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[illegible]

Question 1

(4 marks)

A. Give the names of two *stable* sorting algorithms, together with their worst-case time complexities. Write the names and complexities in the box:

B. Give the names of two *unstable* sorting algorithms, together with their worst-case time complexities. Write the names and complexities in the box:

Question 2

(4 marks)

We are given an array A holding n integers, for some large n . **The array is sorted**, and the values in A range from -2147483648 to 2147483647, evenly distributed. Give Θ expressions for the following tasks:

A. Running the insertion sort algorithm on the array A :

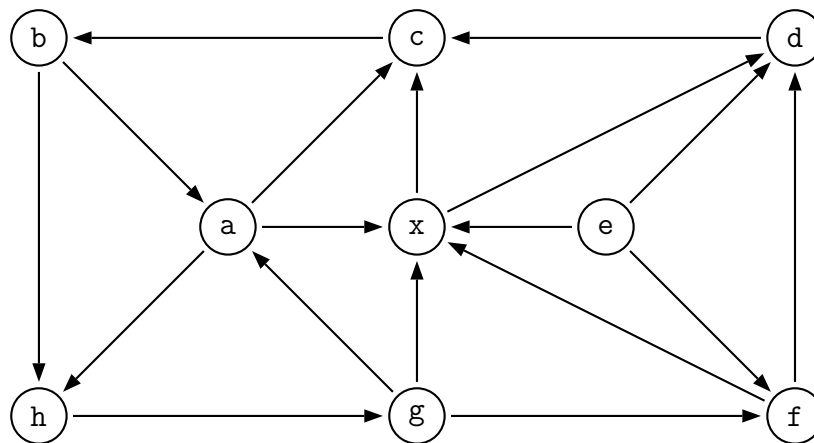
B. Running the selection sort algorithm on the array A :

C. Performing binary search for integer k which is not in A :

D. Performing interpolation search for integer k not in A :

Question 3**(4 marks)**

For the directed graph below, list the order in which the nine nodes are visited during a depth-first (DFS) traversal, as well as the order in which they are visited during a breadth-first (BFS) traversal. As always, assume that any **ties are resolved** by taking nodes in alphabetical order. Write the answers in the boxes given.



DFS sequence:

BFS sequence:

Question 4**(4 marks)**

Given the pattern A T G A and the text

T C A T C A T C C A T G C A C A A T G A C T T T

how many character comparisons will Horspool's algorithm make before locating the pattern in the text? Write the number in the box:

Question 5**(4 marks)**

Assume the array A holds the keys 77, 64, 15, 43, 28, 91, 80, 32, 56 in index positions 1 to 9. Show the heap that results after application of the linear-time bottom-up heap construction algorithm. You may show the heap as a tree or as an array.

Question 6**(4 marks)**

The functions A – D are defined recursively as follows (all divisions round down, to the closest integer):

$$\begin{aligned}
 A(n) &= 2 A(n/3) + 2, & \text{with } A(1) &= 1 \\
 B(n) &= B(n/2) + n/2, & \text{with } B(1) &= 1 \\
 C(n) &= 512 C(n/8) + 4n^2, & \text{with } C(1) &= 4 \\
 D(n) &= 4 D(n/2) + n^2, & \text{with } D(1) &= 2
 \end{aligned}$$

In the following table, for each of the four functions, tick the most precise correct statement about the function's rate of growth:

	$O(n)$	$\Theta(n)$	$O(n \log n)$	$\Theta(n^2)$	$O(n^2 \log n)$	$\Theta(n^2 \sqrt{n})$	$O(n^3)$
A							
B							
C							
D							

Question 7**(4 marks)**

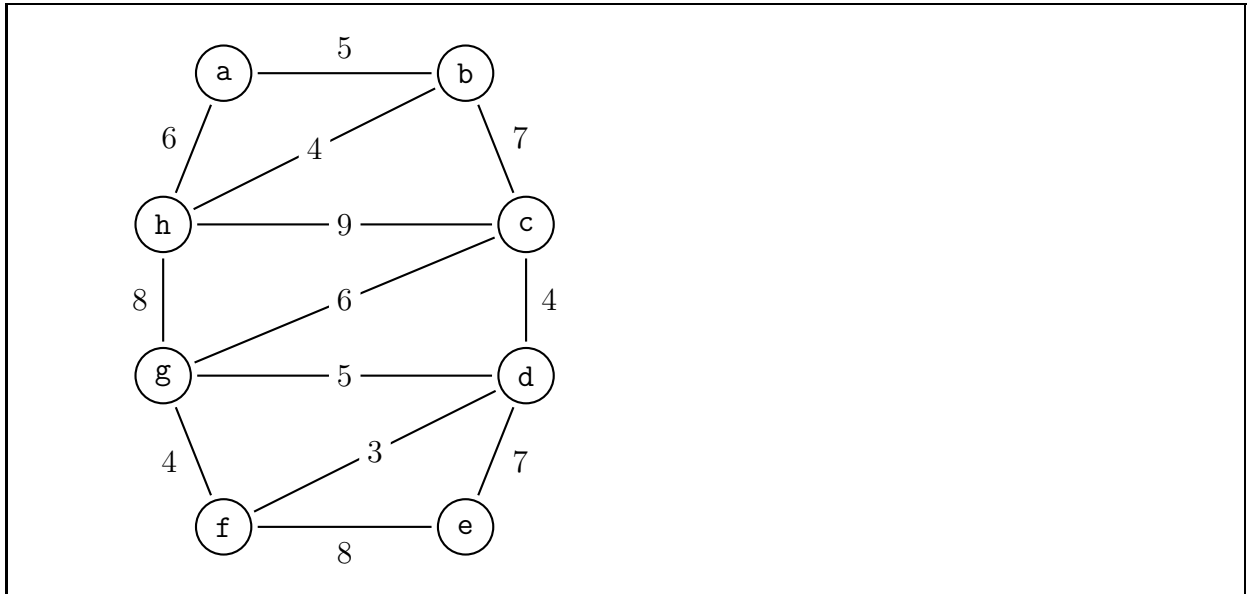
For each of **A–D** below, answer yes or no, and, in each case, briefly explain your reasoning (just a justification of your answer, rather than detailed calculations). A yes/no answer that is not justified will not attract marks, even if correct.

Question	Answer/explanation
A. Is $\sqrt{n} \in \Omega(n)$?	
B. Is $n^{2 \log n} \in O(n^{\log n})$?	
C. Is $\Theta(\log(n^{2 \log n})) = \Theta(\log(n^{\log n}))$?	
D. Is $\Theta(\log(2^n)) = \Theta(\log(3^n))$?	

Question 8

(6 marks)

A. The box below contains a weighted undirected graph with eight nodes. Give a minimum spanning tree for the graph. You may do that either by outlining a minimum spanning tree on the graph itself, or by drawing the tree in the empty space next to the graph.



B. Given a weighted graph $G = \langle V, E \rangle$, a subgraph $\langle V, E' \rangle$ (that is, $E' \subseteq E$) which is a tree with minimal weight is a *maximum spanning tree* for G .

We want a transformation of the graph G so that we can run Prim's algorithm on the transformed graph G' , and the algorithm will find a maximum spanning tree for G .

Describe such a (systematic) transformation from G to G' .

Question 9**(6 marks)**

Consider the function F below. The function takes as input an integer array A , and the size n of A . The array indices run from 1 to n . The division used is integer division, that is, it rounds down to the closest smaller (or equal) integer value.

In the box, give a Θ expression for the function's time complexity.

```
function  $F(A[\cdot], n)$   
   $s \leftarrow 0$   
   $m \leftarrow n$   
  while  $m > 0$  do  
    for  $i \leftarrow 1$  to  $m$  do  
       $s \leftarrow s + A[i]$   
     $m \leftarrow m/2$ 
```

Question 10

(10 marks)

Using pseudo-code, give an algorithm for deleting the smallest element of a binary search tree (a BST). Assume a non-empty binary tree T has attributes $T.left$, $T.right$, and $T.root$ which denote T 's left sub-tree, right sub-tree, and the key of T 's root node, respectively. You can use these tests if they seem useful: $ISLEAF(T)$ tests whether the binary tree T is a leaf, and $ISEMPTY(T)$ tests whether it is empty.

Question 11**(10 marks)**

Consider an array A of n distinct integers (that is, all elements are different). It is known that A was originally sorted in ascending order, but A was then right-rotated r places, where $0 < r < n$. In other words, the last r elements were moved from the end of the array to the beginning, with all other elements being pushed r positions to the right. For example, for $n = 7$ and $r = 3$, the result may look like this:

[43, 46, 58, 12, 20, 29, 34]

For $r = 5$, the result, based on the same original array, would be

[29, 34, 43, 46, 58, 12, 20]

You know that the given $A[0..n-1]$ has this particular form, that is, for some r , the sequence $A[r], \dots, A[n-1], A[0], \dots, A[r-1]$ is in ascending order, but you do not know what r is. Design an algorithm to find the largest integer in A . Full marks are given for an algorithm that works in time $O(\log n)$; half marks are given for a solution that is correct, but less efficient.

Question 12**(10 marks)**

Two programmers face the following problem. Given an array containing n random integers in random order, find the largest integer. The integers are placed in cells $A[1] \dots A[n]$.

Programmer X has come up with the code shown below, on the left. (In the programming language used, arrays are indexed from 0, but X 's method does not use $A[0]$.)

```
function X( $A[\cdot], n$ )
   $max \leftarrow A[1]$ 
   $i \leftarrow 2$ 
  while  $i \leq n$  do
    if  $A[i] > max$  then
       $max \leftarrow A[i]$ 
     $i \leftarrow i + 1$ 
  return  $max$ 
```

```
function Y( $A[\cdot], n$ )
   $i \leftarrow n$ 
  while  $i > 0$  do
     $A[0] \leftarrow A[i]$ 
     $i \leftarrow i - 1$ 
    while  $A[0] > A[i]$  do
       $i \leftarrow i - 1$ 
  return  $A[0]$ 
```

Programmer Y has solved the same problem differently, as shown above on the right.

Compare the two solutions using three criteria: Correctness, time complexity class, and the number of comparisons performed. Write your analysis in the box:

Overflow space

Use this page if you ran out of writing space in some question. Make sure to leave a pointer to this page from the relevant question.