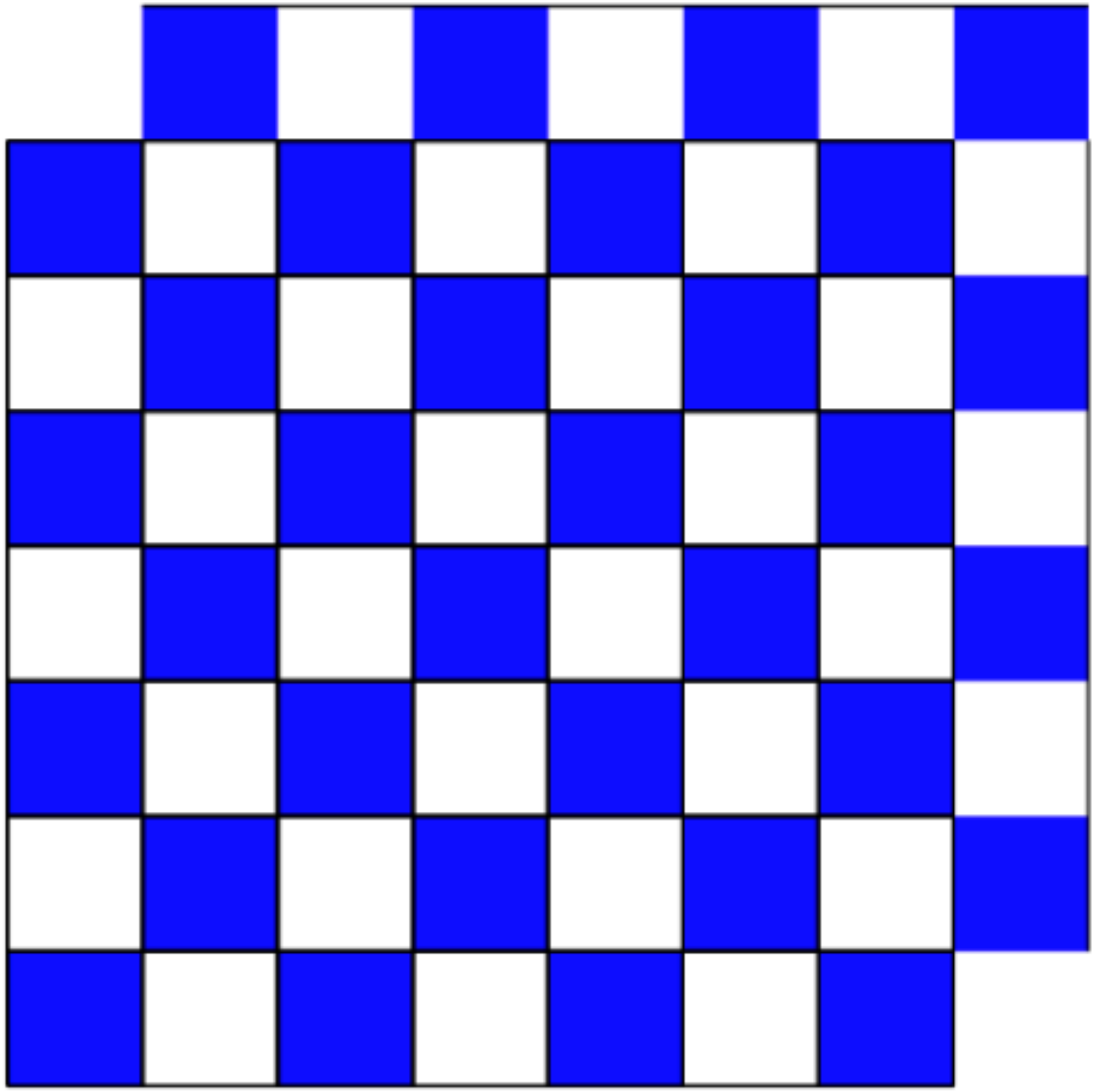

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- **Hint:** Using the way the squares are coloured helps.

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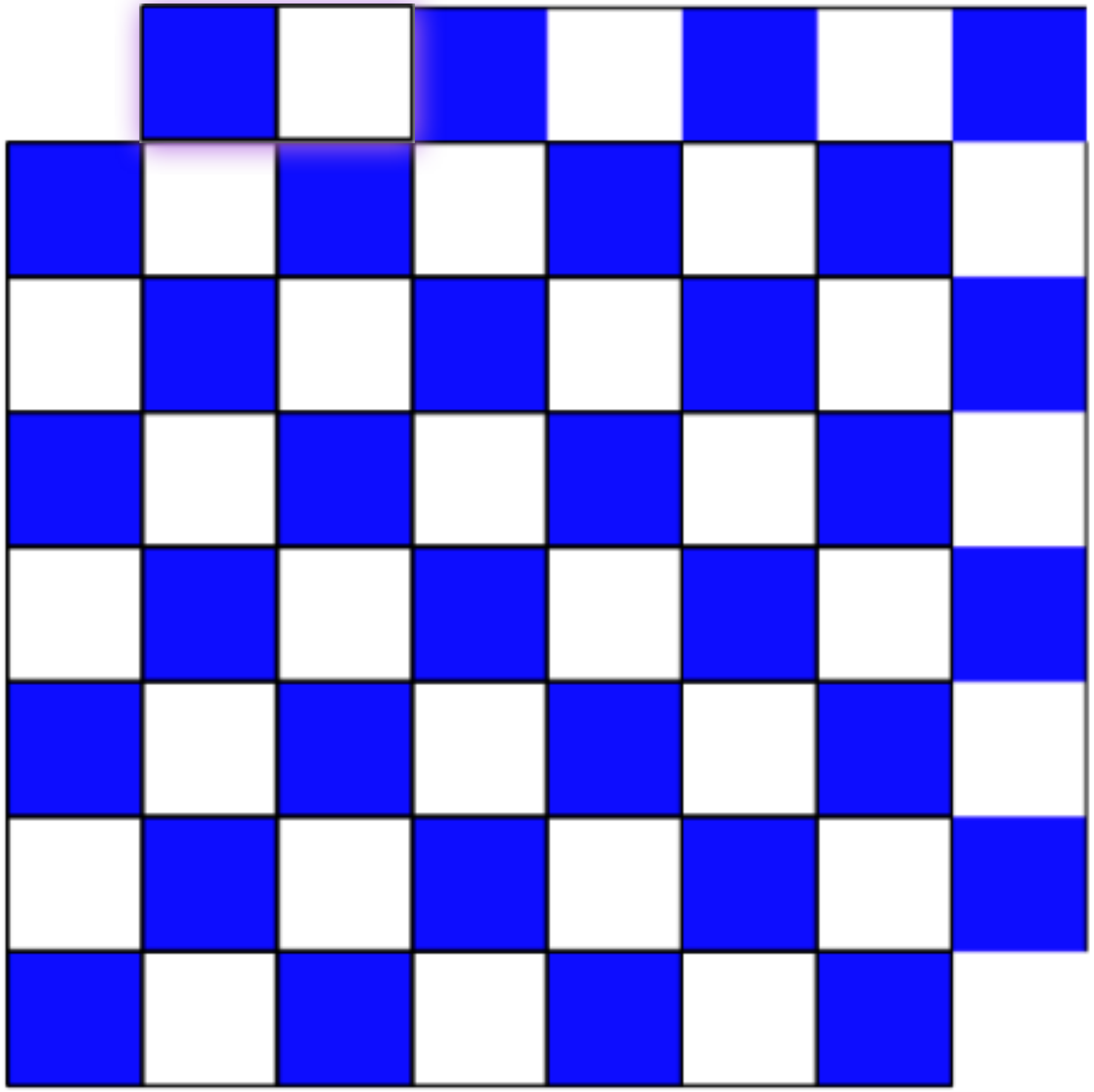

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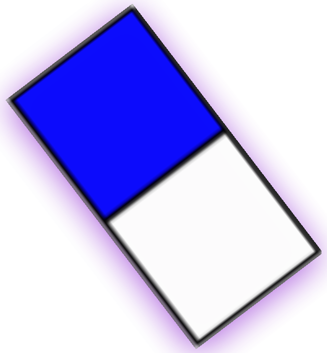

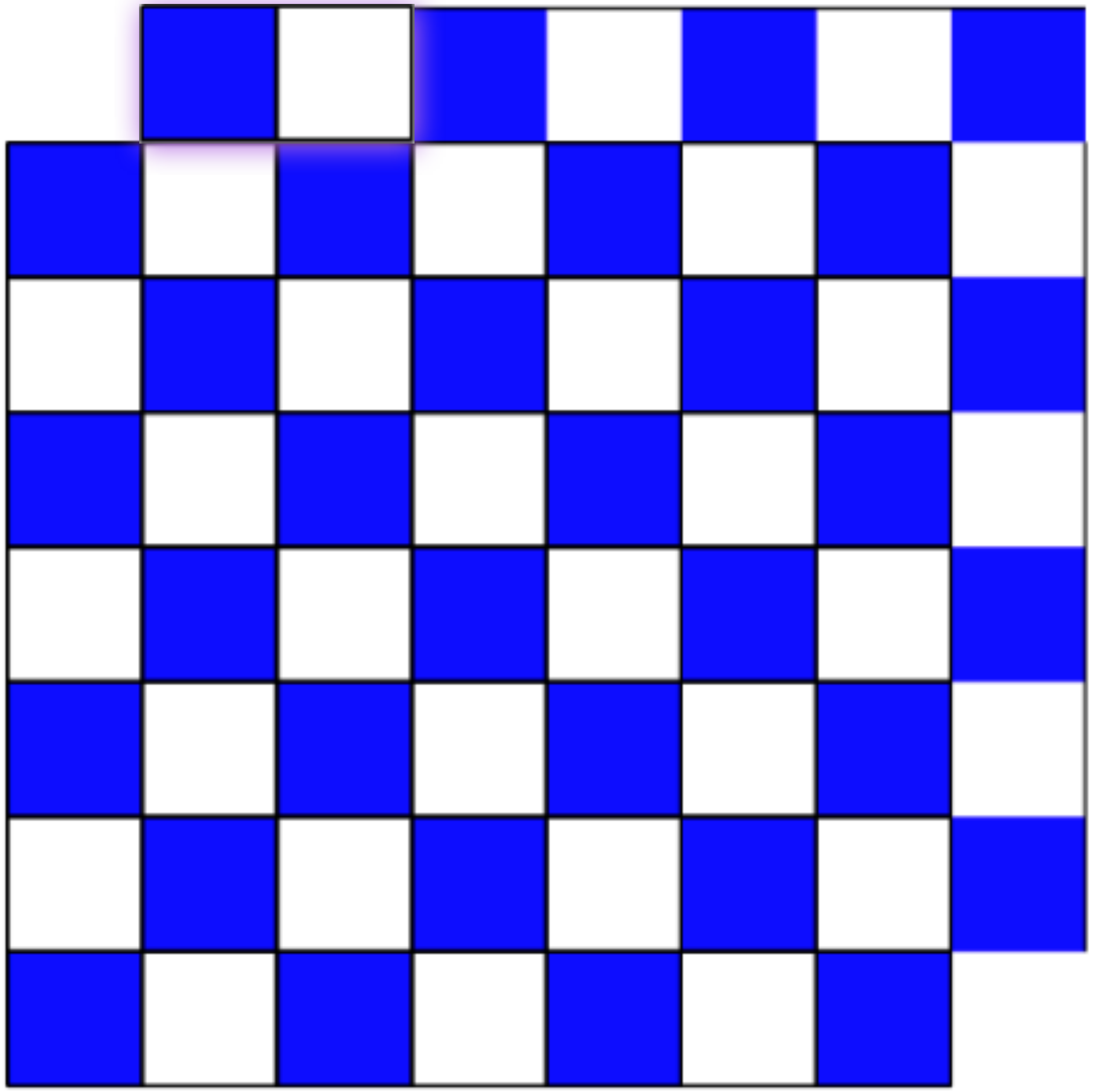

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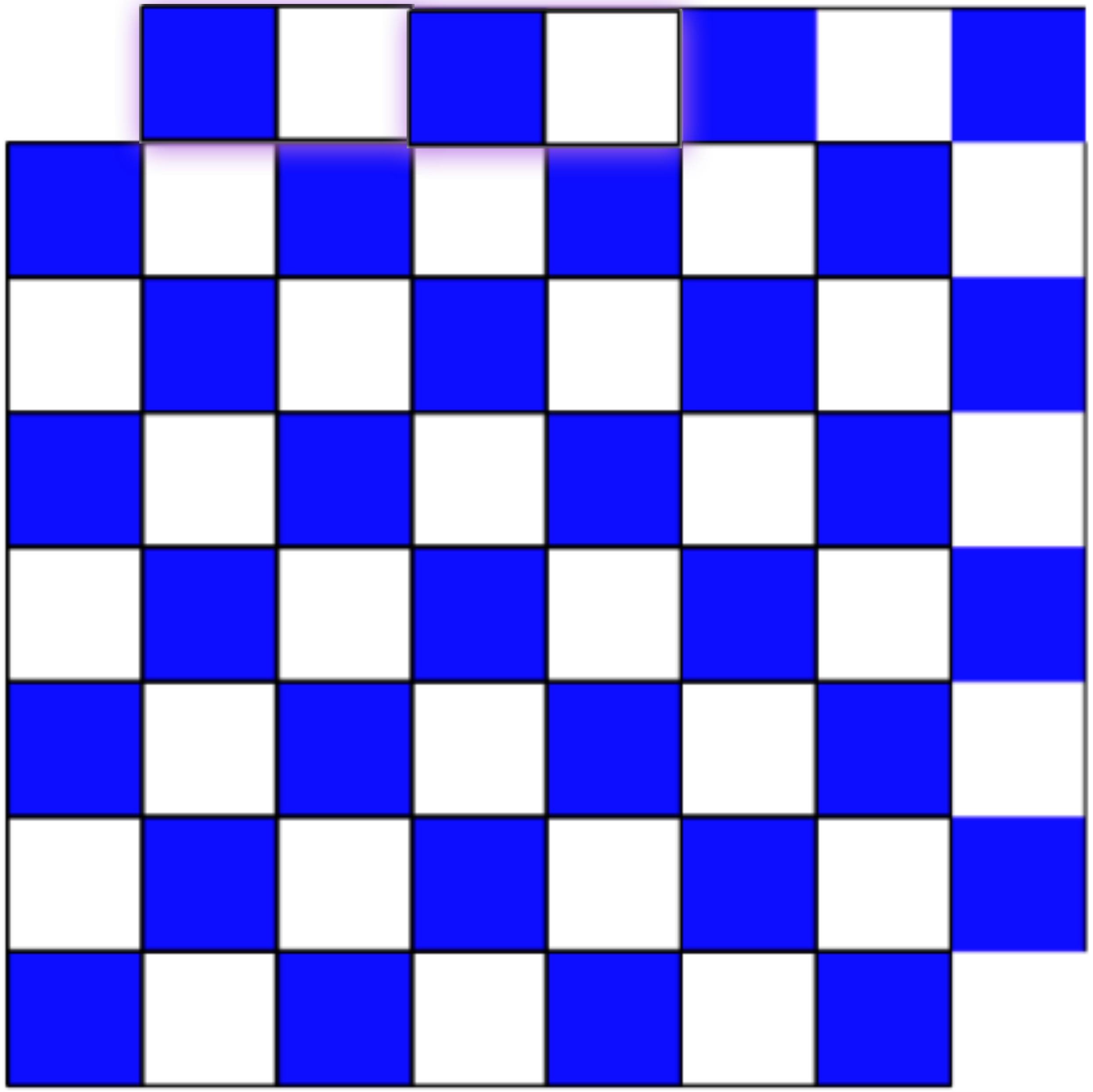

Algorithms and Data Structures

- **Algorithms**: for solving problems, transforming data.
- **Data structures**: for storing data; arranging data in a way that suits an algorithm.
  - **Linear** data structures: stacks and queues
  - Trees and graphs
  - Dictionaries
- Which data structures are you familiar with?

Exercise

- Pick you favourite data structure and describe:
  - How to insert and item into the data structure
  - How to find an item
  - How to handle duplicate items

Primitive Data Structures: The Array

- An array corresponds to a sequence of consecutive cells in memory.
- Depending on programming language: `A[0]` up to `A[n-1]`, or `A[1]` up to `A[n]`.
- Locating a cell, and storing or retrieving data at that cell is very fast.
- The downside of an array is that maintaining a contiguous bank of cells with information can be difficult and time-consuming.

6	9	2	3	7	5	8
0	1	2	3	4	5	6

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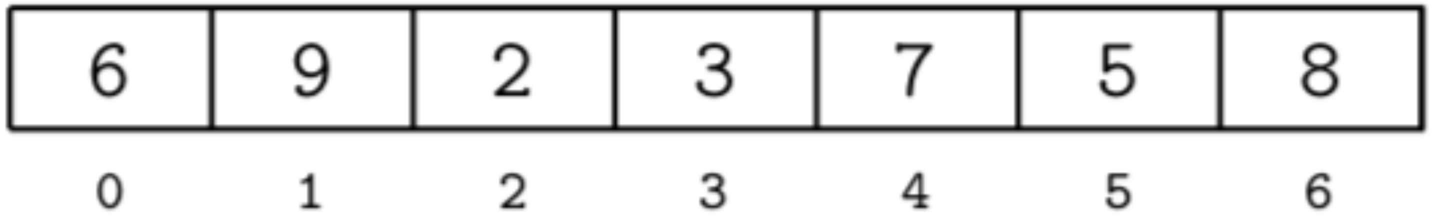

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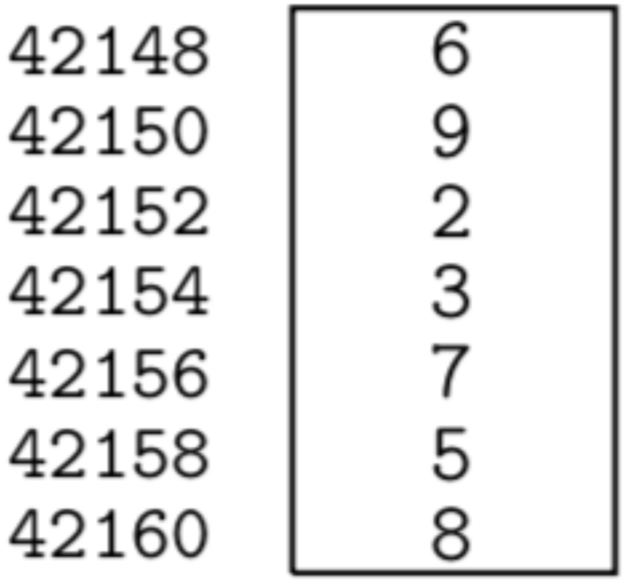

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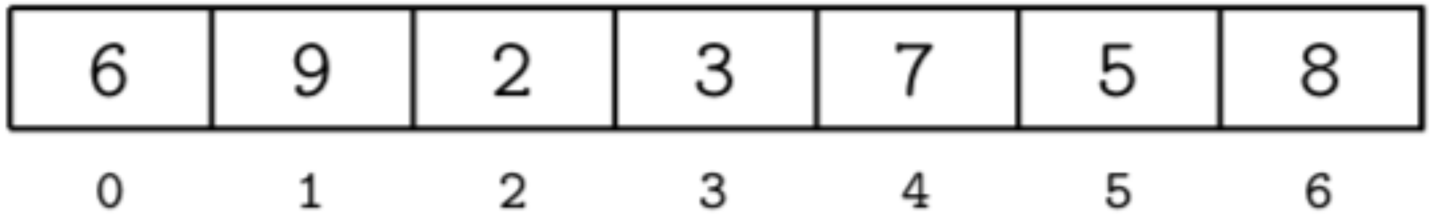

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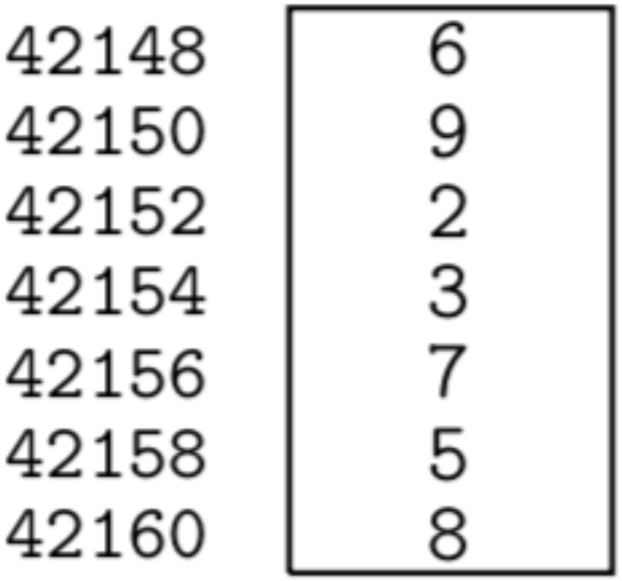

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How many bytes does each integer occupy here?

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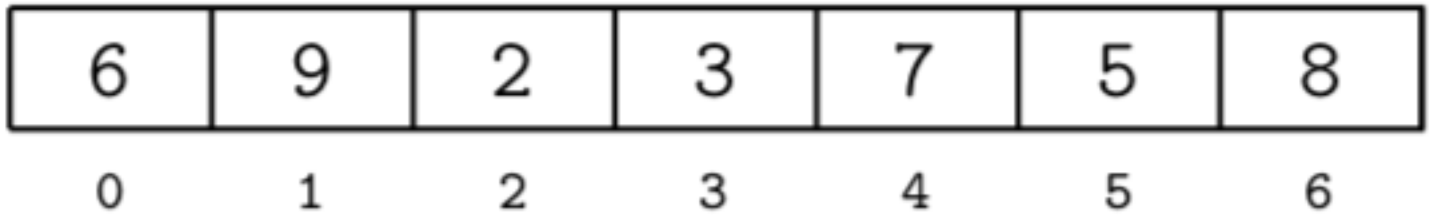

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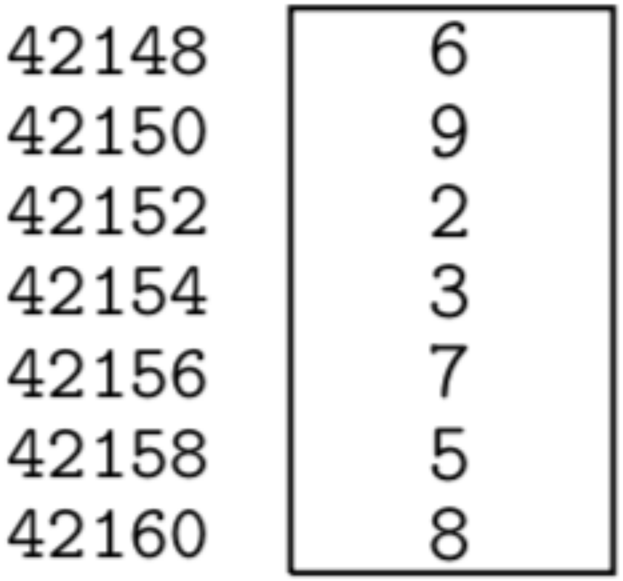

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How many bytes does each integer occupy here?

Answer: 2 (16-bit integers)

Primitive Data Structures: The Linked List

An array `x`:

2	3	5	7

Primitive Data Structures: The Linked List

2

3

5

7

Primitive Data Structures: The Linked List

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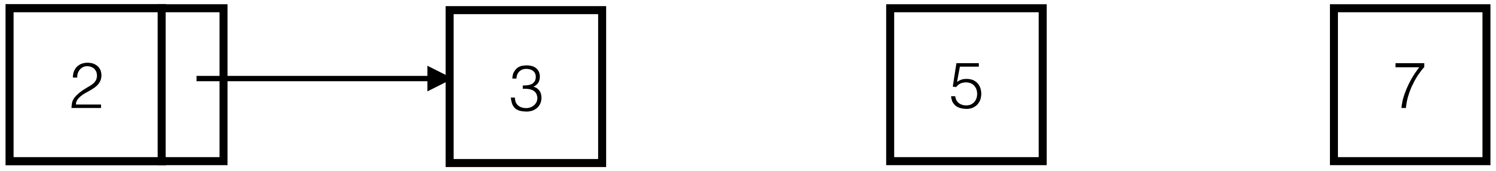

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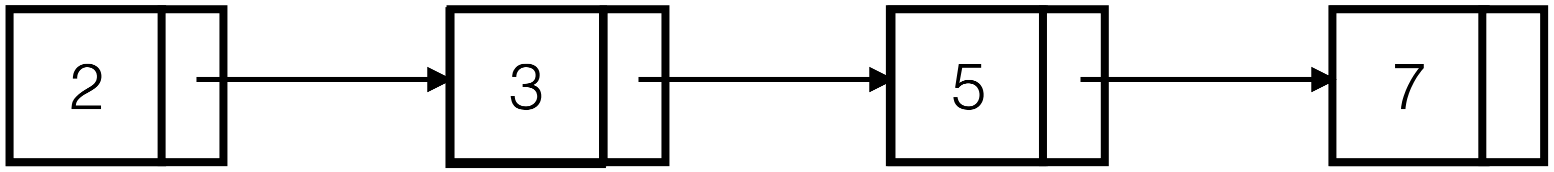

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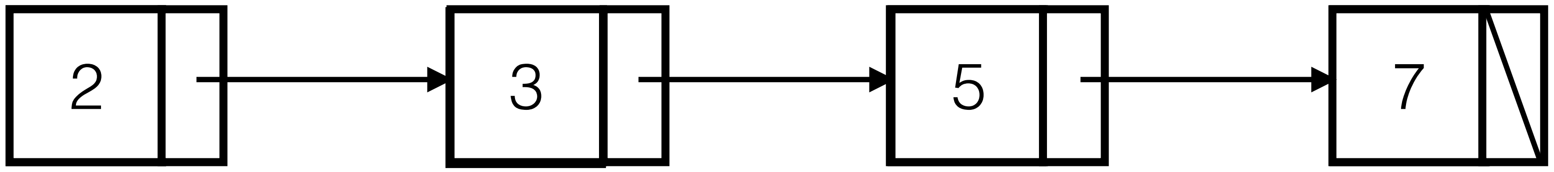

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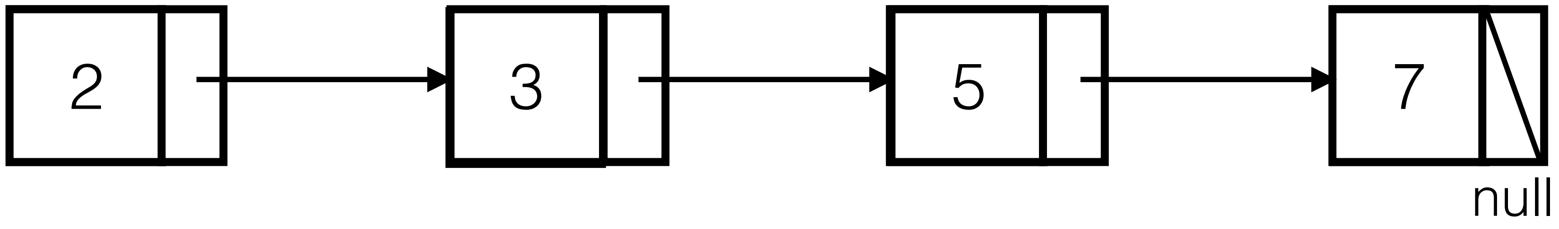
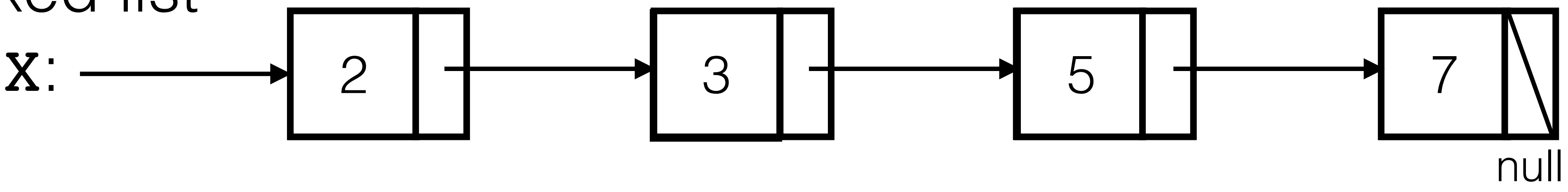


Primitive Data Structures: The Linked List

A linked list



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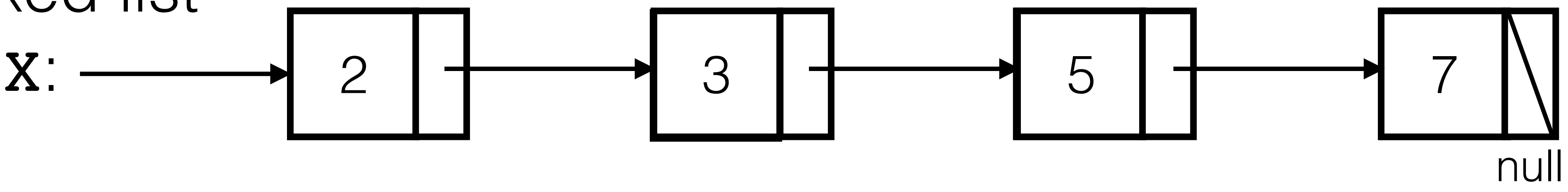

Suppose x corresponds to address 42160, then the list looks like this in memory:

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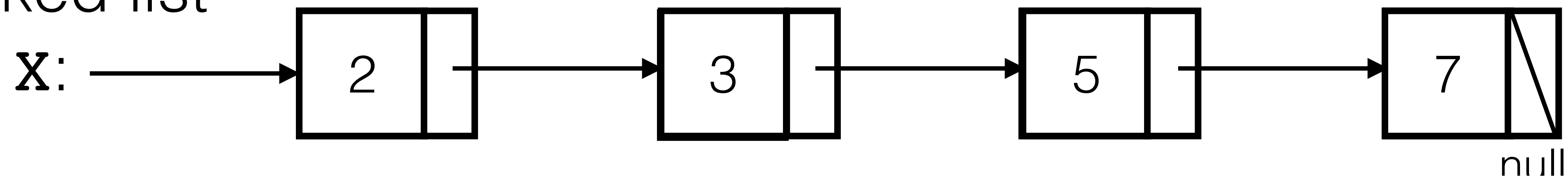

Suppose X corresponds to address 42160, then the list looks like this in memory:

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42158	
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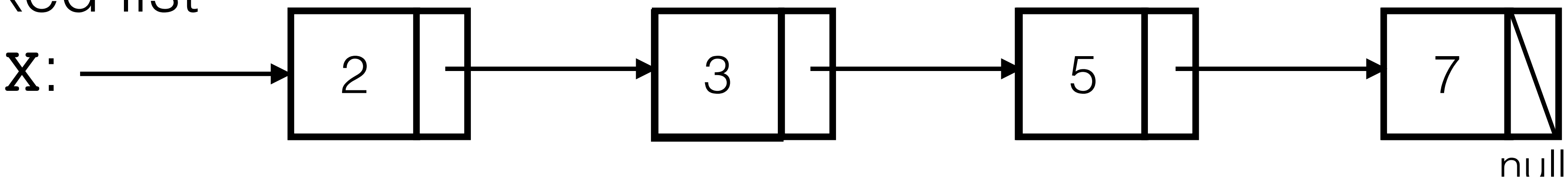
Primitive Data Structures: The Linked List

A linked list



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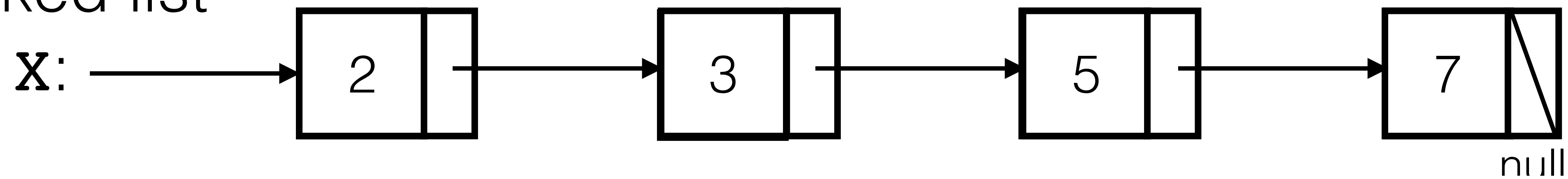

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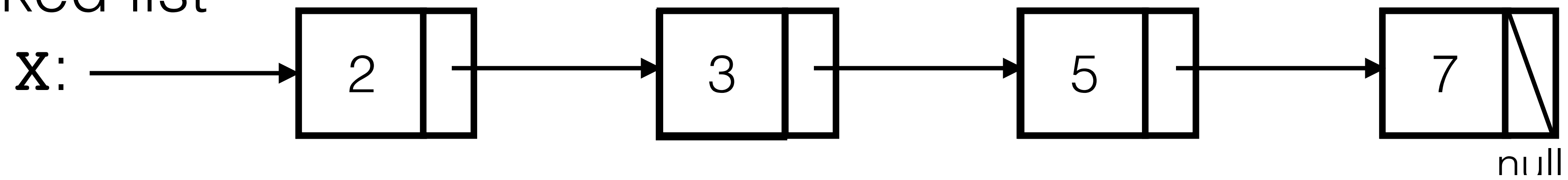

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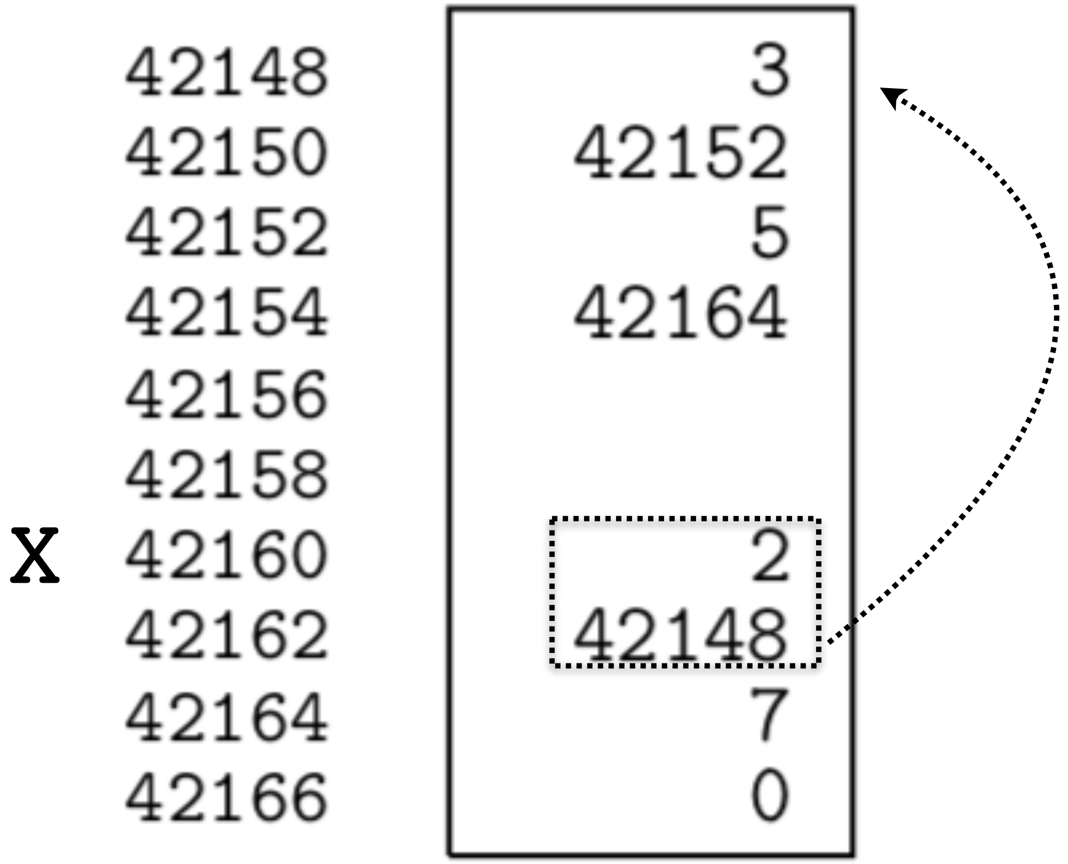

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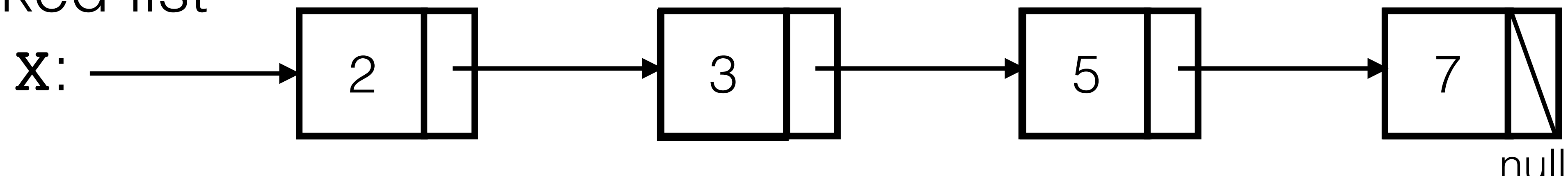

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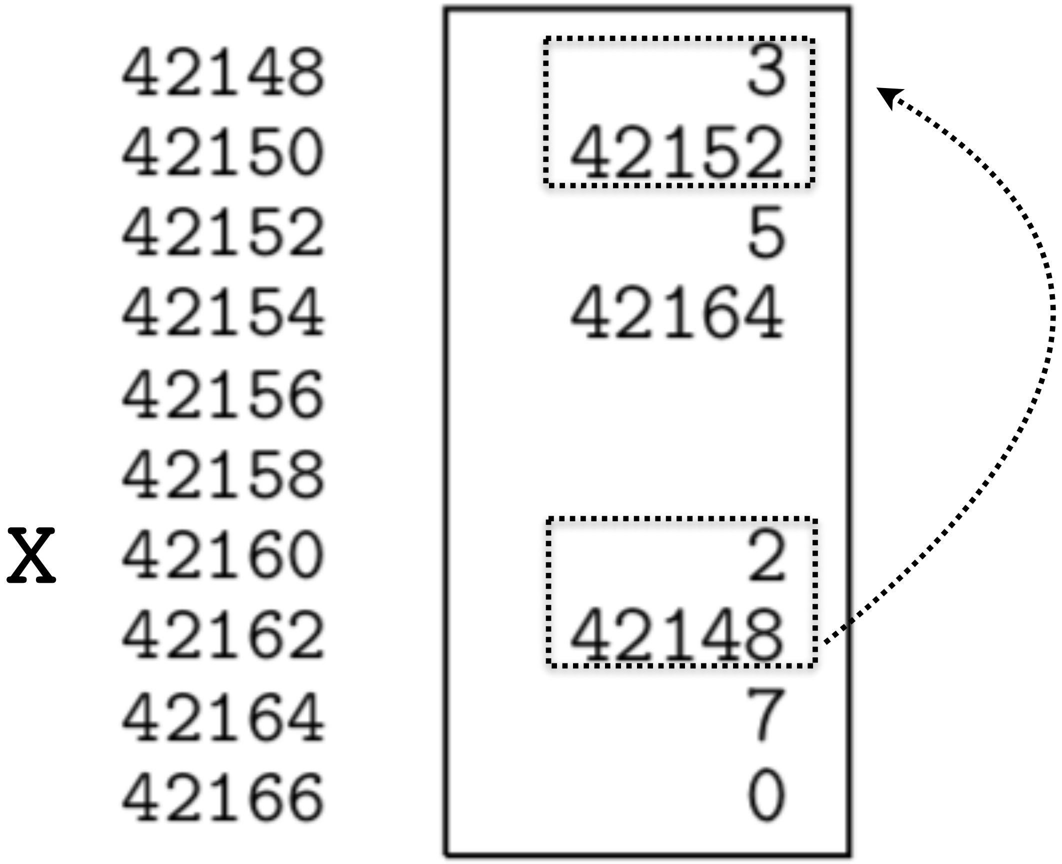

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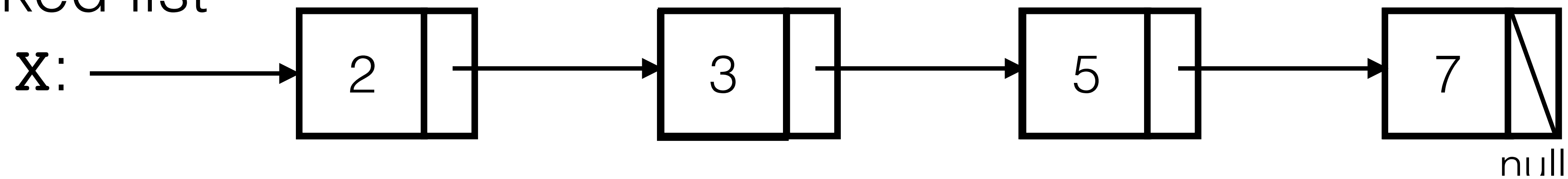

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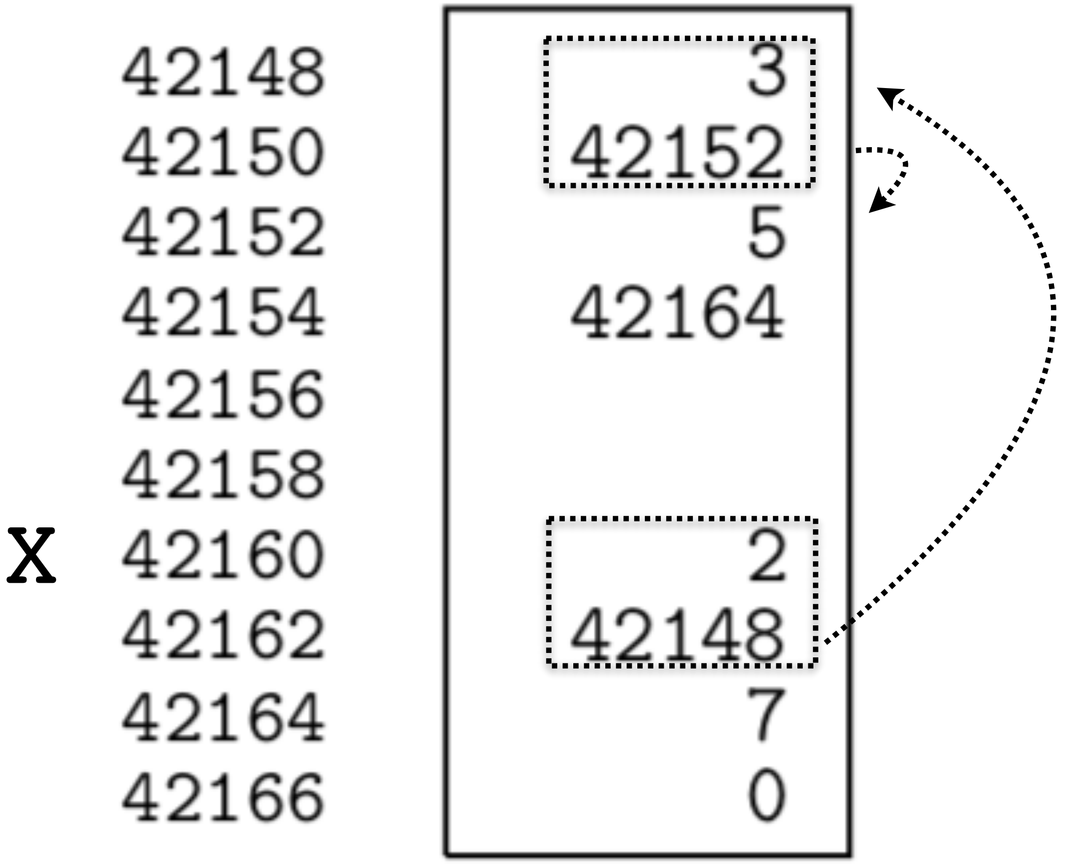

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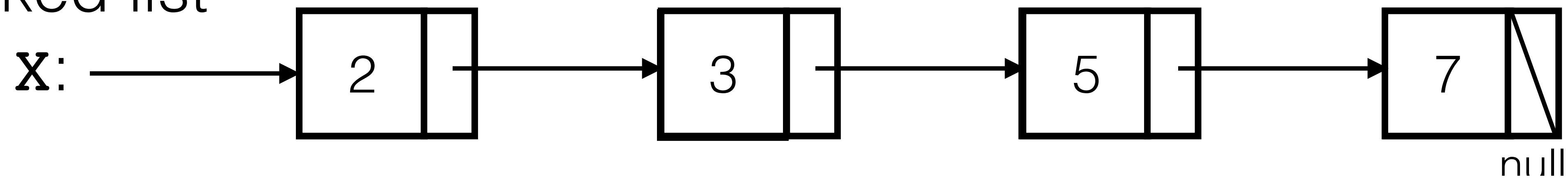

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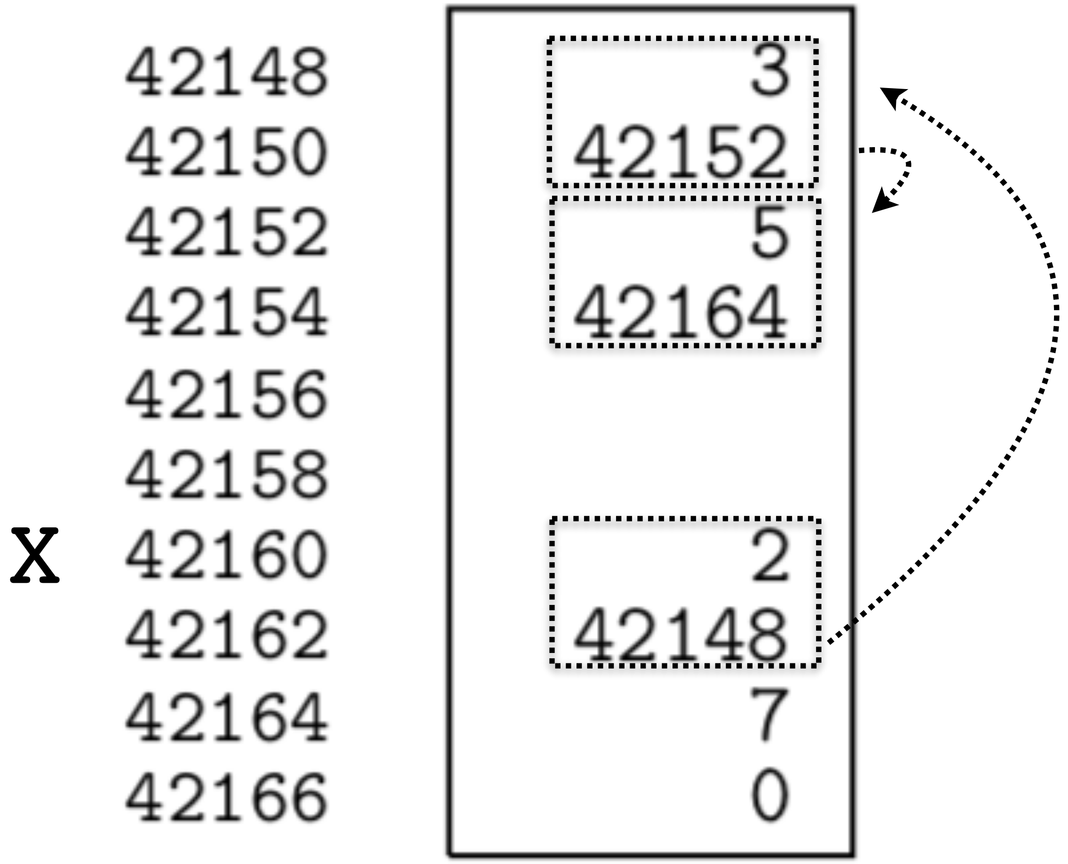

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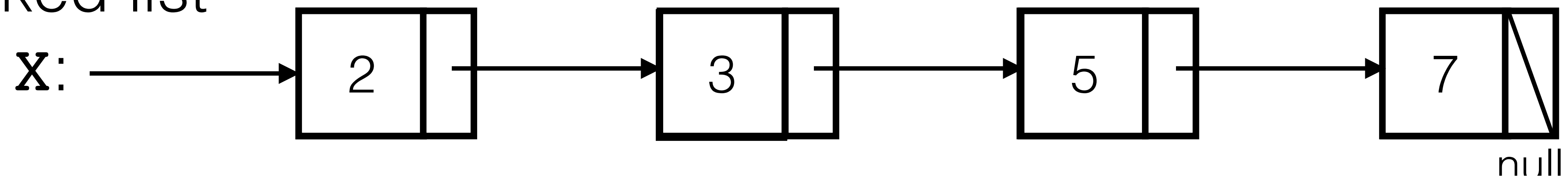

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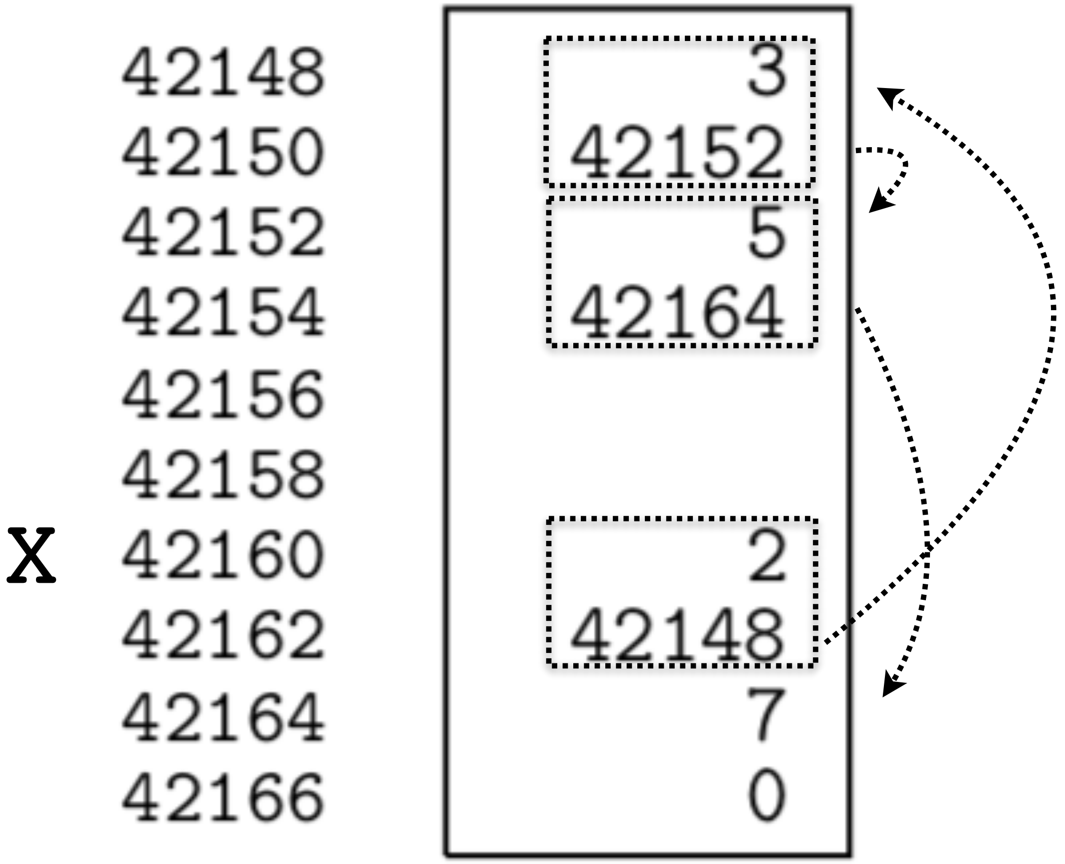

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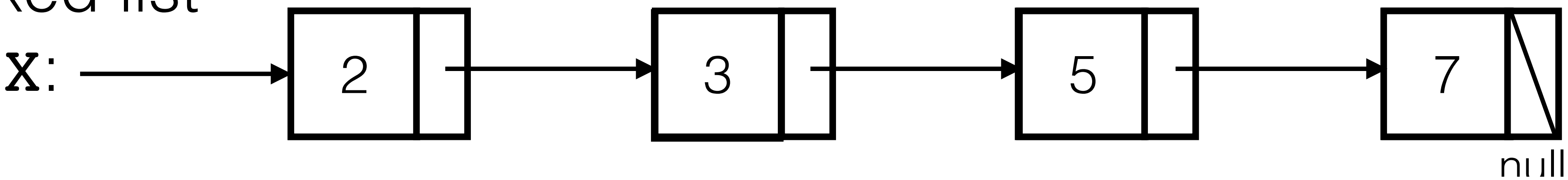

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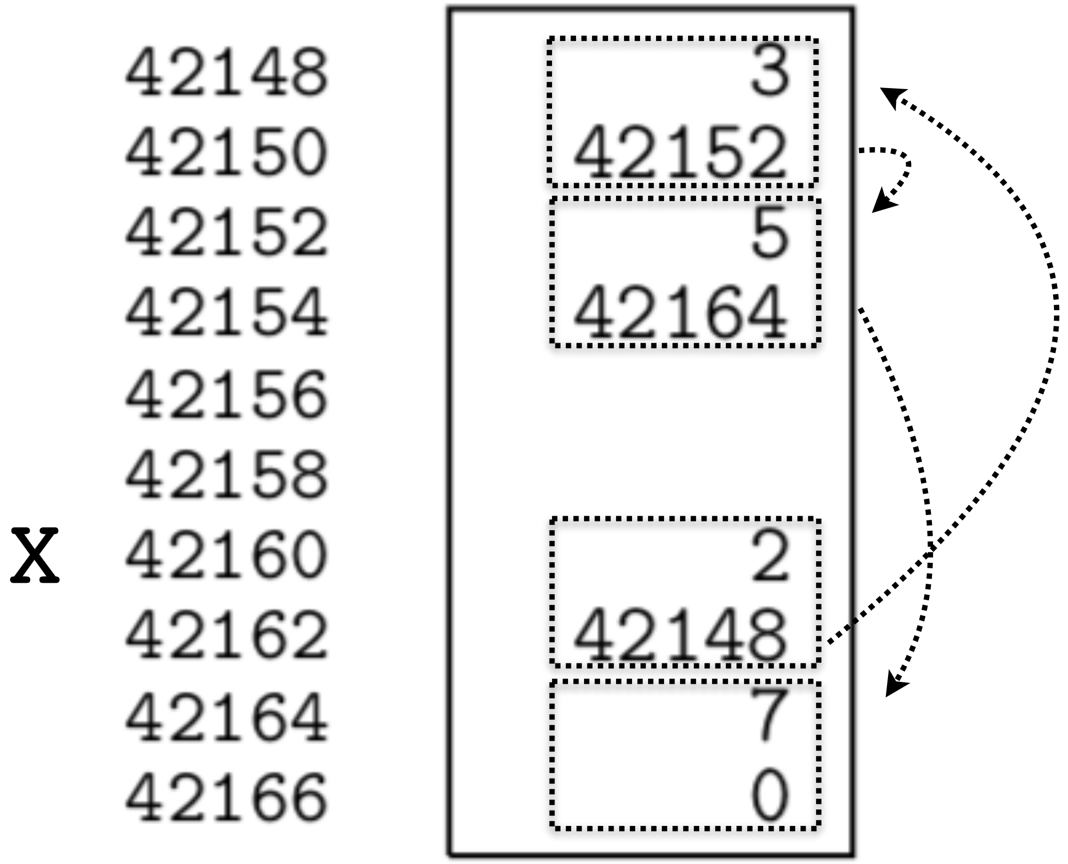

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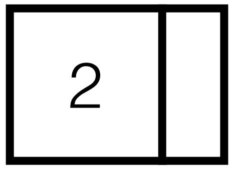

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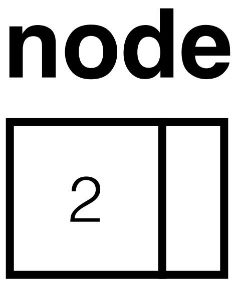
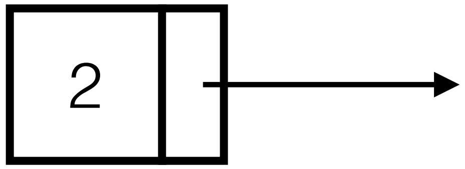


Terminology

node



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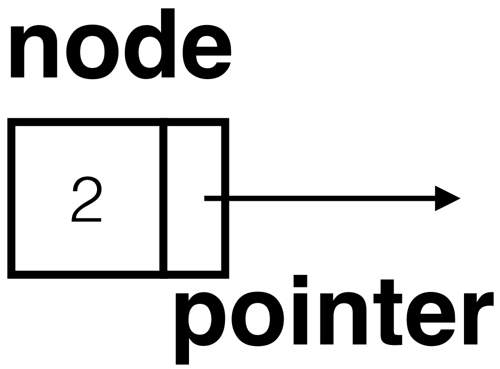
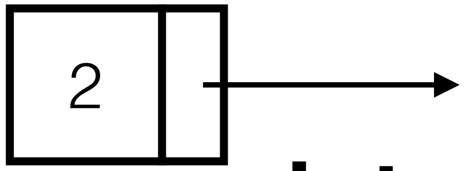


Terminology

node

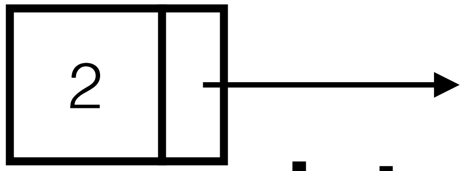


pointer

(in Java: “reference”)

Terminology

node



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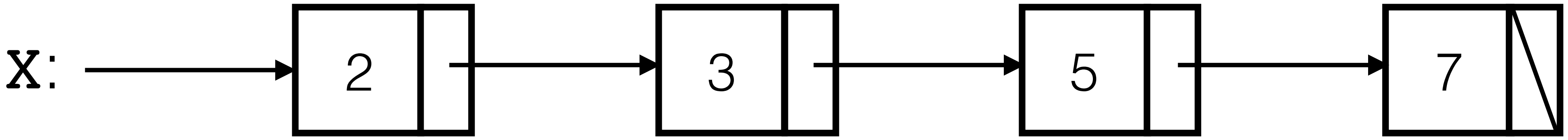
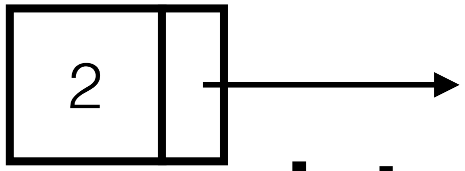


Terminology

node



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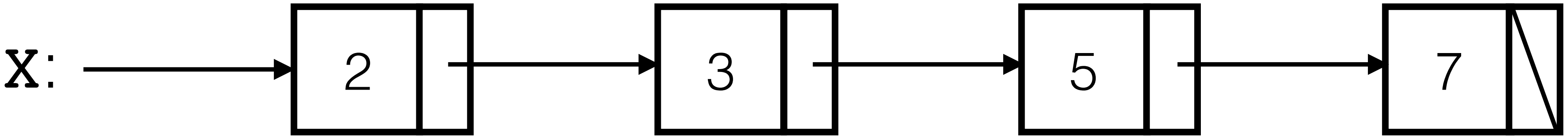
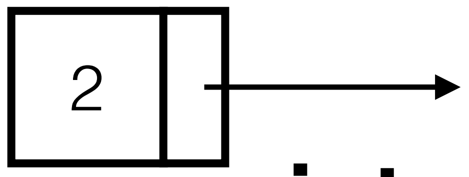


`x` is (a pointer to) the **head node** of the list

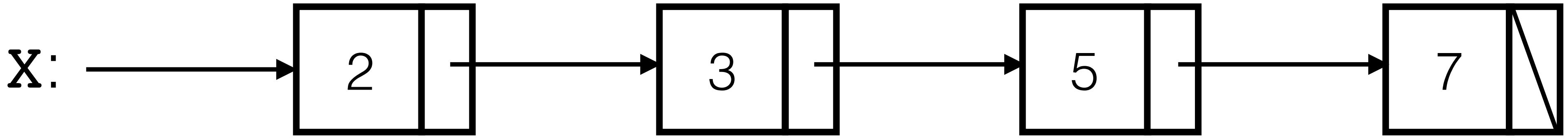
Terminology

node

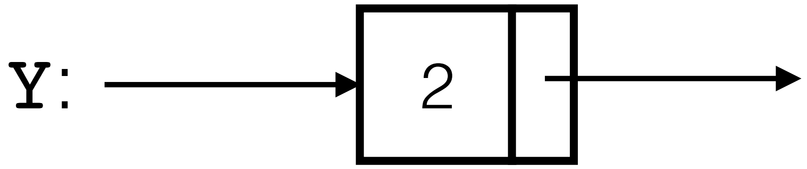


pointer

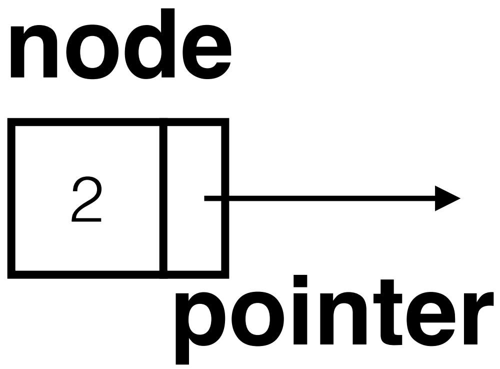
(in Java: “reference”)



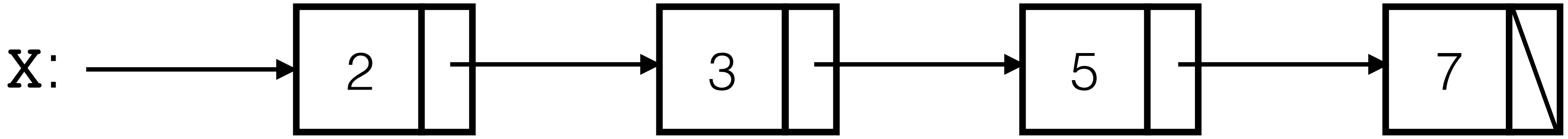
`X` is (a pointer to) the **head node** of the list



Terminology



(in Java: “reference”)



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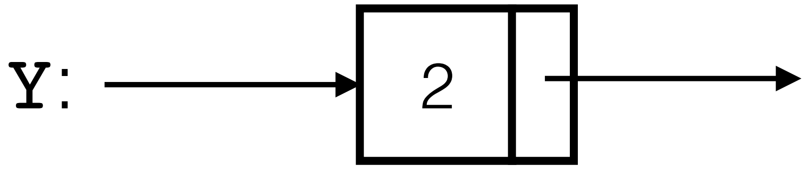
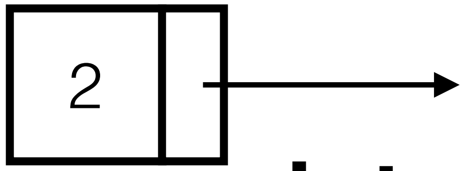


“`Y.val`” *refers to*

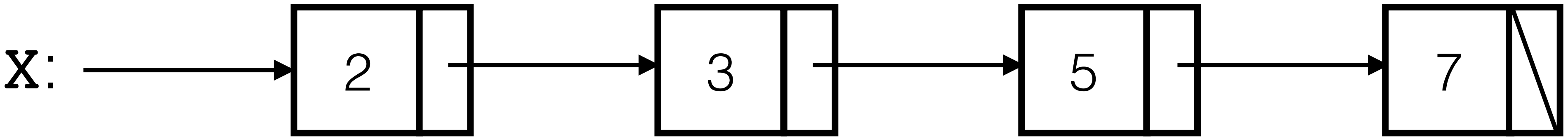
Terminology

node



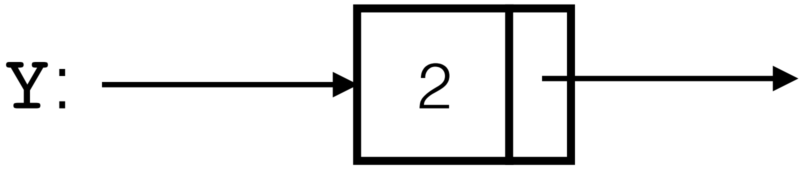
pointer

(in Java: “reference”)



X:

`X` is (a pointer to) the **head node** of the list

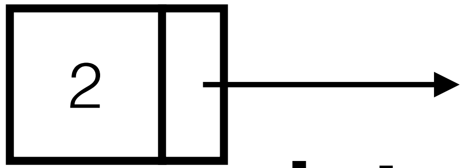


Y:

“`Y.val`” *refers to*

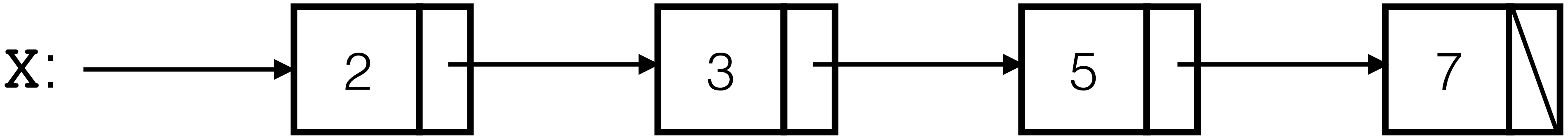
Terminology

node

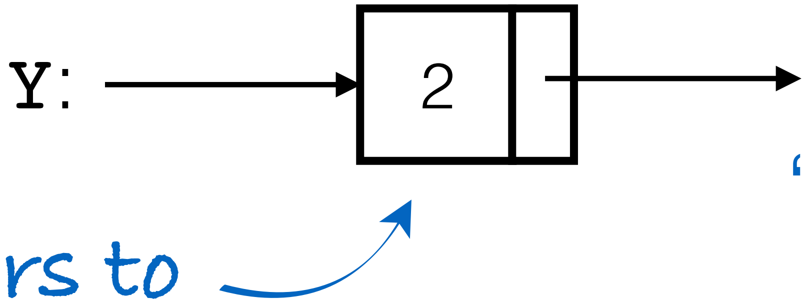


pointer

(in Java: “reference”)



`X` is (a pointer to) the **head node** of the list

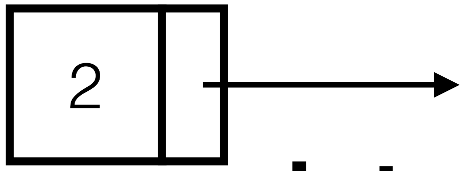


“`Y.val`” *refers to*

“`Y.next`” *refers to*

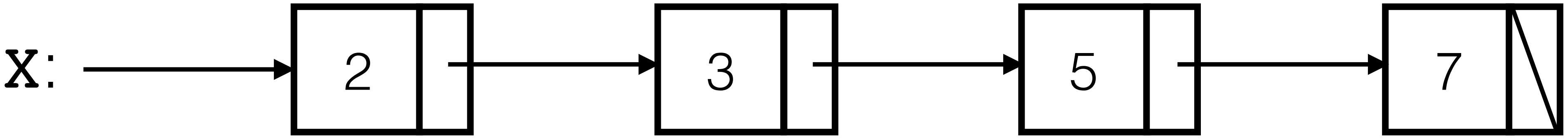
Terminology

node

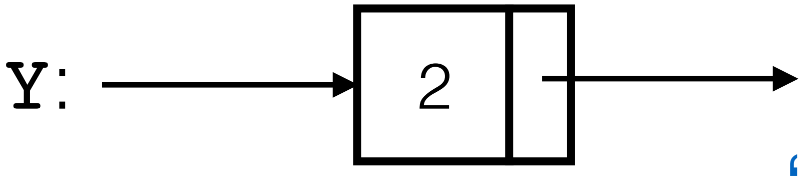


pointer

(in Java: “reference”)



`X` is (a pointer to) the **head node** of the list



“`Y.val`” *refers to*

“`Y.next`” *refers to*

Linked List

- Often we use a dummy head node that points to the first object, or to a special `null` object that represents an empty list. This makes it easier to write functions that insert or delete elements.

- Inserting and deleting elements is very fast: just move a few links around.

- Finding the *i*th element can be time-consuming.

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

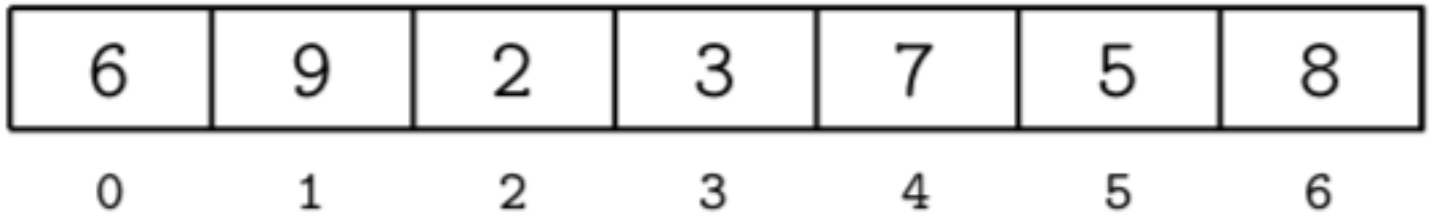
```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



6	9	2	3	7	5	8
0	1	2	3	4	5	6

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **x**.

```
function find(A,x,n)
    $j \leftarrow 0$
    while $j < n$
        if $A[j] = x$
            return j
        $j \leftarrow j+1$
    return -1
```

Y:

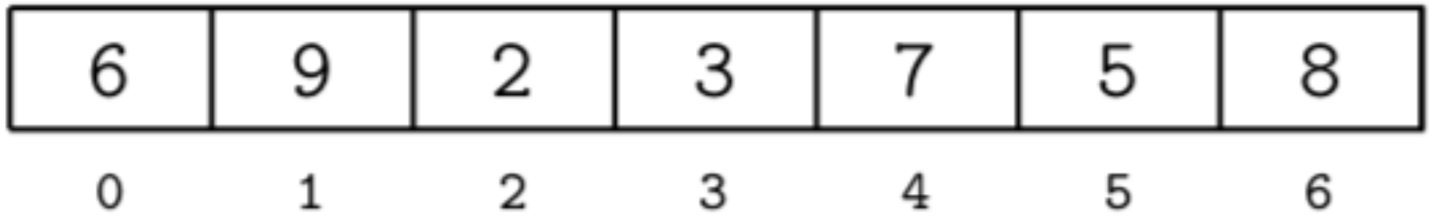
6	9	2	3	7	5	8
0	1	2	3	4	5	6

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item `x`.

```
function find(A,x,n)
    $j \leftarrow 0$
    while $j < n$
        if $A[j] = x$
            return j
        $j \leftarrow j+1$
    return -1
```

Y:



6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

Y:

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y x: 7 n: 6

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 0

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,6)

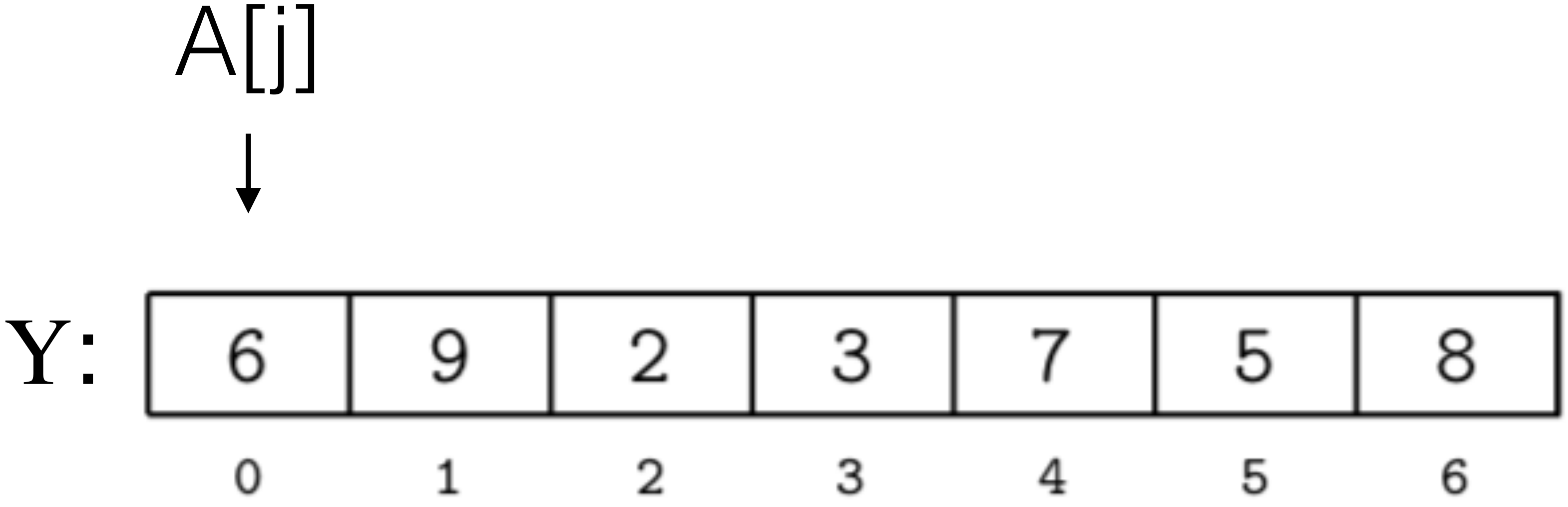
Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y   x: 7   n: 6   j: 0

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

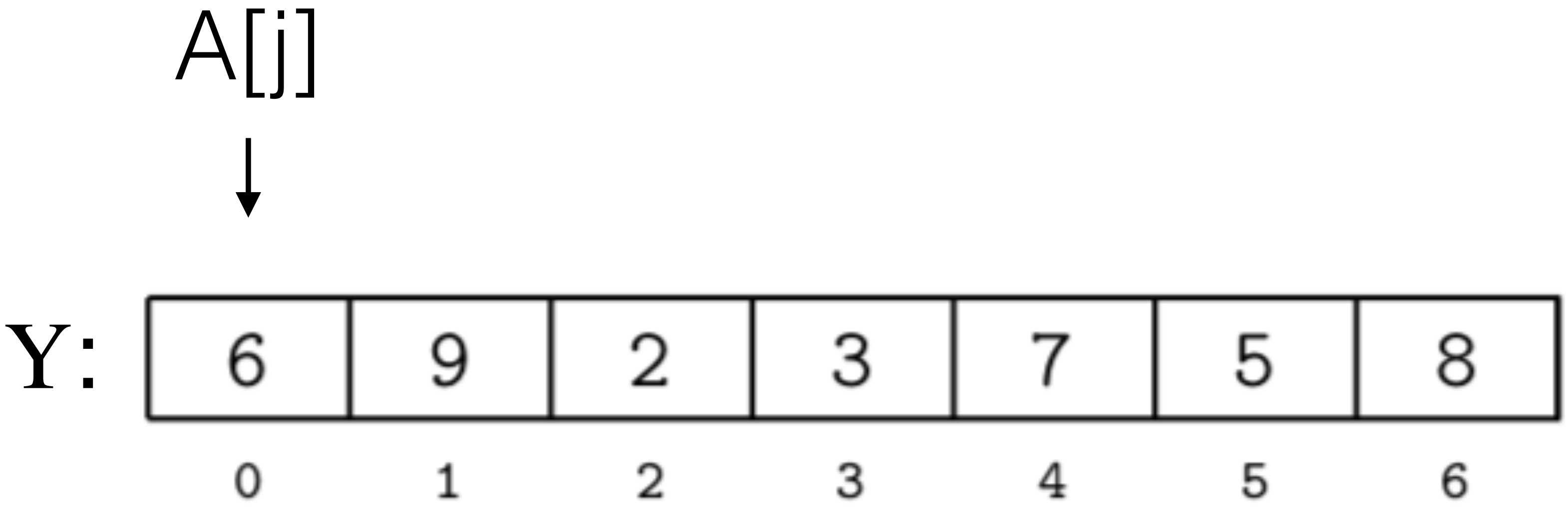
Let’s trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y   x: 7   n: 6   j: 1

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



A[j]

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

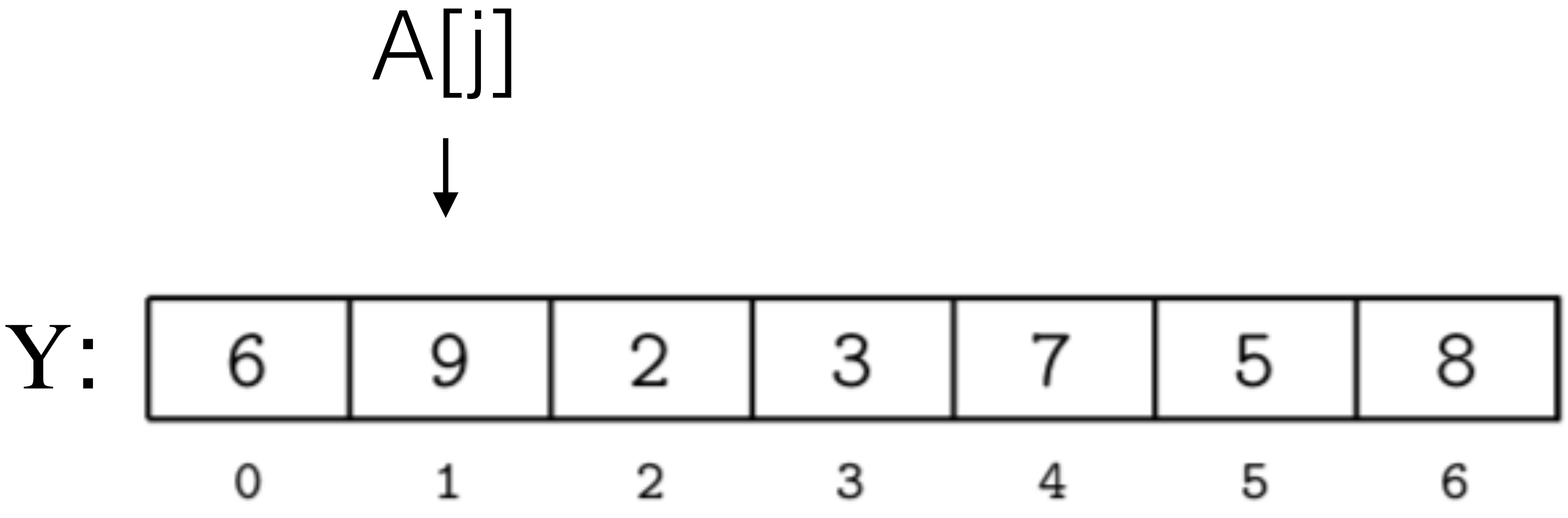
Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 1

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



A[j]

Y:

0	1	2	3	4	5	6
6	9	2	3	7	5	8

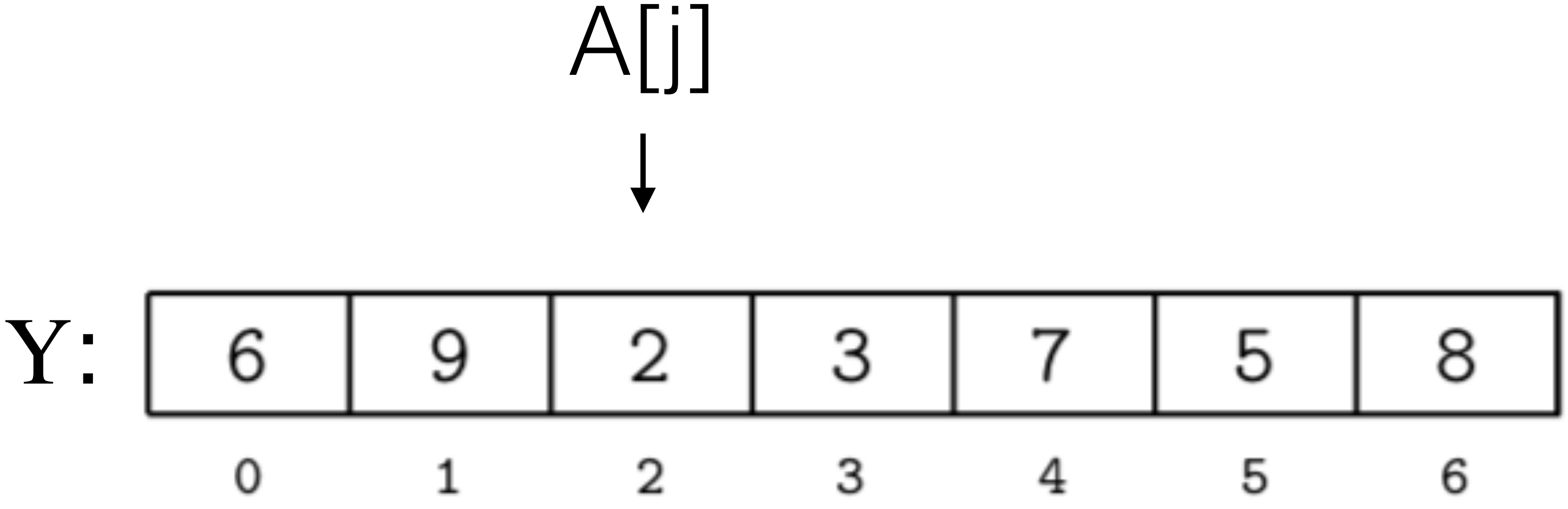
Let’s trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y   x: 7   n: 6   j: 2

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,6)

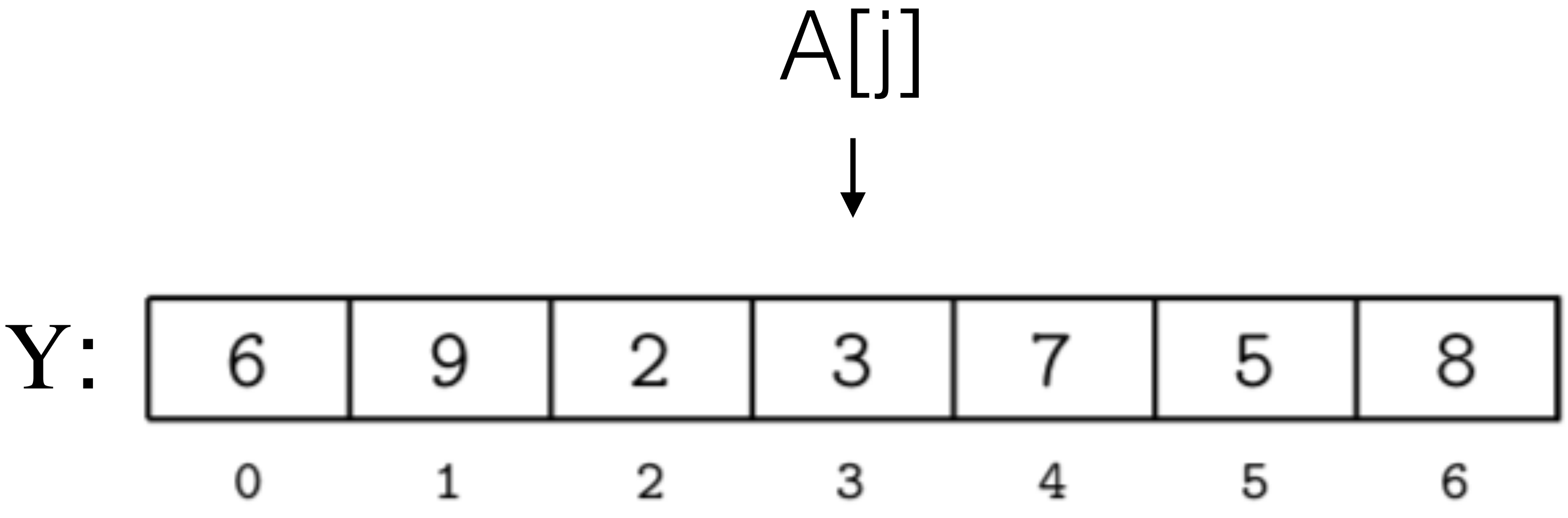
Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 3

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



A[j]

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,6)

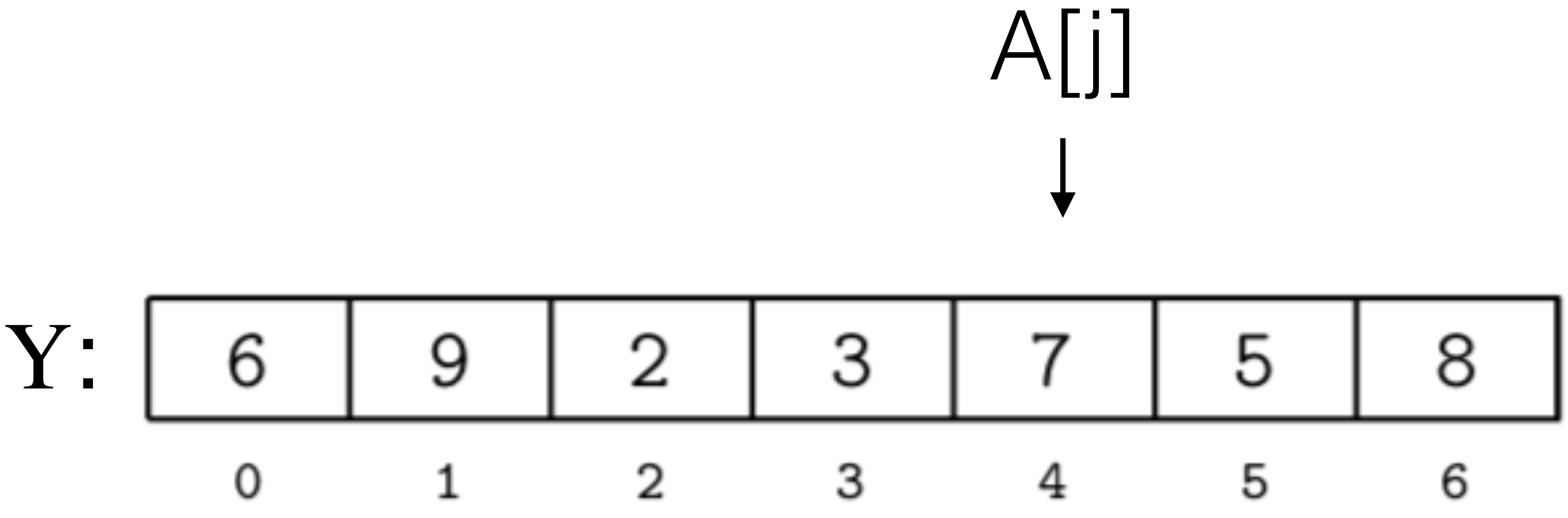
Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y   x: 7   n: 6   j: 4

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



A[j]

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

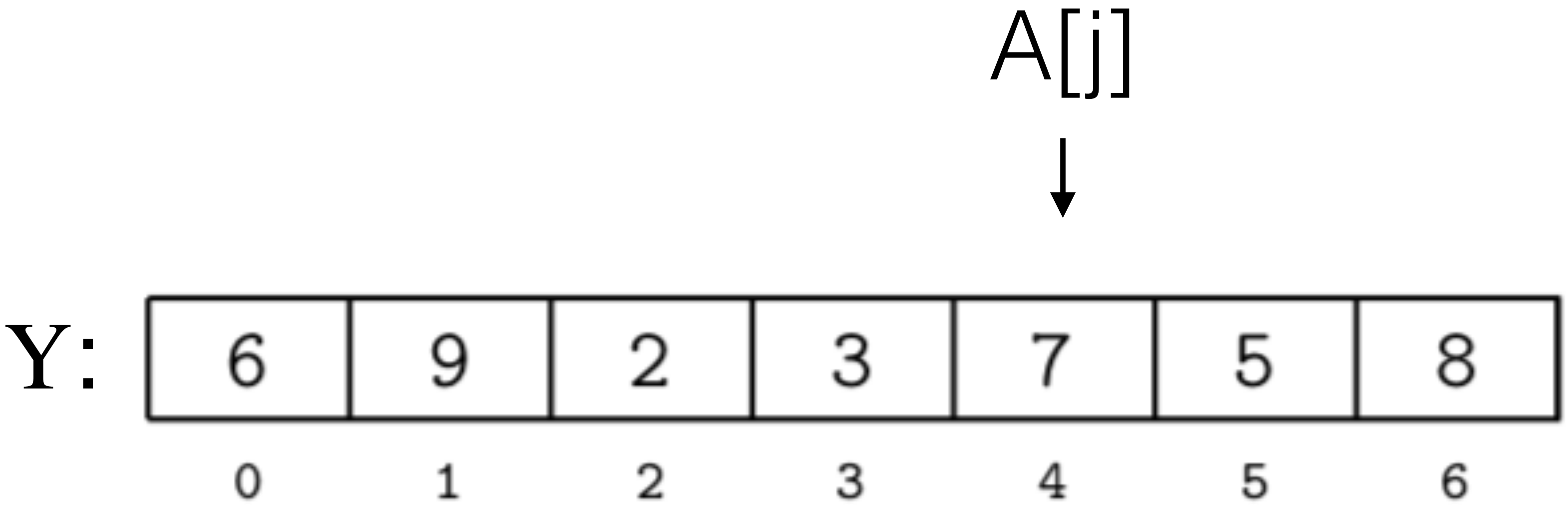
Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 4

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Let’s trace the execution of find(Y,7,6)

(returns 4)

Iterative Processing: List

- Walk through a linked list.

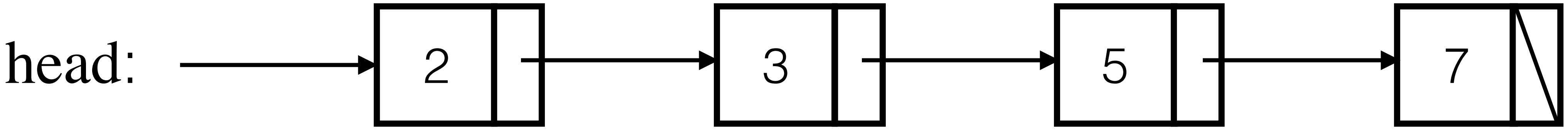
- For example, to locate item `x`.

```
function find(head,x)
    p ← head
    while $p \neq null$
        if $p.val = x$
            return p
      p ← p.next
    return null
```

Iterative Processing: List

- Walk through a linked list.
- For example, to locate item **x**.

```
function find(head,x)
    p ← head
    while p ≠ null
        if p.val = x
            return p
        p ← p.next
    return null
```



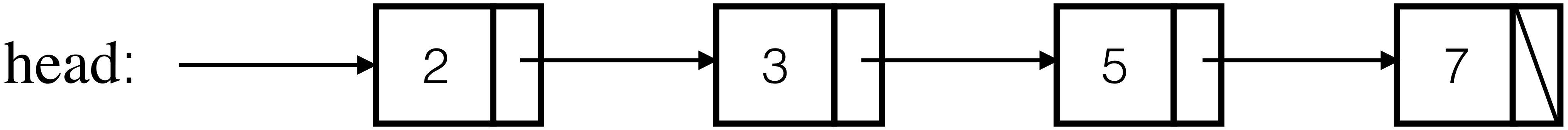
Iterative Processing: List

- Walk through a linked list.
- For example, to locate item **x**.

(note similarity to array version)

```
function find(head,x)
    p ← head
    while p ≠ null
        if p.val = x
            return p
        p ← p.next
    return null
```

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



Iterative Processing: List

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    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
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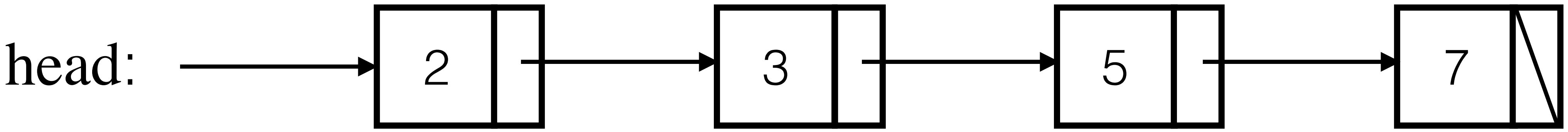
```



Iterative Processing: List

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(note similarity to array version)

```
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    p ← head
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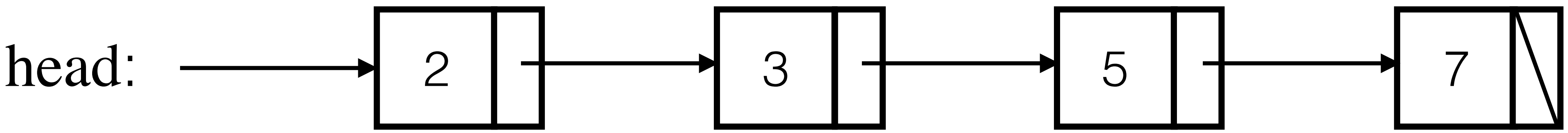
    return null
```

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

p:



Iterative Processing: List

- Walk through a linked list.
- For example, to locate item **x**.

(note similarity to array version)

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    return null
```
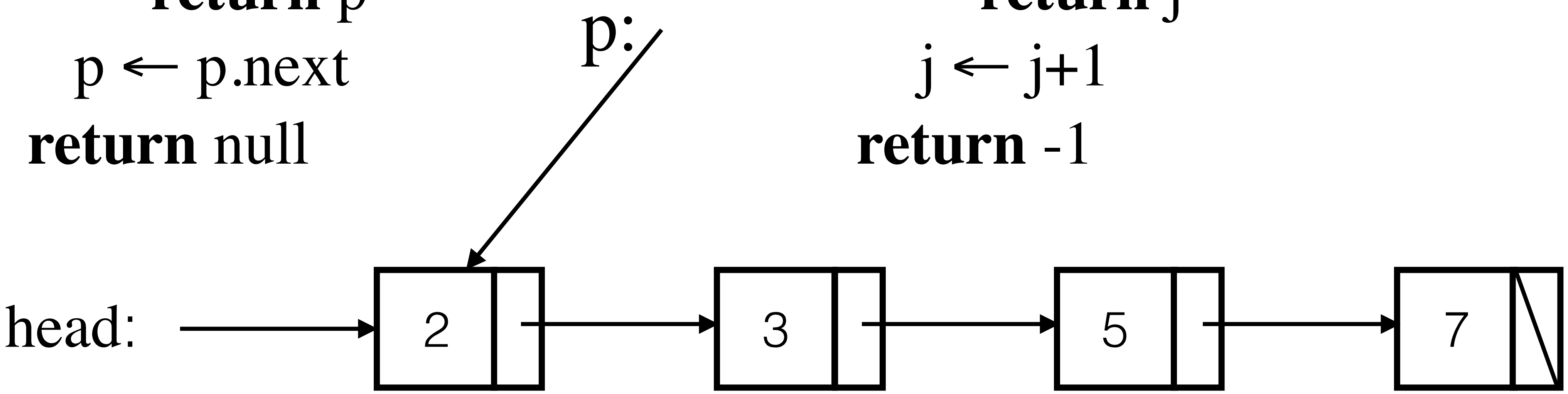

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



Iterative Processing: List

- Walk through a linked list.
- For example, to locate item **x**.

(note similarity to array version)

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```

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
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        j ← j+1
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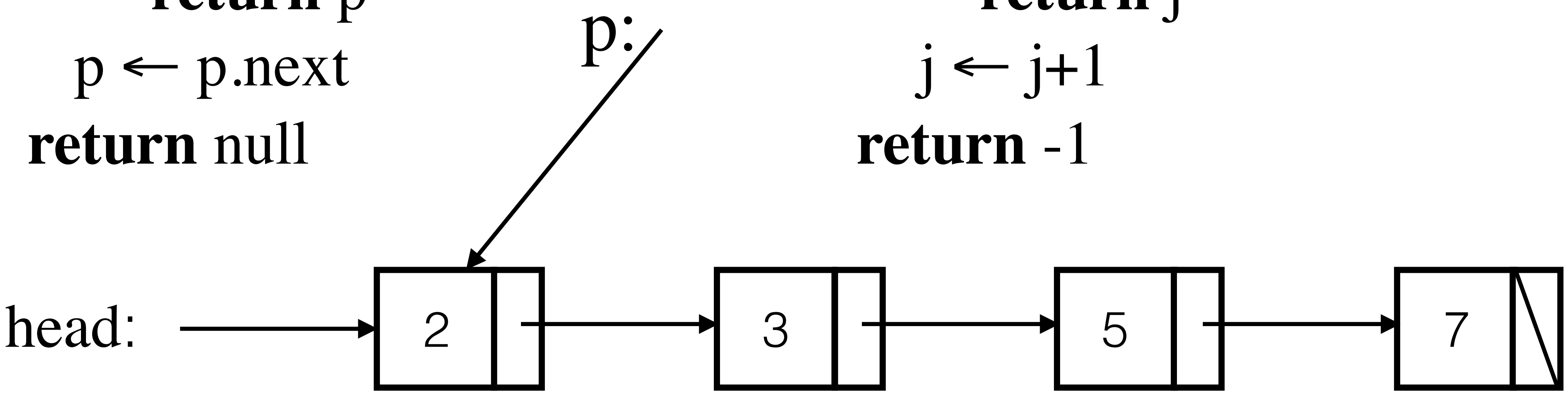
```



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function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```
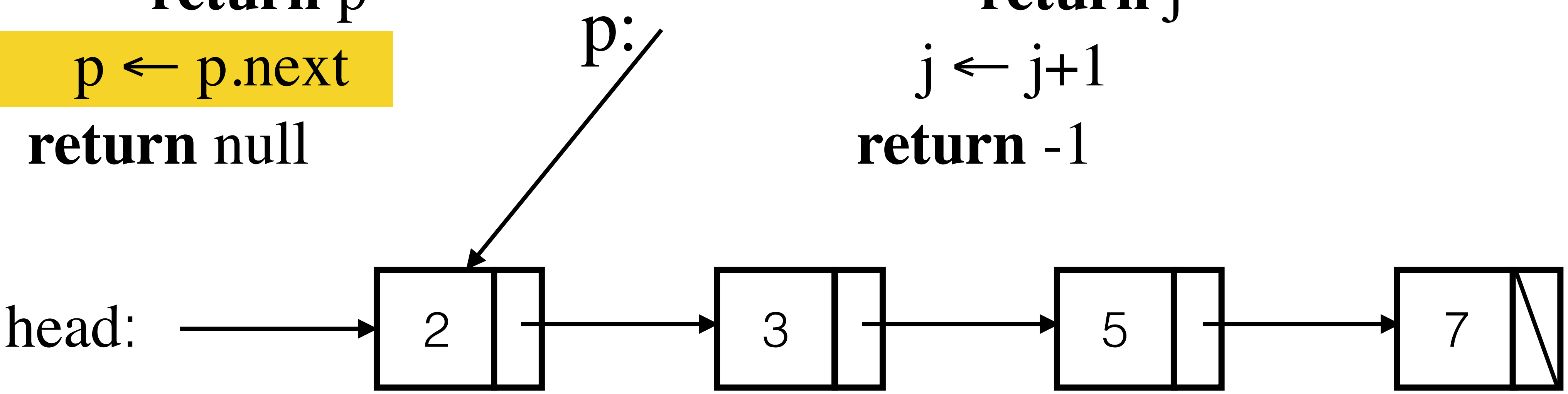



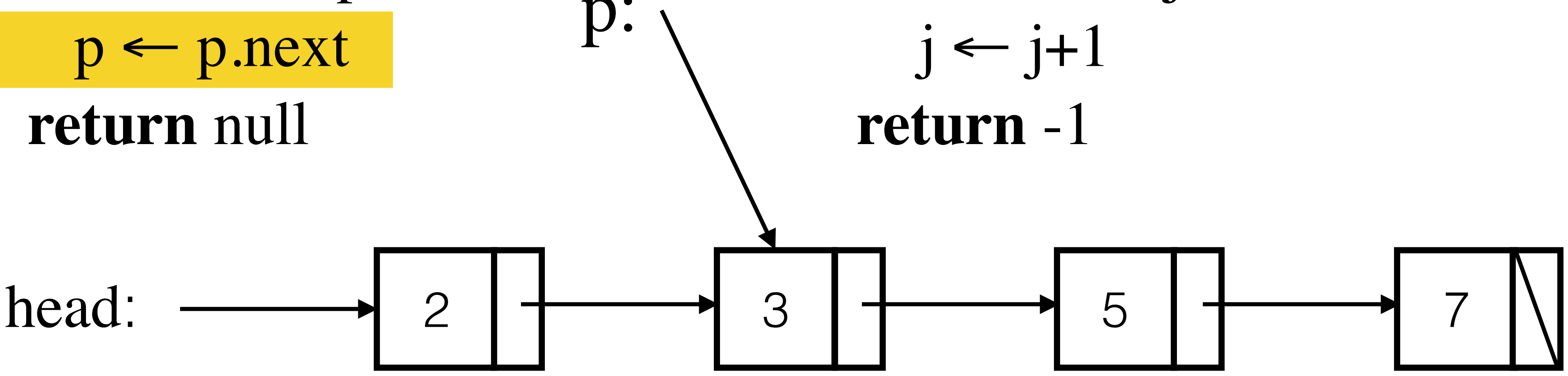
Iterative Processing: List

- Walk through a linked list.
- For example, to locate item **x**.

(note similarity to array version)

```
function find(head,x)
    p ← head
    while p ≠ null
        if p.val = x
            return p
        p ← p.next
    return null
```

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



Iterative Processing: List

- Walk through a linked list.

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```
function find(head,x)
    p ← head
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            return p
        p ← p.next
    return null
```
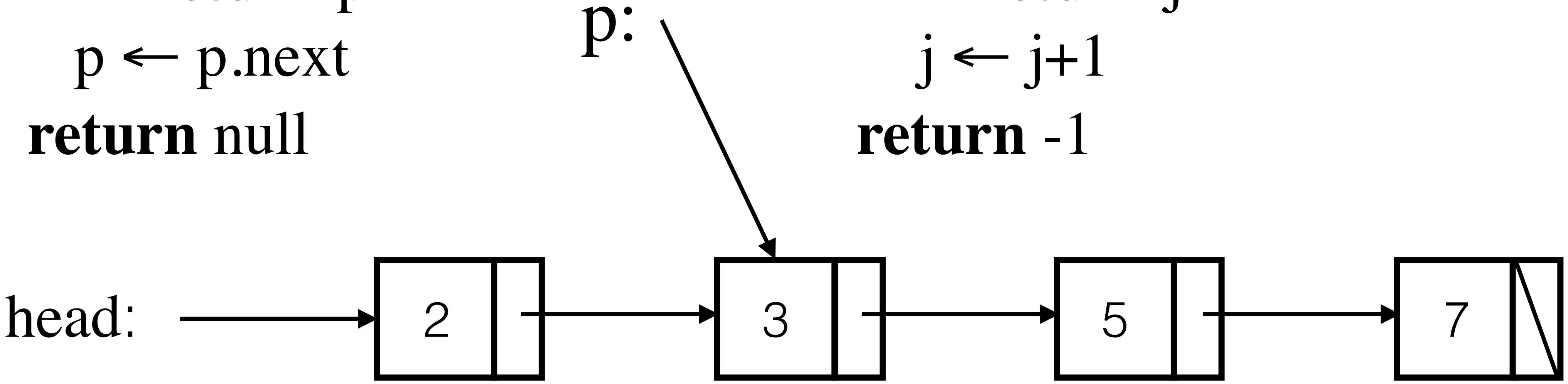

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```



Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Initial call: find(A,x,0,n-1)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

```
function find(A,x,lo,hi)
    if lo > hi
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        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

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Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 0

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance
- For example, to locate item **x**.

A: Y   x: 7   lo: 0   hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

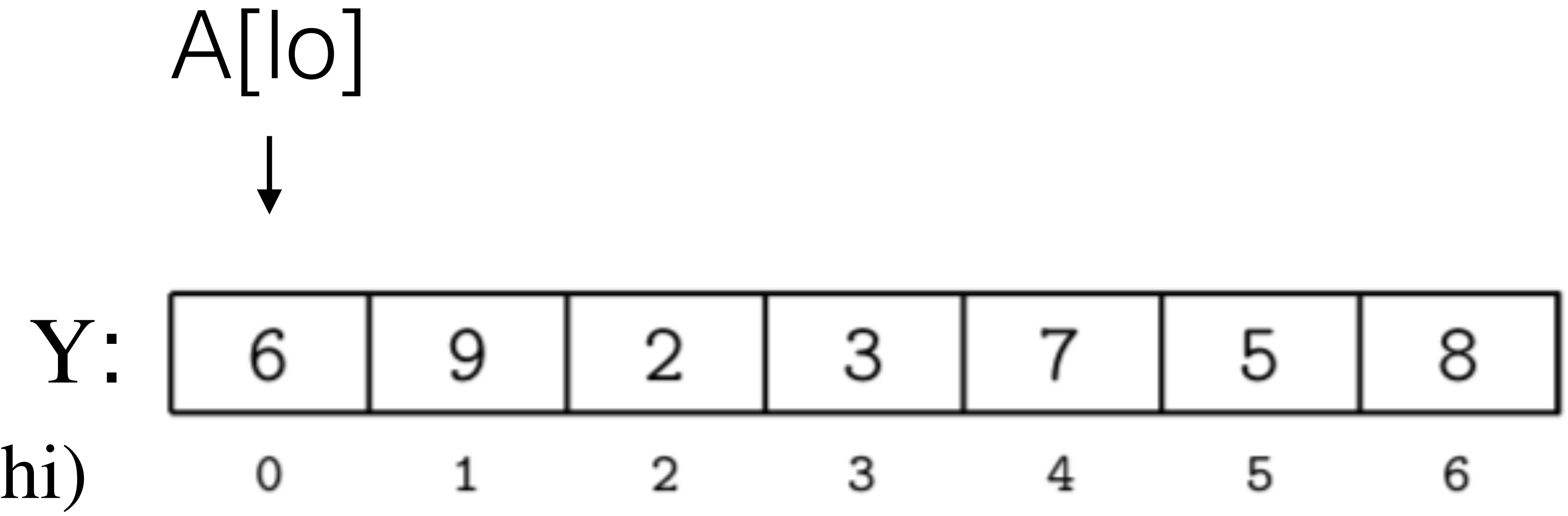
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y   x: 7   lo: 0   hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



A[lo]

Y:

0	1	2	3	4	5	6
6	9	2	3	7	5	8

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

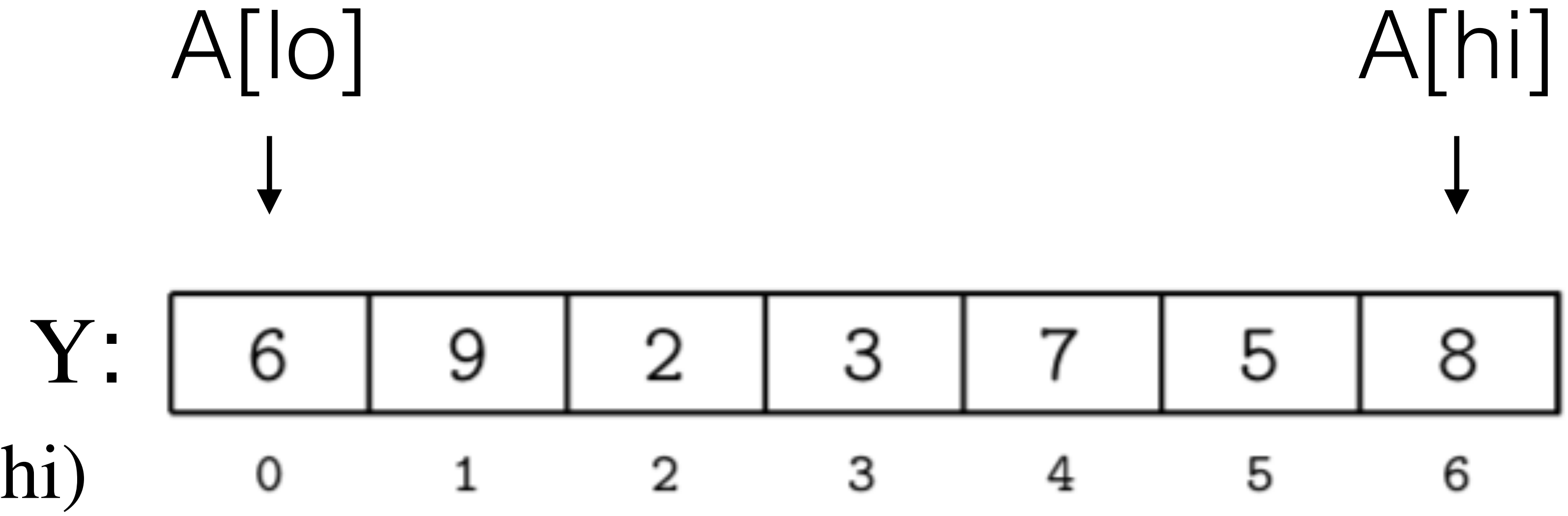
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y   x: 7   lo: 0   hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



A[lo]

A[hi]

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

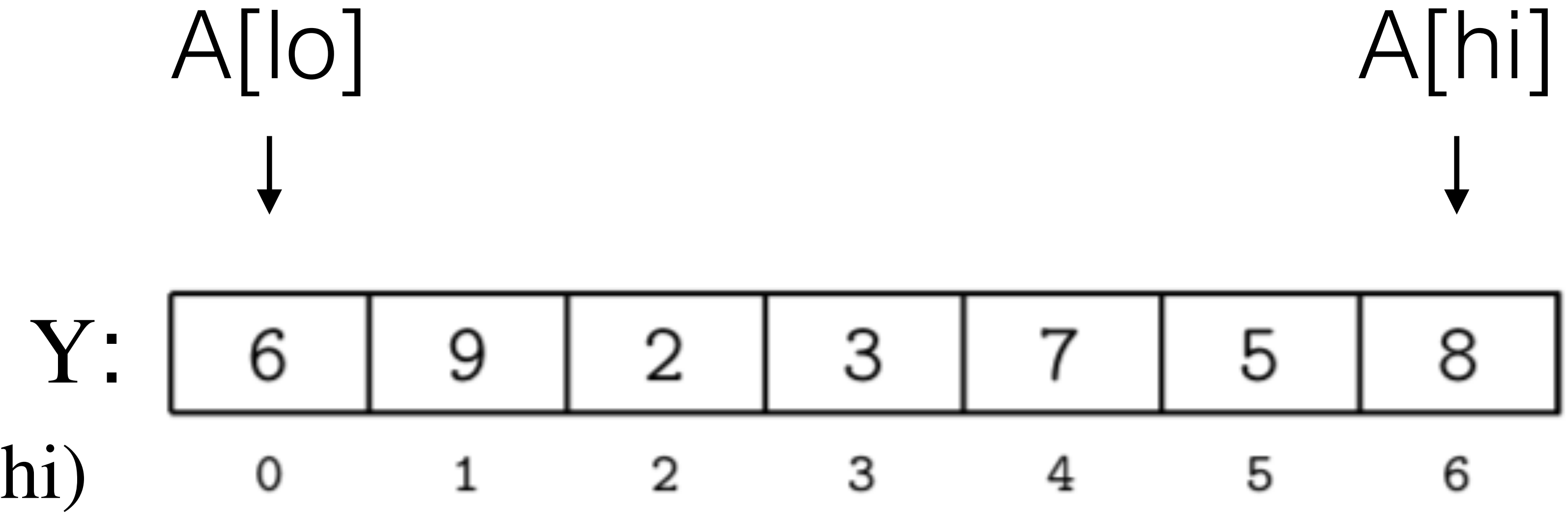
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 1 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

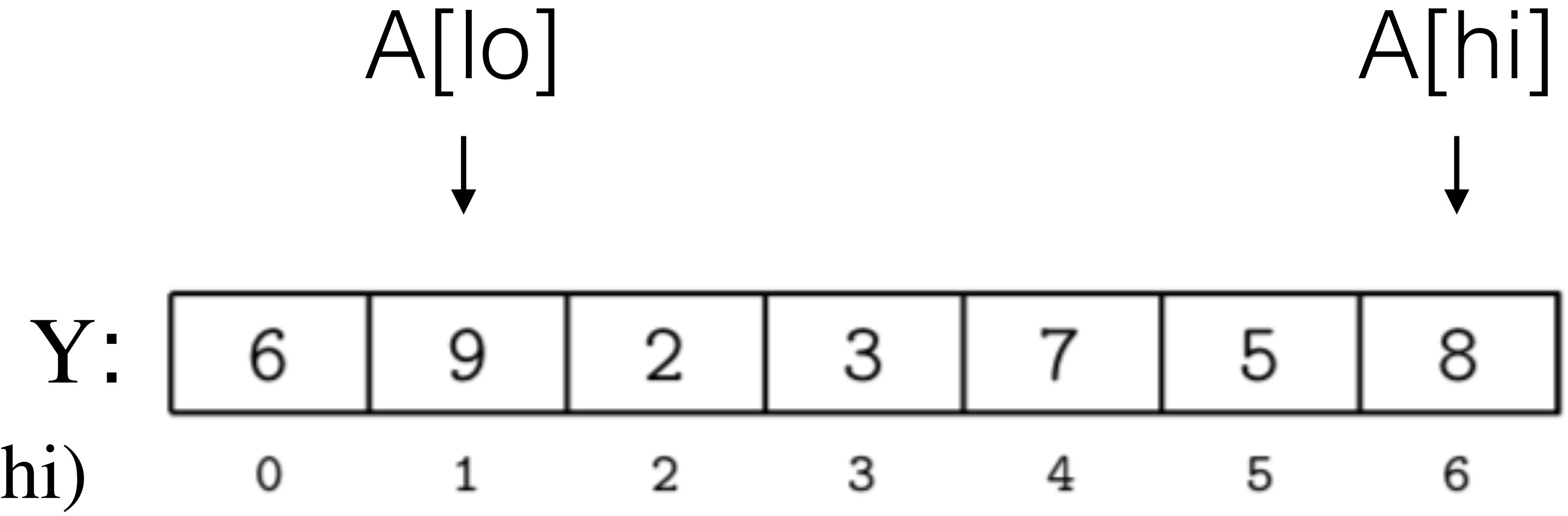
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 1 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



A[lo] A[hi]

Y:

0	1	2	3	4	5	6
6	9	2	3	7	5	8

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

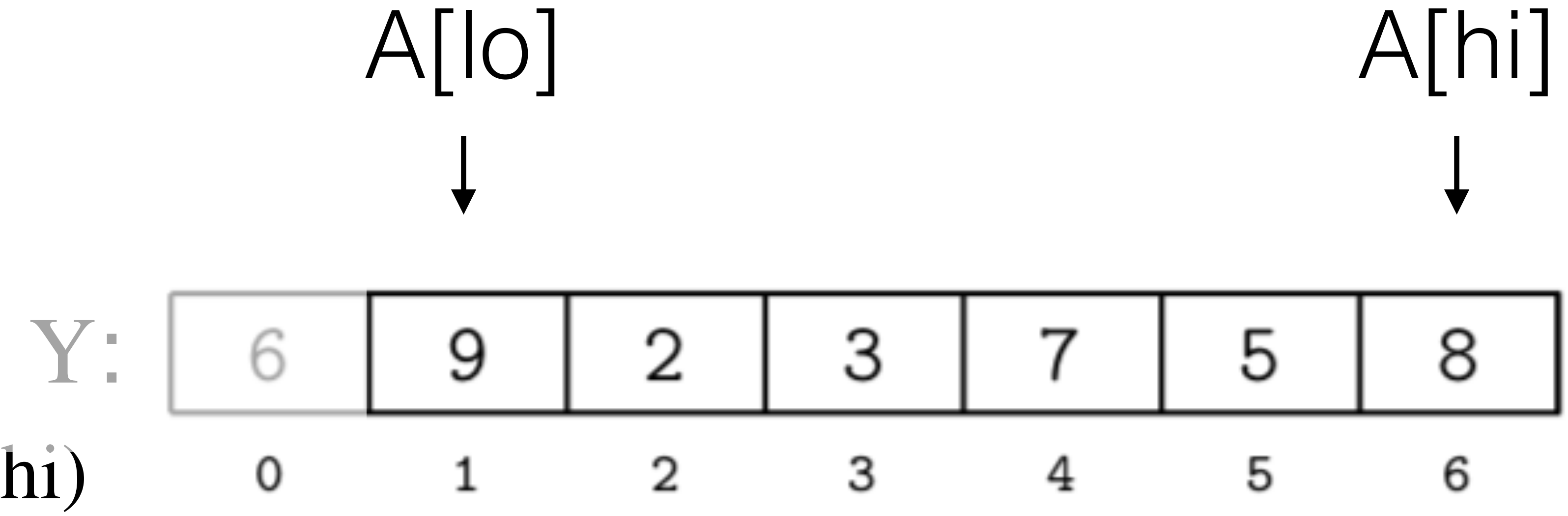
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 1 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

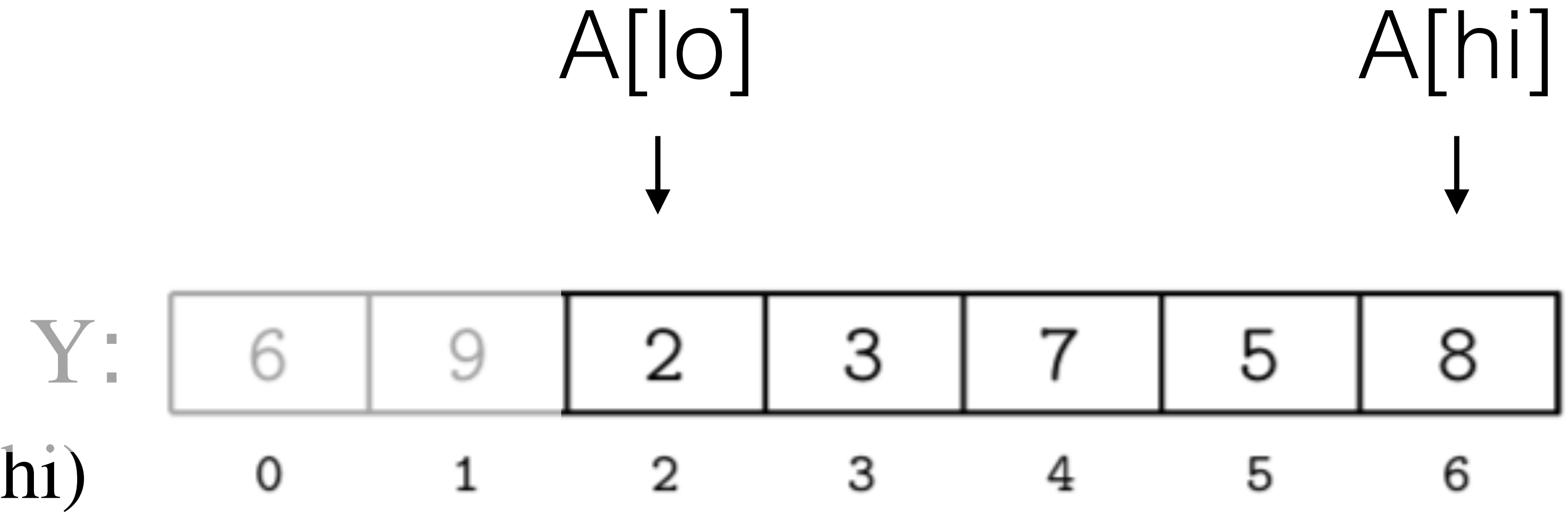
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y x: 7 lo: 2 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



	0	1	2	3	4	5	6
Y:	6	9	2	3	7	5	8

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

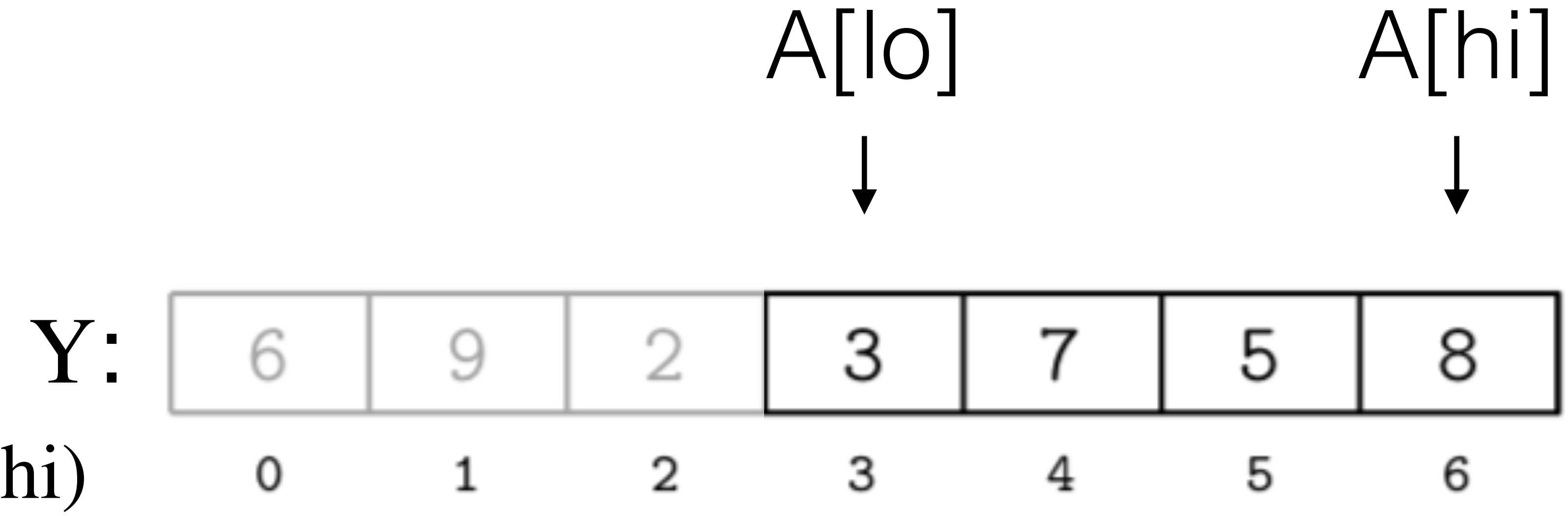
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y   x: 7   lo: 3   hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

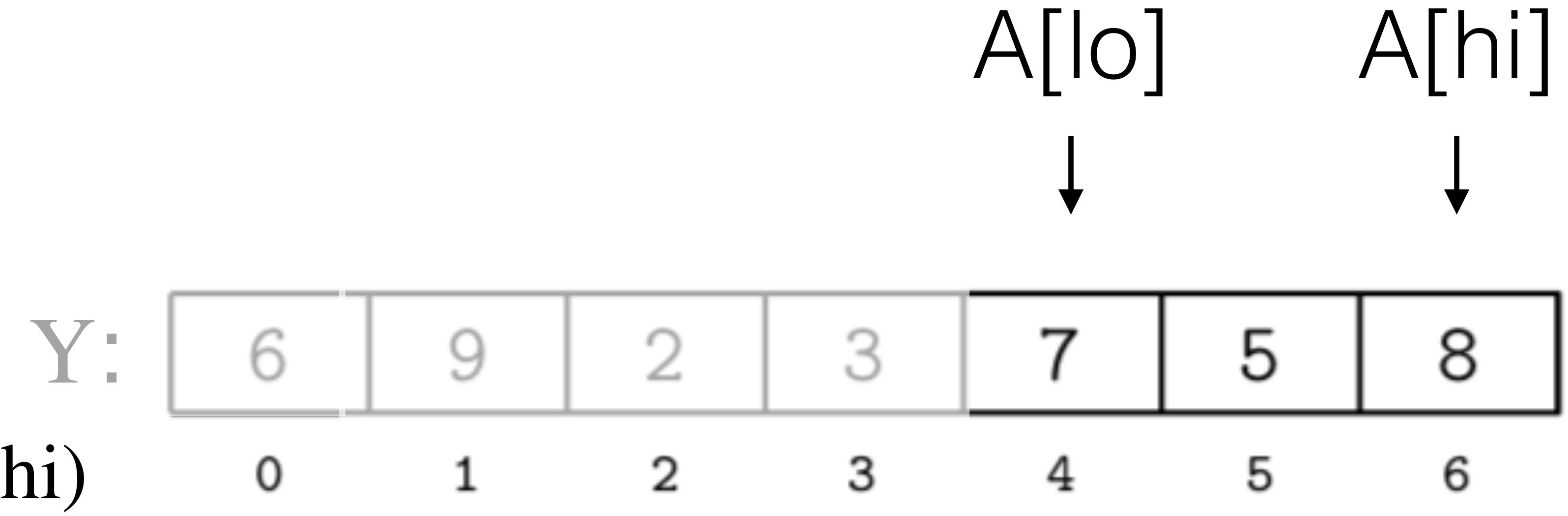
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 4 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



A[lo] A[hi]

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

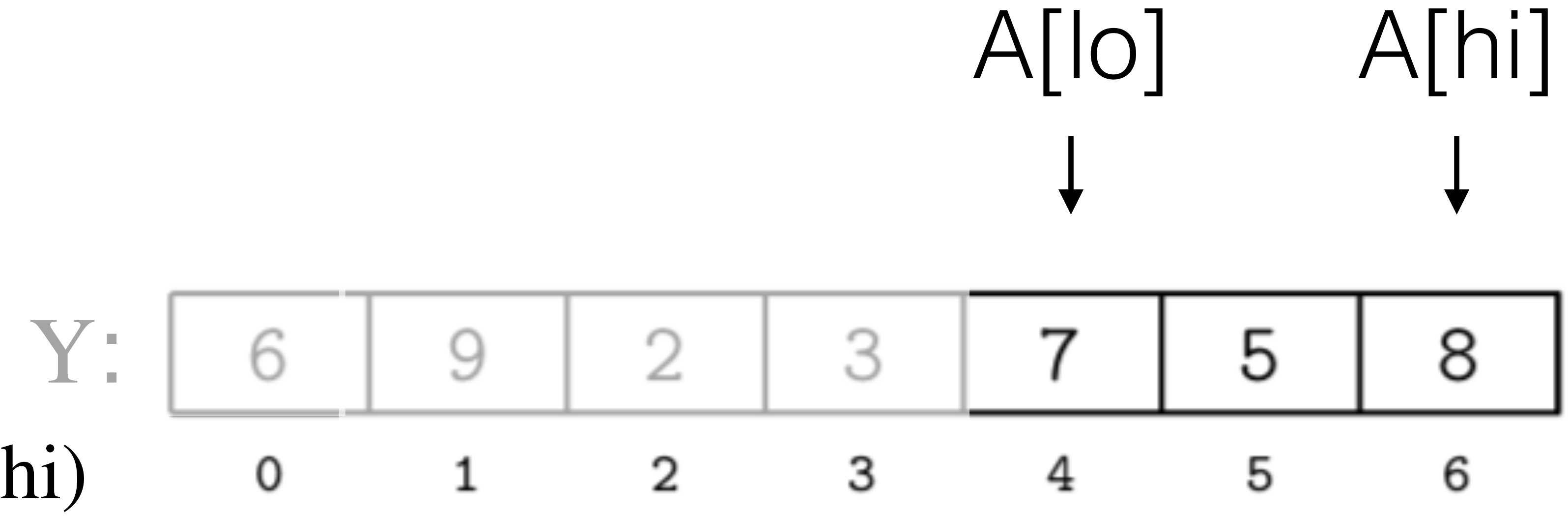
Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item `x`.

A: Y x: 7 lo: 4 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```



A[lo]

A[hi]

Y:	6	9	2	3	7	5	8
	0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

(returns 4)

Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

```
function find(p,x)
    if p = null
        return p
    else if p.val = x
        return p
    else
        return find(p.next,x)
```

Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

```
function find(p,x)
    if p = null
        return p
    else if p.val = x
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```
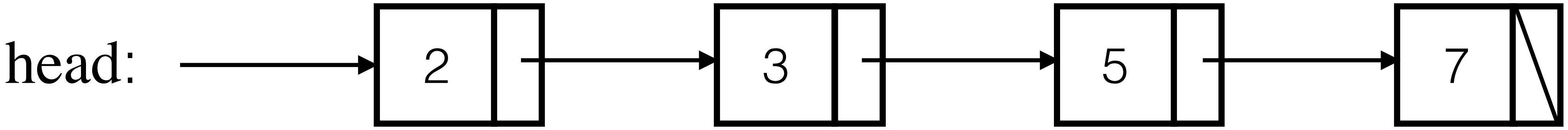
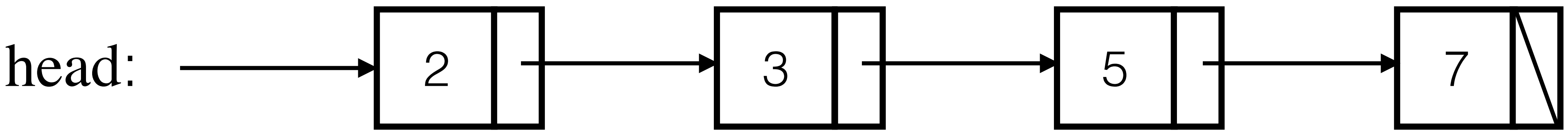



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

```
function find(p,x)
    if p = null
        return p
    else if p.val = x
        return p
    else
        return find(p.next,x)
```

Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

(note similarity to array version)

```
function find(p,x)
    if p = null
        return p
    else if p.val = x
        return p
    else
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```
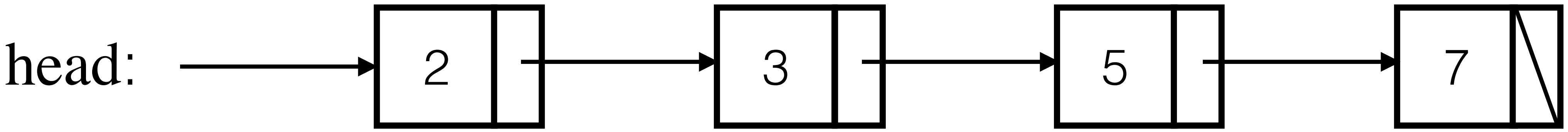

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

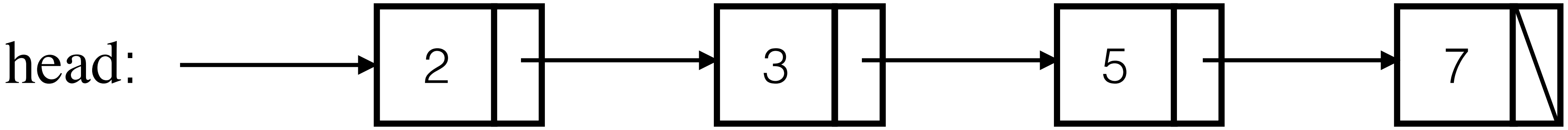
```
function find(p,x)
    if p = null
        return p
    else if p.val = x
        return p
    else
        return find(p.next,x)
```

p:

(note similarity to array version)

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

(note similarity to array version)

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    else if p.val = x
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    else
        return find(p.next,x)
```

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
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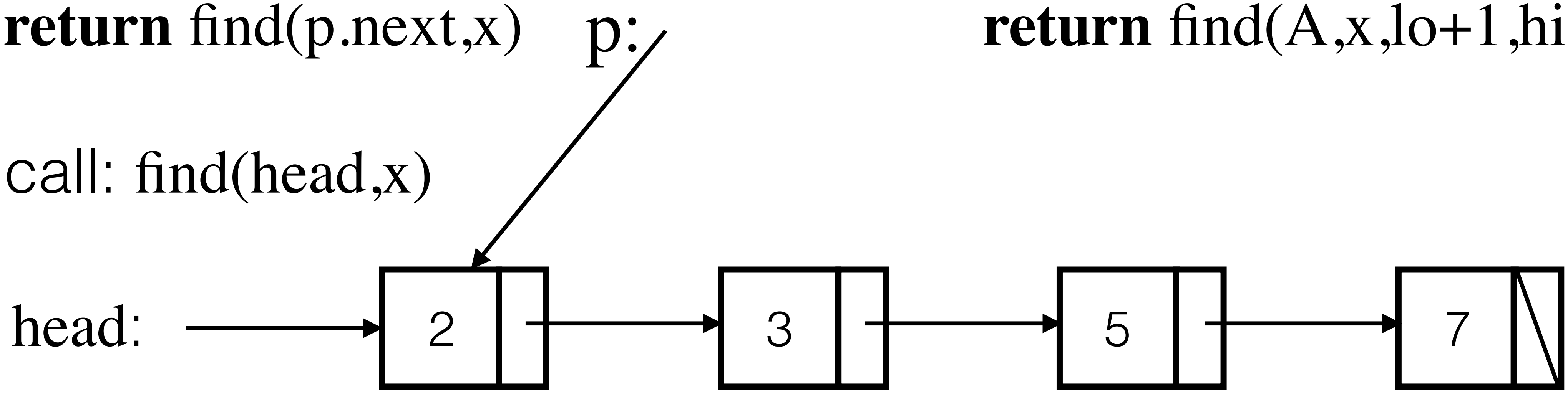
```

Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

(note similarity to array version)

```
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function find(A,x,lo,hi)
    if lo > hi
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    else if A[lo] = x
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    else
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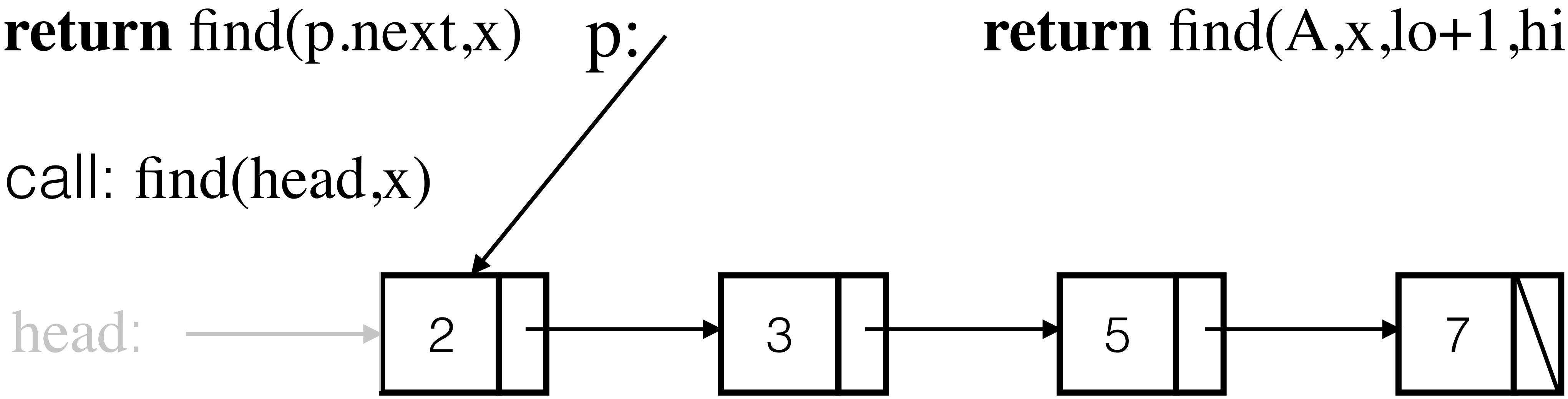
```

Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

(note similarity to array version)

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        return p
    else
        return find(p.next,x)
```
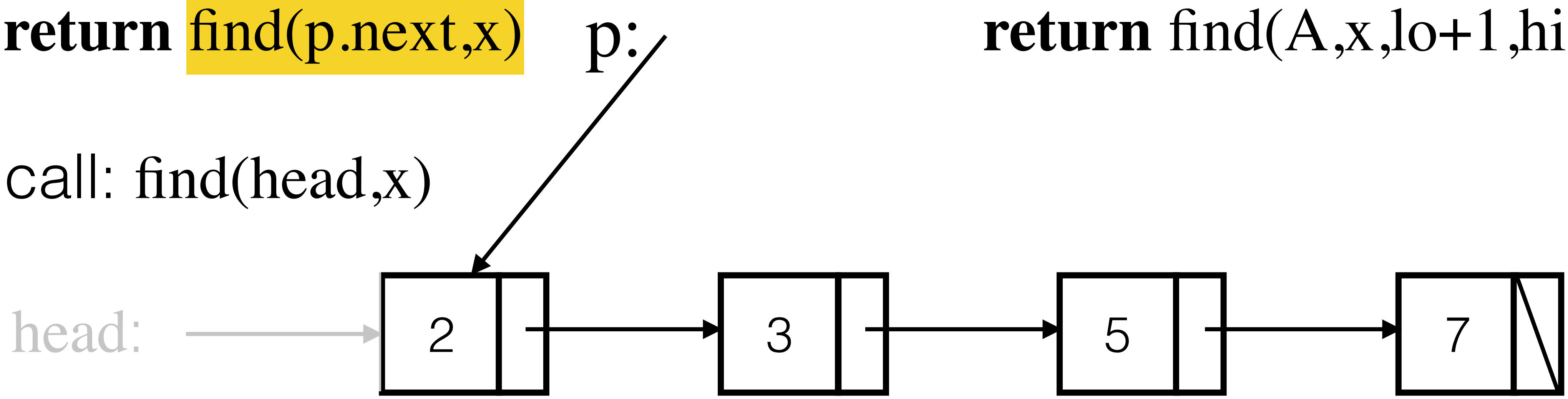

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

(note similarity to array version)

```
function find(p,x)
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        return p
    else if p.val = x
        return p
    else
        return find(p.next,x)
```

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
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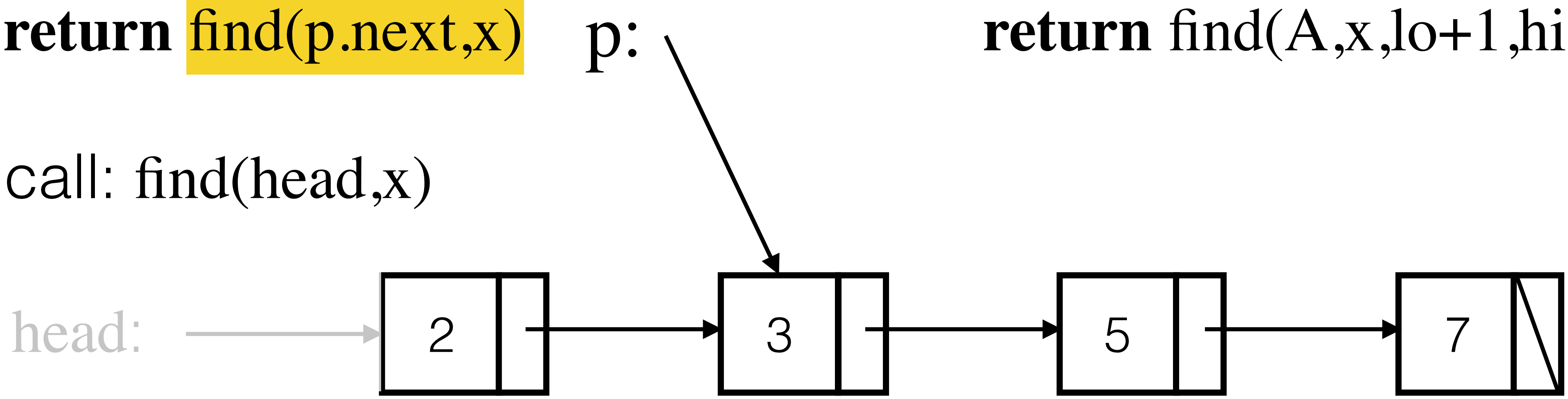
Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

(note similarity to array version)

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function find(A,x,lo,hi)
    if lo > hi
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    else if A[lo] = x
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        return find(A,x,lo+1,hi)
```
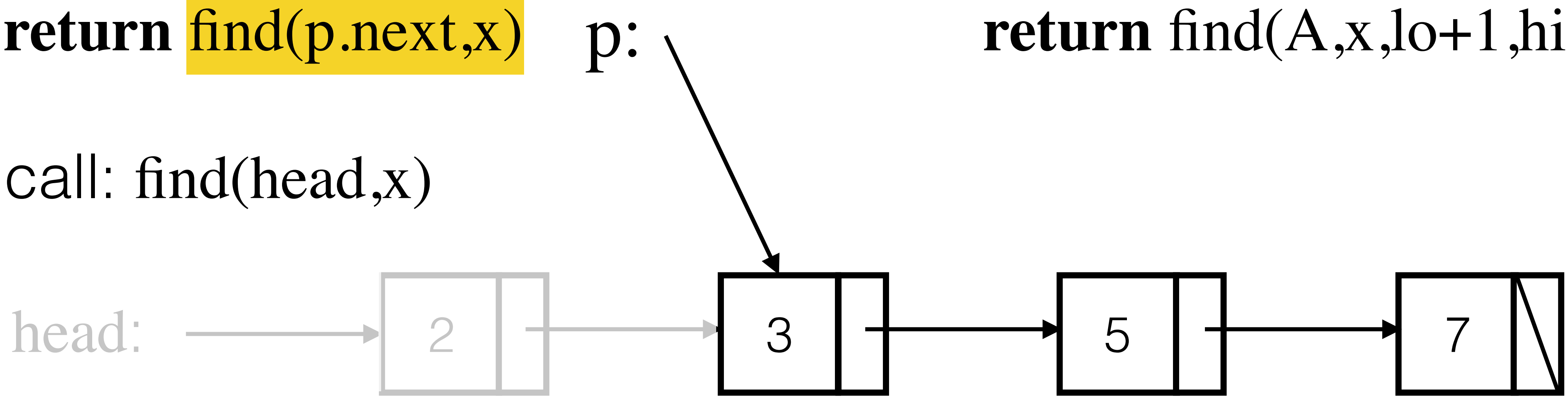

Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

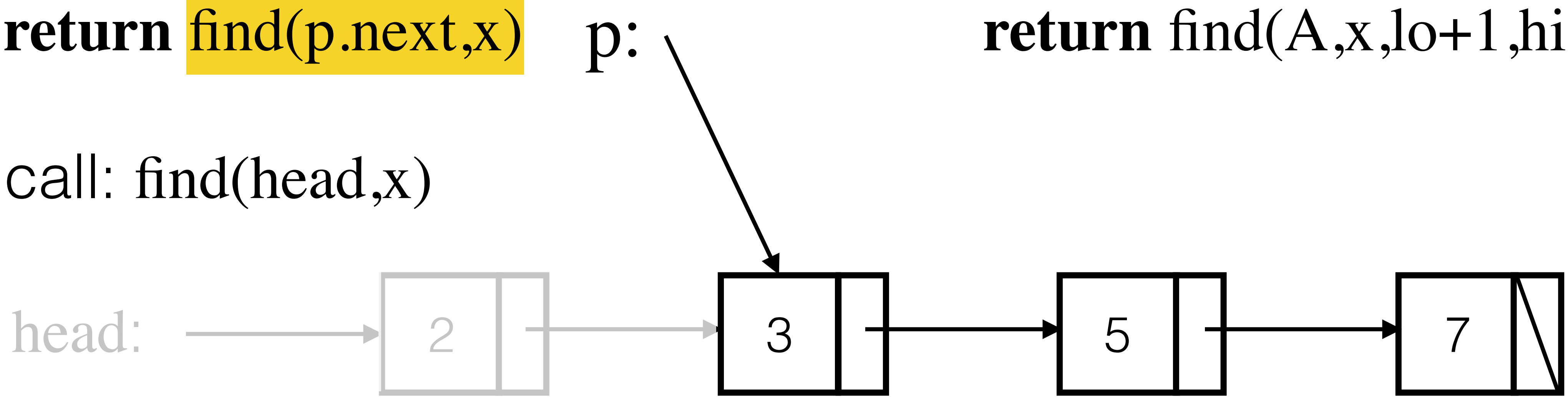
(note similarity to array version)

```
function find(p,x)
    if p = null
        return p
    else if p.val = x
        return p
    else
        return find(p.next,x)
```

We will discuss recursion properly in Week 3

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Initial call: find(head,x)



Abstract DataTypes

- A collection of data items, and a family of operations that operate on that data
- Think of an ADT as a set of contracts, an **interface**
- We must still **implement** these promises, but it is an advantage to separate the implementation of the ADT from the “concept” (i.e. the interface it provides)
- Good programming practice is to support this separation
  - Nothing outside of the definition of the ADT should refer to anything inside, except through function calls and basic operations

Fundamental Data Structure: The Stack

- Last-In-First-Out (LIFO)
- Operations:
  - CreateStack
  - Push
  - Pop
  - Top
  - EmptyStack?
  - …
- Usually implemented as an ADT

Fundamental Data Structure: The Stack

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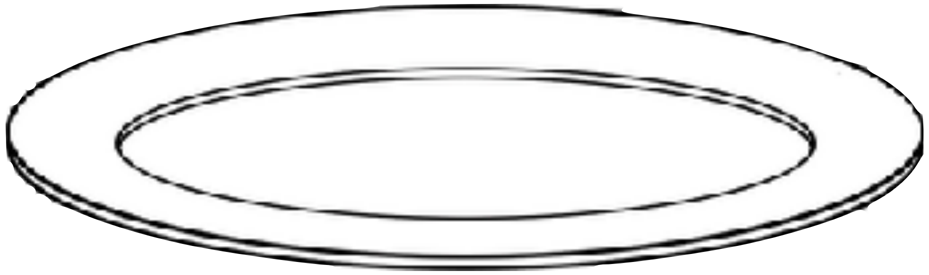

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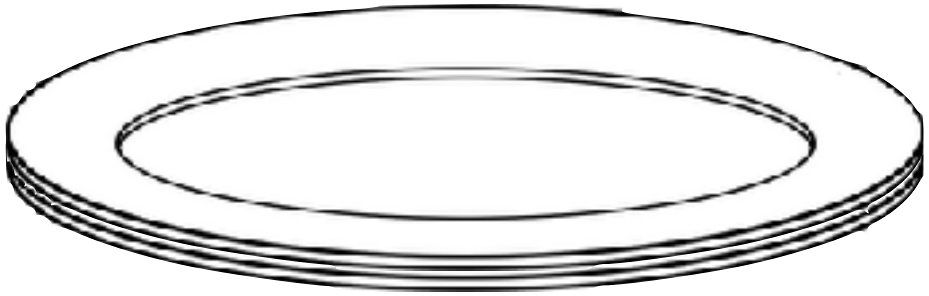

Fundamental Data Structure: The Stack

- Last-In-First-Out (LIFO)
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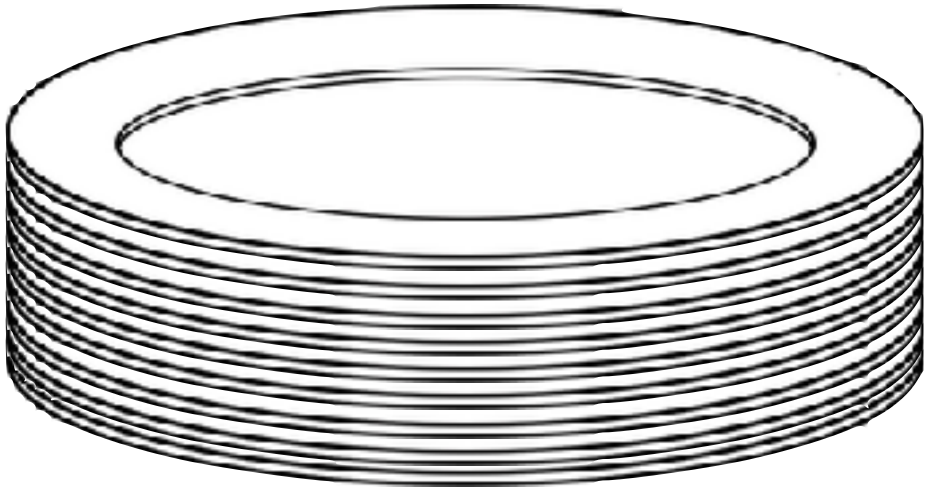

- Usually implemented as an ADT

Fundamental Data Structure: The Stack

- Last-In-First-Out (LIFO)
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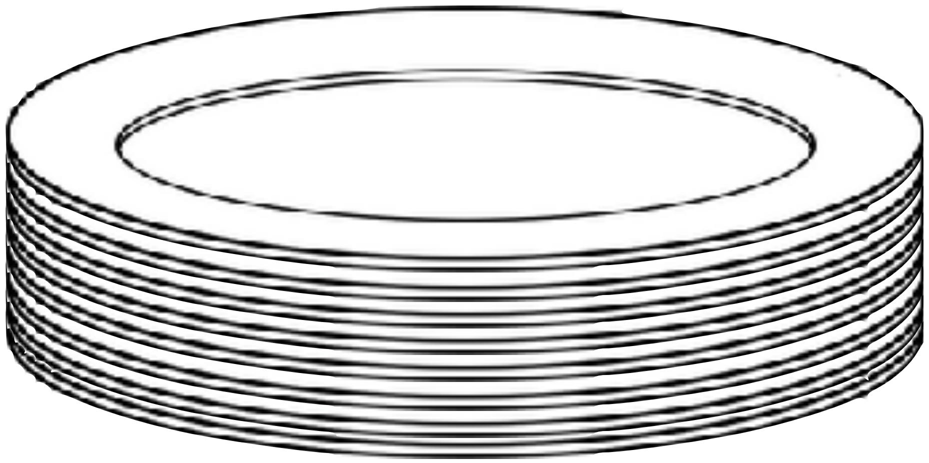

Fundamental Data Structure: The Stack

- Last-In-First-Out (LIFO)
- Operations:
  - CreateStack
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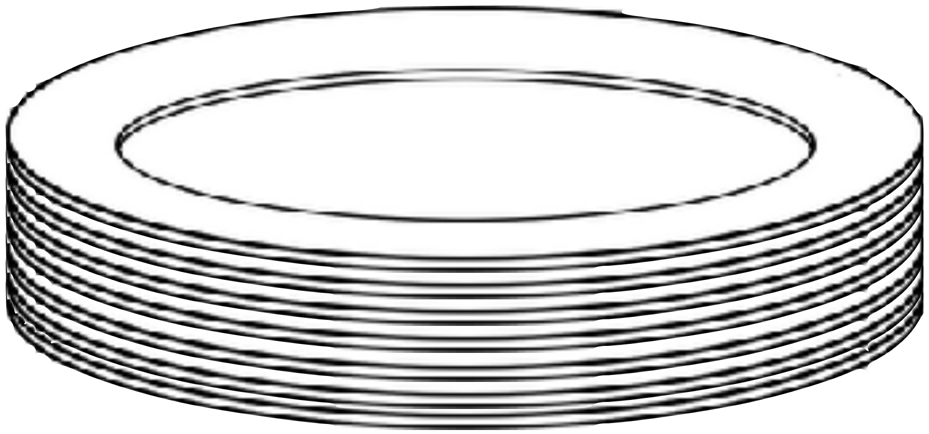

Fundamental Data Structure: The Stack

- Last-In-First-Out (LIFO)
- Operations:
  - CreateStack
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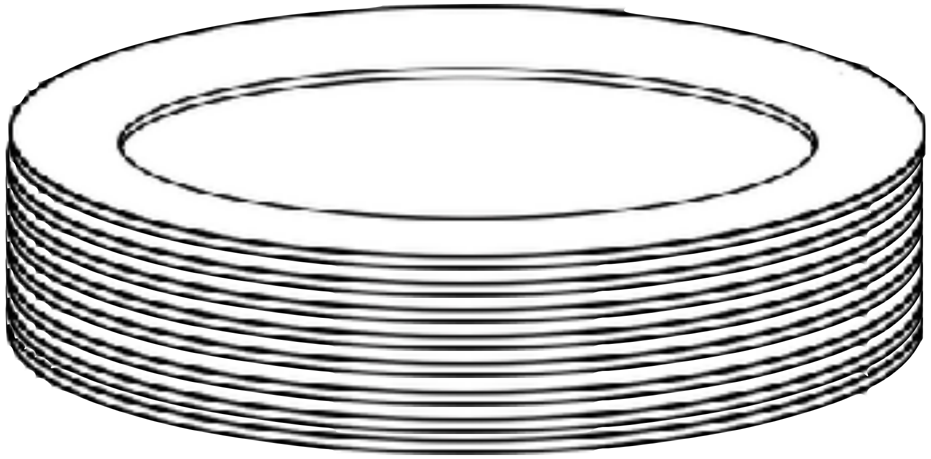

Fundamental Data Structure: The Stack

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    - CreateStack
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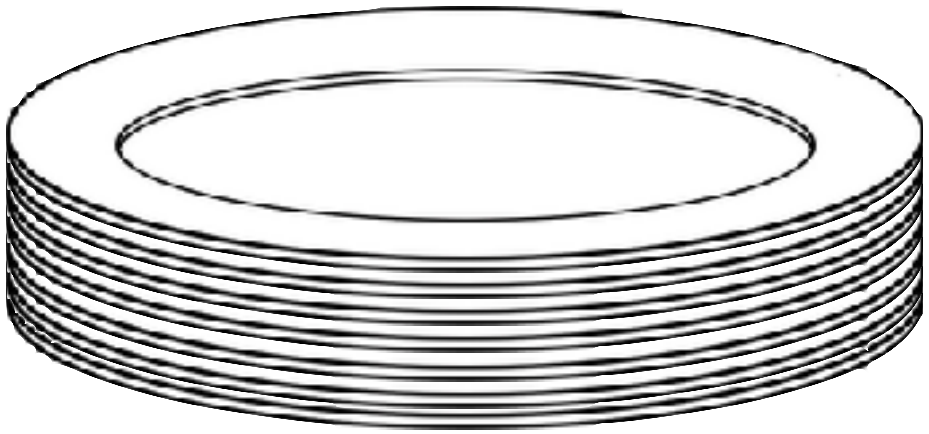

- Usually implemented as an ADT

Fundamental Data Structure: The Stack

- Last-In-First-Out (LIFO)
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  - CreateStack
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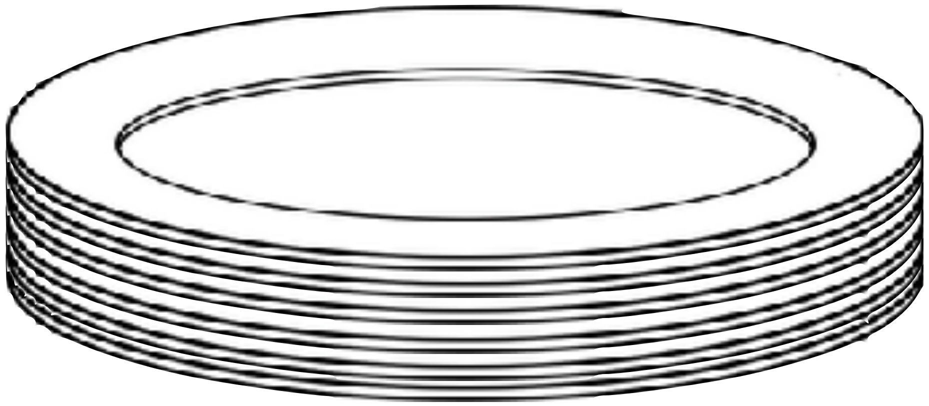


- Usually implemented as an ADT

Stack Implementation: Array

Stack Implementation: Array

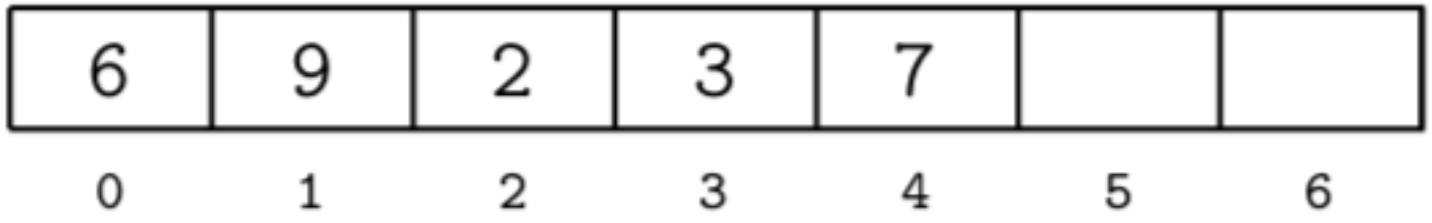
6	9	2	3	7		
0	1	2	3	4	5	6

Stack Implementation: Array

6	9	2	3	7		
0	1	2	3	4	5	6

top: 5

Stack Implementation: Array

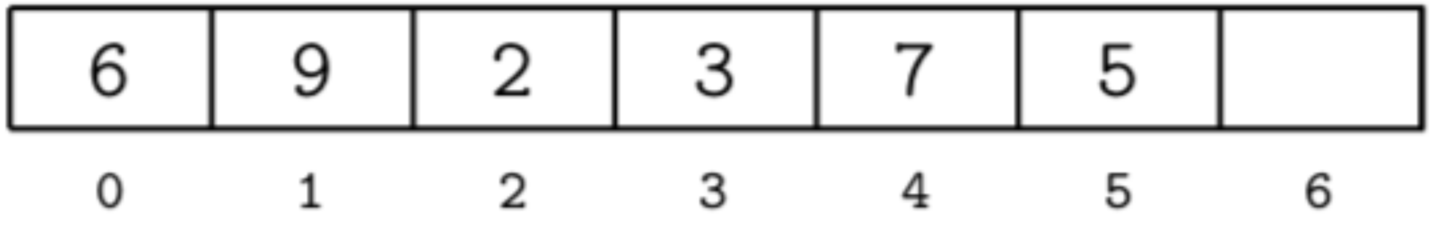


6	9	2	3	7		
0	1	2	3	4	5	6

top: 5

Push(5)

Stack Implementation: Array



6	9	2	3	7	5	
0	1	2	3	4	5	6

top: 5

Push(5)

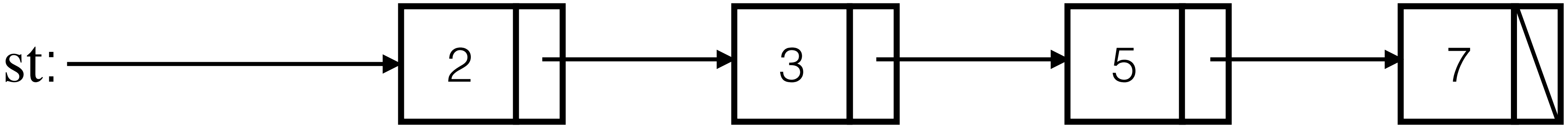
Stack Implementation: Array

6	9	2	3	7	5	
0	1	2	3	4	5	6

top: 6

Push(5)

Stack Implementation: Linked List



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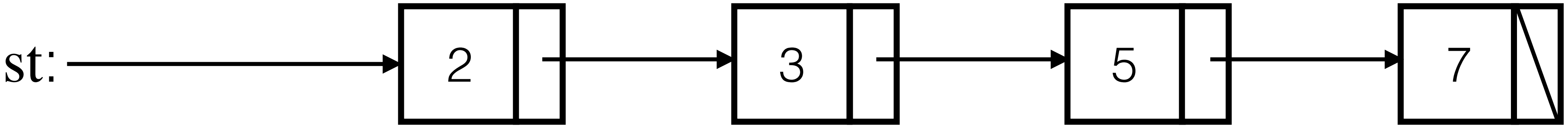

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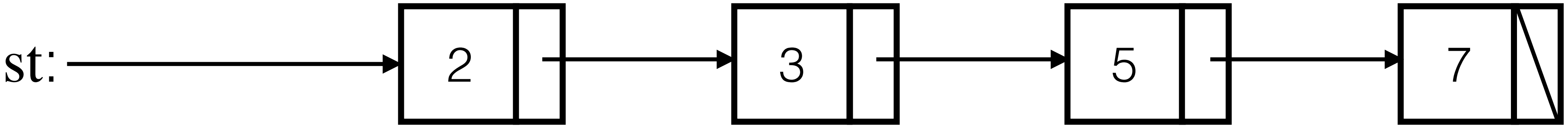

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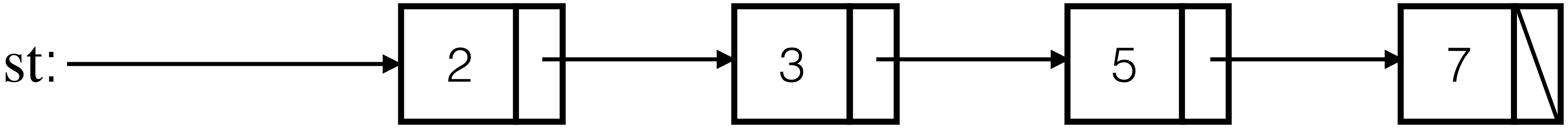

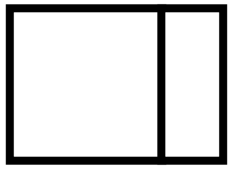

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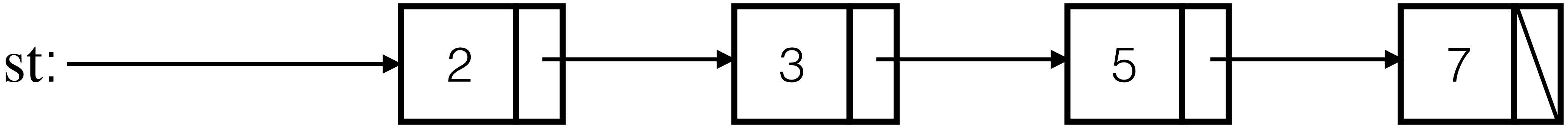

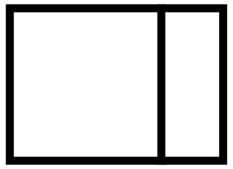

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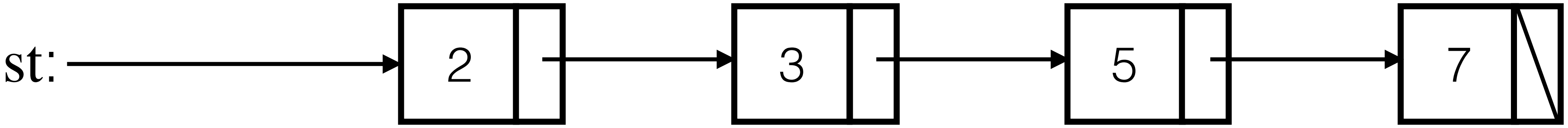

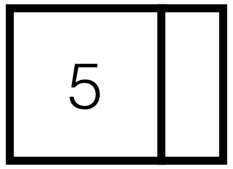

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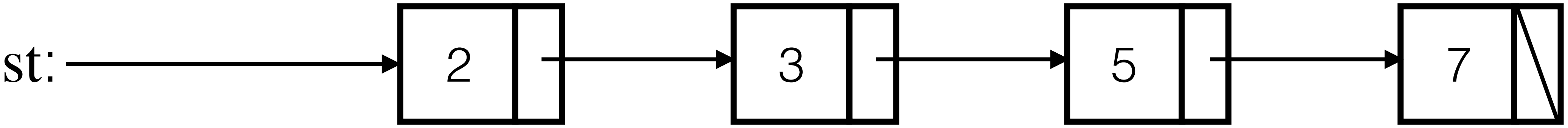

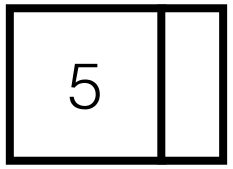

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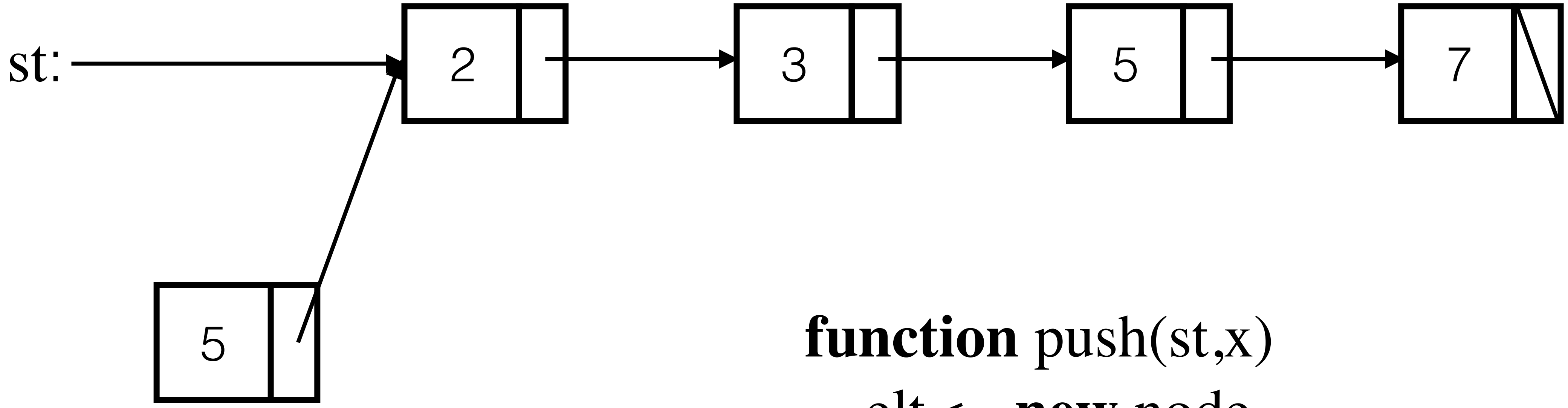

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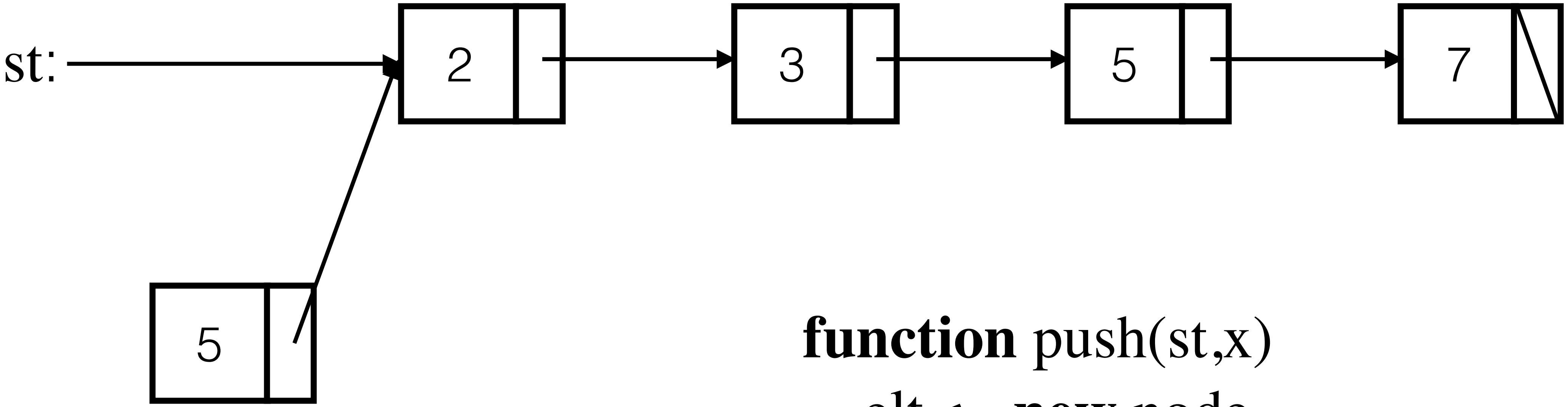

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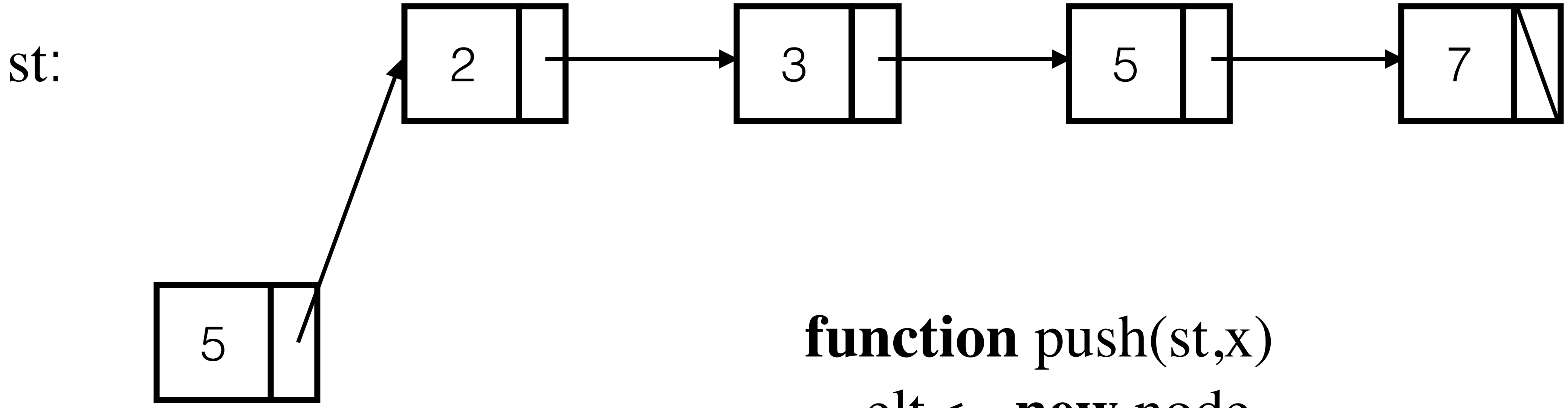

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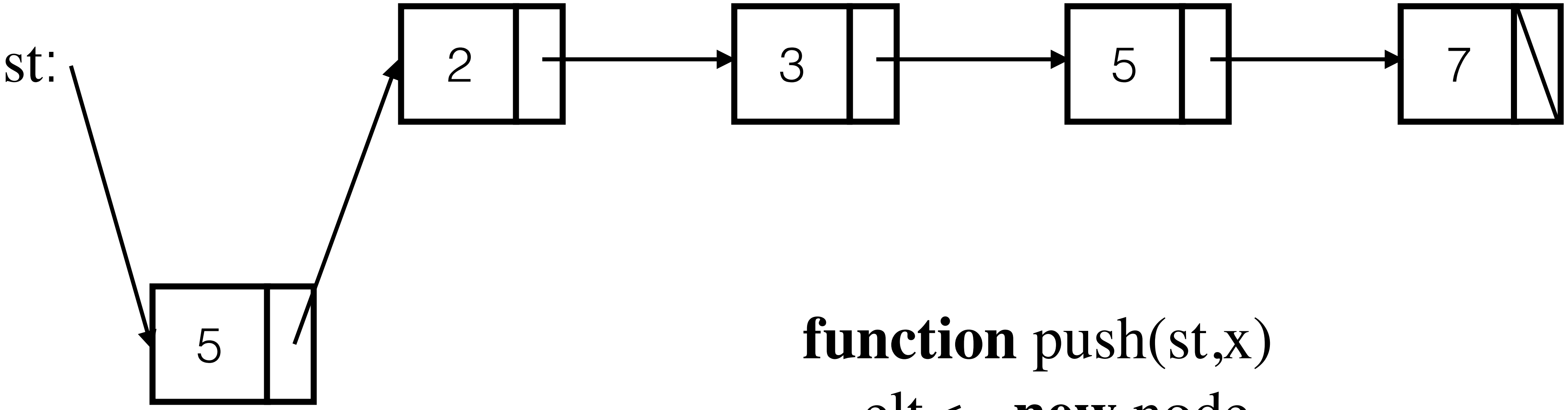

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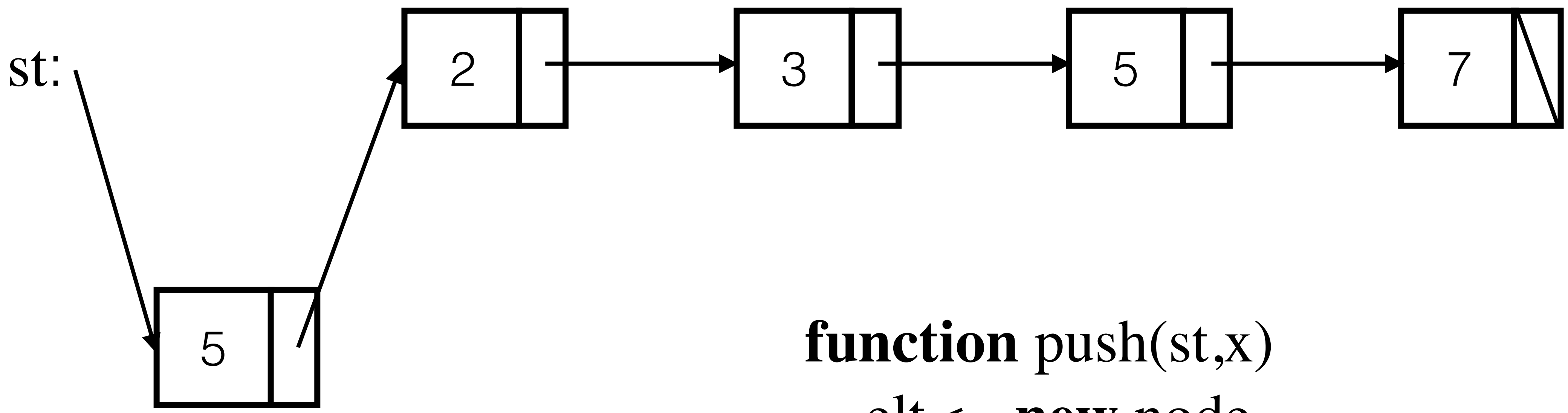

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for more visualisations

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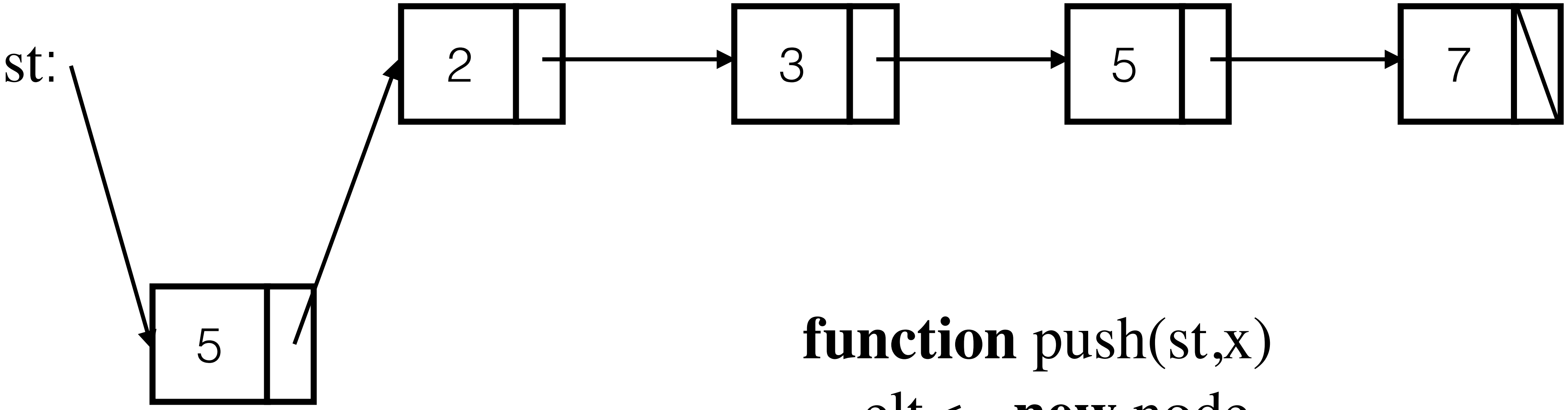

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Pseudo Code

- On the previous slide, we assumed that a “node” has two attributes: a “val” which is its value, and a “next” which points to the rest of the list.

- There is no standard for pseudo-code. Use the examples in Levitin as a guide. Cormen et al. pages 20–22 (in Reading Resources) has a list of standard conventions used with pseudo-code which are good to follow, except we use ← as the assignment operator.

Fundamental Data Structure: Queues

- First-In-First-Out (FIFO)
- Operations:
  - CreateQueue
  - Enqueue
  - Dequeue
  - Head
  - EmptyQueue?
  - …

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Other Data Structures

- We will meet many other (abstract) data structures, e.g.
  - The priority queue
  - Various types of “tree”
  - Various types of “graph”
- If you check out algorithm animation tools or advanced algorithm books, you will meet exotic data structures such as splay trees and skip lists.

Next Week

- Algorithm analysis—how to reason about an algorithm’s resource consumption.