# Obligatory assignment 2 MVE550, autumn 2021

Petter Mostad

November 23, 2021

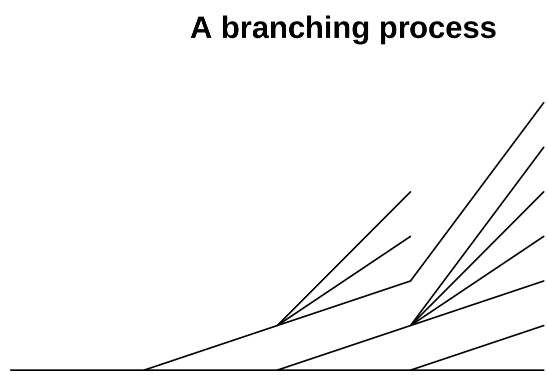


Figure 1: The 5 first steps of the Branching process for question 1

1. Assume a branching process has been observed for $Z_0, \ldots, Z_4$, and it looks like Figure 1. Assume the offspring distribution is Poisson with expectation $\lambda$. Assume that we use the improper prior $\pi(\lambda) \propto_\lambda 1/\lambda$ for $\lambda$. Compute the probability of extinction as follows:
   (a) Using the information about the offspring distribution appearing in Figure 1, compute the posterior distribution for $\lambda$.
   (b) Consider the continuation of the branching process pictured in Figure 1 into generations $Z_5$, $Z_6$, etc. Implement in R a function that takes as input a value for $\lambda$ and outputs the probability that this branching process will become extinct[1].

[1] Hint: Consider the R function `optimize`

(c) Now compute the probability of extinction for the branching process taking the uncertainty in $\lambda$ into account: Write down an integral representing this probability in terms of the results from (a) and (b). Then compute the integral using numerical integration.

(d) In two separate ways of deriving your answer, use simulation to check your result in (c): Note that you need to consider both the uncertainty in $\lambda$ and the uncertainty in the branching process. In your first solution, utilize your code from (a). In your second solution, simulate also the branching process. Compare all results.

(e) What is the maximum likelihood estimate for $\lambda$? What is the probability of extinction for the branching process if you use this estimate for $\lambda$ in our computation?

2. Eliza and Jacob are experimenting with the braking distance for their bike. For 7 different speeds $x_1, \ldots, x_7$ they have observed the braking distances $y_1, \ldots, y_7$ listed in this table:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | 2.3 | 5.4 | 4.6 | 5.7 | 8.9 | 3.2 | 7.1 |
| y | 1.7 | 3.4 | 2.0 | 4.9 | 5.5 | 3.7 | 5.4 |

Given this information, and assuming the speed of the bike is 10, what is the probability that the braking distance will be less than 8?

In order to do Bayesian inference on this problem, we need a complete probabilistic model. We will assume

$$y_i \mid \theta \sim \text{Normal}(\theta_1 + x_i\theta_2, 1)$$

for $i = 1, \ldots, 8$ where we write $x_8$ for the hypothetical speed 10 and $y_8$ for the corresponding braking distance still to be observed. In other words, we assume the braking distance is a linear function of the speed, with some variation around the line.

For the prior for $\theta = (\theta_1, \theta_2)$ we assume

$$\pi(\theta_1, \theta_2) = \text{Normal}(\theta_1; 0, 10^2) \cdot \text{Normal}(\theta_2, 1, 10^2)$$

so that the standard deviations of the normal distributions are 10.

(a) Write an R function that takes as input a parameter vector $\theta$ and outputs the logarithm of the likelihood function for this $\theta$.

(b) Write an R function that takes as input a parameter vector $\theta$ and ouptuts the logarithm of the density of the prior at this $\theta$.

(c) Program an MCMC algorithm in R to answer the following question: When the speed is 10 what is the probability that the braking distance will be less than 8? Use a random walk proposal, with

$$\theta^* = \theta_{i-1} + (\epsilon_1, \epsilon_2)$$

where $\epsilon_1 \sim \text{Normal}(0, 0.2^2)$ and $\epsilon_2 \sim \text{Normal}(0, 0.1^2)$.