COMP90049 Knowledge Technologies

Data Mining
Association Analysis (Lecture **Set 8)**

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Some of slides are derived from Prof Vipin Kumar and modified, http://www-users.cs.umn.edu/~kumar/

Association Rule Mining

Given a set of (several millions of) transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

{Diaper} → {Beer},
{ Bread} → {Milk},

{ Bread, Milk} → {Diaper}

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

Itemset

- A collection of one or more items

  Example: {Milk, Bread, Diaper}
- k-itemset

  An itemset that contains k items

Support count (σ)

- Frequency of occurrence of an itemset
- E.g. σ({Bread, Milk, Diaper}) = 2

Support

- Fraction of transactions that contain an itemset
- E.g. s({Milk, Bread, Diaper}) = 2/5

Frequent Itemset

- An itemset whose support is greater than or equal to a *minsup* threshold

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Milk, Diaper, Beer, Bread
5	Milk, Diaper, Bread, Coke

Association Rule

- An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
- Example:
  $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$

Rule Evaluation Metrics

- Support (s)

  Fraction of transactions that contain both X and Y
- Confidence (c)

  Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\,\text{T}\,|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

Given a set of transactions T, the goal of association rule mining is to find all rules having

- support ≥ *minsup* threshold
- confidence ≥ *minconf* threshold

Brute-force approach:

- List all possible association rules
- Compute the support and confidence for each rule
- Prune rules that fail the *minsup* and *minconf* thresholds

$\Rightarrow$ Computationally prohibitive!

Mining Association Rules

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

{Milk,Diaper} → {Beer} (s=2/5, c=2/3)
{Milk,Beer} → {Diaper} (s=2/5, c=2/2)
{Diaper,Beer} → {Milk} (s=2/5, c=2/3)
{Beer} → {Milk,Diaper} (s=2/5, c=2/3)
{Diaper} → {Milk,Beer} (s=2/5, c=2/4)
{Milk} → {Diaper,Beer} (s=2/5, c=2/4)

Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

Two-step approach:

1. Frequent Itemset Generation

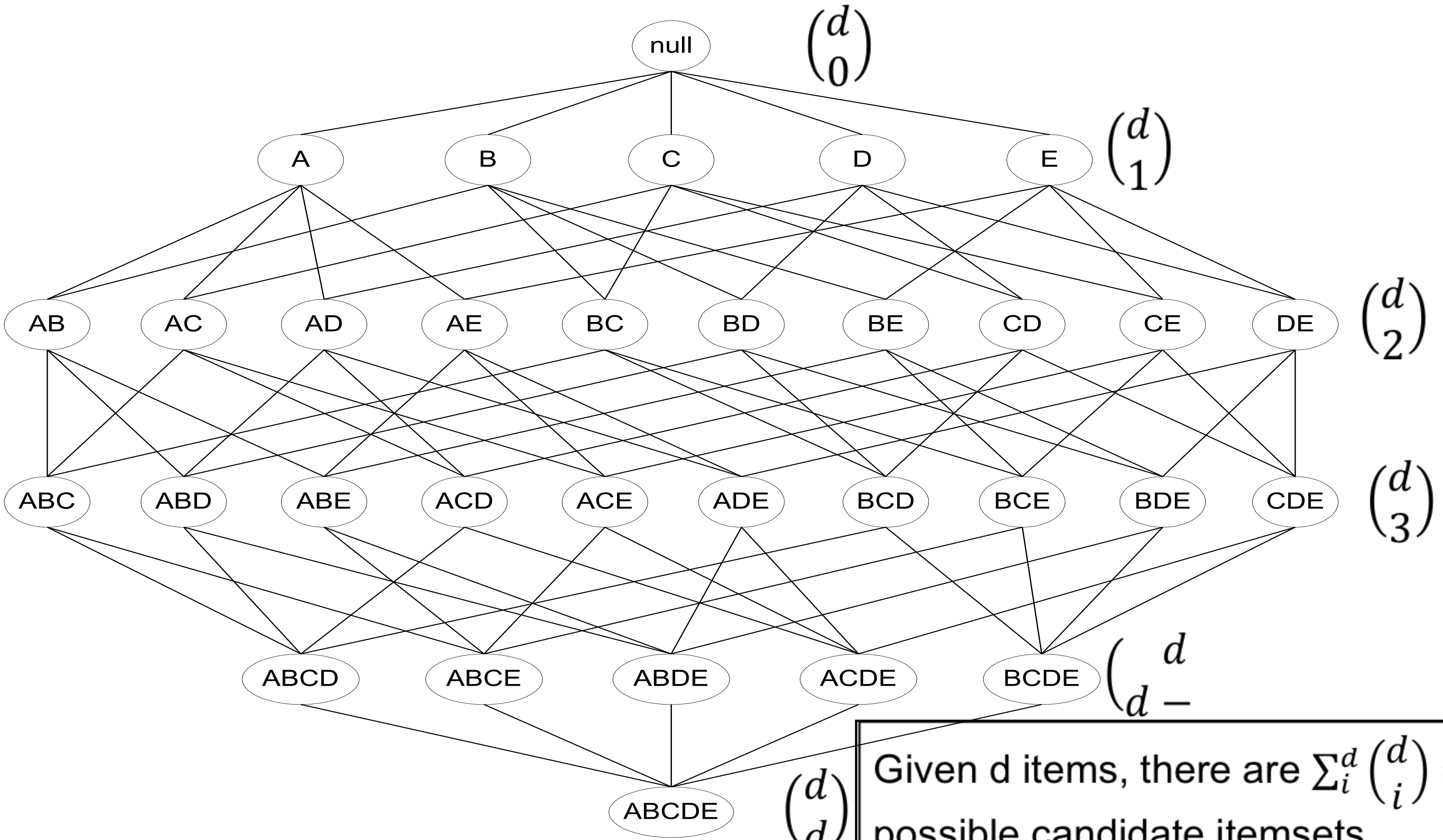
   Generate all itemsets whose support $\geq$ minsup

2. Rule Generation

   Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

Frequent itemset generation is still computationally expensive

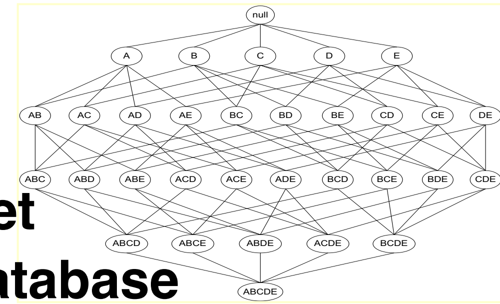
Frequent Itemset Generation



Given d items, there are $\sum_i^d \binom{d}{i} = (1+1)^d = 2^d$ possible candidate itemsets

Frequent Itemset Generation

Lattice



Brute-force approach:

- **Each itemset in the lattice is a candidate frequent itemset**
- **Count the support of each candidate by scanning the database**

Transactions

N

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
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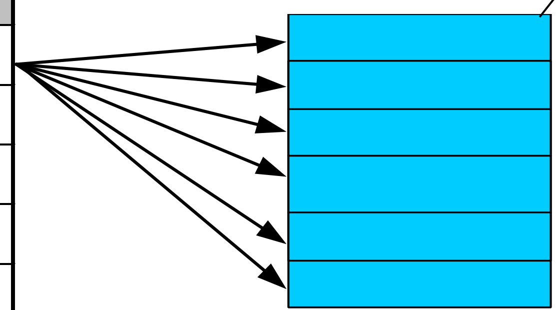
5	Bread, Milk, Diaper, Coke

w

List of Candidates



M

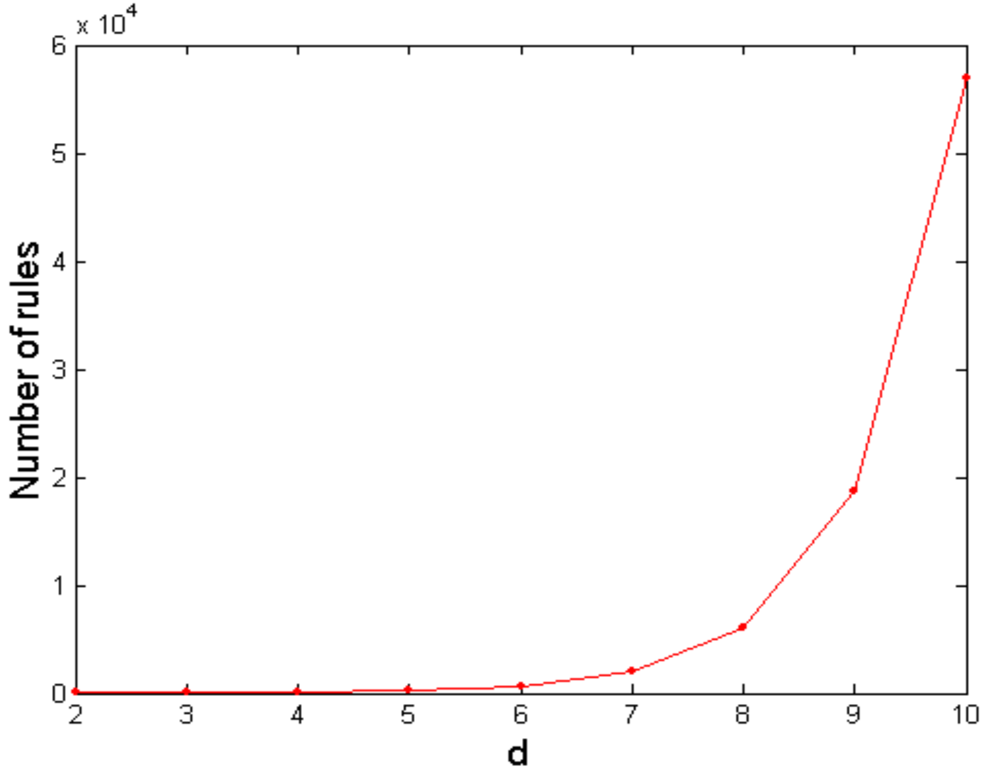
{Bread (Br)}
{Milk (M)}
…
({Br, M}
{Br,D}
…
{Br, M, D}
…
{Br, M, D, Be}
…
{Br, M, D, Be, C}
…
{Br, M, D, Be, C, E}

- Match each transaction against every candidate
- Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Computational Complexity

Given d unique items:

- Total number of itemsets = 2^d
- Total number of possible association rules:



#ways left side items can be chosen out of d items

#ways right side items can be chosen using the remaining d-k items

$$R = \sum_{k=1}^{d-1} \left[\binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \right]$$

$$= 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

An example d= 3 and item set = {abc}

$\{a\} \rightarrow \{b\}$ $\{a\} \rightarrow \{c\}$ $\{a\} \rightarrow \{bc\}$

$\{b\} \rightarrow \{a\}$ $\{b\} \rightarrow \{c\}$ $\{b\} \rightarrow \{ac\}$

$\{c\} \rightarrow \{a\}$ $\{c\} \rightarrow \{b\}$ $\{c\} \rightarrow \{ab\}$

$\{ab\} \rightarrow \{c\}$ $\{ac\} \rightarrow \{b\}$ $\{bc\} \rightarrow \{a\}$

Frequent Itemset Generation Strategies

Reduce the number of candidates (M)

- Complete search: $M=2^d$
- Use pruning techniques to reduce M

Reduce the number of transactions (N)

- Reduce size of N as the size of itemset increases
- Used by DHP (Direct Hashing and Pruning) and vertical-based mining algorithms

Reduce the number of comparisons (NM)

- Use efficient data structures to store the candidates or transactions
- No need to match every candidate against every transaction

Reducing Number of Candidates

Apriori principle:

- If an itemset is frequent, then all of its subsets must also be frequent

Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

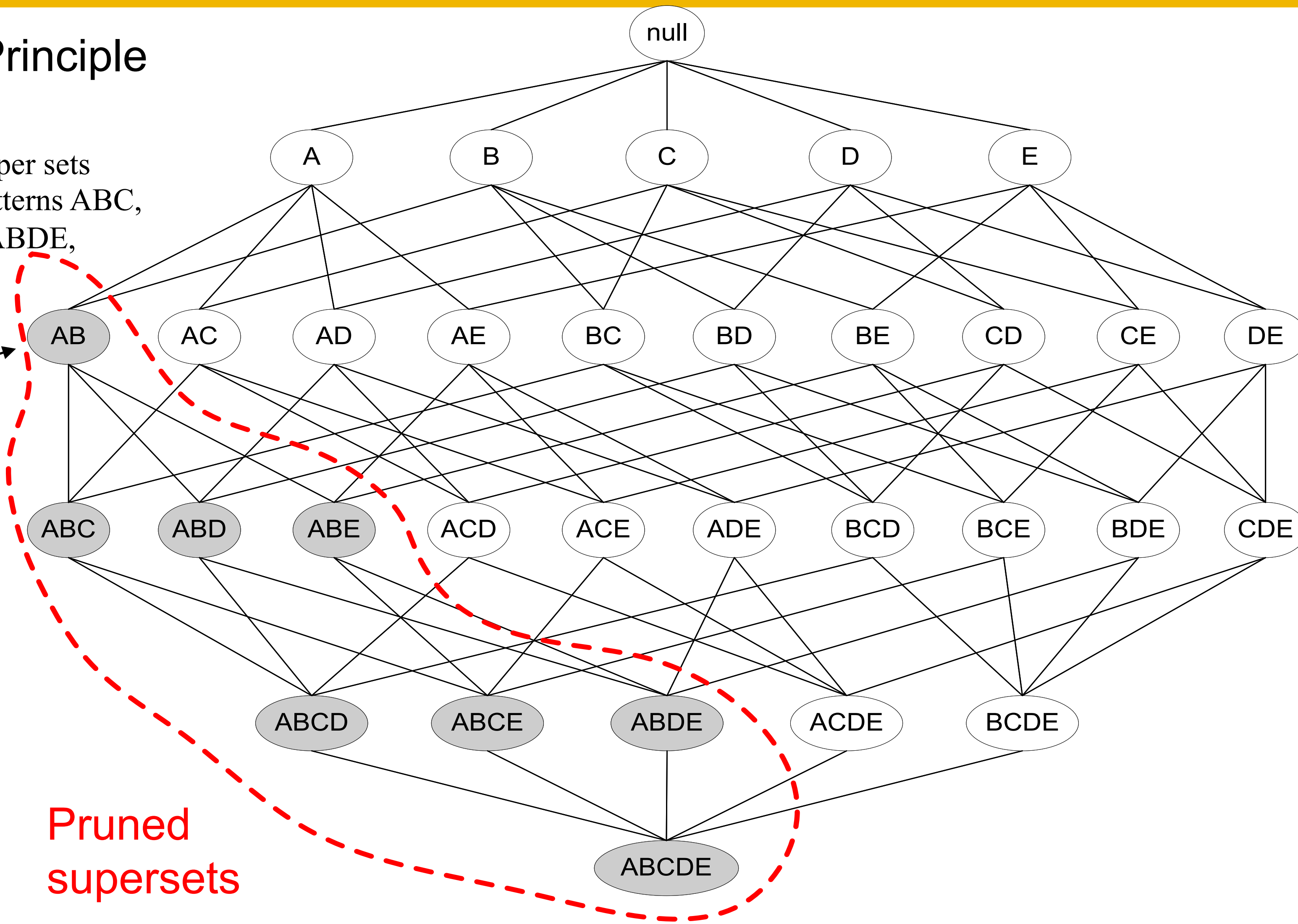
$$Where\ s(X)\ \text{support of}\ X$$

- Support of an itemset never exceeds the support of its subsets
- This is known as the anti-monotone property of support

Illustrating Apriori Principle

If AB is infrequent all its super sets are in frequent and hence patterns ABC, ABD, ABE,ABCD,ABCE, ABDE, ABCDE are all infrequent.

Found to be Infrequent



Illustrating Apriori Principle

TID	Items
1	Bread, Milk
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Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs as min support = 3)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	2

If every subset up to 3 itemsets are considered, Number of subsets $= {}^6C_1$(itemset size of 1)$+ {}^6C_2$ (itemset size of 2)$+ {}^6C_3$ (itemset size of 3)$=$ 41

With support-based pruning (see tables above),

6 + 6 + 1 = 13

Apriori Algorithm

Method:

- Let k=1
- Generate frequent itemsets of length k
- Repeat until no new frequent itemsets are identified
  - Generate length (k+1) candidate itemsets from length k frequent itemsets
  - Prune candidate itemsets containing subsets of length k+1 that are infrequent
  - Count the support of each candidate by scanning the DB
  - Eliminate candidates that are infrequent, leaving only those that are frequent

Reducing Number of Comparisons

Candidate counting:

- Scan the database of transactions to determine the support of each candidate itemset
- To reduce the number of comparisons, store the candidates in a hash structure

  Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

Transactions

N

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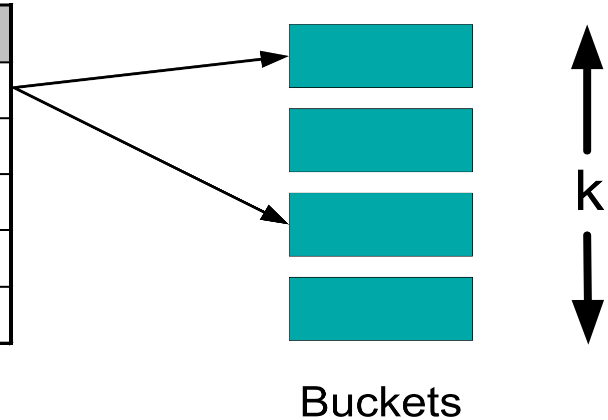
Hash Structure



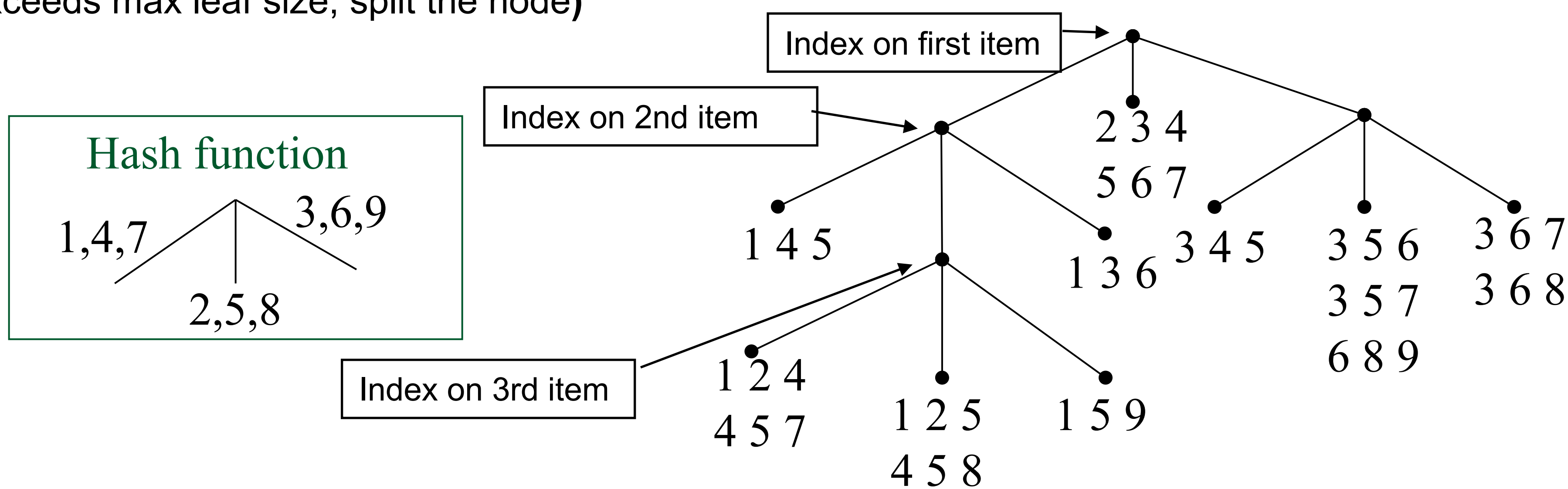
Generate Hash Tree

Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

We need:

- Hash function
- Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)

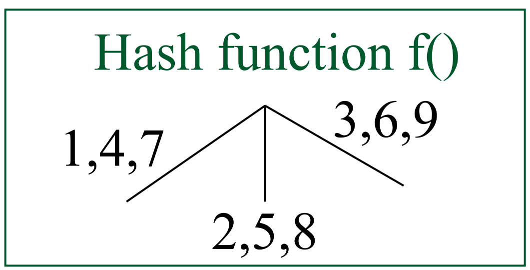


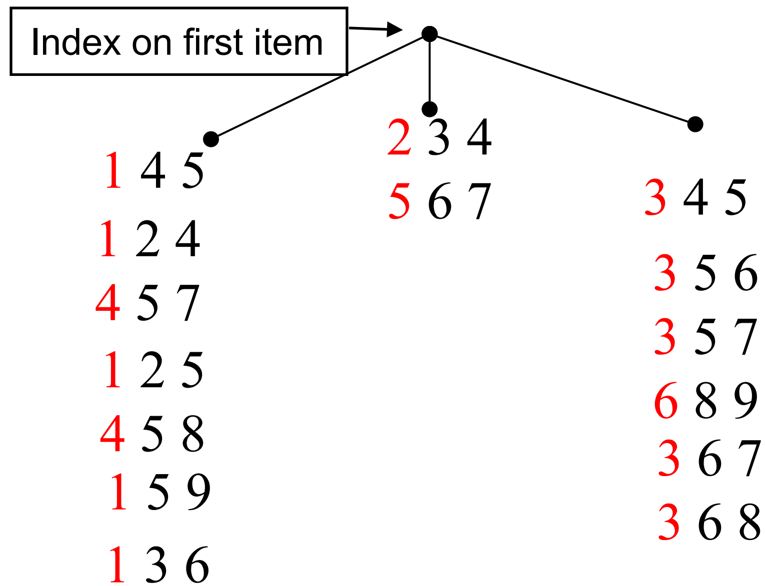
Generate Hash Tree

Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

f(1 or 4 or 7) = left branch
f(2 or 5 or 8) = middle branch
f(3 or 6 or 9) = right branch

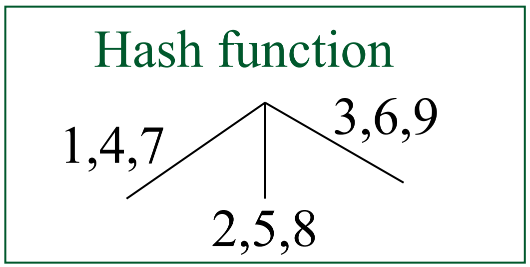


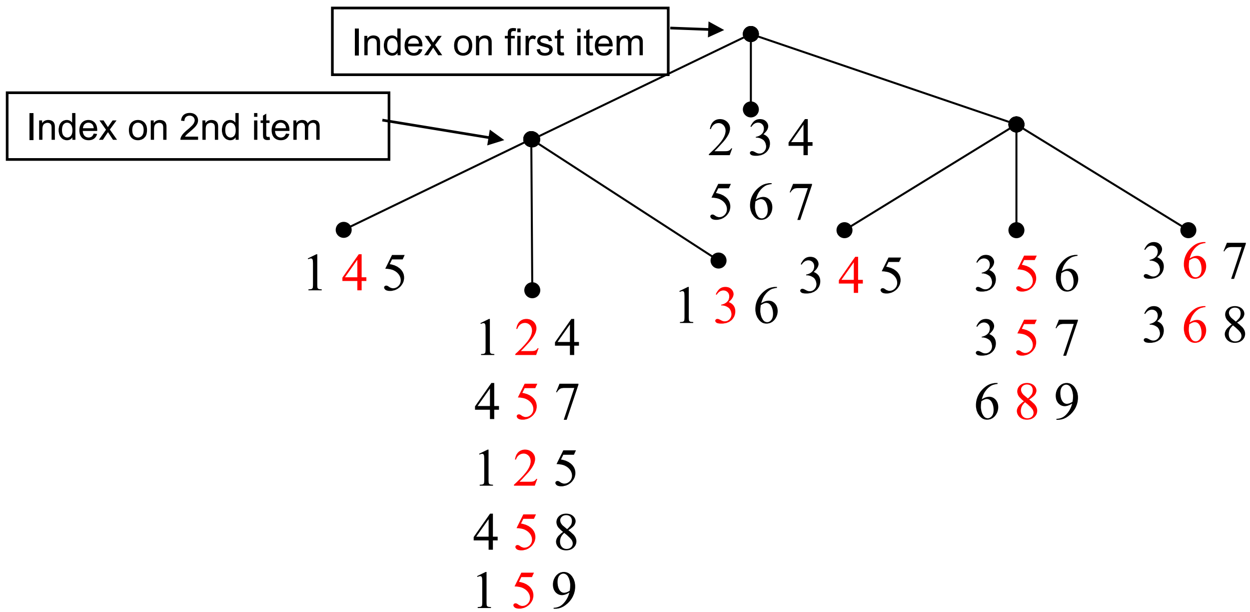


Generate Hash Tree

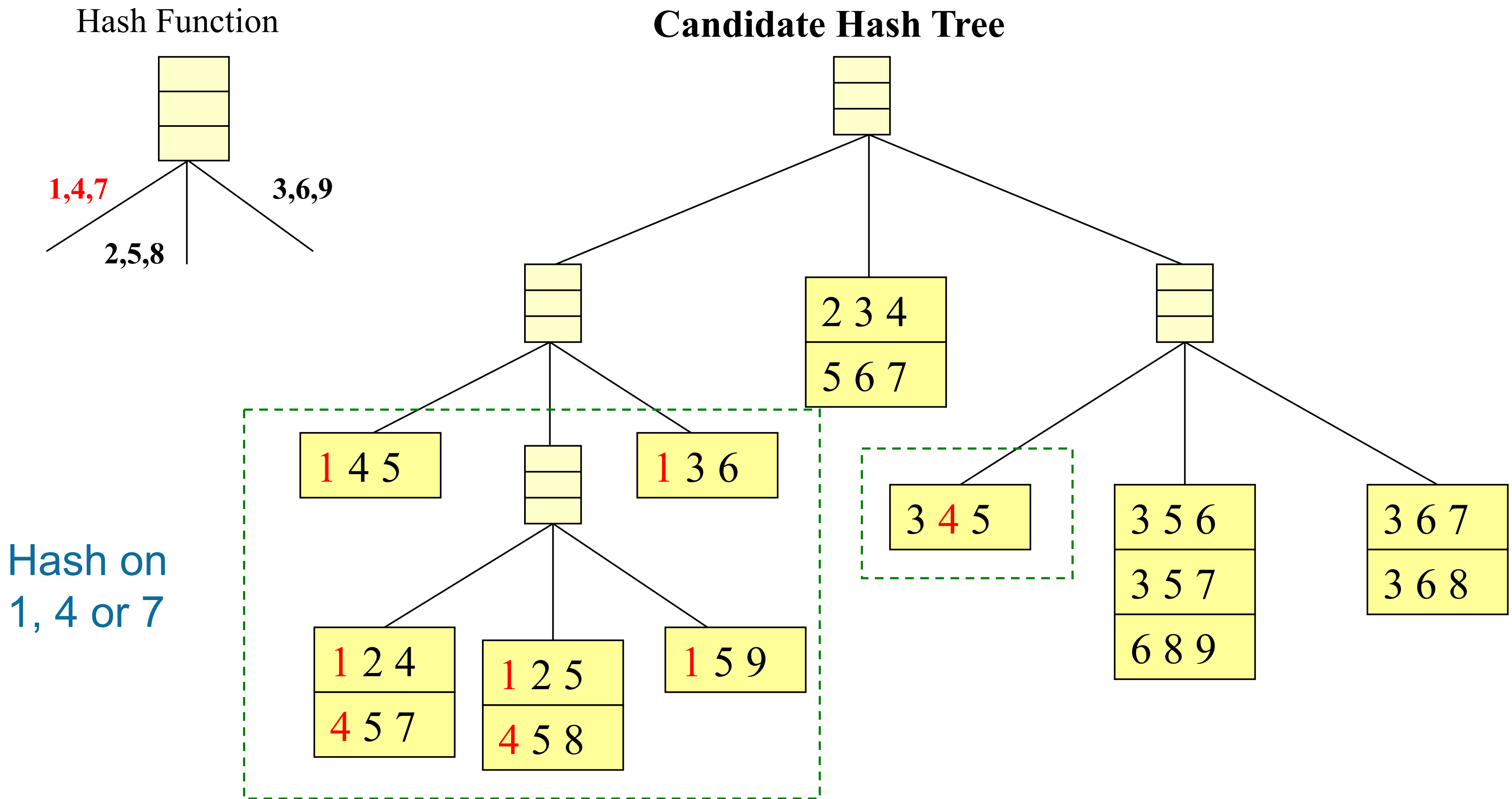
Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

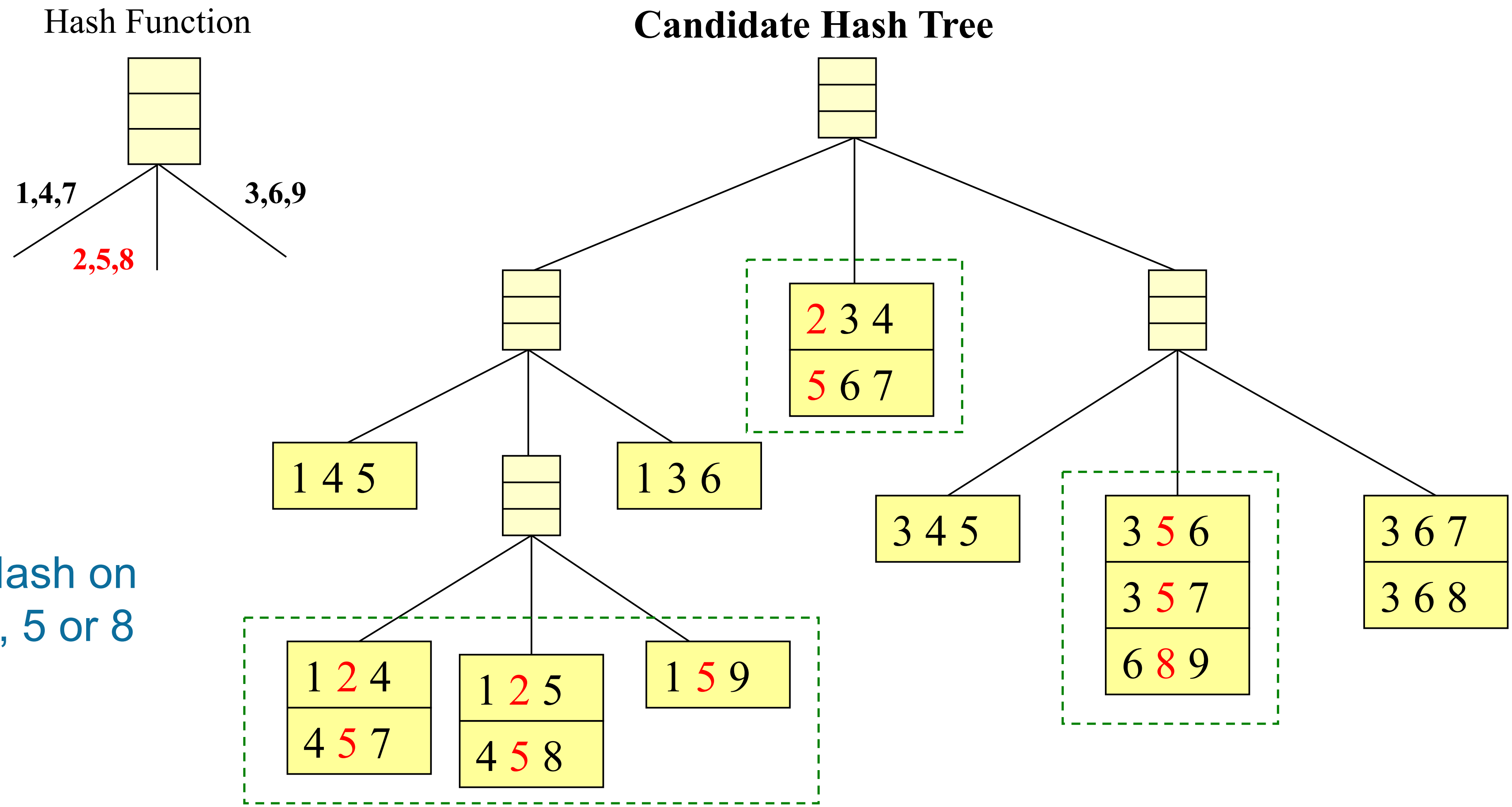




Association Rule Discovery: Hash tree

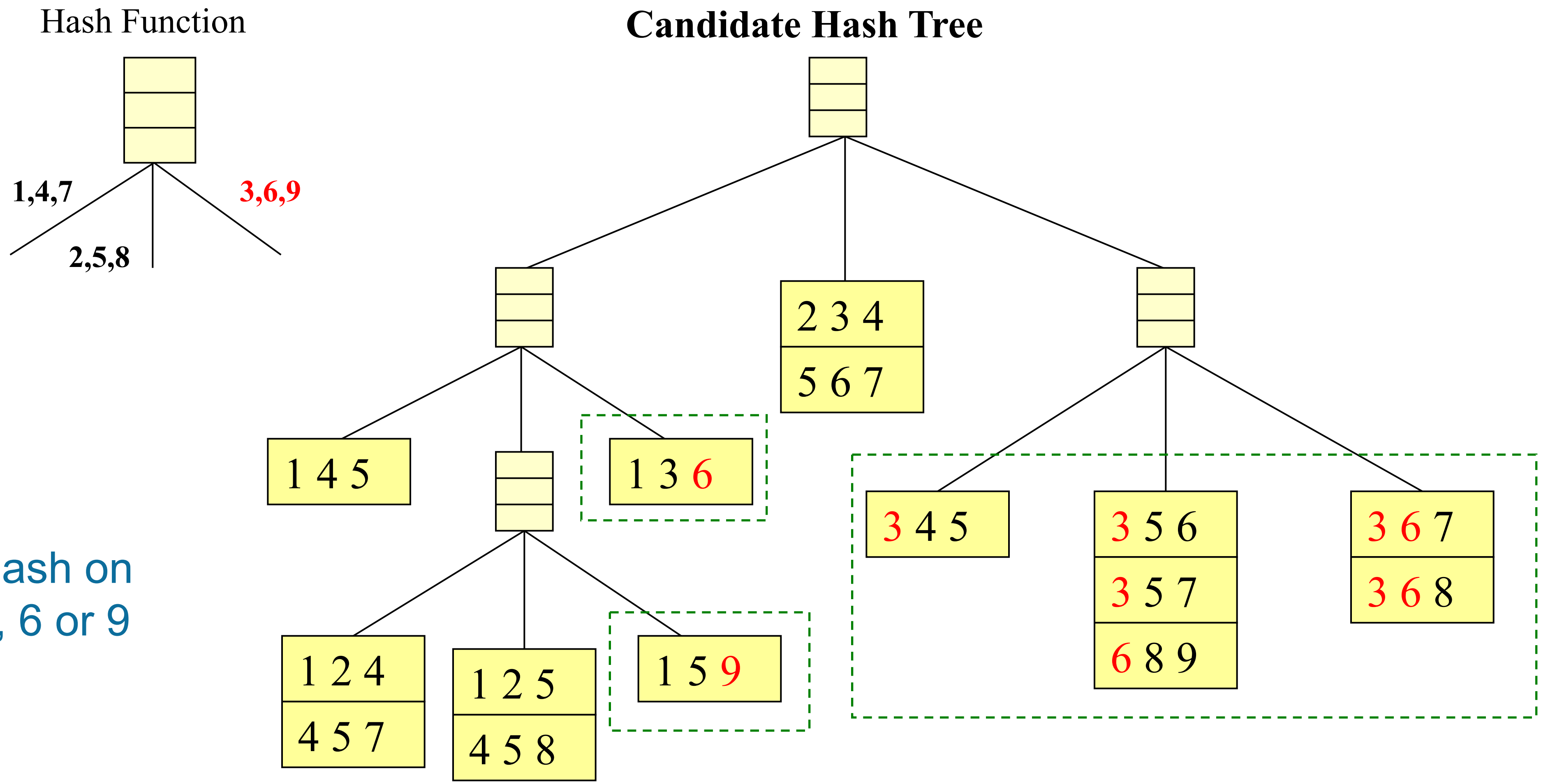


Association Rule Discovery: Hash tree



Hash on
2, 5 or 8

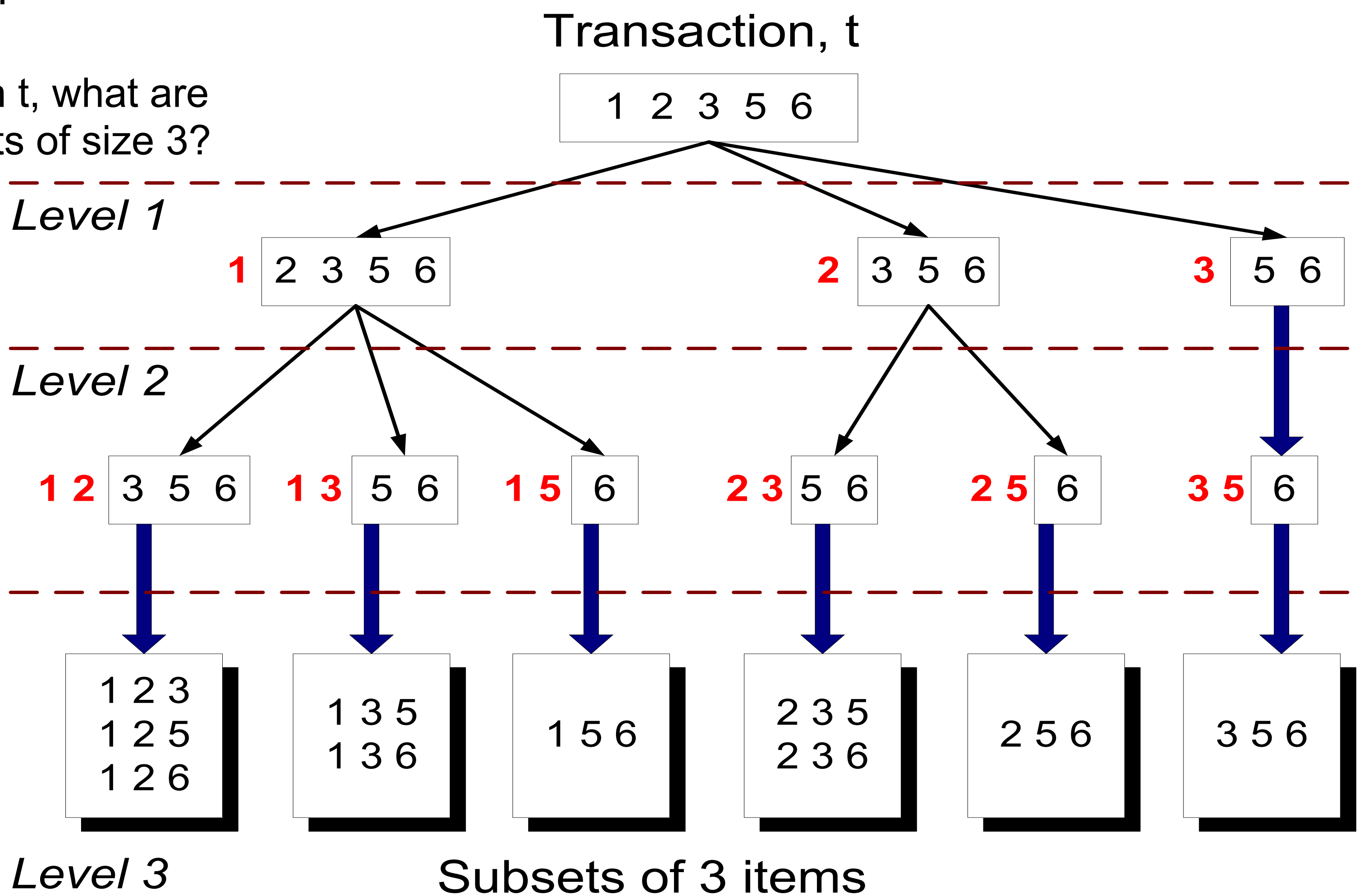
Association Rule Discovery: Hash tree



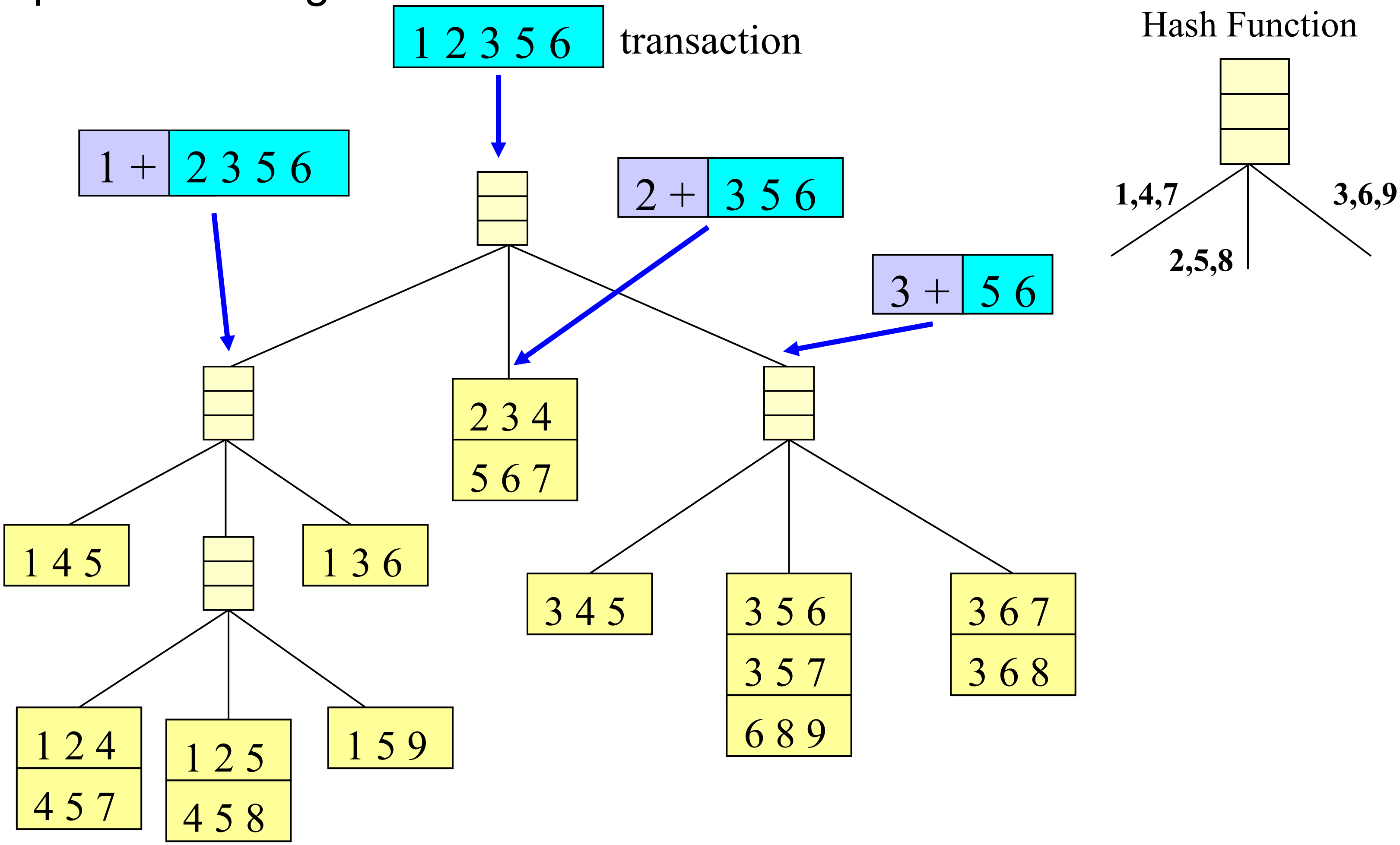
Hash on
3, 6 or 9

Subset Operation

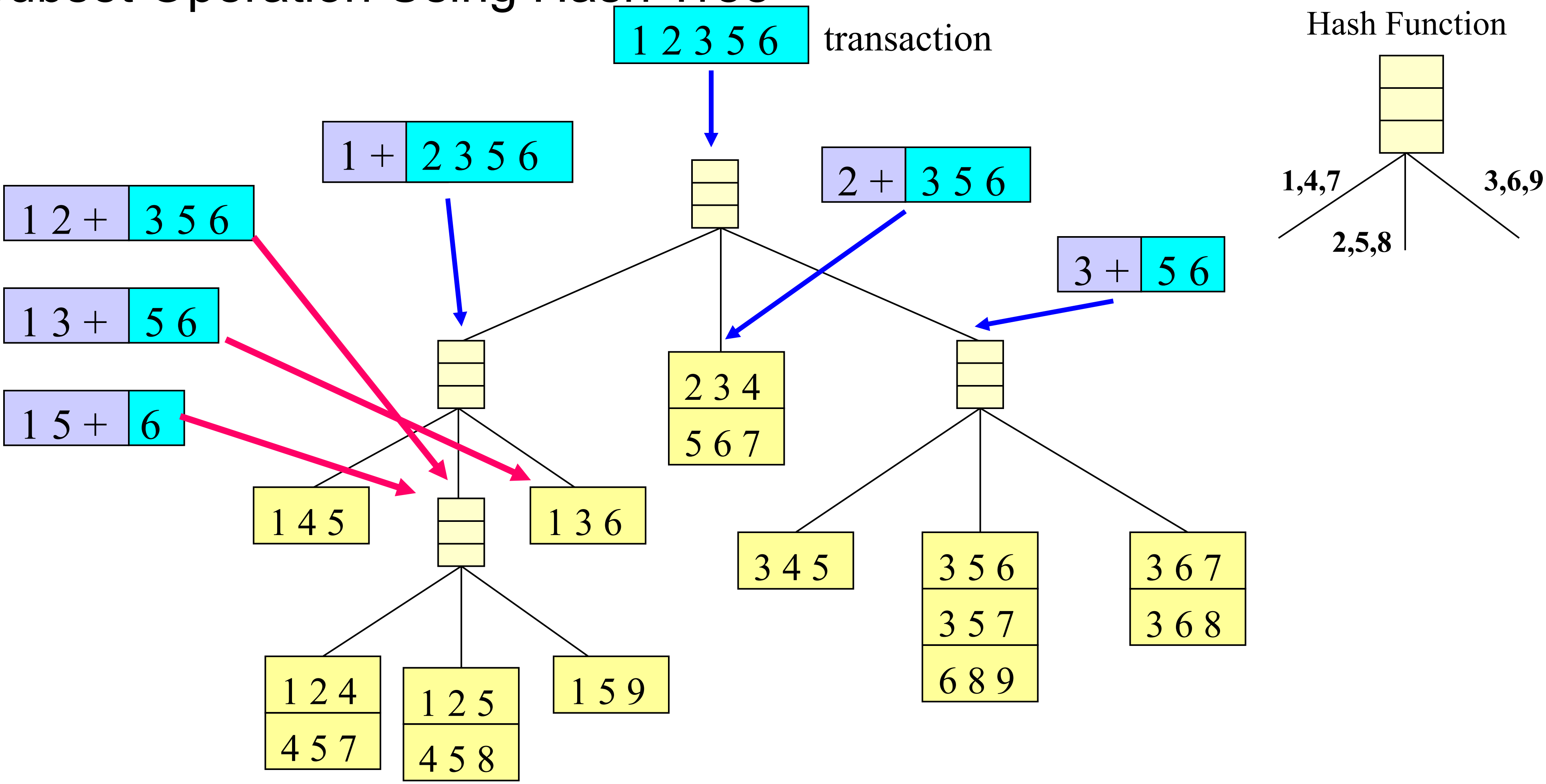
Given a transaction t, what are the possible subsets of size 3?



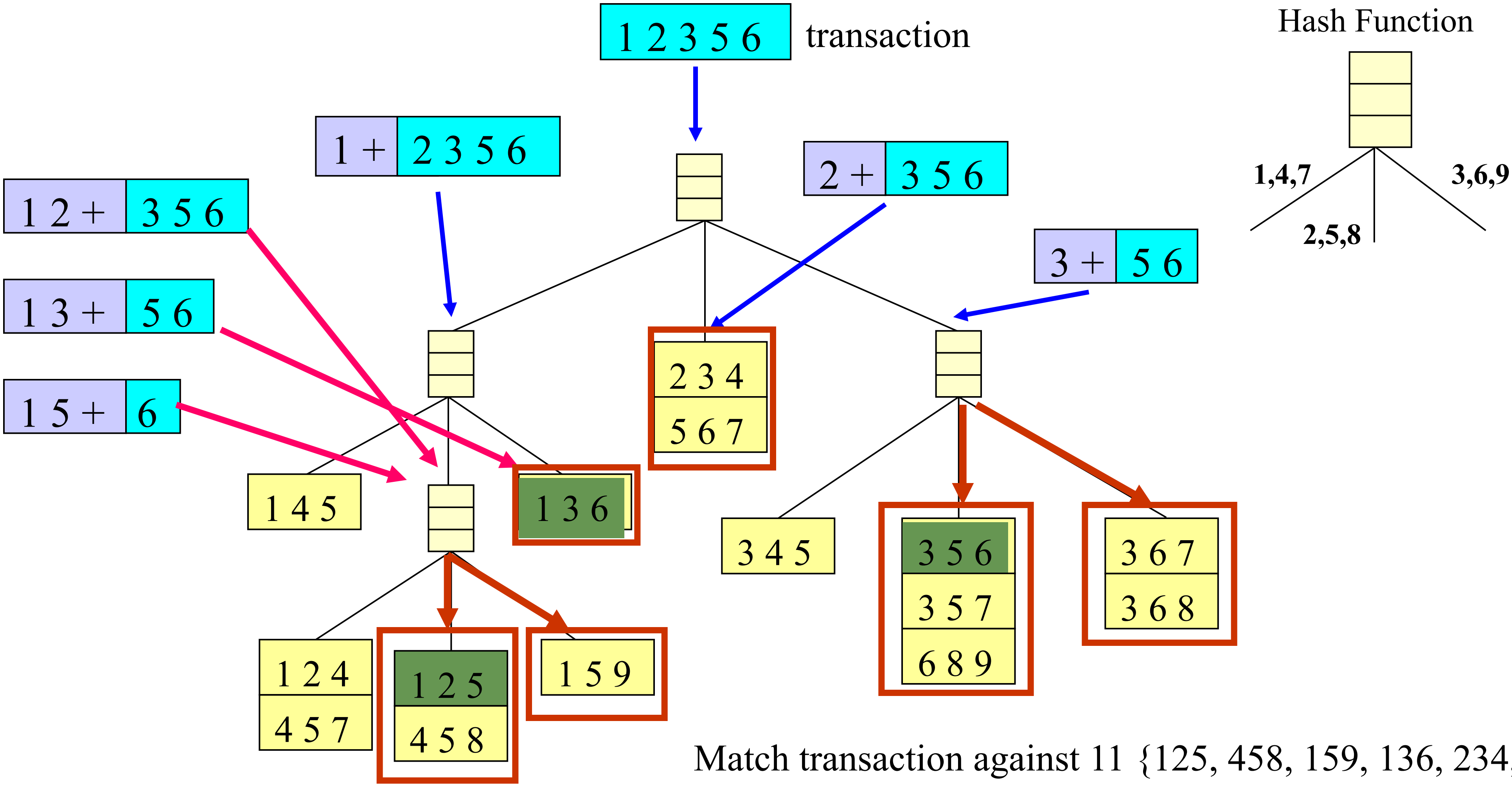
Subset Operation Using Hash Tree



Subset Operation Using Hash Tree



Subset Operation Using Hash Tree



Match transaction against 11 {125, 458, 159, 136, 234, 567, 356, 357, 689, 367, 368} out of 15 candidates.

Rule Generation

Given a frequent itemset L, find all non-empty subsets $F \subset L$ such that $F \rightarrow L - F$ satisfies the minimum confidence requirement

- If {A,B,C,D} is a frequent itemset, candidate rules:

$A \rightarrow BCD$, $B \rightarrow ACD$, $C \rightarrow ABD$, $D \rightarrow ABC$

$AB \rightarrow CD$, $AC \rightarrow BD$, $AD \rightarrow BC$, $BC \rightarrow AD$,
$BD \rightarrow AC$, $CD \rightarrow AB$,

$ABC \rightarrow D$, $ABD \rightarrow C$, $ACD \rightarrow B$, $BCD \rightarrow A$,

If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \varnothing$ and $\varnothing \rightarrow L$)

Rule Generation

$$Support_D(X) = \frac{|\{x \mid X \subseteq x, x \in D\}|}{|D|} = p(X) \le 1$$ Support of X in D is the proportion of records in D that have itemset X

$$\mathrm{Confidence}_\mathrm{D}(X \to Y) = p(Y \mid X) = \frac{Support_D(X \cup Y)}{Support_D(X)} \le 1$$

How to efficiently generate rules from frequent itemsets?

- In general, confidence does not have an anti-monotone property

  c(ABC $\to$D) can be larger or smaller than c(AB $\to$D)

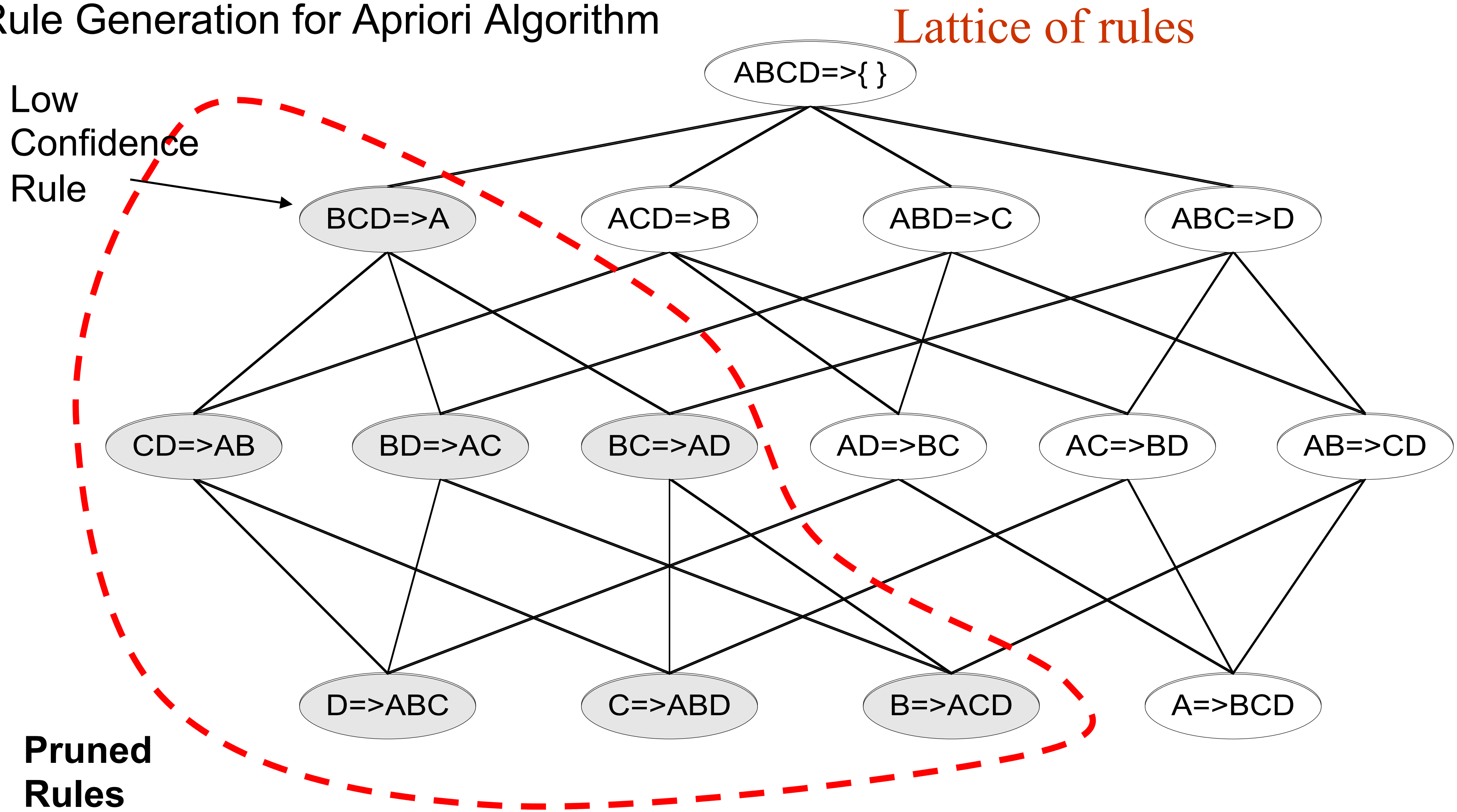
- But confidence of rules generated from the same itemset has an anti-monotone property
- e.g., L = {A,B,C,D}:

c(ABC $\to$ D) = sup(ABCD)/sup(ABC) $\ge$ c(AB $\to$ CD) = sup(ABCD)/sup(AB) $\ge$ c(A $\to$ BCD) = sup(ABCD)/sup(A)

sup(ABC) <= sup(AB) <= sup(A)

Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm

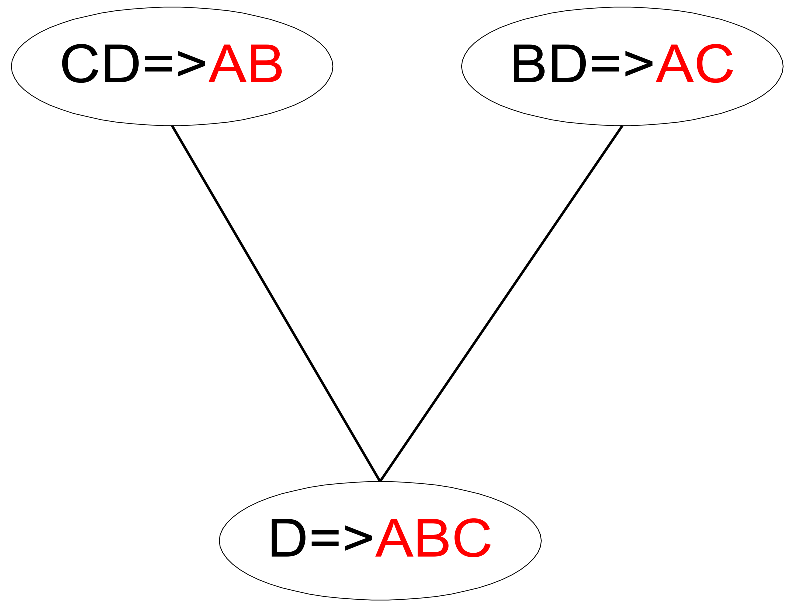


Rule Generation for Apriori Algorithm

Candidate rule is generated by merging two rules that share the same prefix in the rule consequent

join(CD=>AB,BD=>AC)
would produce the candidate
rule D => ABC

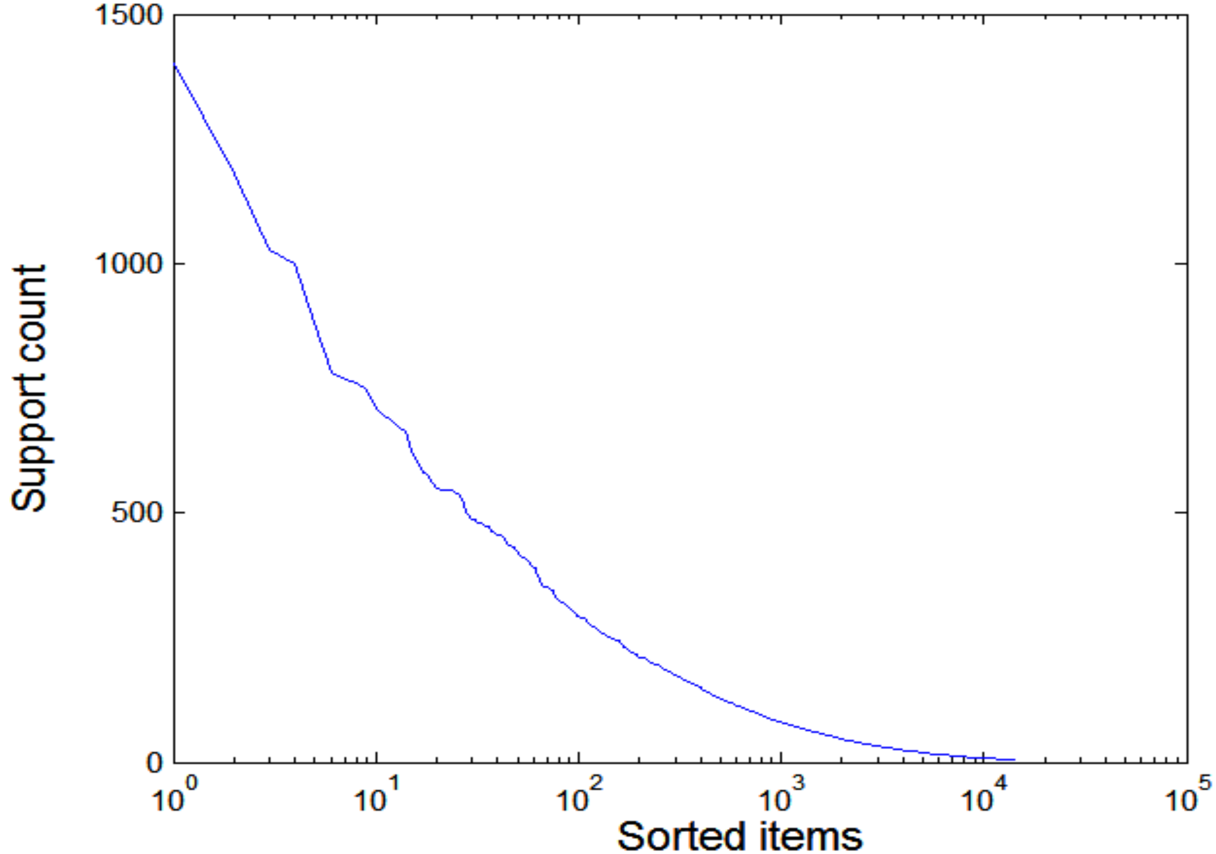
Prune rule D=>ABC if its
subset AD=>BC does not have
high confidence



Effect of Support Distribution

Many real data sets have skewed support distribution

Support distribution of a retail data set



The rest of slides would be of great interest for you to read but due to lack of time the rest of the slides are not covered in the lectures and are also not examinable. If you have time and interest and I recommend you to study these slides!

Please watch the following video with regard to AI and Data Mining on

19 May 2017 by Google CEO: Sundar Pichai https://www.recode.net/2017/5/19/15666704/google-lens-key-example-ai-first-computer-vision

- I will be away on 6 June 2017
- I am available generally from next week: Monday morning; Tuesday afternoon; Wednesday morning; Friday morning for consultation. Please come in groups as it will be useful for many.

Effect of Support Distribution

How to set the appropriate *minsup* threshold?

- If *minsup* is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products)

- If *minsup* is set too low, it is computationally expensive and the number of itemsets is very large

Using a single minimum support threshold may not be effective

Factors Affecting Complexity

Choice of minimum support threshold

- lowering support threshold results in more frequent itemsets
- this may increase number of candidates and max length of frequent itemsets

Dimensionality (number of items) of the data set

- more space is needed to store support count of each item
- if number of frequent items also increases, both computation and I/O costs may also increase

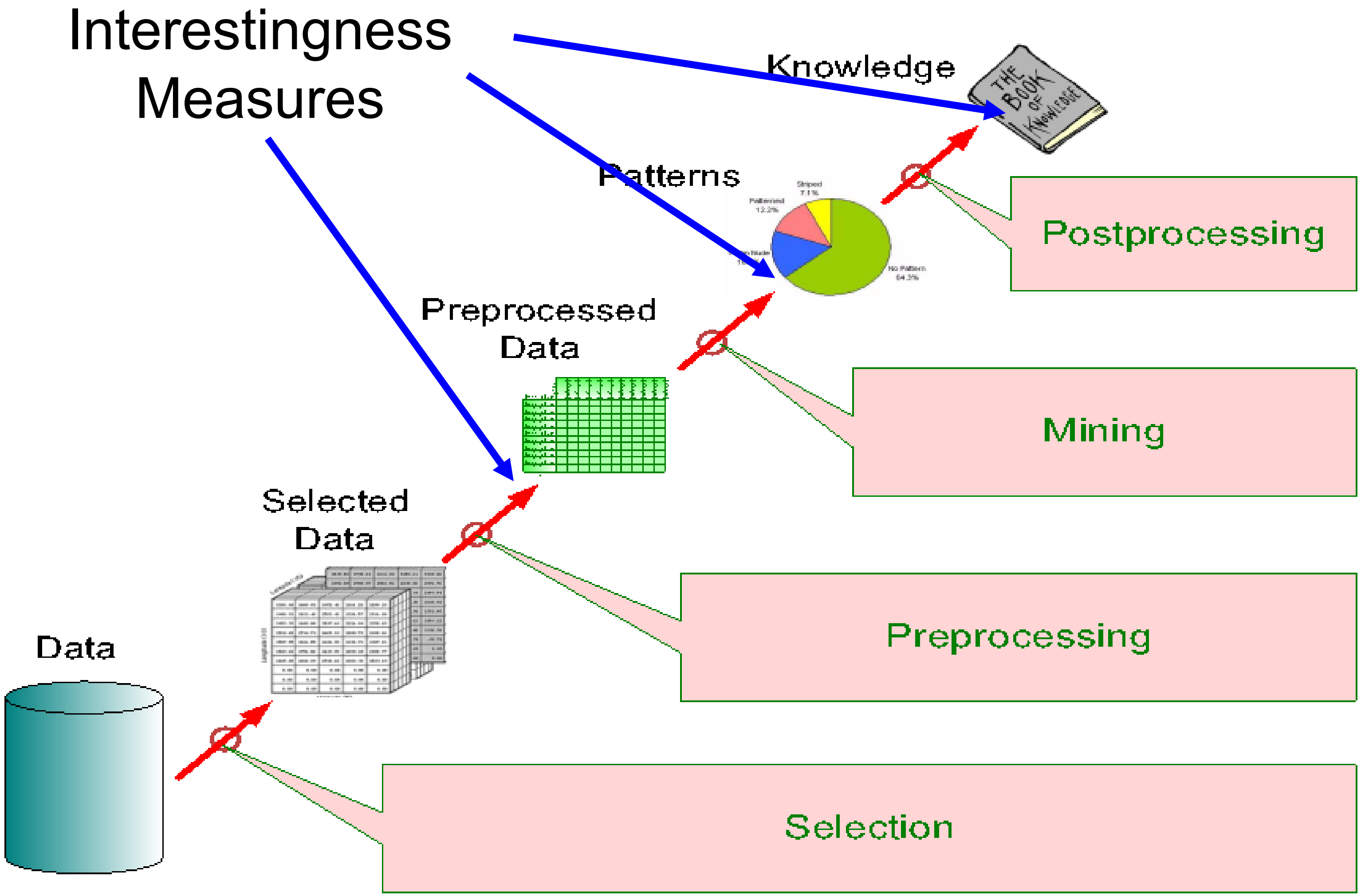
Size of database

- since Apriori makes multiple passes, run time of algorithm may increase with number of transactions

Average transaction width

- transaction width increases with denser data sets
- This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

Application of Interestingness Measure



Pattern Evaluation

Association rule algorithms tend to produce too many rules

- many of them are uninteresting or redundant
- Redundant if $\{A,B,C\} \rightarrow \{D\}$ and $\{A,B\} \rightarrow \{D\}$ have same support & confidence

Interestingness measures can be used to prune/rank the derived patterns

In the original formulation of association rules, support & confidence are the only measures used

Computing Interestingness Measure

Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	$\|T\|$

f_{11}: support of X and Y
f_{10}: support of X and $\overline{Y}$
f_{01}: support of $\overline{X}$ and Y
f_{00}: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

Drawback of Confidence

$$Support_D(X) = \frac{|\{x \mid X \subseteq x, x \in D\}|}{|D|} = p(X) \leq 1$$ Support of X in D is the proportion of records in D that have itemset X

$$\text{Confidence}_D(X \rightarrow Y) = p(Y \mid X) = \frac{Support_D(X \cup Y)}{Support_D(X)} \leq 1$$

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

$\Rightarrow$ Although confidence is high, rule is misleading

$\Rightarrow$ $P(\text{Coffee}|\overline{\text{Tea}}) = 0.9375$

Statistical Independence

Population of 1000 students

- 600 students know how to swim (S)
- 700 students know how to bike (B)
- 420 students know how to swim and bike (S,B)

- $P(S \wedge B) = 420/1000 = 0.42$
- $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

- $P(S \wedge B) = P(S) \times P(B)$ => Statistical independence
- $P(S \wedge B) > P(S) \times P(B)$ => Positively correlated
- $P(S \wedge B) < P(S) \times P(B)$ => Negatively correlated

Statistical-based Measures

Measures that take into account statistical dependence

$$Lift \text{ also called } Interest = \frac{P(Y \mid X)}{P(Y)} = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

Compact Representation of Frequent Itemsets

Some itemsets are redundant because they have identical support as their supersets

TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

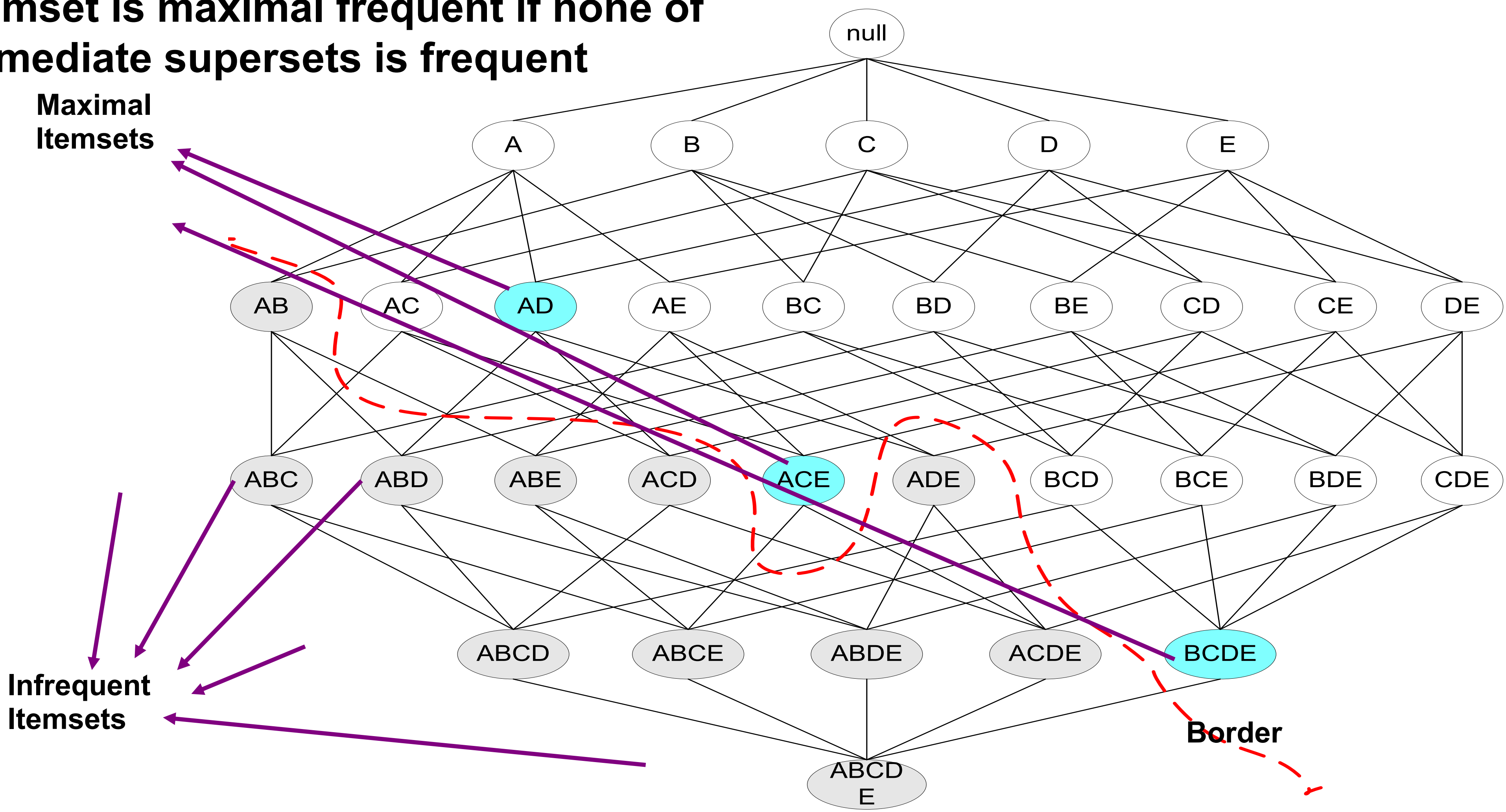
Number of frequent itemsets

$$= 3 \times \sum_{k=1}^{10} \binom{10}{k}$$

Need a compact representation

Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets is frequent



Closed Itemset

An itemset is closed if none of its immediate supersets has the same support as the itemset

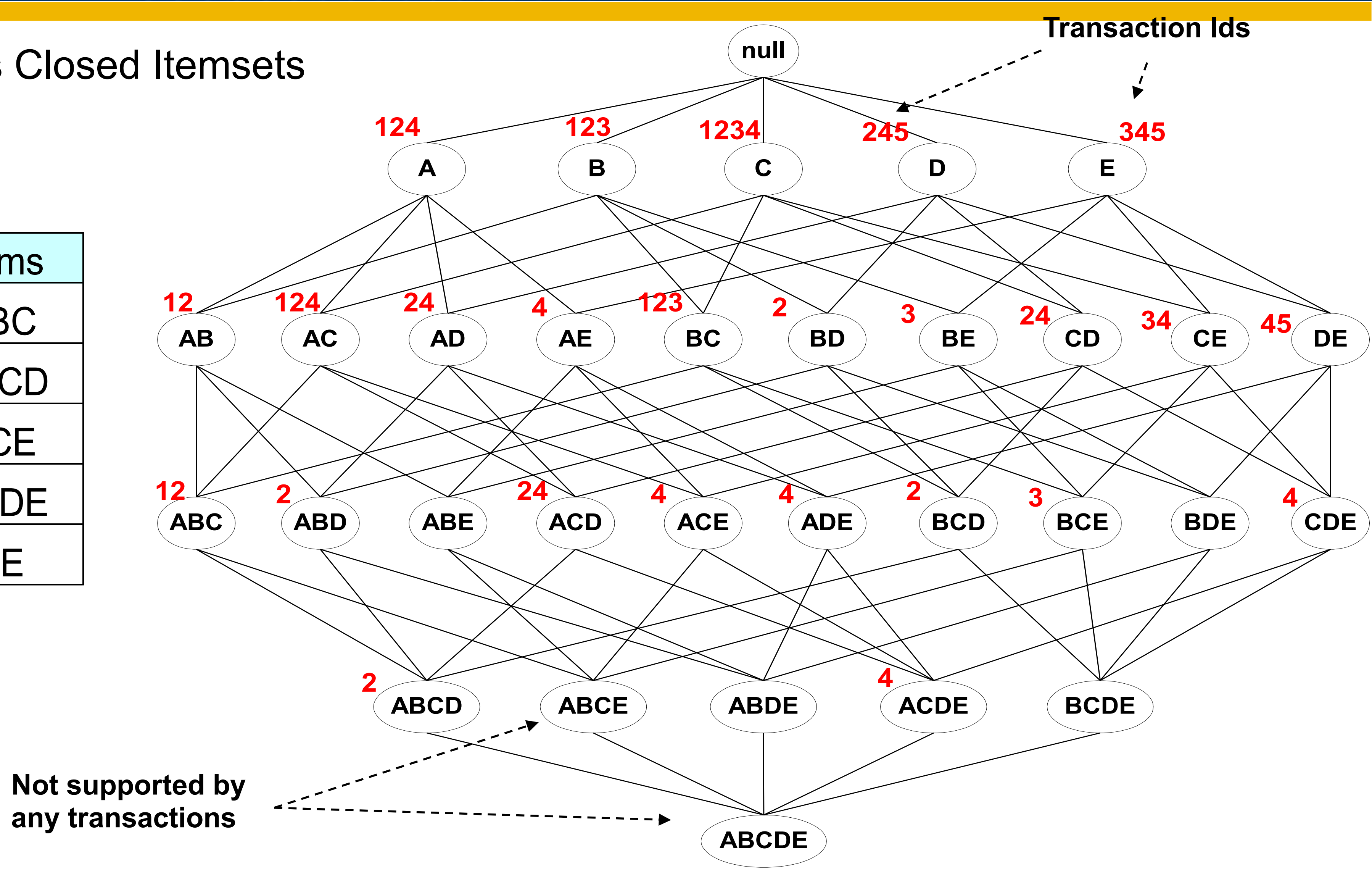
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

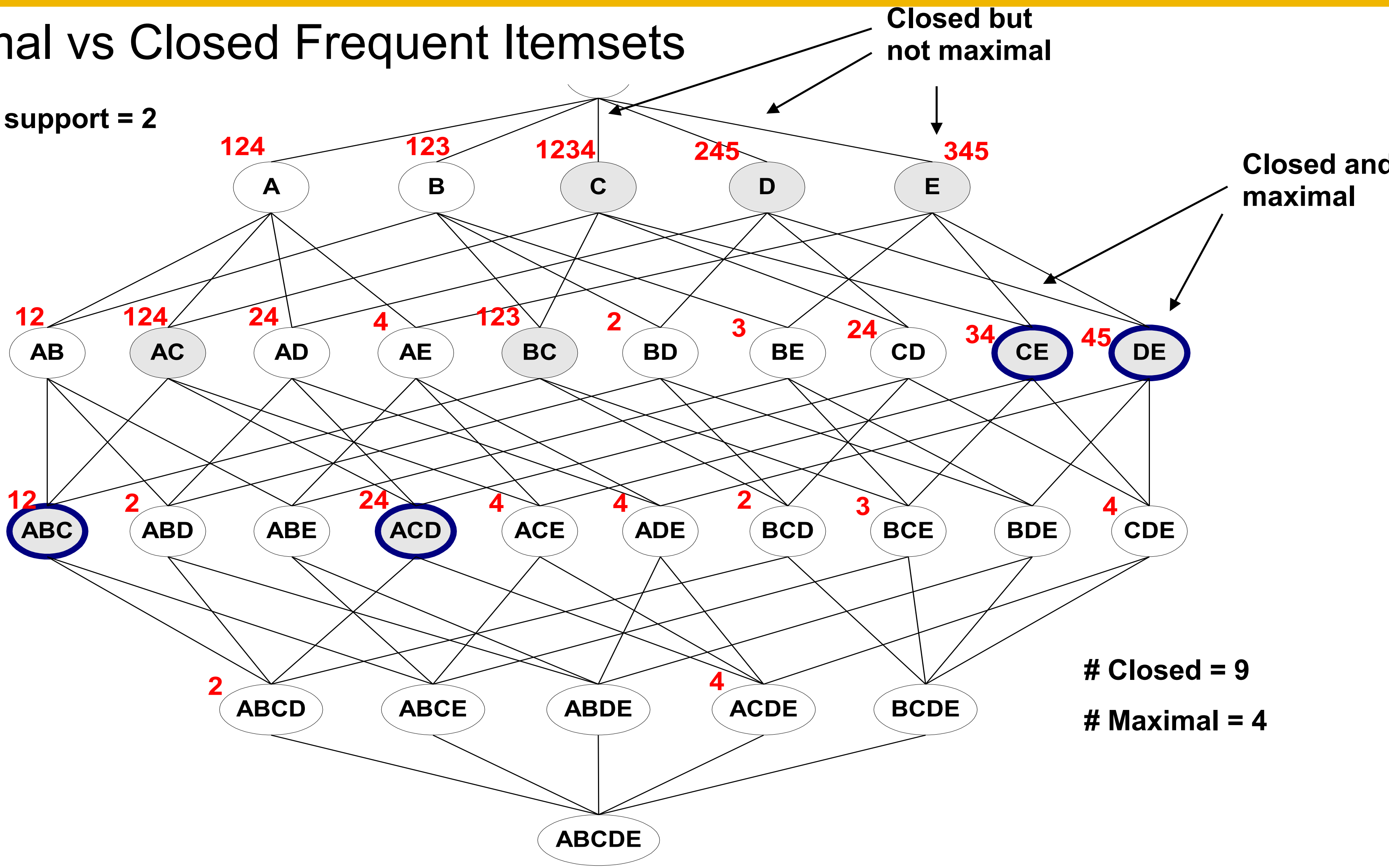
Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal vs Closed Frequent Itemsets

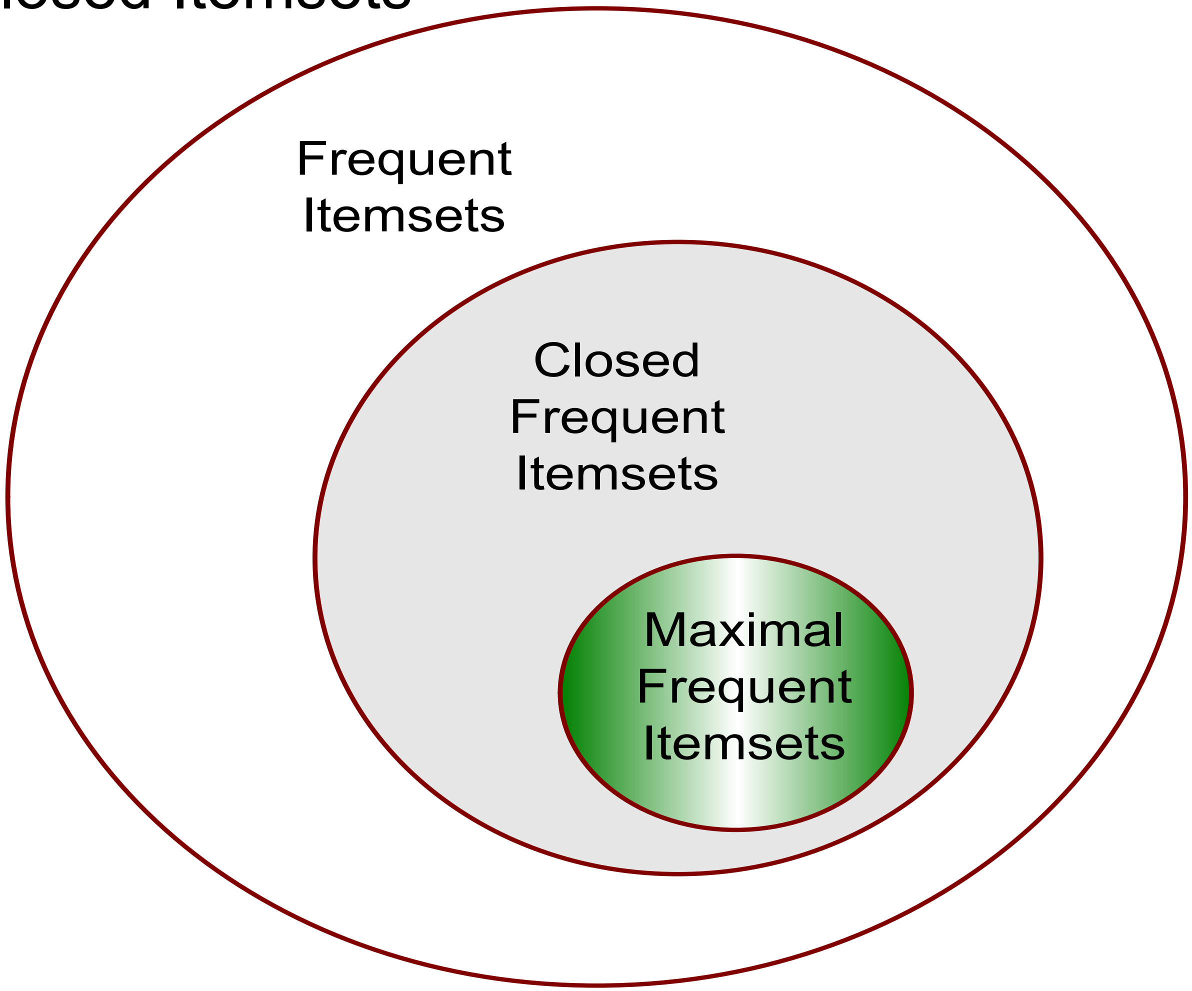
Minimum support = 2



 # Closed = 9

 # Maximal = 4

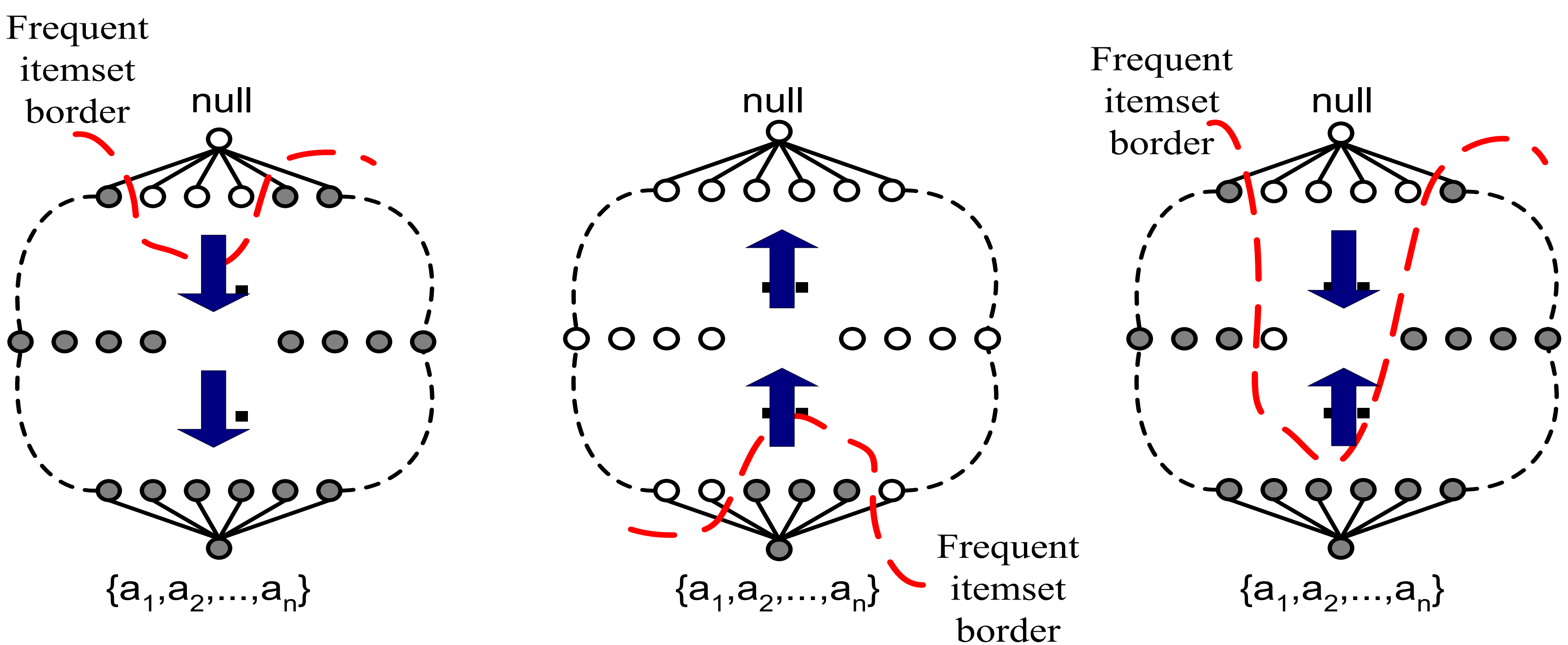
Maximal vs Closed Itemsets



Alternative Methods for Frequent Itemset Generation

Traversal of Itemset Lattice

- General-to-specific vs Specific-to-general

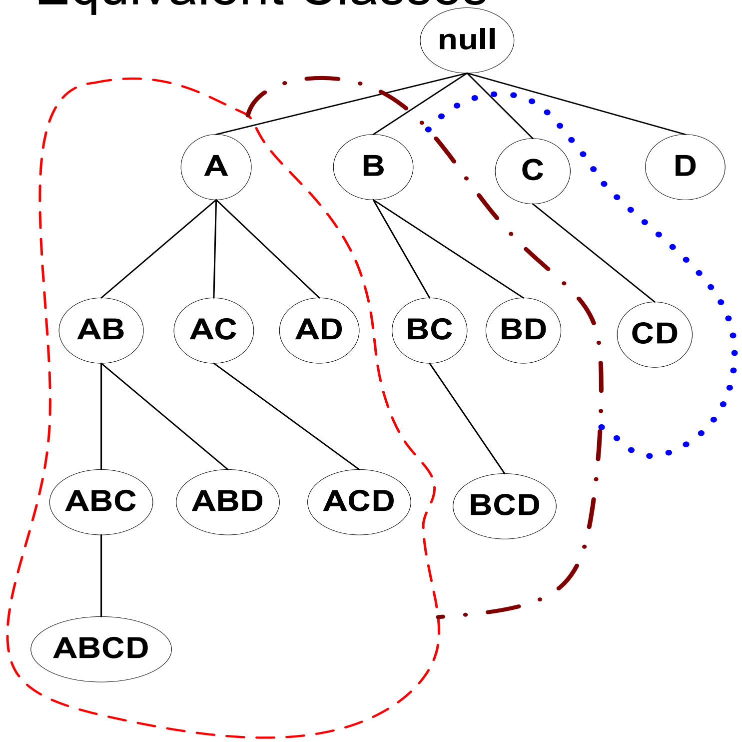


(a) General-to-specific (b) Specific-to-general (c) Bidirectional

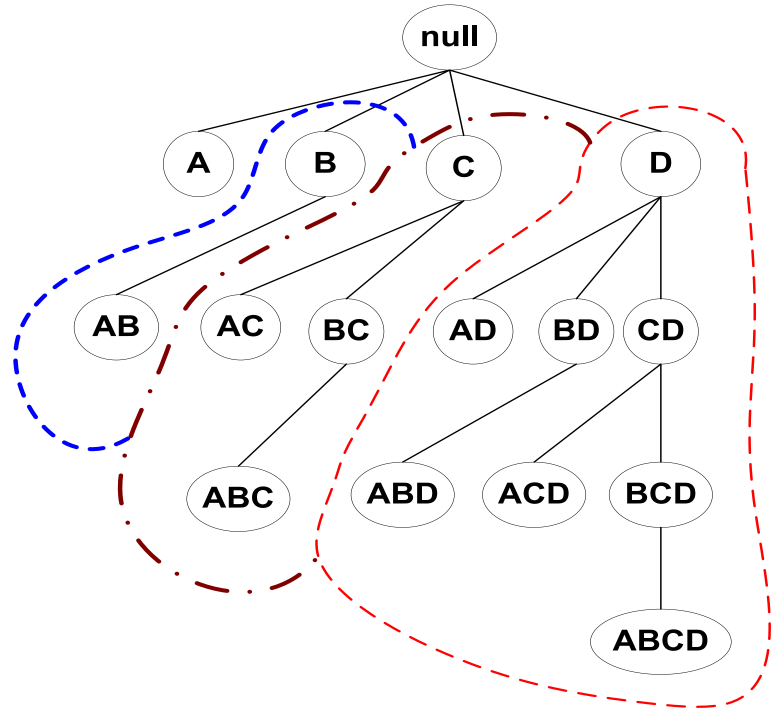
Alternative Methods for Frequent Itemset Generation

Traversal of Itemset Lattice

- Equivalent Classes



(a) Prefix tree

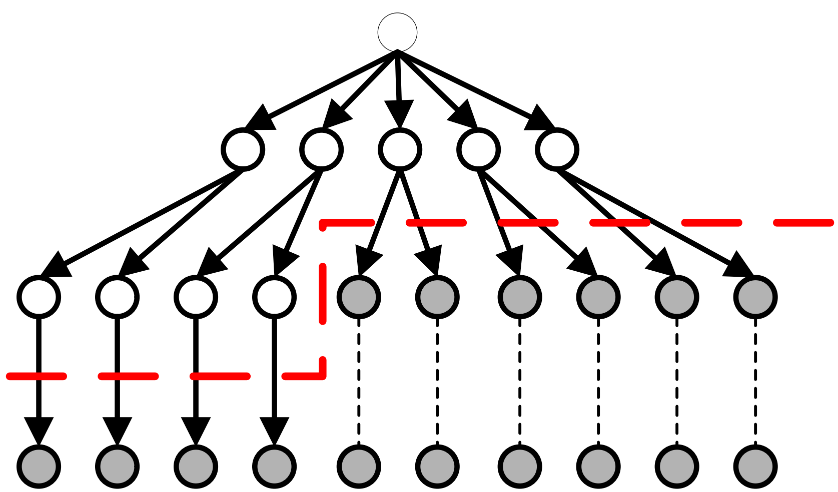


(b) Suffix tree

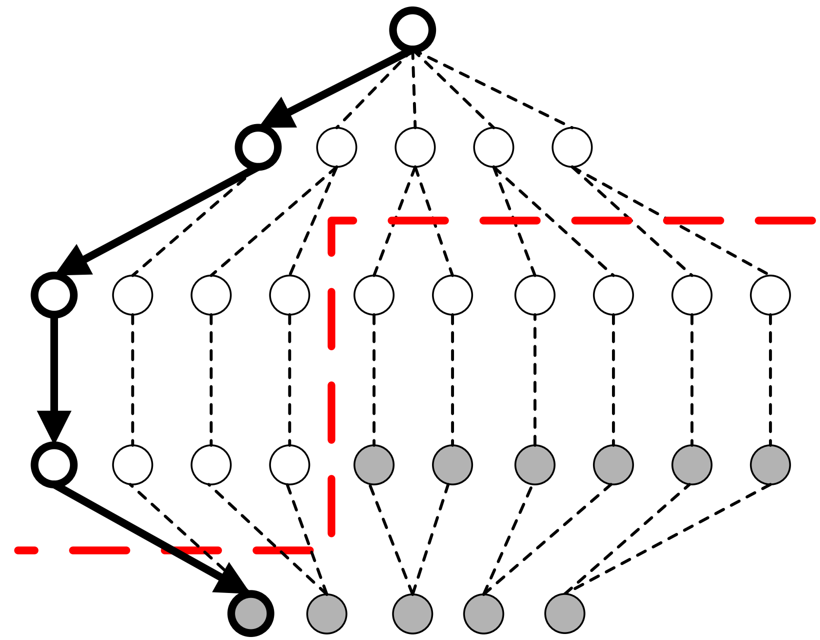
Alternative Methods for Frequent Itemset Generation

Traversal of Itemset Lattice

- Breadth-first vs Depth-first



(a) Breadth first



(b) Depth first

Alternative Methods for Frequent Itemset Generation

Representation of Database

- horizontal vs vertical data layout

Horizontal
Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

FP-growth Algorithm

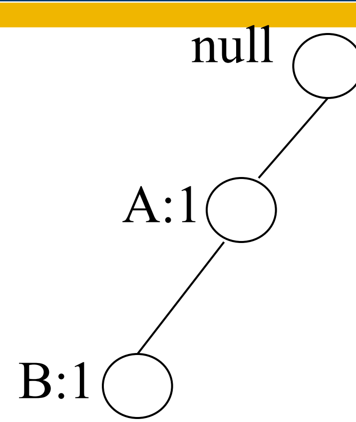
Use a compressed representation of the database using an FP-tree

Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets

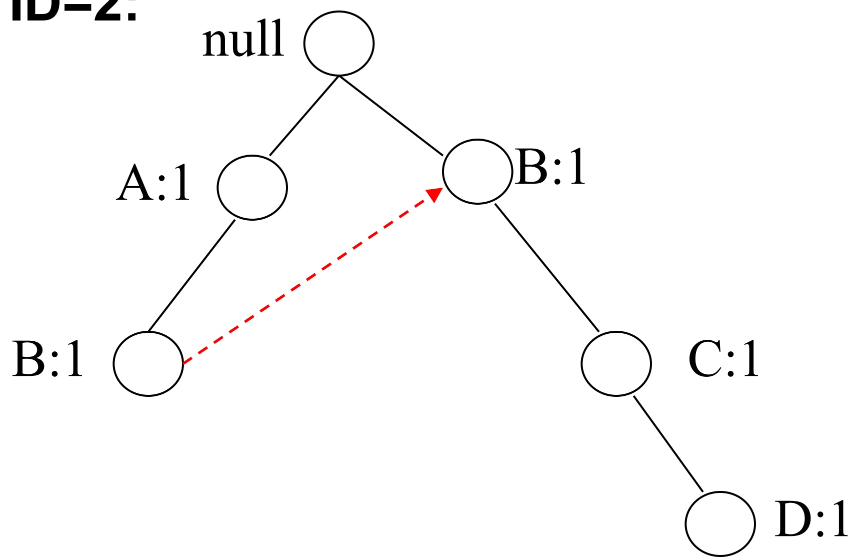
FP-tree construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

After reading TID=1:



After reading TID=2:



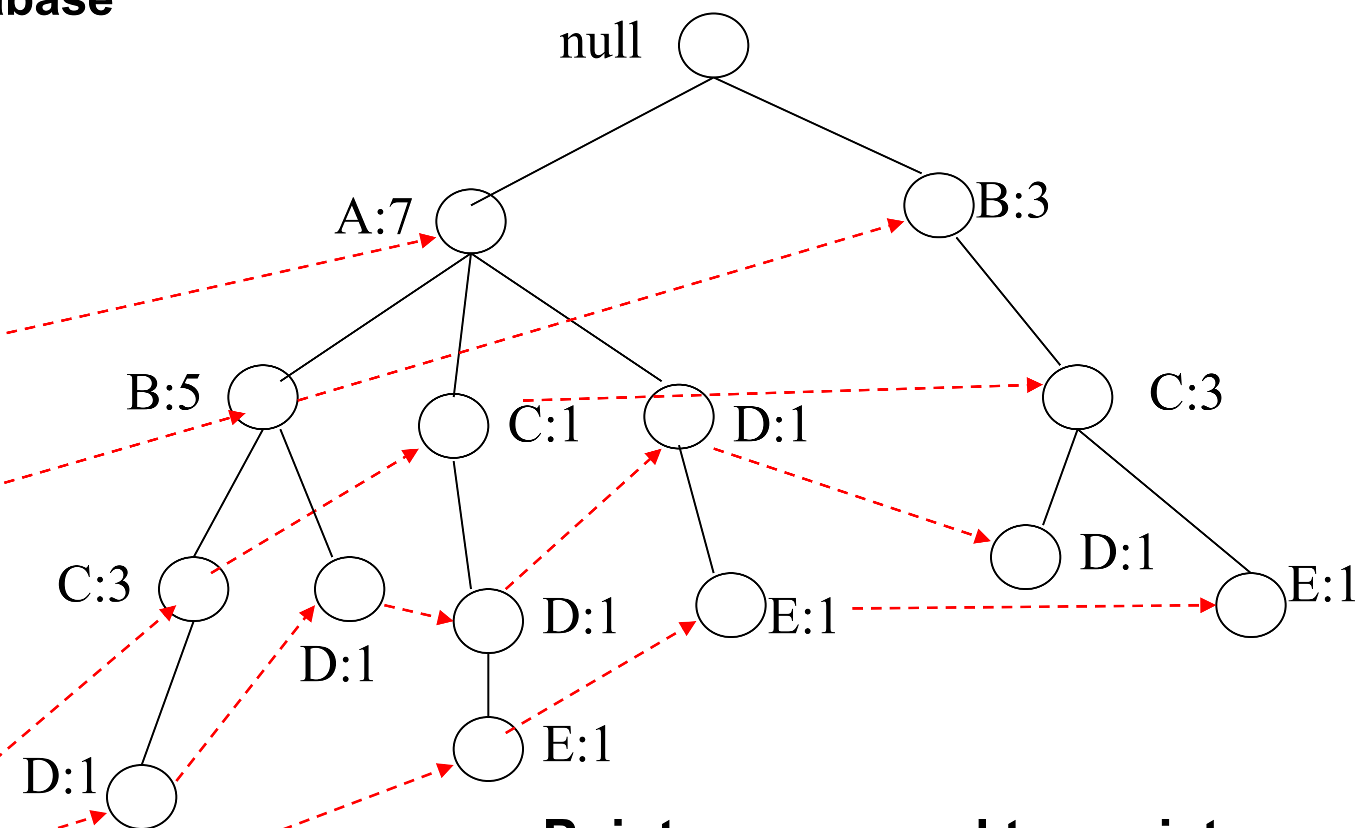
FP-Tree Construction

Transaction Database

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

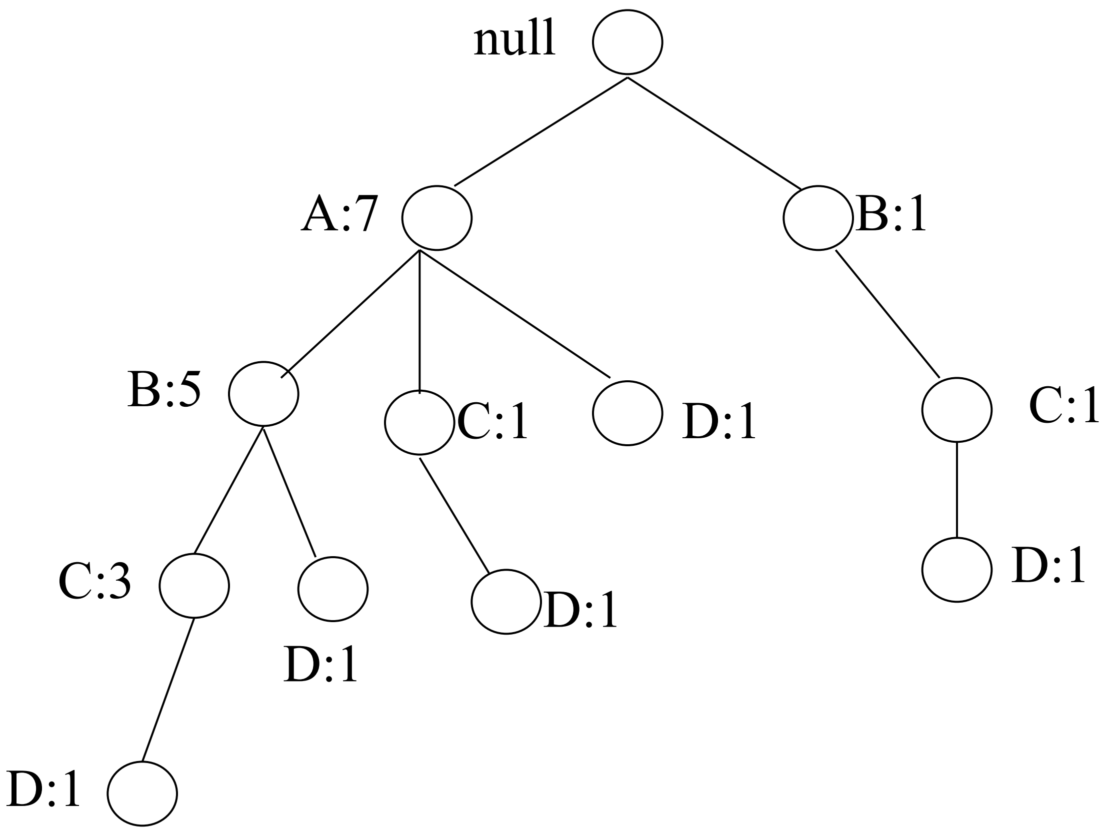
Header table

Item	Pointer
A	
B	
C	
D	
E	



Pointers are used to assist frequent itemset generation

FP-growth



Conditional Pattern base for D:

**P = {(A:1,B:1,C:1),
(A:1,B:1),
(A:1,C:1),
(A:1),
(B:1,C:1)}**

Recursively apply FP-growth on P

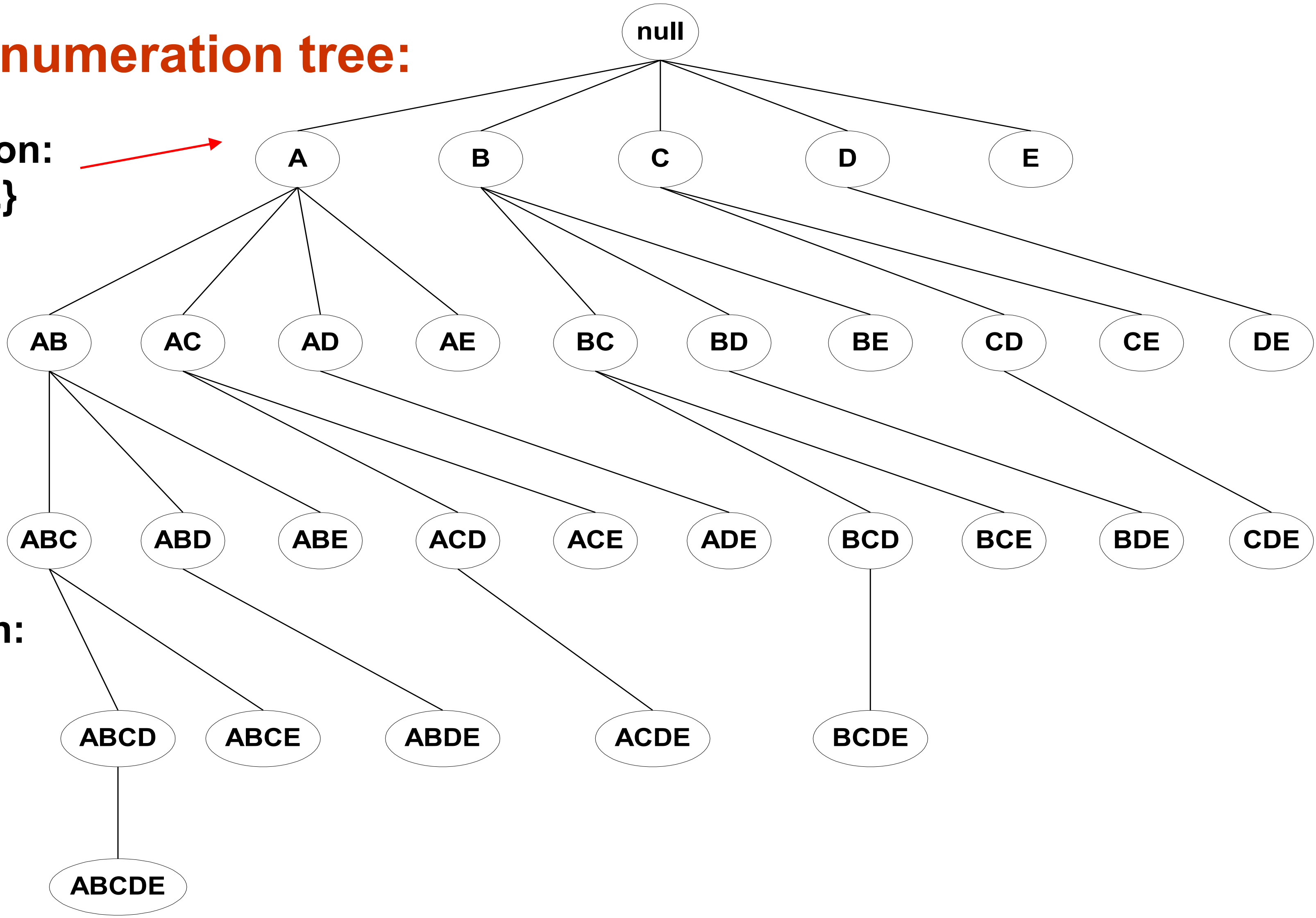
Frequent Itemsets found (with sup > 1):
AD(4), BD(3), CD(3), ACD(2), BCD(2), ABD(2)

Tree Projection

Set enumeration tree:

Possible Extension:
E(A) = {B,C,D,E}

Possible Extension:
E(ABC) = {D,E}



Tree Projection

Items are listed in lexicographic order

Each node P stores the following information:

- Itemset for node P
- List of possible lexicographic extensions of P: E(P)
- Pointer to projected database of its ancestor node
- Bitvector containing information about which transactions in the projected database contain the itemset

Projected Database

Original Database:

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Projected Database for node A:

TID	Items
1	{B}
2	{}
3	{C,D,E}
4	{D,E}
5	{B,C}
6	{B,C,D}
7	{}
8	{B,C}
9	{B,D}
10	{}

For each transaction T, projected transaction at node A is

$$T - A \text{ If } \mathrm{A} \in \mathrm{T}$$

$$\{\} \text{ Otherwise}$$

ECLAT

For each item, store a list of transaction ids (tids)

Horizontal Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

TID-list

ECLAT

Determine support of any k-itemset by intersecting tid-lists of two of its (k-1) subsets.

A
1
4
5
6
7
8
9

$\wedge$

B
1
2
5
7
8
10

$\rightarrow$

AB
1
5
7
8

3 traversal approaches:

- top-down, bottom-up and hybrid

Advantage: very fast support counting

Disadvantage: intermediate tid-lists may become too large for memory

Multiple Minimum Support

How to apply multiple minimum supports?

- MS(i): minimum support for item i
- e.g.: MS(Milk)=5%, MS(Coke) = 3%,
  MS(Broccoli)=0.1%, MS(Salmon)=0.5%
- MS({Milk, Broccoli}) = min (MS(Milk), MS(Broccoli))
  = 0.1%

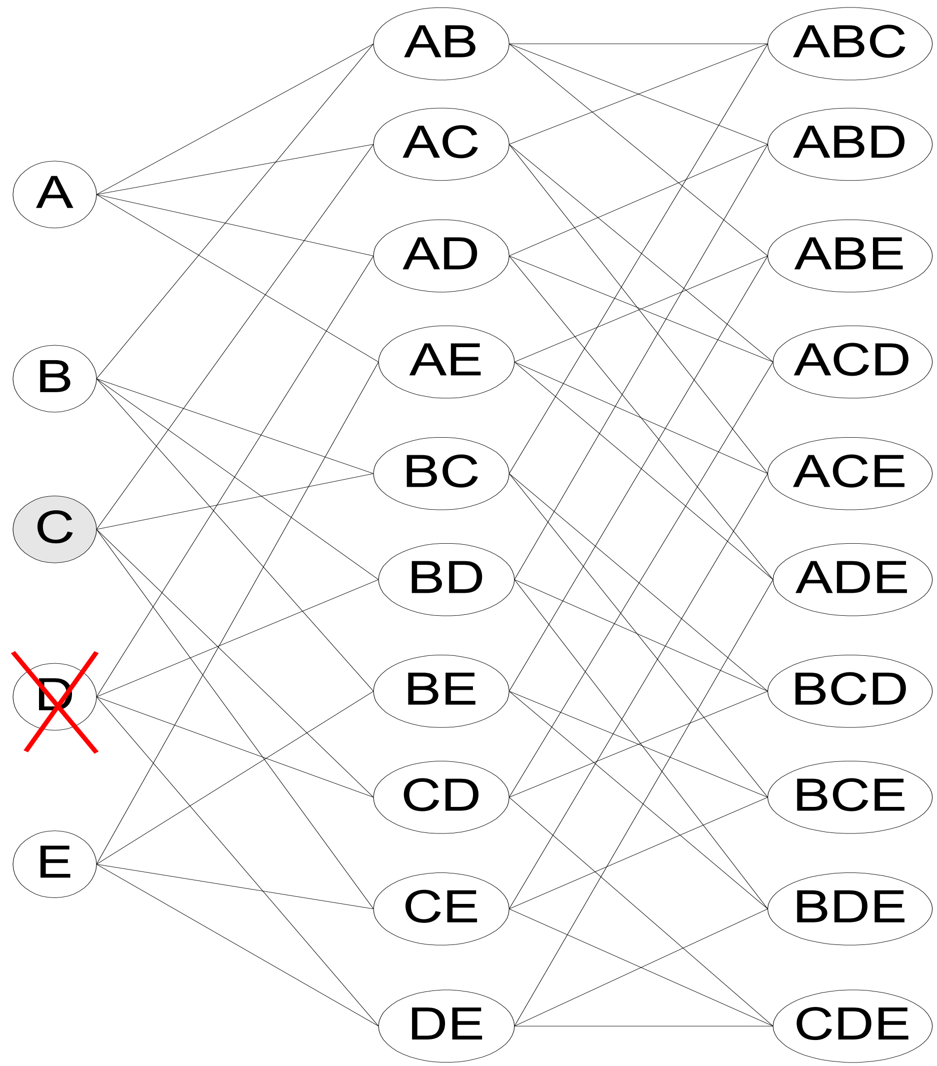
- Challenge: Support is no longer anti-monotone

  Suppose: Support(Milk, Coke) = 1.5% and
  Support(Milk, Coke, Broccoli) = 0.5%

  {Milk,Coke} is infrequent but {Milk,Coke,Broccoli} is frequent due to different minimum support requirements!

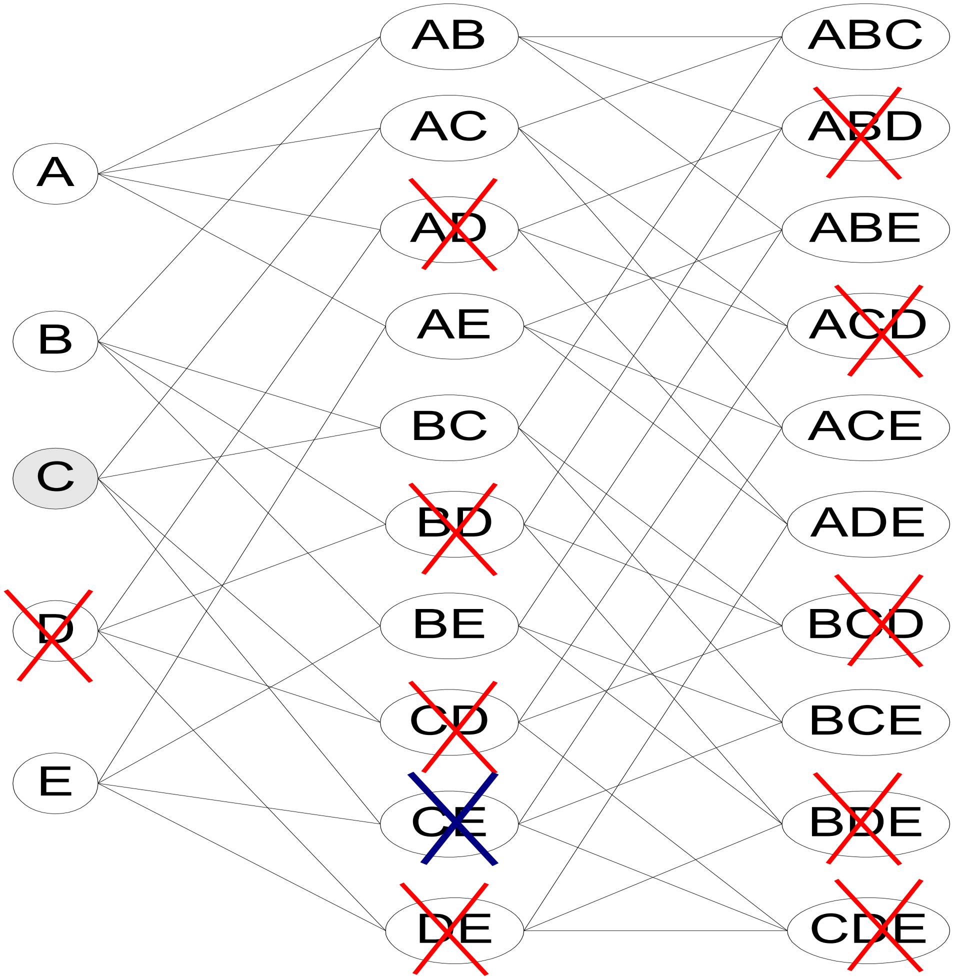
Multiple Minimum Support

Item	MS(I)	Sup(I)
A	0.10%	0.25%
B	0.20%	0.26%
C	0.30%	0.29%
D	0.50%	0.05%
E	3%	4.20%



Multiple Minimum Support

Item	MS(I)	Sup(I)
A	0.10%	0.25%
B	0.20%	0.26%
C	0.30%	0.29%
D	0.50%	0.05%
E	3%	4.20%



Multiple Minimum Support (Liu 1999)

Order the items according to their minimum support (in ascending order)

- e.g.: MS(Milk)=5%, MS(Coke) = 3%,
  MS(Broccoli)=0.1%, MS(Salmon)=0.5%
- Ordering: Broccoli, Salmon, Coke, Milk

Need to modify Apriori such that:

- L_1 : set of frequent items
- F_1 : set of items whose support is $\geq$ MS(1)
  where MS(1) is $\min_i$(MS(i))
- C_2 : candidate itemsets of size 2 is generated from F_1
  instead of L_1

Multiple Minimum Support (Liu 1999)

Modifications to Apriori:

- In traditional Apriori,
  - A candidate (k+1)-itemset is generated by merging two frequent itemsets of size k
  - The candidate is pruned if it contains any infrequent subsets of size k
- Pruning step has to be modified:
  - Prune only if subset contains the first item
  - e.g.: Candidate={Broccoli, Coke, Milk} (ordered according to minimum support)
  - {Broccoli, Coke} and {Broccoli, Milk} are frequent but {Coke, Milk} is infrequent
    - Candidate is not pruned because {Coke,Milk} does not contain the first item, i.e., Broccoli.

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

$\Rightarrow$ Lift = 0.75/0.9= 0.8333 (< 1, therefore is negatively associated)

Drawback of Lift & Interest

	Y	$\overline{Y}$	
X	10	0	10
$\overline{X}$	0	90	90
	10	90	100

$$Lift = \frac{1.0}{0.1} = 10$$

	Y	$\overline{Y}$	
X	90	0	90
$\overline{X}$	0	10	10
	90	10	100

$$Lift = \frac{1.0}{0.9} = 1.11$$

$$Lift \text{ also called } Interest = \frac{P(Y \mid X)}{P(Y)} = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

Statistical independence:

If P(X,Y)=P(X)P(Y) => Lift = 1

There are lots of measures proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support based pruning? How does it affect these measures?

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)}=\frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}}=\frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i),-\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\Big(P(A,B)\log(\frac{P(B\|A)}{P(B)})+P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),$ $P(A,B)\log(\frac{P(A\|B)}{P(A)})+P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\Big)$
9	Gini index (G)	$\max\Big(P(A)[P(B\|A)^2+P(\overline{B}\|A)^2]+P(\overline{A})[P(B\|\overline{A})^2+P(\overline{B}\|\overline{A})^2]$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\|B)^2+P(\overline{A}\|B)^2]+P(\overline{B})[P(A\|\overline{B})^2+P(\overline{A}\|\overline{B})^2]$ $-P(A)^2-P(\overline{A})^2\Big)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\|A),P(A\|B))$
12	Laplace (L)	$\max\Big(\frac{NP(A,B)+1}{NP(A)+2},\frac{NP(A,B)+1}{NP(B)+2}\Big)$
13	Conviction (V)	$\max\Big(\frac{P(A)P(\overline{B})}{P(A\overline{B})},\frac{P(B)P(\overline{A})}{P(B\overline{A})}\Big)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B)-P(A)P(B)$
17	Certainty factor (F)	$\max\Big(\frac{P(B\|A)-P(B)}{1-P(B)},\frac{P(A\|B)-P(A)}{1-P(A)}\Big)$
18	Added Value (AV)	$\max(P(B\|A)-P(B),P(A\|B)-P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})}\times\frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\|A)-P(B),P(A\|B)-P(A))$

Properties of A Good Measure

Piatetsky-Shapiro:
3 properties a good measure M must satisfy:

- M(A,B) = 0 if A and B are statistically independent
- M(A,B) increase monotonically with P(A,B) when P(A) and P(B) remain unchanged
- M(A,B) decreases monotonically with P(A) [or P(B)] when P(A,B) and P(B) [or P(A)] remain unchanged

Comparing Different Measures

10 examples of contingency tables:

Example	f_{11}	f_{10}	f_{01}	f_{00}
E1	8123	83	424	1370
E2	8330	2	622	1046
E3	9481	94	127	298
E4	3954	3080	5	2961
E5	2886	1363	1320	4431
E6	1500	2000	500	6000
E7	4000	2000	1000	3000
E8	4000	2000	2000	2000
E9	1720	7121	5	1154
E10	61	2483	4	7452

Rankings of contingency tables using various measures:

#	ϕ	λ	α	Q	Y	κ	M	J	G	s	c	L	V	I	IS	PS	F	AV	S	ζ	K
E1	1	1	3	3	3	1	2	2	1	3	5	5	4	6	2	2	4	6	1	2	5
E2	2	2	1	1	1	2	1	3	2	2	1	1	1	8	3	5	1	8	2	3	6
E3	3	3	4	4	4	3	3	8	7	1	4	4	6	10	1	8	6	10	3	1	10
E4	4	7	2	2	2	5	4	1	3	6	2	2	2	4	4	1	2	3	4	5	1
E5	5	4	8	8	8	4	7	5	4	7	9	9	9	3	6	3	9	4	5	6	3
E6	6	6	7	7	7	7	6	4	6	9	8	8	7	2	8	6	7	2	7	8	2
E7	7	5	9	9	9	6	8	6	5	4	7	7	8	5	5	4	8	5	6	4	4
E8	8	9	10	10	10	8	10	10	8	4	10	10	10	9	7	7	10	9	8	7	9
E9	9	9	5	5	5	9	9	7	9	8	3	3	3	7	9	9	3	7	9	9	8
E10	10	8	6	6	6	10	5	9	10	10	6	6	5	1	10	10	5	1	10	10	7

Property under Variable Permutation

	B	$\overline{B}$
A	p	q
$\overline{A}$	r	s

	A	$\overline{A}$
B	p	r
$\overline{B}$	q	s

Does M(A,B) = M(B,A)?

Symmetric measures:

- support, lift, collective strength, cosine, Jaccard, etc

Asymmetric measures:

- confidence, conviction, Laplace, J-measure, etc

Property under Row/Column Scaling

Grade-Gender Example (Mosteller, 1968):

	Male	Female	
High	2	3	5
Low	1	4	5
	3	7	10

	Male	Female	
High	4	30	34
Low	2	40	42
	6	70	76

2x 10x

Mosteller:

Underlying association should be independent of the relative number of male and female students in the samples

Property under Inversion Operation

	A	B	C	D	E	F
Transaction 1	1	0	0	1	0	0
▪	0	0	1	1	1	0
	0	0	1	1	1	0
▪	0	0	1	1	1	0
	0	1	1	0	1	1
▪	0	0	1	1	1	0
	0	0	1	1	1	0
▪	0	0	1	1	1	0
	0	0	1	1	1	0
▪ Transaction N	1	0	0	1	0	0
	(a)		(b)		(c)	

Example: ϕ-Coefficient

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

ϕ-coefficient is analogous to correlation coefficient for continuous variables

	Y	$\bar{Y}$	
$\bar{X}$	60	10	70
X	10	20	30
	70	30	100

	Y	$\bar{Y}$	
X	20	10	30
$\bar{X}$	10	60	70
	30	70	100

$$\phi = \frac{0.6 - 0.7 \times 0.7}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}}$$
$$= 0.5238$$

$$\phi = \frac{0.2 - 0.3 \times 0.3}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}}$$
$$= 0.5238$$

ϕ Coefficient is the same for both tables

Property under Null Addition

	B	$\overline{B}$
A	p	q
$\overline{A}$	r	s

	B	$\overline{B}$
A	p	q
$\overline{A}$	r	s + k

Invariant measures:

- support, cosine, Jaccard, etc

Non-invariant measures:

- correlation, Gini, mutual information, odds ratio, etc

Different Measures have Different Properties

Symbol	Measure	Range	P1	P2	P3	O1	O2	O3	O3'	O4
Φ	Correlation	-1 … 0 … 1	Yes	Yes	Yes	Yes	No	Yes	Yes	No
λ	Lambda	0 … 1	Yes	No	No	Yes	No	No*	Yes	No
α	Odds ratio	0 … 1 … ∞	Yes*	Yes	Yes	Yes	Yes	Yes*	Yes	No
Q	Yule's Q	-1 … 0 … 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Y	Yule's Y	-1 … 0 … 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
κ	Cohen's	-1 … 0 … 1	Yes	Yes	Yes	Yes	No	No	Yes	No
M	Mutual Information	0 … 1	Yes	Yes	Yes	Yes	No	No*	Yes	No
J	J-Measure	0 … 1	Yes	No	No	No	No	No	No	No
G	Gini Index	0 … 1	Yes	No	No	No	No	No*	Yes	No
s	Support	0 … 1	No	Yes	No	Yes	No	No	No	No
c	Confidence	0 … 1	No	Yes	No	Yes	No	No	No	Yes
L	Laplace	0 … 1	No	Yes	No	Yes	No	No	No	No
V	Conviction	0.5 … 1 … ∞	No	Yes	No	Yes**	No	No	Yes	No
I	Interest	0 … 1 … ∞	Yes*	Yes	Yes	Yes	No	No	No	No
IS	IS (cosine)	0 .. 1	No	Yes	Yes	Yes	No	No	No	Yes
PS	Piatetsky-Shapiro's	-0.25 … 0 … 0.25	Yes	Yes	Yes	Yes	No	Yes	Yes	No
F	Certainty factor	-1 … 0 … 1	Yes	Yes	Yes	No	No	No	Yes	No
AV	Added value	0.5 … 1 … 1	Yes	Yes	Yes	No	No	No	No	No
S	Collective strength	0 … 1 … ∞	No	Yes	Yes	Yes	No	Yes*	Yes	No
ζ	Jaccard	0 .. 1	No	Yes	Yes	Yes	No	No	No	Yes
K	Klosgen's	$\left(\sqrt{\frac{2}{\sqrt{3}}}-1\right)\left(2-\sqrt{3}-\frac{1}{\sqrt{3}}\right)\ldots 0 \ldots \frac{2}{3\sqrt{3}}$	Yes	Yes	Yes	No	No	No	No	No

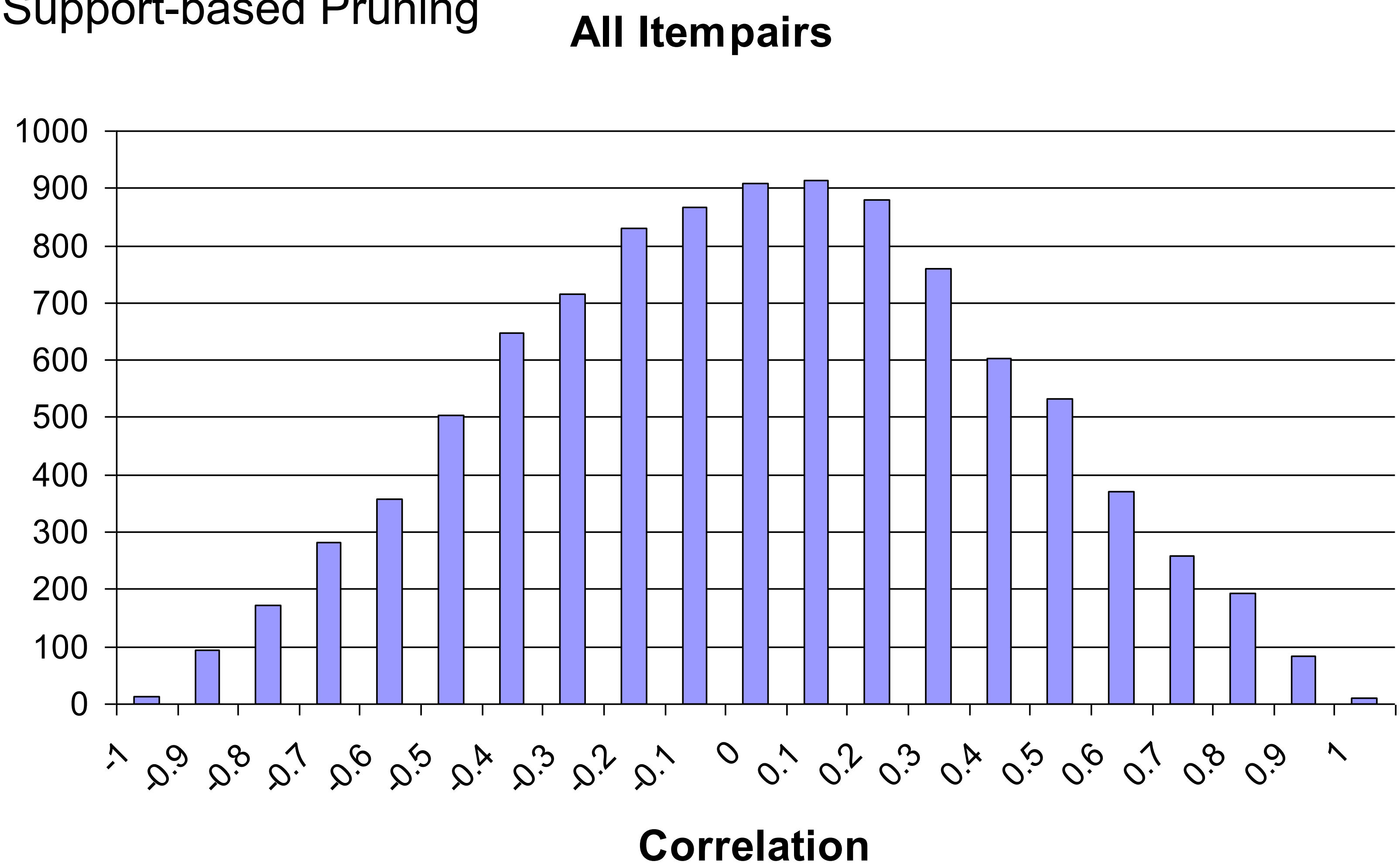
Support-based Pruning

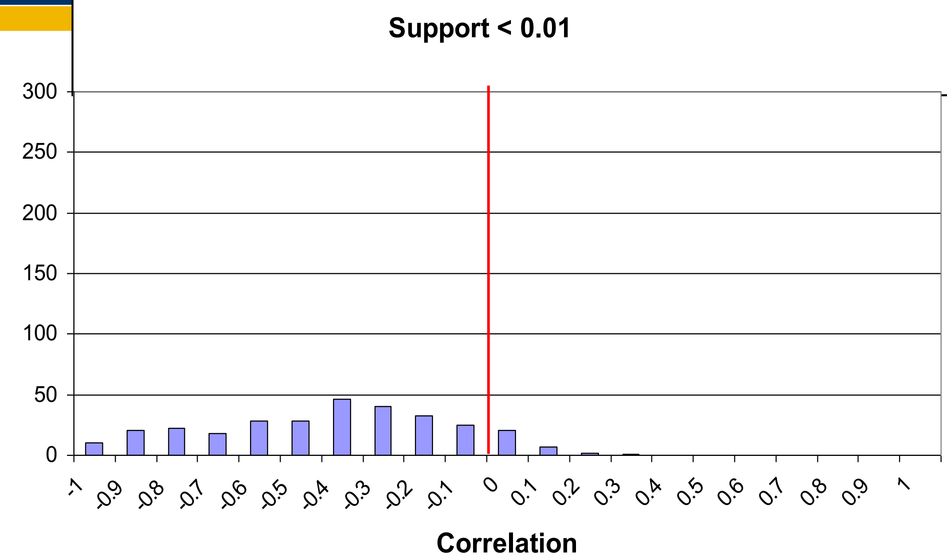
Most of the association rule mining algorithms use support measure to prune rules and itemsets

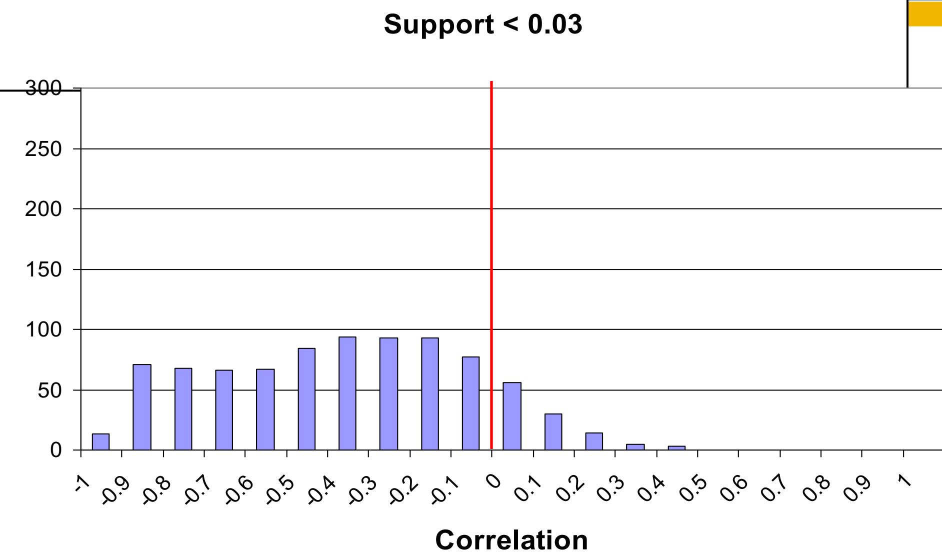
Study effect of support pruning on correlation of itemsets

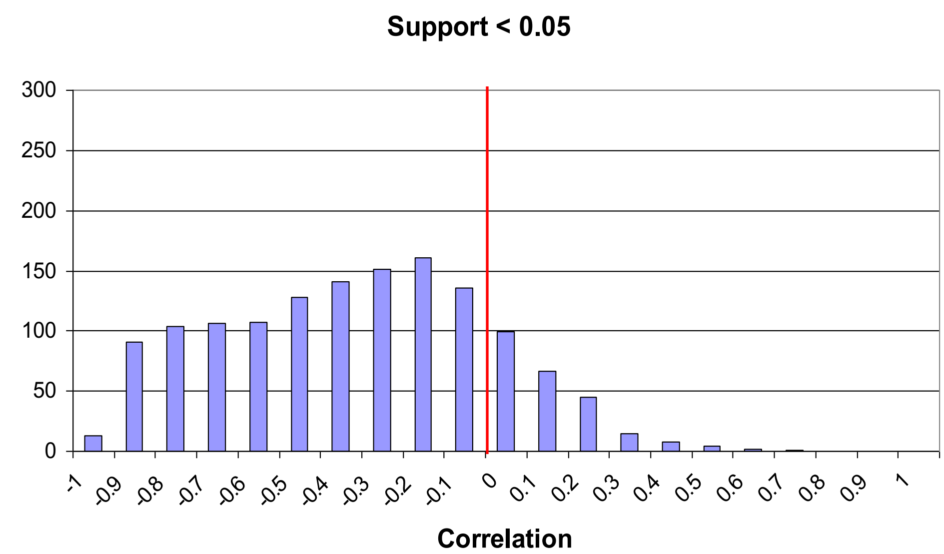
- Generate 10000 random contingency tables
- Compute support and pairwise correlation for each table
- Apply support-based pruning and examine the tables that are removed

Effect of Support-based Pruning









Support-based pruning eliminates mostly negatively correlated itemsets

Effect of Support-based Pruning

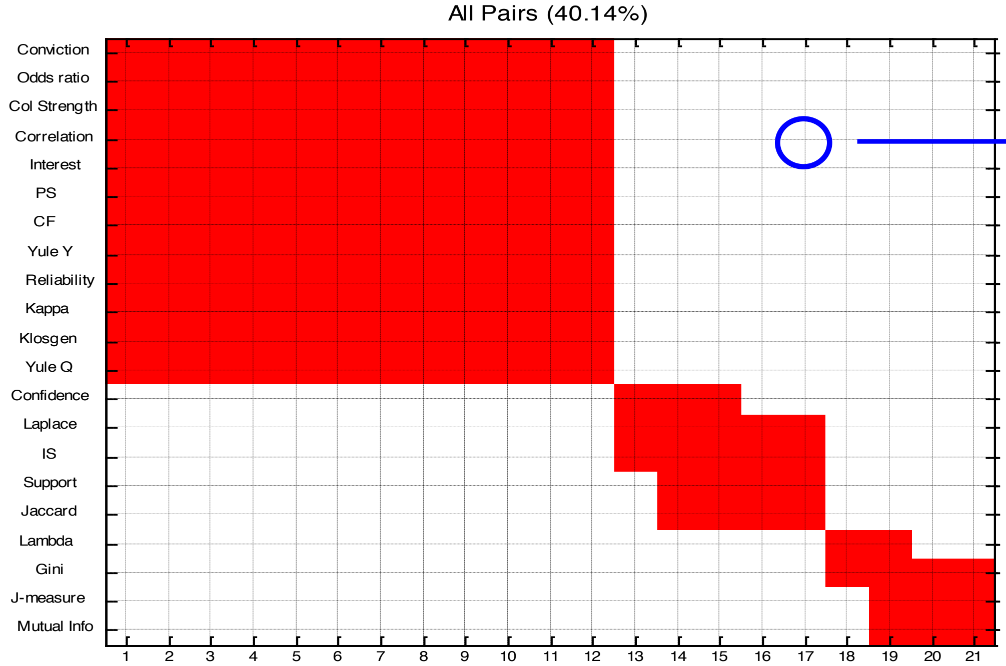
Investigate how support-based pruning affects other measures

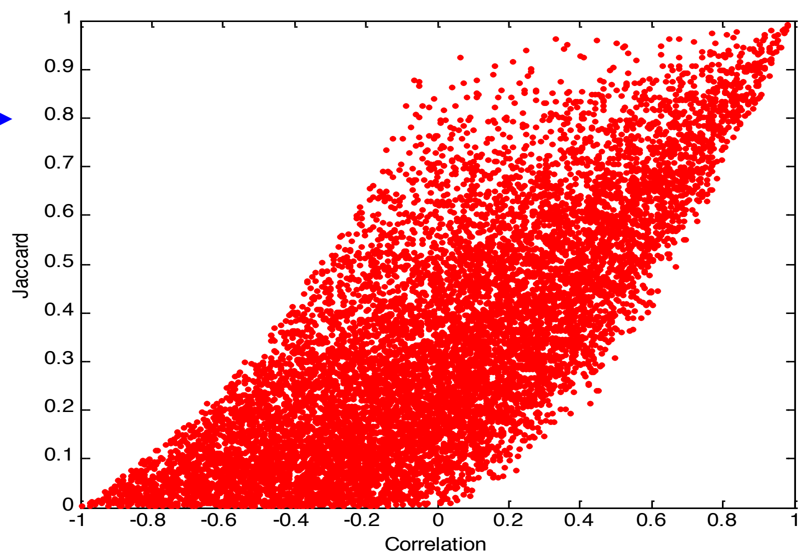
Steps:

- Generate 10000 contingency tables
- Rank each table according to the different measures
- Compute the pair-wise correlation between the measures

Effect of Support-based Pruning

- Without Support Pruning (All Pairs)



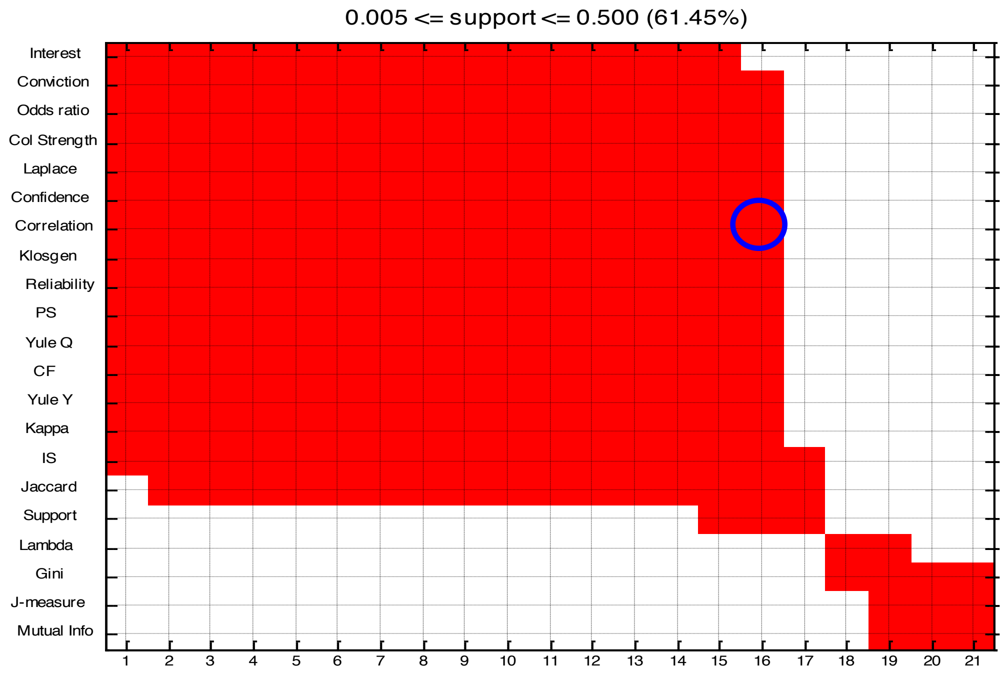


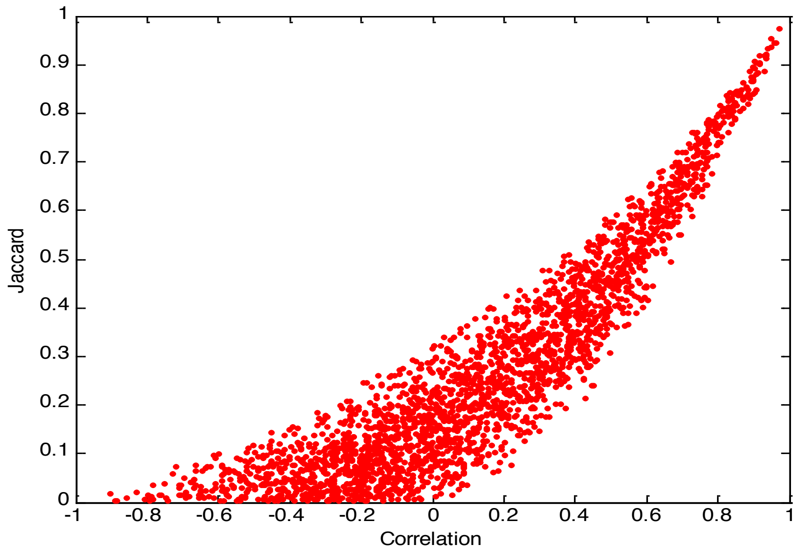
Scatter Plot between Correlation & Jaccard Measure

- Red cells indicate correlation between the pair of measures > 0.85
- 40.14% pairs have correlation > 0.85

Effect of Support-based Pruning

- 0.5% ≤ support ≤ 50%



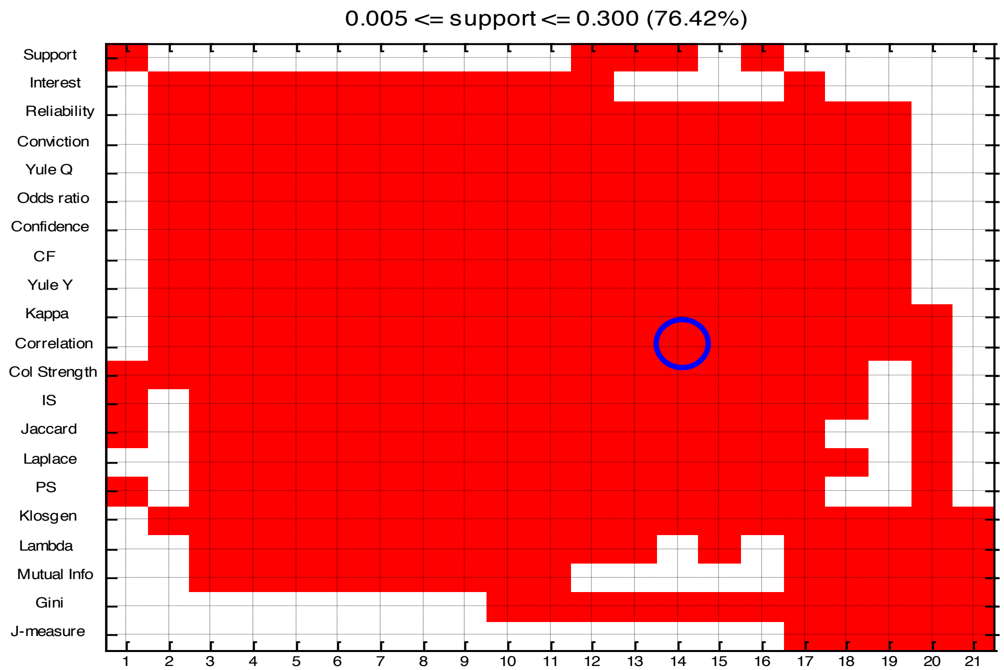


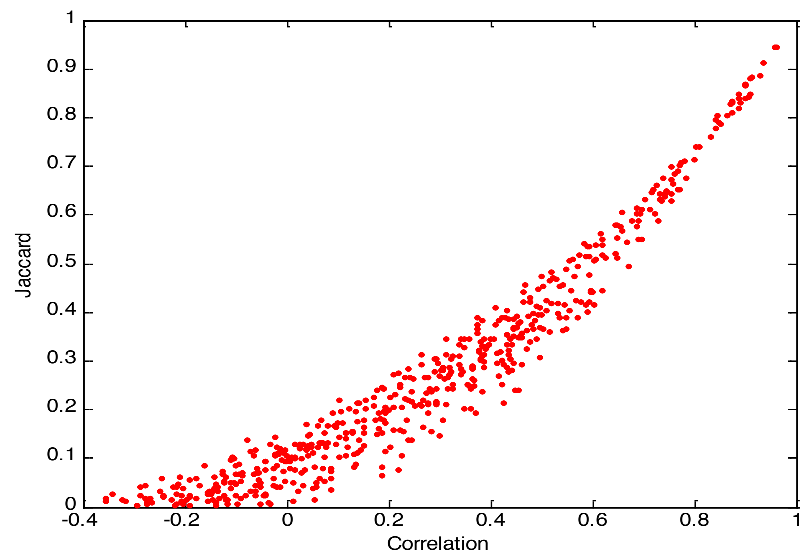
Scatter Plot between Correlation & Jaccard Measure:

- 61.45% pairs have correlation > 0.85

Effect of Support-based Pruning

- 0.5% ≤ support ≤ 30%





Scatter Plot between Correlation & Jaccard Measure

- 76.42% pairs have correlation > 0.85

Subjective Interestingness Measure

Objective measure:

- Rank patterns based on statistics computed from data
- e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).

Subjective measure:

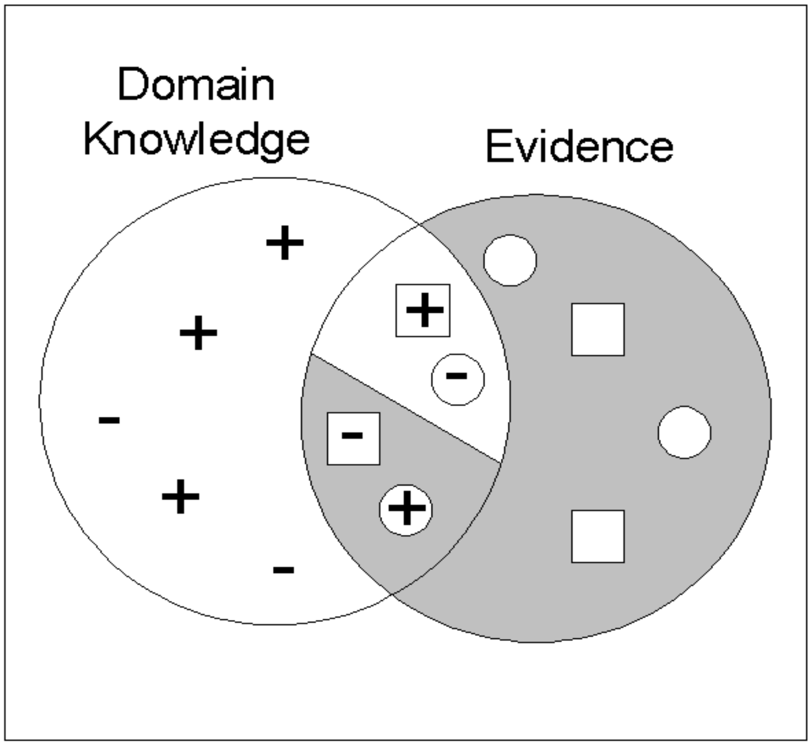
- Rank patterns according to user's interpretation

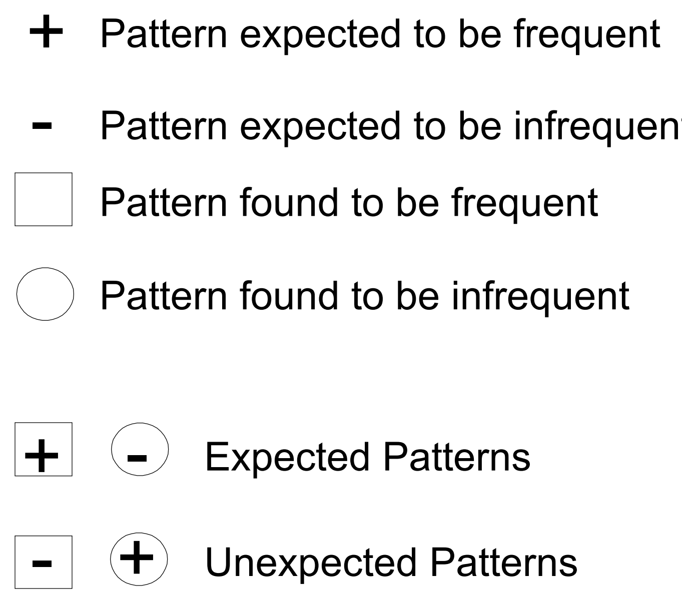
  A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)

  A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)

Interestingness via Unexpectedness

Need to model expectation of users (domain knowledge)





Need to combine expectation of users with evidence from data (i.e., extracted patterns)

Interestingness via Unexpectedness

Web Data (Cooley et al 2001)

- Domain knowledge in the form of site structure
- Given an itemset F = $\{X_1, X_2, \ldots, X_k\}$ (X_i : Web pages)

  L: number of links connecting the pages

  lfactor = L / (k $\times$ k-1)

  cfactor = 1 (if graph is connected), 0 (disconnected graph)

- Structure evidence = cfactor $\times$ lfactor
- Usage evidence $= \dfrac{P(X_1 \cap X_2 \cap \ldots \cap X_k)}{P(X_1 \cup X_2 \cup \ldots \cup X_k)}$
- Use Dempster-Shafer theory to combine domain knowledge and evidence from data