Institute for Astronomical and Physical Geodesy

ESPACE: Orbit Mechanics

Exercise 3:

«Integration of satellite orbits with different force models»

WS14/15

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Contents

Given data 4
Brief overview 4
Plotting. Solution A and B 6
Plotting. Solution C and B 7
Plotting. Solution D and A 8
Plotting. Solution E and A 9
Plotting. Solution F and A 10
Plotting. Solution G and B 11
Plotting. Solution H and B 12
Plotting. Solution I and C 13
Plotting. Solution I and J 14
Plotting. Solution K and L 15
Plotting. Solution M and L 16
Plotting. Solution N and K 17
Plotting. Solution N and D (only surface Forces) 18
Plotting. Solution O and K 19
Plotting. Solution O and E (only surface Forces) 20
Plotting. Solution P and K 21
Plotting. Solution Q and L 22
Plotting. Solution R and L 23
Plotting. Solution S and M 24
Plotting. Solution T and S 25
Plotting. Solution U and A 26

Plotting. Solution V and A 27

Conclusion 29

MATLAB CODE.. 30

Given data

IAPG developed software for orbit integration. Table with instructions about which forces to consider and compare.

Solution	Satellite	Gravity Field	Degree Order	Atmospheric Drag	Solar Pressure	Direct Tides Sun & Moon	Indirect Solid Earth Tides	Indirect Ocean Tides	Compare to Solution
A	GRACE	GRIM5-C1	0	-	-	-	-	-	-
B	GRACE	GRIM5-C1	50	-	-	-	-	-	A
C	GRACE	GRIM5-C1	120	-	-	-	-	-	B
D	GRACE	GRIM5-C1	0	+	-	-	-	-	A
E	GRACE	GRIM5-C1	0	-	+	-	-	-	A
F	GRACE	GRIM5-C1	0	-	-	+	-	-	A
G	GRACE	GRIM5-C1	50	-	-	-	+	-	B
H	GRACE	GRIM5-C1	50	-	-	-	-	+	B
I	GRACE	GRIM5-C1	120	+	+	+	+	+	C
J	GRACE	EIGEN-1S	119	+	+	+	+	+	I
K	GPS	GRIM5-C1	0	-	-	-	-	-	-
L	GPS	GRIM5-C1	50	-	-	-	-	-	K
M	GPS	GRIM5-C1	120	-	-	-	-	-	L
N	GPS	GRIM5-C1	0	+	-	-	-	-	K & D*
O	GPS	GRIM5-C1	0	-	+	-	-	-	K & E*
P	GPS	GRIM5-C1	0	-	-	+	-	-	K
Q	GPS	GRIM5-C1	50	-	-	-	+	-	L
R	GPS	GRIM5-C1	50	-	-	-	-	+	L
S	GPS	GRIM5-C1	120	+	+	+	+	+	M
T	GPS	EIGEN-1S	119	+	+	+	+	+	S
U[1)]	GRACE	GRIM5-C1	0	-	-	-	-	-	A
V[2)]	GRACE	GRIM5-C1	0	-	-	-	-	-	A

* here only the surface forces (Atmospheric Drag or Solar Pressure) shall be compared in the orbit system as they are provided by the software in accelerations [m/s^2]. Here orbits (position, velocities) do not have to be compared.

1) change the initial conditions of the position by adding +1m to x,y,z. Initial conditions of velocities are unchanged (kept as in A to J).

2) change the initial conditions of the velocities by adding +1mm/sec to x,y,z. Initial conditions of positions are unchanged (kept as in A to J.

Brief overview

We use the numerical orbit integration software, developed by IAPG, which is able to deal with all gravitational and non-gravitational forces. The software is operated with a comfortable graphical user interface. As numerical integrator an Adams-Moulton multi-step procedure is applied. In this exercise, orbits of different satellites applying different force models shall be computed and compared against each other. The impact of the individual forces on the satellite orbits shall be visualized. In particular the following tasks shall be completed:

1) We have to compute the orbits for the GRACE and GPS satellites with different force models for one day. The following general parameters have to be used:

For GRACE

- Initial Conditions: Generate initial conditions with Default-button!
- Change time to: 1.8.2002
- Start-, Stop Time: 0 – 1 day in [sec]
- Do not change position vector, velocity vector and integrator specifications.
- Step size: 10 sec
- Precision: 1e-16

For GPS

- Initial Conditions: Change position- [m] and velocity vectors [m/s] to:

position: 13147200.00000, 22771618.37727, 0.00000

velocity: -1943.65825355171, 1122.17161590072, 3205.25431294238

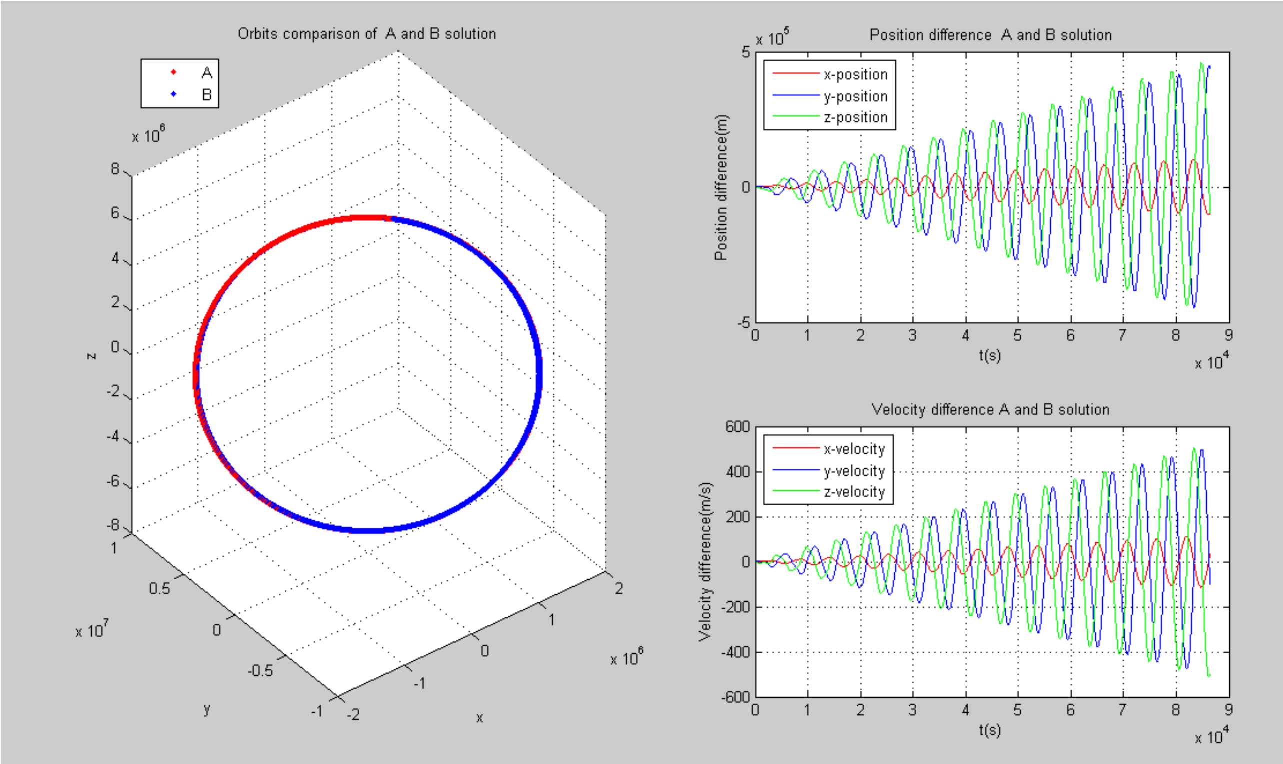
2) We have to compute the differences between the orbits for each component (x,y,z) according to the specification in the table on page 3 and analyze them with an appropriate graphical tool (e.g. Matlab). Then we interpret the differences and the individual impact of the forces on the orbit.

   Additionally we are supposed to compare the surface forces (in the orbit system) between the GRACE and GPS satellites (only for the test cases indicated in the table). After that we have to plot the results and interpret them.

3) Finally, we prepare a table for GRACE and GPS showing the size of the individual effects (e.g. indicate minimum, maximum, RMS of the differences for one day).

The tutorial is carried out under the supervision of Prof. Dr. Urs Hugentobler and Ye Hao.

Plotting. Solution A and B

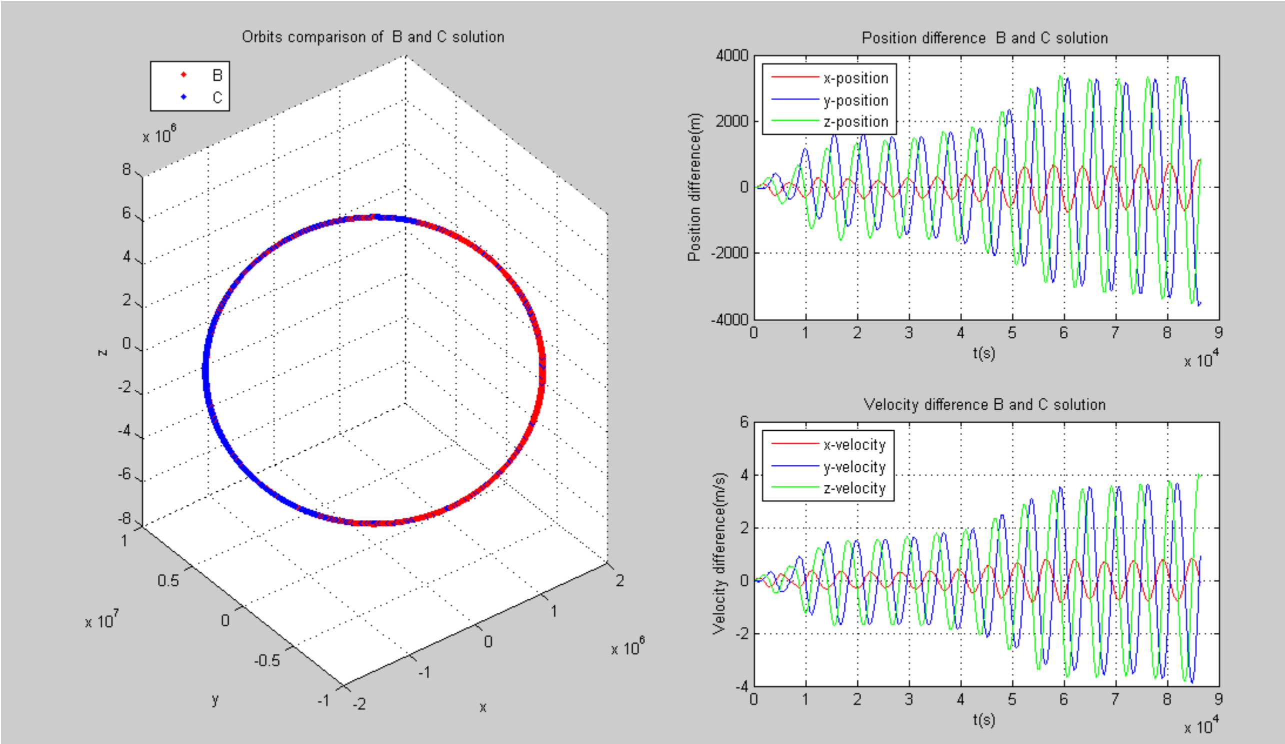


Picture 1: Solutions A and B

A: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0

B: GRACE satellite. Gravity model GRIM5-C1, Degree order – 50

Plotting. Solution C and B

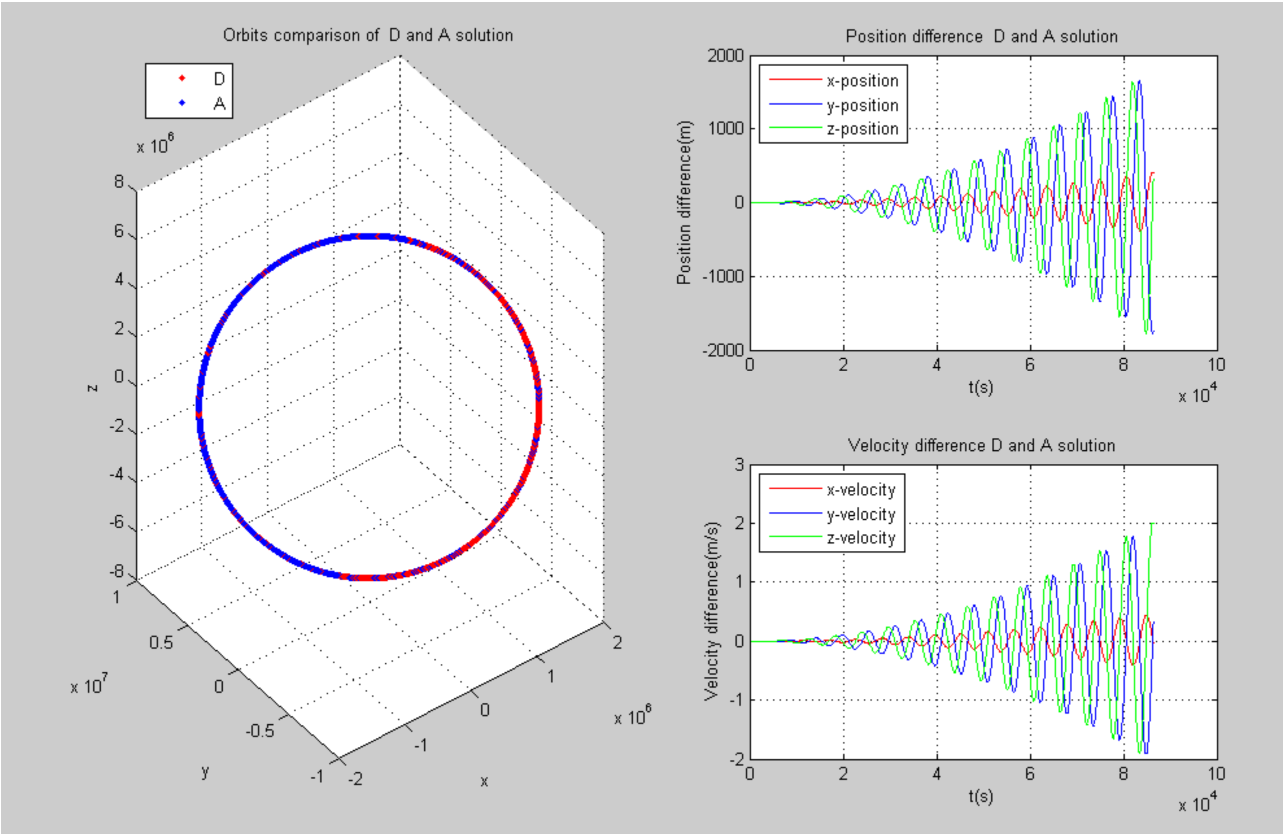


Picture 2: Solutions B and C

B: GRACE satellite. Gravity model GRIM5-C1, Degree order – 50

C: GRACE satellite. Gravity model GRIM5-C1, Degree order – 120

Plotting. Solution D and A

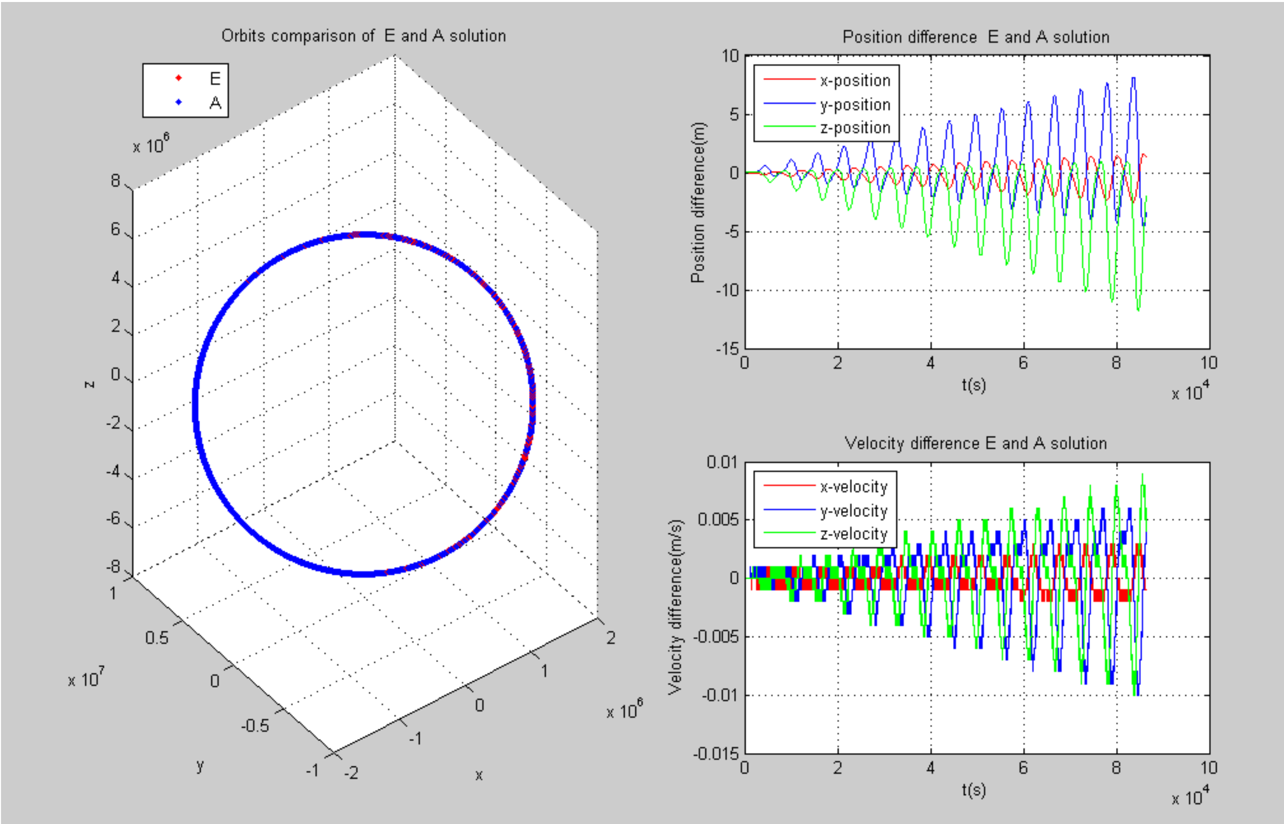


Picture 3: Solutions D and A

A: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0

D: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0, Atmospheric Drag is considered

Plotting. Solution E and A

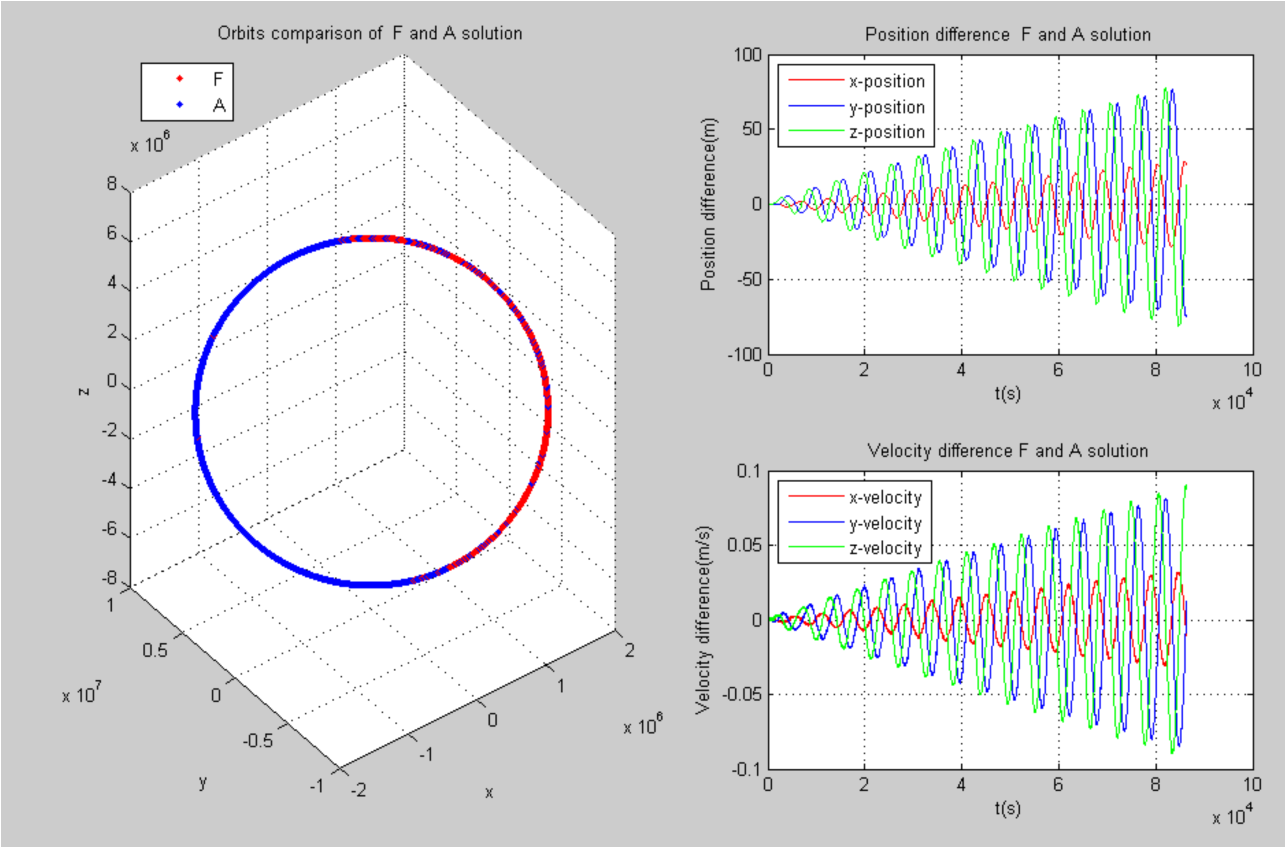


Picture 4: Solutions E and A

A: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0

D: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0, Solar pressure is considered

Plotting. Solution F and A

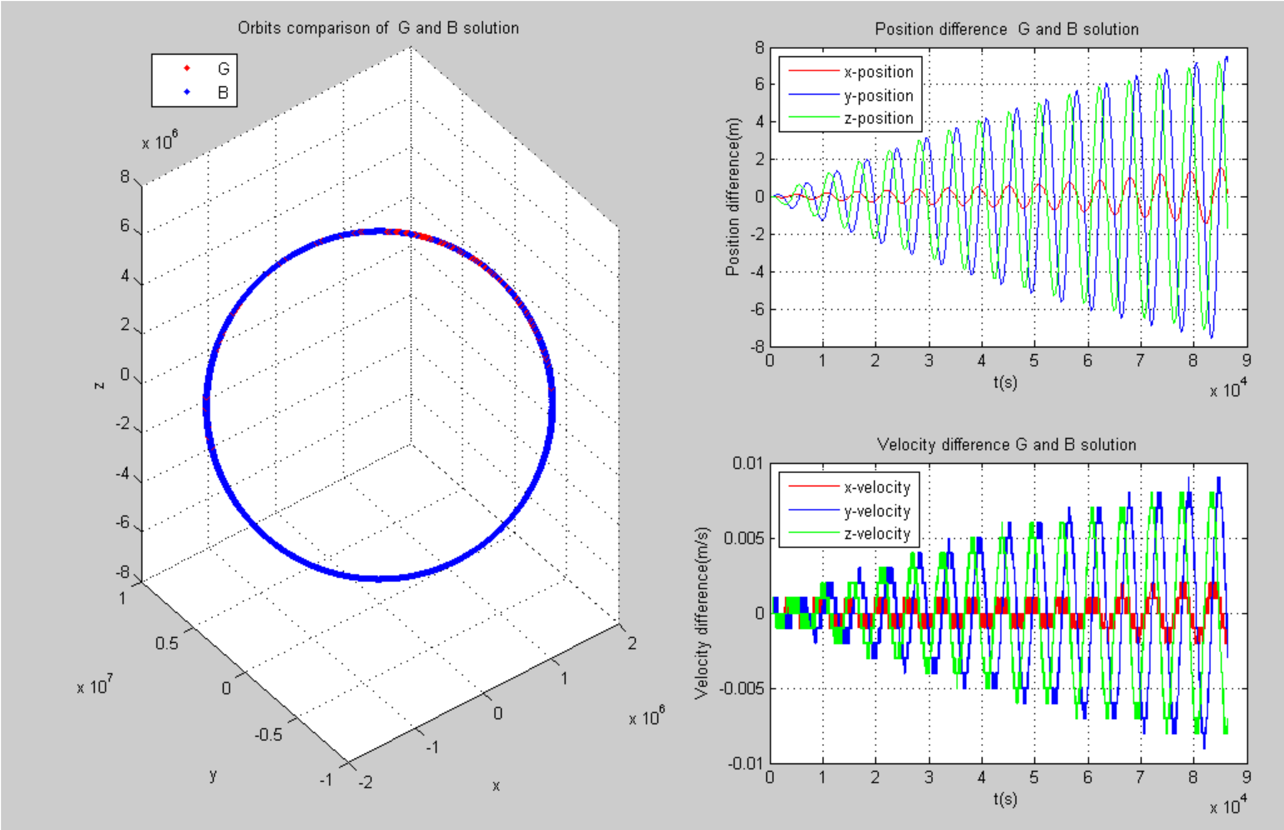


Picture 5: Solutions F and A

A: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0

F: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0, Direct tides (Sun and Moon) are considered

Plotting. Solution G and B

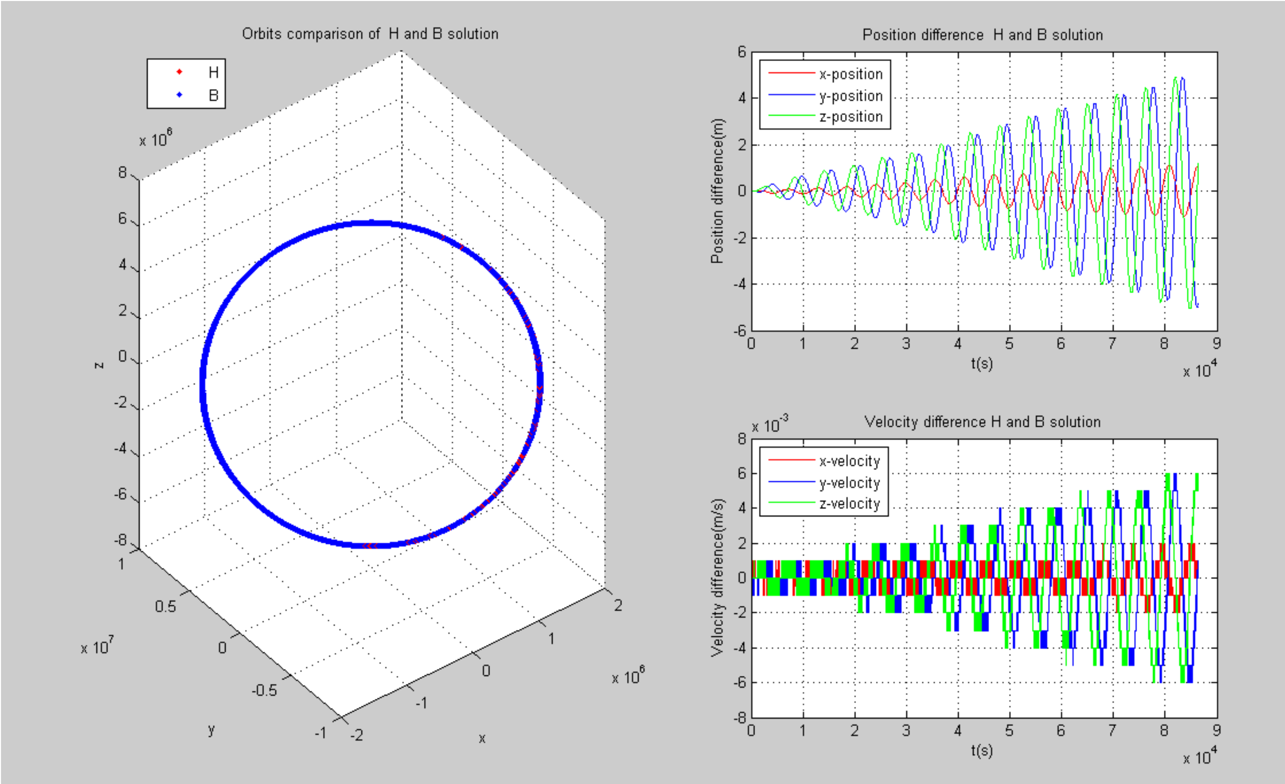


Picture 6: Solutions G and B

B: GRACE satellite. Gravity model GRIM5-C1, Degree order – 50

G: GRACE satellite. Gravity model GRIM5-C1, Degree order – 50, Indirect solid Earth tides are considered

Plotting. Solution H and B

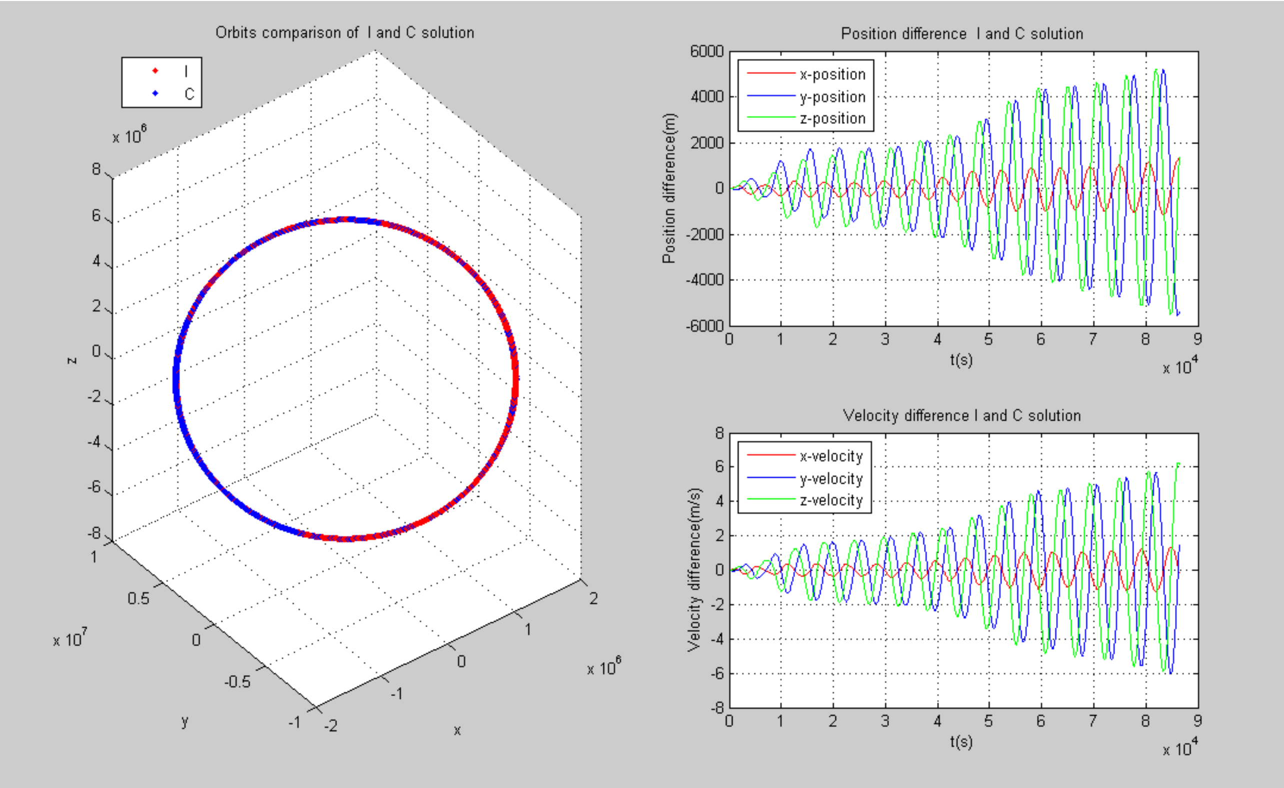


Picture 7: Solutions H and B

B: GRACE satellite. Gravity model GRIM5-C1, Degree order – 50

G: GRACE satellite. Gravity model GRIM5-C1, Degree order – 50, Indirect ocean tides are considered

Plotting. Solution I and C

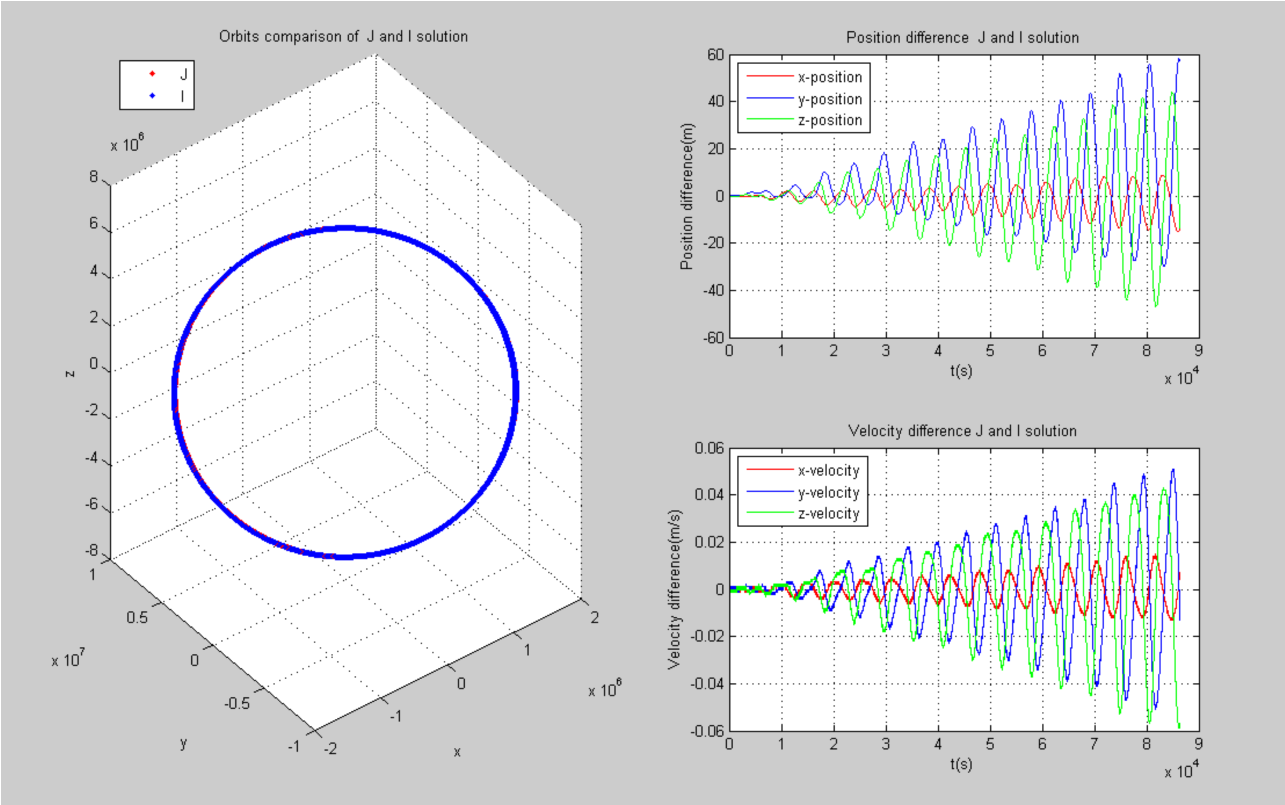


Picture 8: Solutions I and C

C: GRACE satellite. Gravity model GRIM5-C1, Degree order – 120

I: GRACE satellite. Gravity model GRIM5-C1, Degree order – 120, Atmospheric Drag, Solar pressure, Direct tides from Sun and Moon, Indirect tides from solid Earth and Indirect ocean tides are considered.

Plotting. Solution I and J

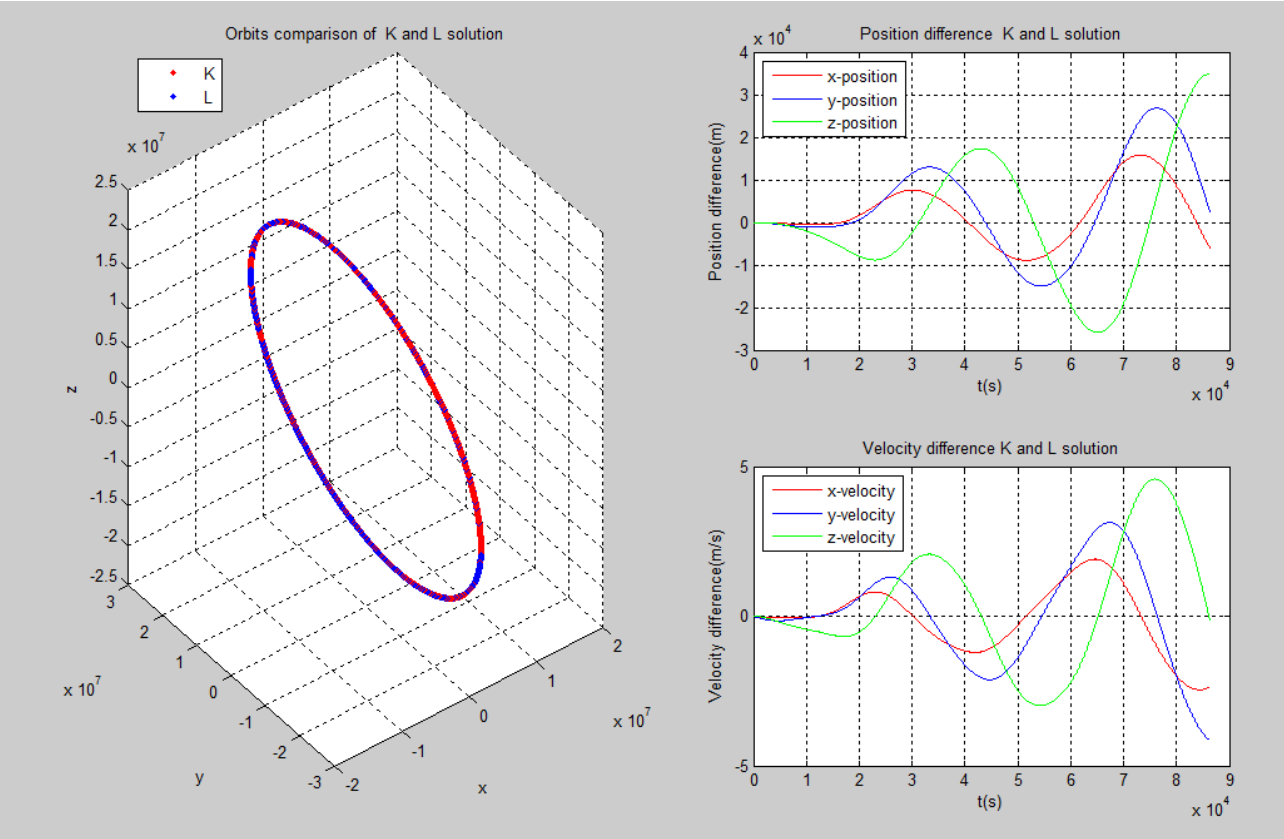


Picture 9: Solutions I and J

J: GRACE satellite. Gravity model EIGEN-1S, Degree order – 119, Atmospheric Drag, Solar pressure, Direct tides from Sun and Moon, Indirect tides from solid Earth and Indirect ocean tides are considered.

I: GRACE satellite. Gravity model GRIM5-C1, Degree order – 120, Atmospheric Drag, Solar pressure, Direct tides from Sun and Moon, Indirect tides from solid Earth and Indirect ocean tides are considered.

Plotting. Solution K and L

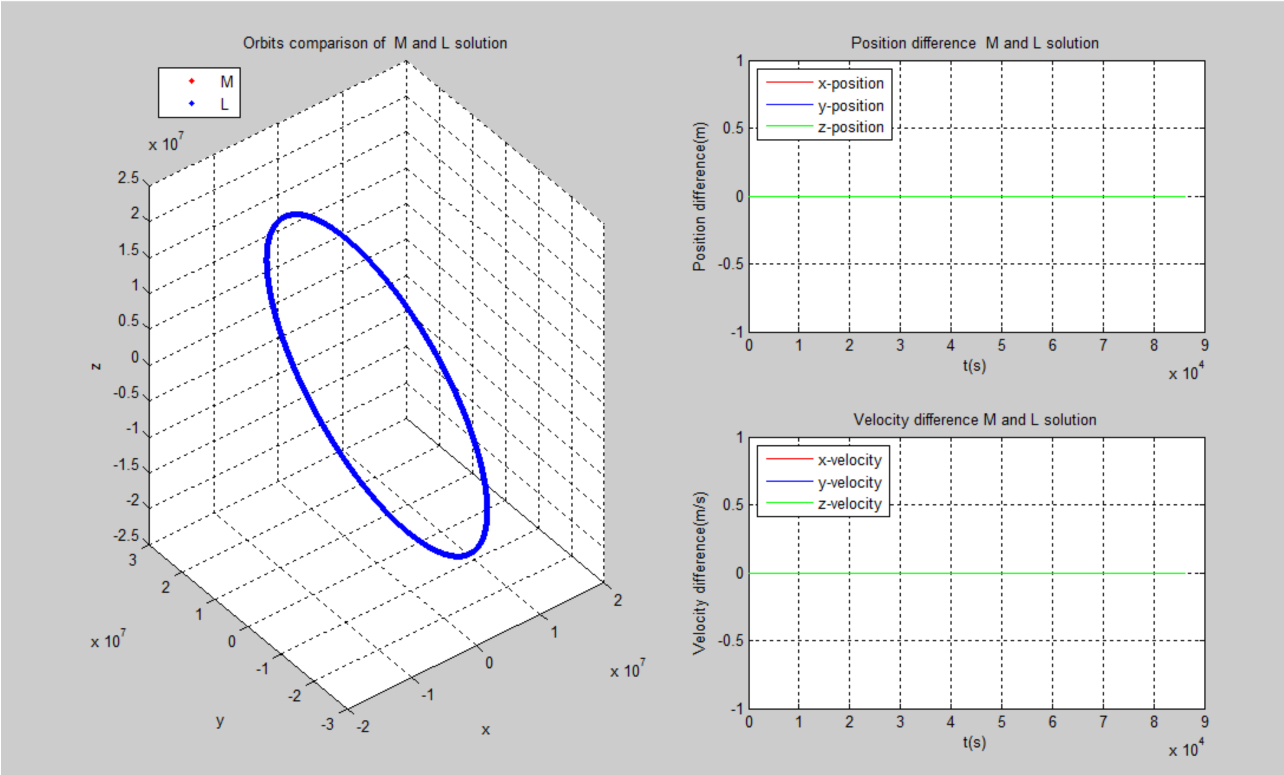


Picture 10: Solutions K and L

K: GPS satellite. Gravity model GRIM5-C1, Degree order – 0

L: GPS satellite. Gravity model GRIM5-C1, Degree order – 50

Plotting. Solution M and L

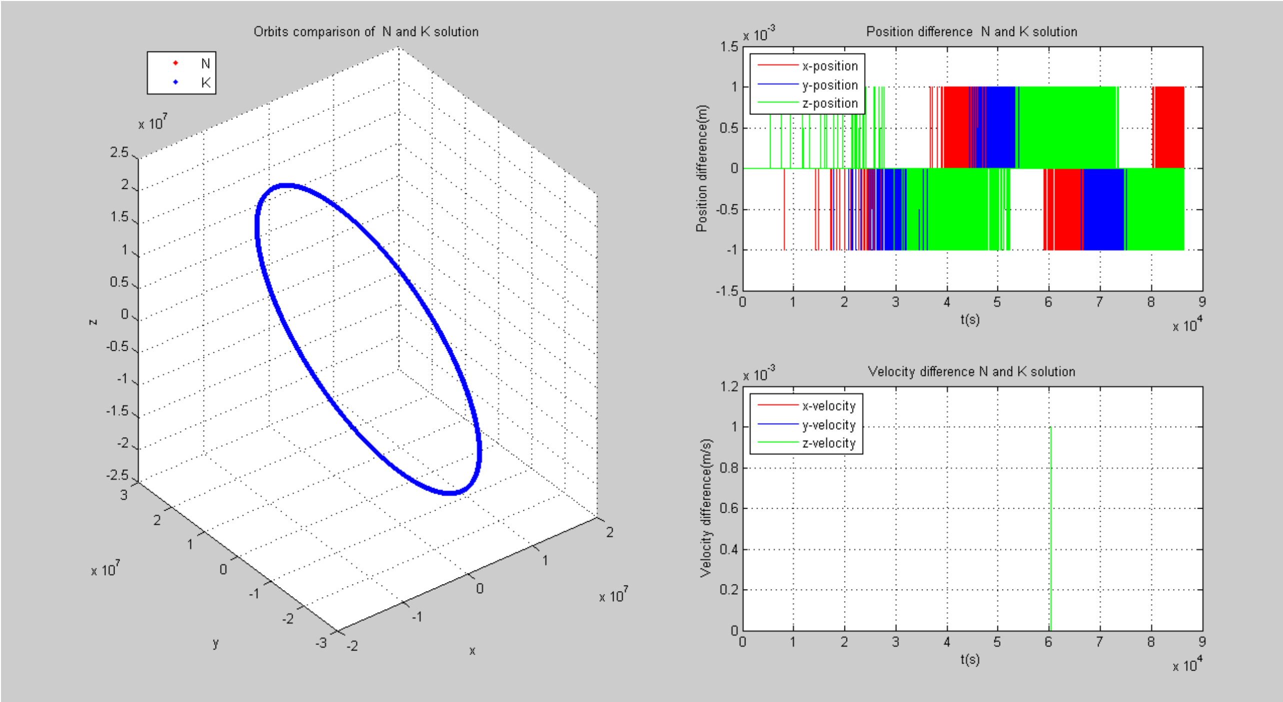


Picture 11: Solutions M and L

M: GPS satellite. Gravity model GRIM5-C1, Degree order – 120

L: GPS satellite. Gravity model GRIM5-C1, Degree order – 50

Plotting. Solution N and K

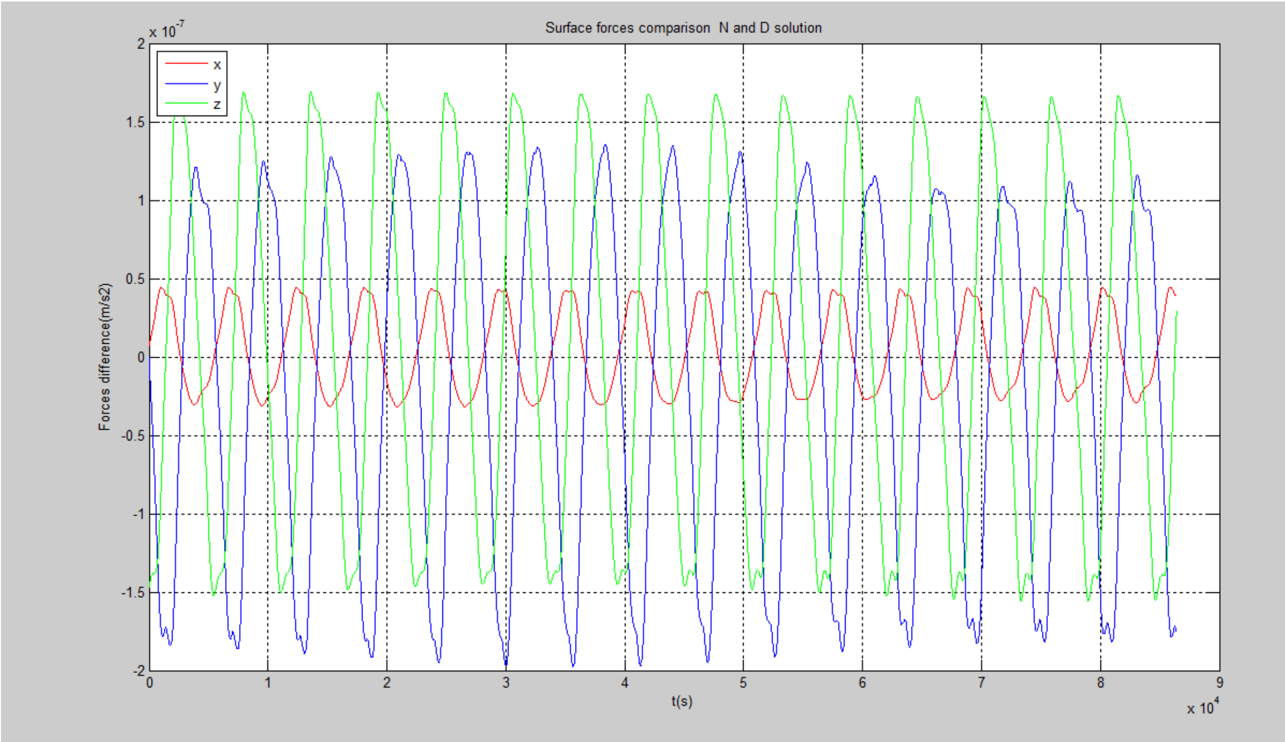


Picture 12: Solutions N and K

K: GPS satellite. Gravity model GRIM5-C1, Degree order – 0

N: GPS satellite. Gravity model GRIM5-C1, Degree order – 0, Atmospheric Drag included

Plotting. Solution N and D (only surface Forces)

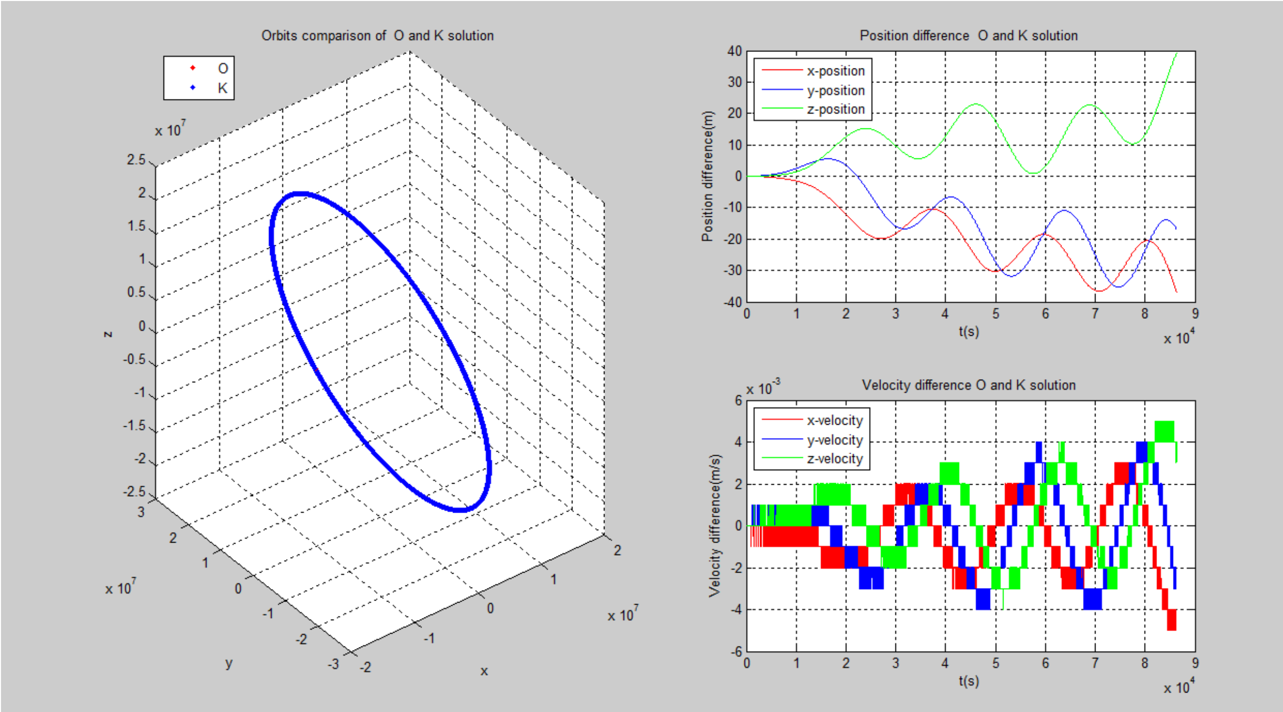


Picture 13: Solutions N and D (only surface forces)

D: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0, Atmospheric Drag is considered

N: GPS satellite. Gravity model GRIM5-C1, Degree order – 0, Atmospheric Drag included

Plotting. Solution O and K

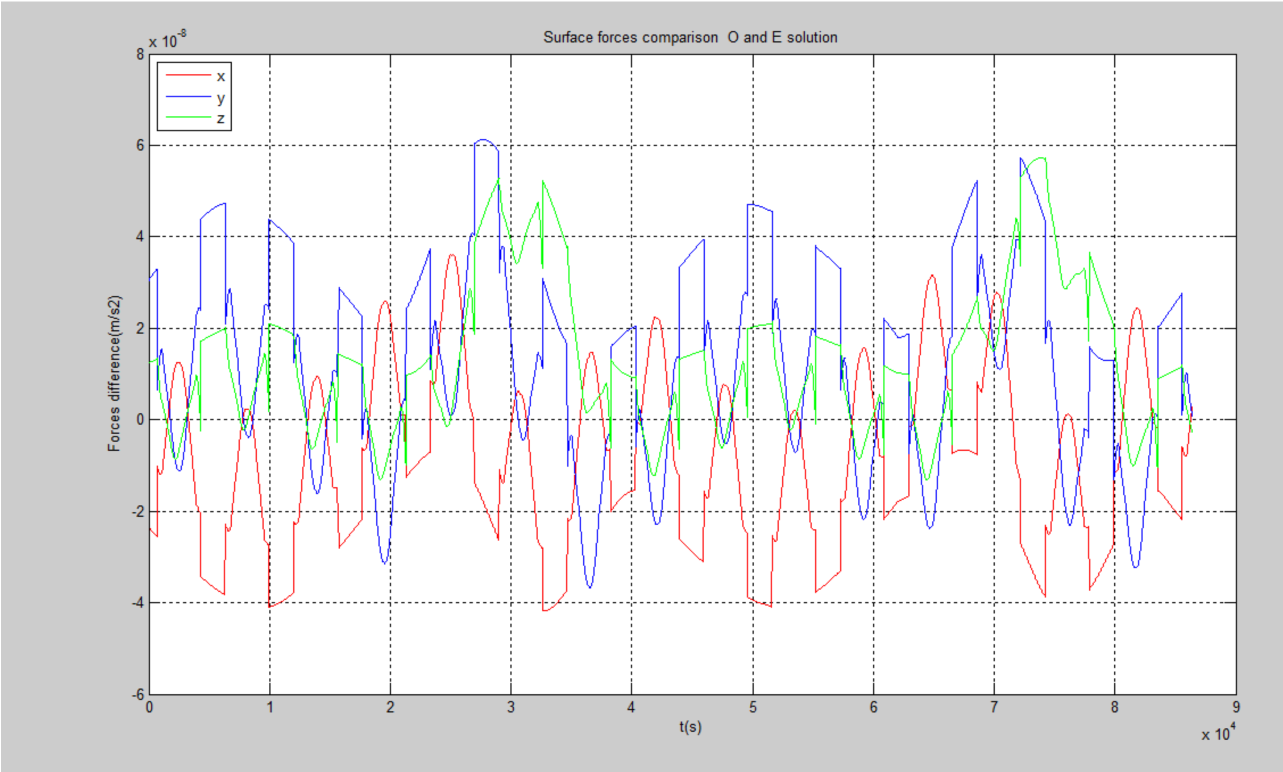


Picture 14: Solutions O and K

O: GPS satellite. Gravity model GRIM5-C1, Degree order – 0, Solar Pressure is considered

K: GPS satellite. Gravity model GRIM5-C1, Degree order – 0

Plotting. Solution O and E (only surface Forces)

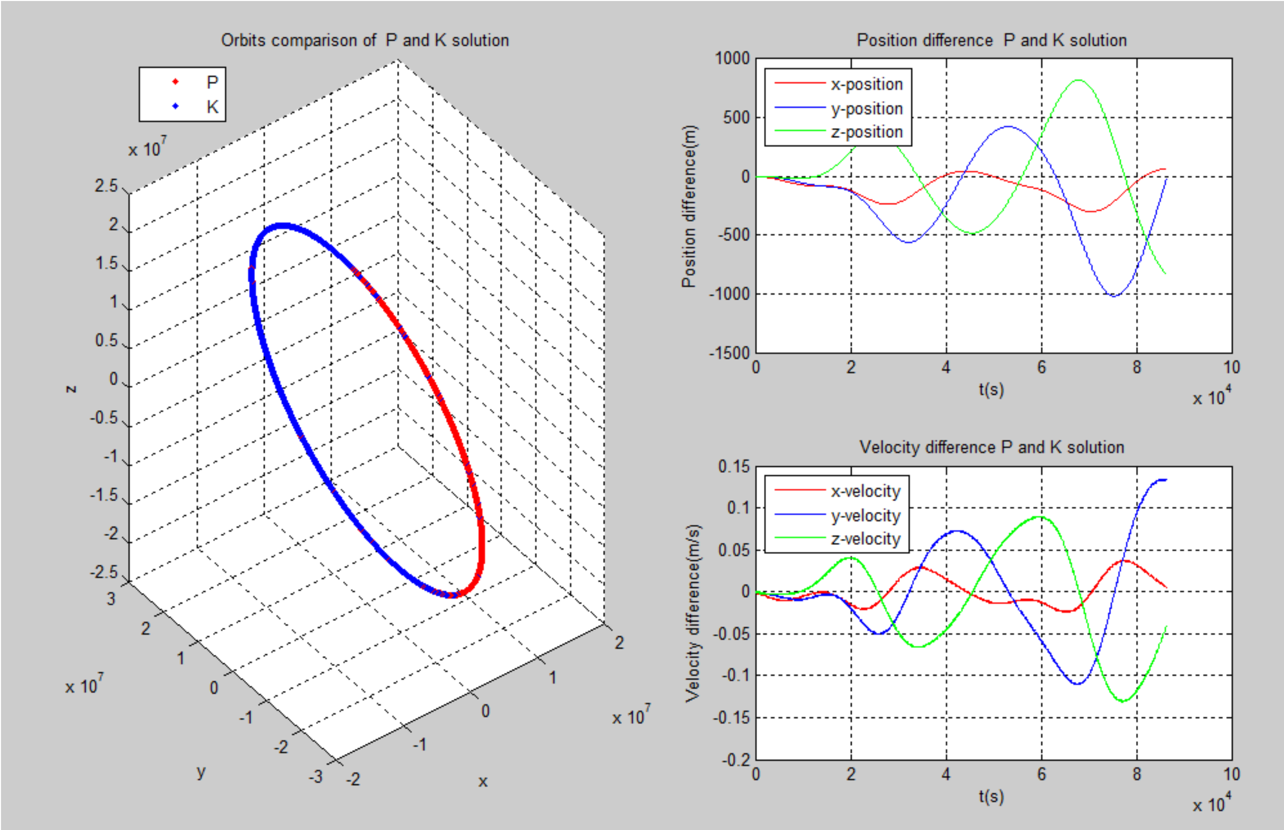


Picture 15: Solutions O and E

O: GPS satellite. Gravity model GRIM5-C1, Degree order – 0, Solar Pressure is considered

E: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0, Solar Pressure is considered

Plotting. Solution P and K

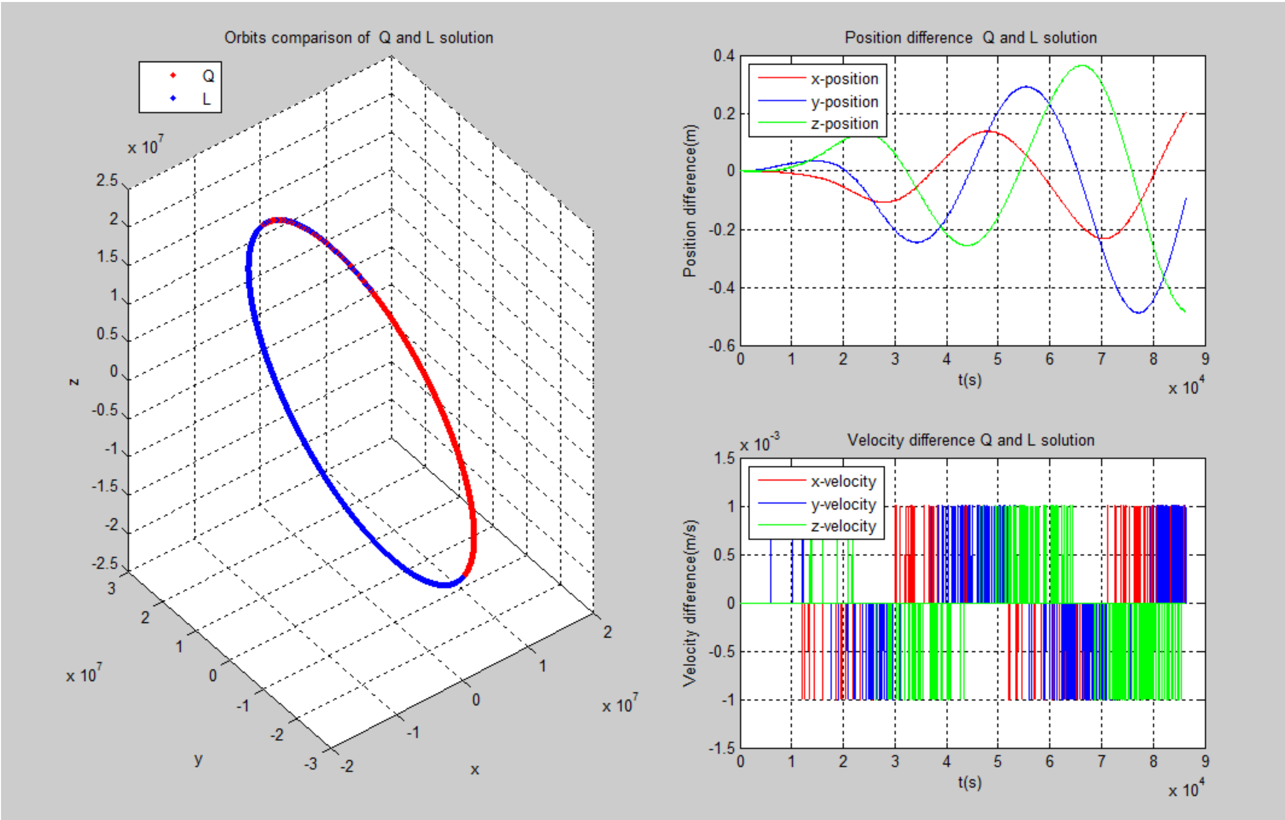


Picture 16: Solutions P and K

P: GPS satellite. Gravity model GRIM5-C1, Degree order – 0, Direct Tides from Sun and Moon are considered

K: GPS satellite. Gravity model GRIM5-C1, Degree order – 0

Plotting. Solution Q and L

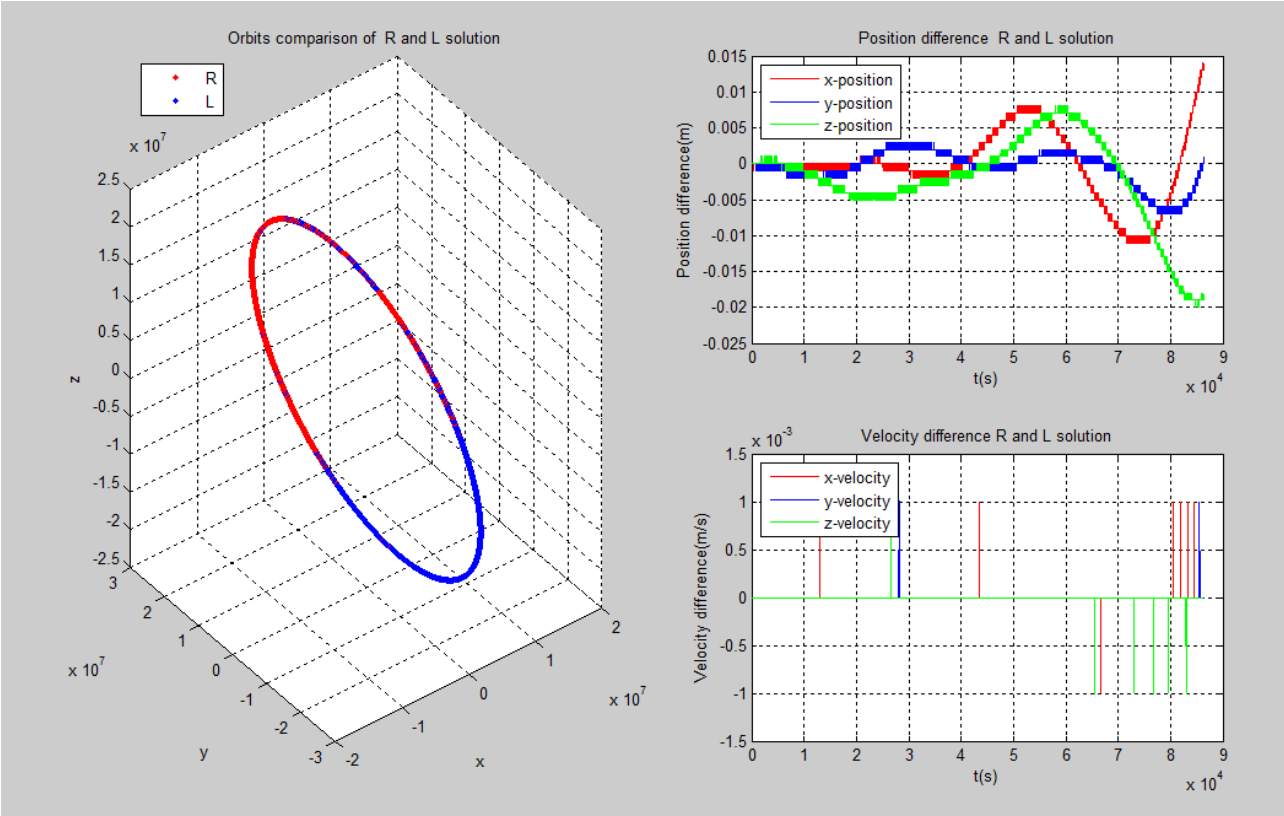


Picture 17: Solutions Q and L

Q: GPS satellite. Gravity model GRIM5-C1, Degree order – 50, Indirect Solid Earth Tides are considered

L: GPS satellite. Gravity model GRIM5-C1, Degree order – 50

Plotting. Solution R and L

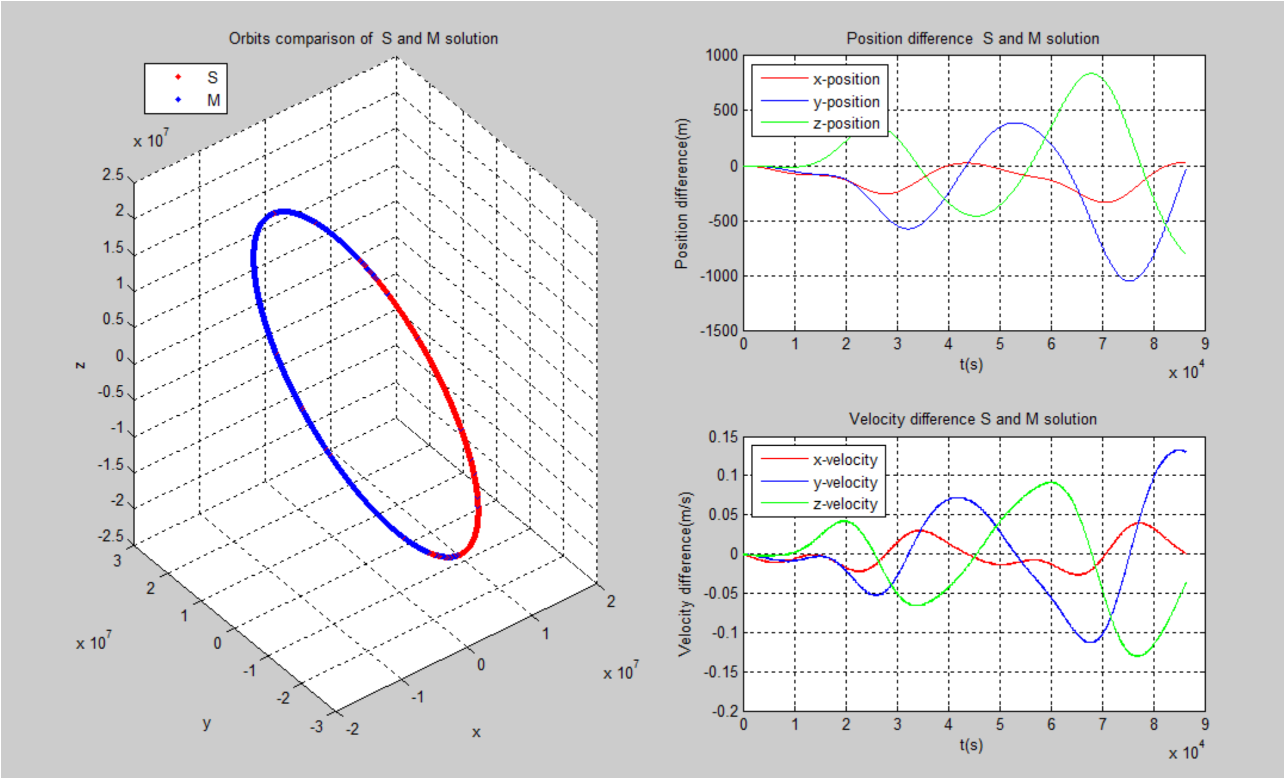


Picture 18: Solutions R and L

R: GPS satellite. Gravity model GRIM5-C1, Degree order – 50, Indirect Ocean Tides are considered

L: GPS satellite. Gravity model GRIM5-C1, Degree order – 50

Plotting. Solution S and M

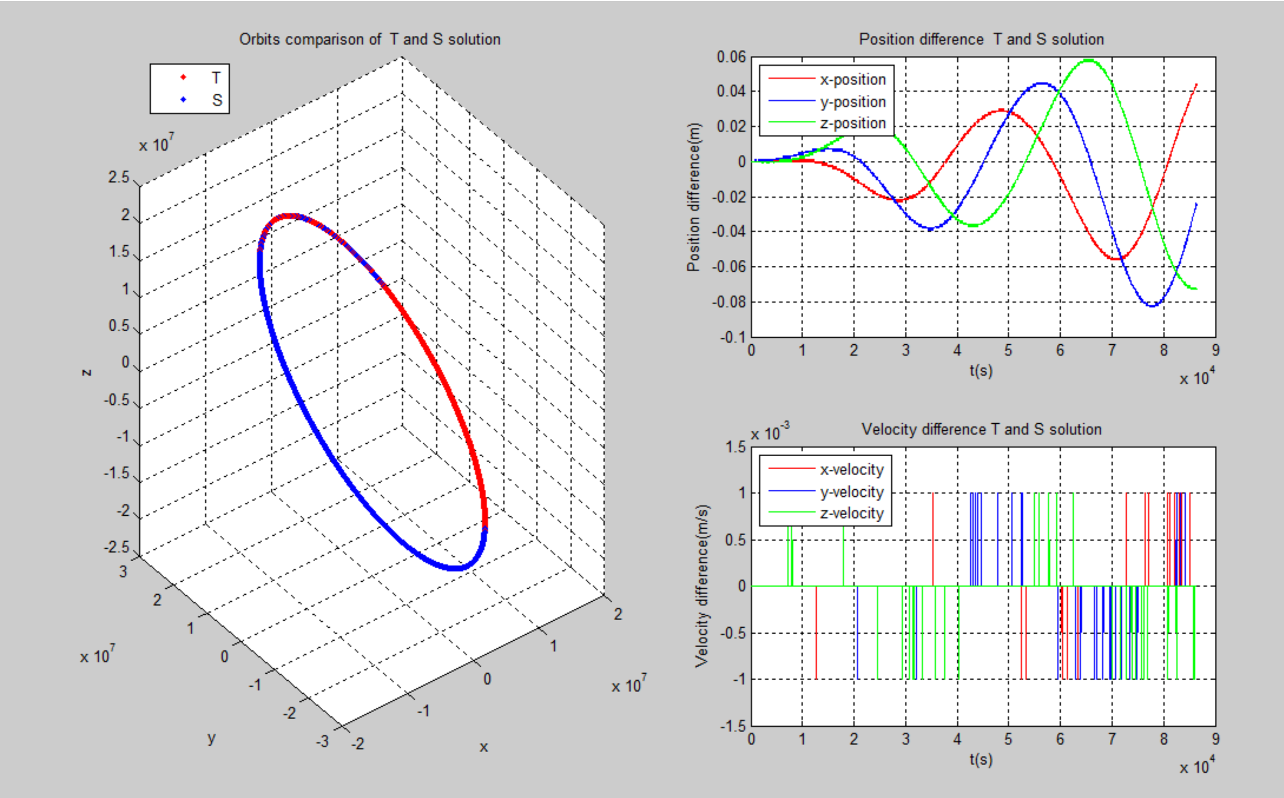


Picture 19: Solutions S and M

S: GPS satellite. Gravity model GRIM5-C1, Degree order – 120, Atmospheric Tides, Solar Pressure, Direct Tides from Sun and Moon, Indirect Solid Earth Tides, Indirect Ocean Tides are considered

M: GPS satellite. Gravity model GRIM5-C1, Degree order – 120

Plotting. Solution T and S

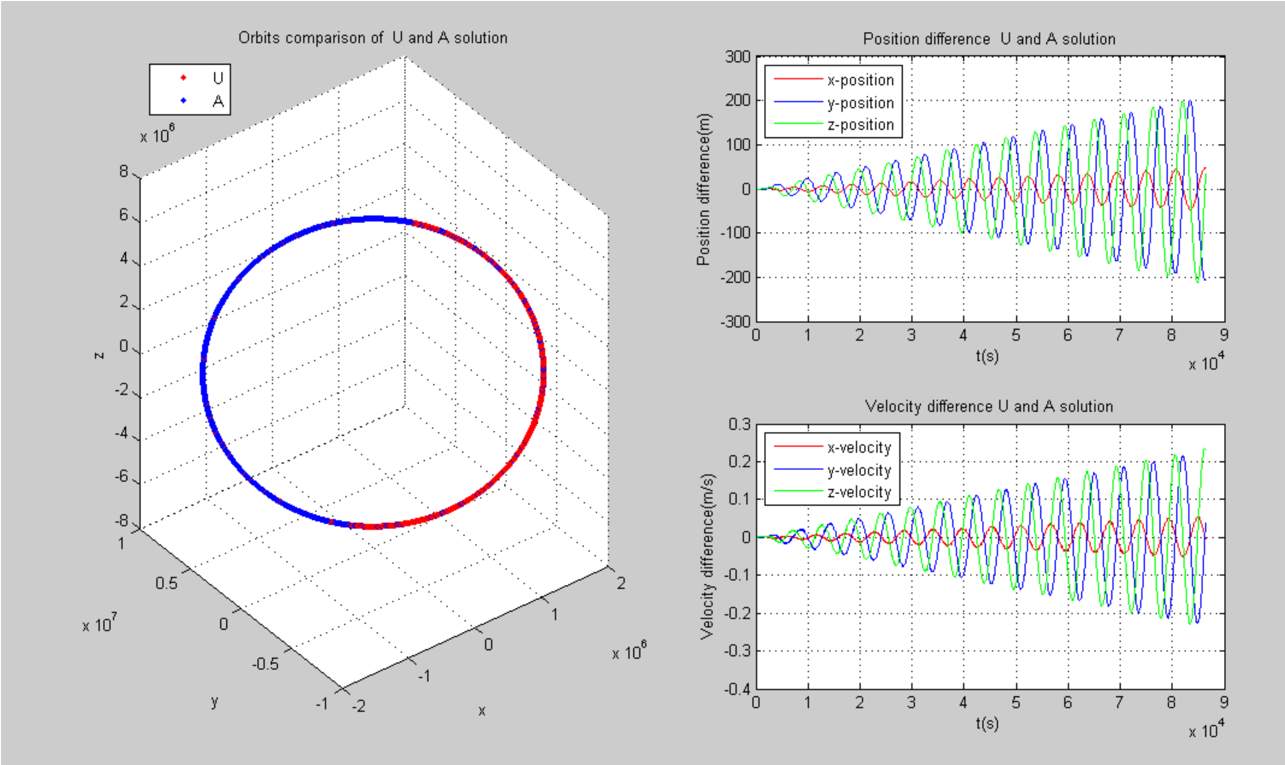


Picture 20: Solutions S and M

S: GPS satellite. Gravity model GRIM5-C1, Degree order – 120, Atmospheric Tides, Solar Pressure, Direct Tides from Sun and Moon, Indirect Solid Earth Tides, Indirect Ocean Tides are considered

T: GPS satellite. Gravity model EIGEN -1S, Degree order – 119, Atmospheric Tides, Solar Pressure, Direct Tides from Sun and Moon, Indirect Solid Earth Tides, Indirect Ocean Tides are considered

Plotting. Solution U and A

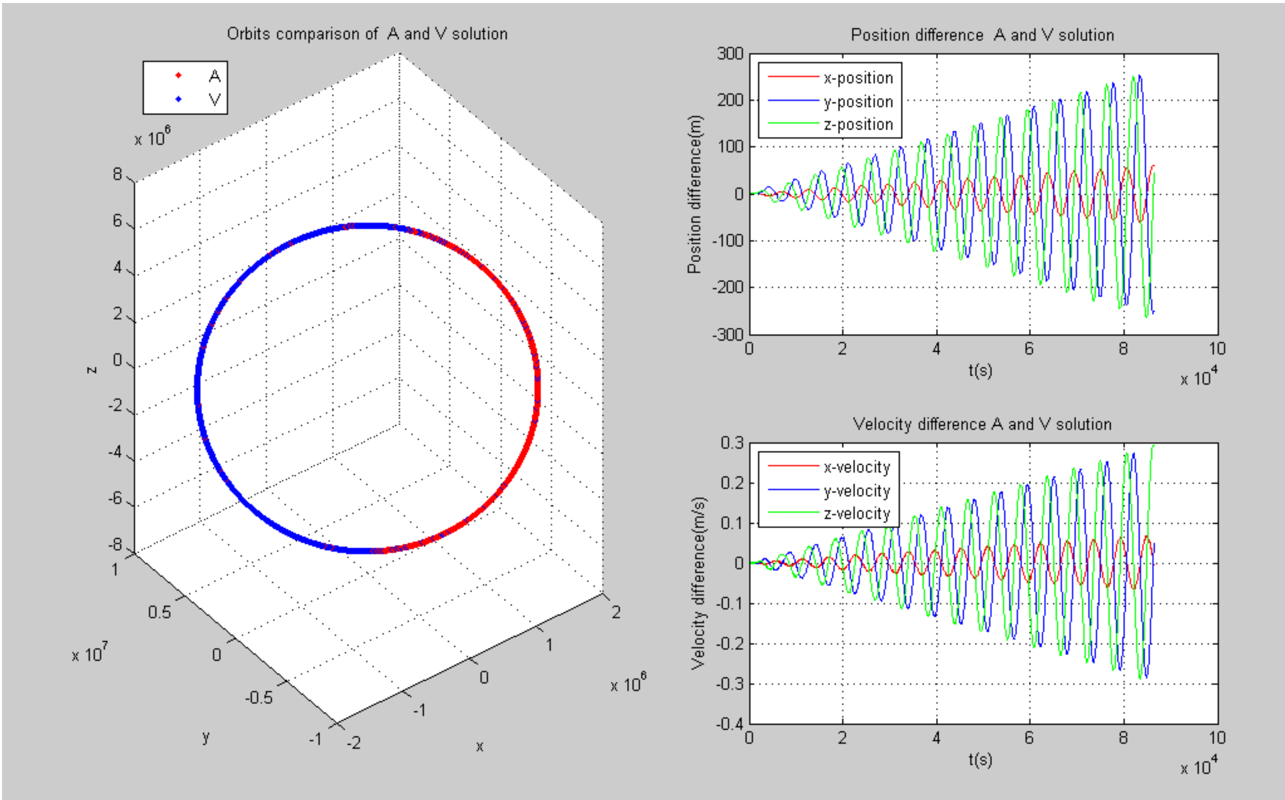


Picture 21: Solutions U and A

A: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0

U: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0, +1m to x,y,z.

Plotting. Solution V and A



Picture 22: Solutions V and A

A: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0

V: GRACE satellite. Gravity model GRIM5-C1, Degree order – 0, +1mm/sec to x,y,z.

	X			Y			Z			X_V			Y_V			Z_V		
	MAX	MIN	RMS	MAX	MIN	RMS	MAX	MIN	RMS	MAX	MIN	RMS	MAX	MIN	RMS	MAX	MIN	RMS
A-B	102578	-102472	42758	445369,9	-445329	185845	460068,7	-439324	189508,2	109,148	-114,582	47,108821	498,452	-476,544	205,1904	503,384	-512,11	211,909
C-B	818,345	-788,373	365,78	3307,45	-3589,23	1636,94	3388,899	-3531,8	1663,389	0,816	-0,832	0,4043168	3,672	-3,867	1,805818	4,033	-3,788	1,858446
A-D	384,604	-411,005	130,87	1774,742	-1659,15	567,07	1771,477	-1637,21	576,346	0,411	-0,445	0,1446361	1,92	-1,772	0,624151	1,893	-2,023	0,646107
A-E	2,522	-1,61	0,8811	4,599	-8,235	2,71874	11,755	-0,956	3,678582	0,002	-0,003	0,0010651	0,01	-0,006	0,003104	0,01	-0,009	0,003259
A-F	28,697	-28,535	11,872	76,268	-74,76	32,5324	77,39	-81,516	34,47312	0,032	-0,031	0,0130632	0,081	-0,085	0,035958	0,09	-0,089	0,038466
G-B	1,44	-1,541	0,5526	7,575	-7,512	3,43151	7,092	-7,198	3,300392	0,002	-0,002	0,0007274	0,009	-0,009	0,003807	0,008	-0,008	0,003716
H-B	1,136	-1,121	0,4891	4,909	-5,001	2,02034	4,895	-5,097	2,056289	0,002	-0,002	0,0006653	0,006	-0,006	0,002267	0,006	-0,006	0,002333
I-C	1284,47	-1138,16	509,41	5174,957	-5573,4	2253,83	5217,042	-5525,4	2289,683	1,324	-1,233	0,5627559	5,666	-6,025	2,484078	6,299	-5,919	2,563406
I-J	15,171	-8,844	5,1101	29,87	-58,21	18,5967	47,088	-44,149	17,98401	0,013	-0,015	0,0054768	0,051	-0,051	0,01949	0,059	-0,043	0,02039
A-U	46,784	-47,599	19,602	205,22	-201,253	84,5651	211,097	-198,996	86,32375	0,051	-0,053	0,0217491	0,228	-0,216	0,093416	0,229	-0,235	0,096356
A-V	60,15	-59,908	25,133	253,679	-255,012	105,786	250,544	-263,228	108,1076	0,067	-0,065	0,0278247	0,272	-0,285	0,116845	0,293	-0,288	0,120704
L-K	8905,25	-15868,8	6968,5	14953,6	-26862,2	11836,2	25834,66	-34729,2	14500,35	2,458	-1,905	1,0843853	4,132	-3,133	1,656339	2,989	-4,583	2,061265
M-L	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
N-K	0,001	-0,001	0,0004	0,001	-0,001	0,00043	0,001	-0,001	0,000407	0	0	0	0	0	0	0,001	0	1,08E-05
O-K	0	-37,288	20,999	5,519	-35,276	17,7026	39,482	0	14,2029	0,003	-0,005	0,0016655	0,004	-0,004	0,001922	0,005	-0,004	0,001911
P-K	67,664	-299,769	140,47	423,186	-1018,99	439,535	818,723	-838,893	390,122	0,037	-0,024	0,0170375	0,134	-0,111	0,05942	0,09	-0,131	0,060729
Q-L	0,205	-0,233	0,107	0,292	-0,488	0,21941	0,365	-0,485	0,199492	0,001	-0,001	0,0001174	0,001	-0,001	0,000157	0,001	-0,001	0,000158
R-L	0,014	-0,011	0,0049	0,003	-0,007	0,00247	0,008	-0,02	0,006563	0,001	-0,001	2,85E-05	0,001	0	1,52E-05	0,001	-0,001	2,64E-05
S-M	30,18	-336,289	156,35	391,061	-1053,14	448,349	840,274	-799,236	390,1771	0,04	-0,027	0,0176749	0,133	-0,114	0,059937	0,092	-0,131	0,060529
T-S	0,044	-0,056	0,0245	0,045	-0,083	0,03589	0,058	-0,073	0,031294	0,001	-0,001	4,69E-05	0,001	-0,001	5,59E-05	0,001	-0,001	5,89E-05

Table 1: Maximum, minimum and RMS of the differences

	MAX	MIN	RMS	MAX	MIN	RMS	MAX	MIN	RMS
N-D	4,46E-08	-3,17E-08	2,68E-08	1,35E-07	-1,98E-07	1,15E-07	1,69E-07	-1,56E-07	1,15E-07
O-E	3,61E-08	-4,18E-08	2,12E-08	6,13E-08	-3,67E-08	2,62E-08	5,73E-08	-1,32E-08	2,13E-08

Table 2: Maximum, minimum and RMS of the applied surface forces

Conclusion

For GPS satellites solutions M and L we can see no differences neither in position nor in velocities, so for GPS satellite degree order causes no effect.

Whereas for GRACE satellite the degree order plays significant role, the differences in position and velocities are the highest among all solutions (forces are not included in calculation of orbits).

As we can see, changes in initial conditions cause considerable effect on the satellites orbits. This can be derived from comparison of solutions A with U and V.

Solutions (M-S) with all forces included show that those forces have a significant effect on positions and velocities.

As we can see from comparison of solution P and K, direct tides from Sun and Moon alone give strong effect on the GPS satellites, significantly changing their trajectories.

As we can see from N-K comparison, atmospheric drag doesn’t have any effect on the velocities at all and barely changes GPS satellites trajectory. While solar pressure influences both the position and velocity, and this effect is considerable compared to other solutions comparisons.

For GRACE satellite, atmospheric drag significantly influences on the position, while the changes in velocities are relatively small.

Solar pressure affects the GRACE satellite as well, but not as much as atmospheric drag does - about 50 times less changes in position can be observed and around 100 times less changes in velocities.

MATLAB CODE

```
clc
clear all
close all


%% Input solution letter
prompt = 'Enter solution letter 1  ';
sol_letter_1 = input(prompt,'s');
data1 = load(['Solution' sol_letter_1 '.txt']);

prompt = 'Enter solution letter 2  ';
sol_letter_2  = input(prompt,'s');
data2 = load(['Solution' sol_letter_2 '.txt']);

t=data1(:,1);
%% Two cases with air drag and solar pressure comparison
if (sol_letter_1 == 'N' &&  sol_letter_2 == 'D') || (sol_letter_1 == 'O' &&  sol_letter_2 == 'E')
[p_sol_1, v_sol_1, force1] = get_data_forces(data1);
[p_sol_2, v_sol_2, force2] = get_data_forces(data2);
dif_forces_matrix = force2 - force1;
figure(3)
plot(t,dif_forces_matrix(:,1),'r', t,dif_forces_matrix(:,2),'b', t,dif_forces_matrix(:,3),'g');
figure(3)
title(['Surface forces comparison  ', num2str(sol_letter_1),' and ',num2str(sol_letter_2),' solution']);
legend('x','y','z','Location','northwest');
xlabel('t(s)');
ylabel('Forces difference(m/s2)');
grid on

maximum_xf = max(dif_forces_matrix(:,1));
maximum_yf = max(dif_forces_matrix(:,2));
maximum_zf = max(dif_forces_matrix(:,3));
minimum_xf = min(dif_forces_matrix(:,1));
minimum_yf = min(dif_forces_matrix(:,2));
minimum_zf = min(dif_forces_matrix(:,3));
rms_xf = rms(dif_forces_matrix(:,1));
rms_yf = rms(dif_forces_matrix(:,2));
rms_zf = rms(dif_forces_matrix(:,3));

super_table_forces = [maximum_xf, minimum_xf, rms_xf, maximum_yf, minimum_yf, rms_yf, maximum_zf, minimum_zf, rms_zf];

end

%% Getting required matrices
[p_sol_1, v_sol_1] = get_data(data1);
[p_sol_2, v_sol_2] = get_data(data2);
```

```
difference_matrix_position = p_sol_2 - p_sol_1;
difference_matrix_velocity = v_sol_2 - v_sol_1;

%% Plot the orbits
figure(2)
subplot(2,2,[1 3])
plot3(p_sol_1(:,1),p_sol_1(:,2),p_sol_1(:,3),'.r');
hold on
plot3(p_sol_2(:,1),p_sol_2(:,2),p_sol_2(:,3),'.b');

%% Plot the differences
figure(2)
subplot(2,2,2)
plot(t,difference_matrix_position(:,1),'r', t,difference_matrix_position(:,2),'b',
t,difference_matrix_position(:,3),'g');
hold on
subplot(2,2,4)
plot(t,difference_matrix_velocity(:,1),'r', t,difference_matrix_velocity(:,2),'b',
t,difference_matrix_velocity(:,3),'g');
%% Plots information
figure(2)
subplot(2,2,[1 3])
title(['Orbits comparison of  ', num2str(sol_letter_1),' and ',num2str(sol_letter_2),'
solution']);
legend(num2str(sol_letter_1), num2str(sol_letter_2), 'Location','northwest');
xlabel('x');
ylabel('y');
zlabel('z');
grid on
subplot(2,2,2)
title(['Position difference  ', num2str(sol_letter_1),' and ',num2str(sol_letter_2),' solution']);
legend('x-position','y-position','z-position','Location','northwest');
xlabel('t(s)');
ylabel('Position difference(m)');
grid on
subplot(2,2,4)
title(['Velocity difference ', num2str(sol_letter_1),' and ',num2str(sol_letter_2),' solution']);
legend('x-velocity','y-velocity','z-velocity','Location','northwest');
xlabel('t(s)');
ylabel('Velocity difference(m/s)');
grid on

%% Table of values

maximum_x = max(difference_matrix_position(:,1));
maximum_y = max(difference_matrix_position(:,2));
maximum_z = max(difference_matrix_position(:,3));
maximum_xv = max(difference_matrix_velocity(:,1));
maximum_yv = max(difference_matrix_velocity(:,2));
maximum_zv = max(difference_matrix_velocity(:,3));
```

```
minimum_x = min(difference_matrix_position(:,1));
minimum_y = min(difference_matrix_position(:,2));
minimum_z = min(difference_matrix_position(:,3));
minimum_xv = min(difference_matrix_velocity(:,1));
minimum_yv = min(difference_matrix_velocity(:,2));
minimum_zv = min(difference_matrix_velocity(:,3));

rms_x = rms(difference_matrix_position(:,1));
rms_y = rms(difference_matrix_position(:,2));
rms_z = rms(difference_matrix_position(:,3));
rms_xv = rms(difference_matrix_velocity(:,1));
rms_yv = rms(difference_matrix_velocity(:,2));
rms_zv = rms(difference_matrix_velocity(:,3));

super_table = [maximum_x, minimum_x, rms_x, maximum_y, minimum_y, rms_y,
maximum_z, minimum_z, rms_z, maximum_xv, minimum_xv, rms_xv, maximum_yv,
minimum_yv, rms_yv, maximum_zv, minimum_zv, rms_zv]
```