Obligatory assignment 1 MVE550, autumn 2021

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November 8, 2021

1. Lisa owns a consulting business. We assume the number of customers that contact her business each day is Poisson(λ) distributed, where λ is the expected number of daily customers. There are 4 customers on Monday and one customer on Tuesday. Lisa would like to predict the number of customers on Wednesday.[1]

   (a) Based on experience, Lisa assumes the prior probability density for λ is Gamma$(6, 2)$. Compute the posterior density for λ given the observations on Monday and Tuesday. Also, given this data, compute the predicted probability for getting $0, 1, 2, \ldots, 9$, or 10 or more customers on Wednesday.

   (b) Re-compute the predicted probability for getting exactly 3 customers on Wednesday by using a discretization of the λ variable, similar to section 1.5 of the compendium.

   (c) Re-compute the predicted probability for getting exactly 3 customers on Wednesday by using numerical integration, similar to section 1.6 of the compendium.

   (d) Bobby has done some more research on the number of customers that can be expected at this type of business, and he believes the correct prior density to use would be a normal density with expectation 3 and standard deviation 2, truncated at zero. (In other words, it is a normal density with expectation 3 and standard deviation 2, cut off so that the density for negative values is zero, and re-scaled so that the integral below the curve is 1). Re-compute the predicted probability of getting exactly 3 customers on Thursday when using this prior.

   (e) Now go back to using the prior from (a) and perform a simulation study as follows: Simulate one million random λ values from the Gamma$(6, 2)$ distribution. Then, for each lambda, simulate the number of customers for Monday and for Tuesday using the Poisson distribution. Now, compute the proportion of the simulated λs for

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[1]NOTE: In this task, you may have use for R functions like `rpois`, `dpois`, `ppois`, `qpois`, `rgamma`, `dgamma`, `pgamma`, `qgamma`, `rnorm`, `dnorm`, `pnorm`, `qnorm` etc. Use the help function in R to learn more.

which the corresponding simulated values for Monday and Tuesday are equal to 4 and 1, respectively. Finally, compute the proportion of the simulated λs for which the simulated values for Monday, Tuesday, and Wednesday, are 4, 1, and 3, respectively. How do your results connect to the results above?

2. Answer question 3.52 in Dobrow.

The question is about the "Snakes and Ladder" game described in Section 3.8 of Dobrow. Make sure you understand the rules of the game. For example, the finishing square must be reached by an exact roll of the die. If you don't get the exact number, you stay put on the same square (see for example the Snake and Ladders simulation in Dobrow).