USAD Mathematics Reference

Created for the LT ACADEC Team

2025-2026

1 Introduction

This Reference will cover a variety of topics and mathematical rules that are often need of additional practice to better succeed at USAD Competitions. The first part of this Reference will elaborate on various basic mathematics concepts, ideas, complete with supplementary visuals.

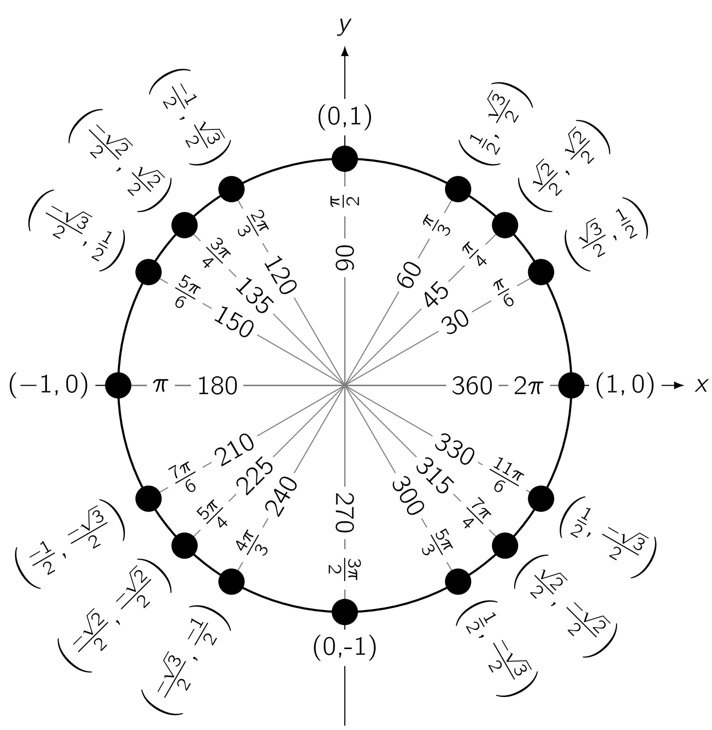
Note that this reference sheet makes use of basic mathematics symbols and set notation. Here is quick overview of what each used symbol means.

$\Leftrightarrow$	equivalence
$a \in \mathbb{X}$	a is an element of the set $\mathbb{X}$, where $\mathbb{X}$ is a list of numbers (a set) that a can be found within
θ	an unknown angle, used for the same purpose as x but in the context of angles
$\therefore$	therefore
:	such that

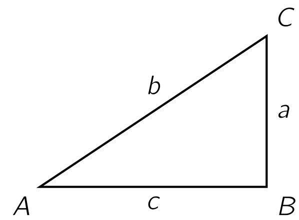
2 Useful Mathematics Concepts, Rules, and Identities

2.1 Radians, Degrees and the Unit Circle

1. Two mathematical systems are used to describe a given angle : degrees and radians.
   (a) The degree system divides one full rotation into 360 degrees, denoted as $x°$.
   (b) The radian system divides one full rotation into 2π.
   (c) Both systems employ a 'standard angle reference' on the unit circle, which is the positive X axis. This means that 0 degrees is along the positive x axis.
2. Below is the unit circle. The first value is the angle from the 'standard reference angle' in degrees, radians and then the coordinate location of that angle's intersection with the circumference of the circle.



2.2 Right Triangle Relationships



1. In the right triangle above, the capital letters represent the angle, and the lowercase letters sides. Your knowledge of *"SOH CAH TOA"* is assumed.

   In a right triangle, such as shown above, the following identities hold true :

$$\sin(A) = \cos(B) = \frac{a}{c} \tag{1}$$

$$\cos(A) = \cos(B) = \frac{b}{c} \tag{2}$$

$$\tan(A) = \cot(B) = \frac{a}{b} \tag{3}$$

$$\cot(A) = \tan(B) = \frac{b}{a} \tag{4}$$

2.3 Useful Equations

1. **Circle Equation**

   The equation for a circle is as follows, where r is the radius of the circle, a is the horizontal offset and b is the vertical offset from (0,0):

$$r^2 = (x + a)^2 + (y + b)^2 \tag{5}$$

2. **Heron's Formula**

   Heron's formula allows you to solve for the area of any triangle with 3 given sides, *a, b* and *c*, using the semiperimeter of the triangle, which is defined as half the perimeter.

$$A = \sqrt{s(s-a)(s-b)(s-c)} \tag{6}$$

3. **Distance Formula**

   The distance formula is a simple formula for getting the distance travelled by an object given a speed r over a given time period, t. Keep in mind, r and t must use the same base units, such as kilometers. The formula is as follows :

$$d = rt \tag{7}$$

   One can additionally use other methods of finding distance travelled, such as through the usage of *integration*. This will be briefly explained later on.

2.4 Rational Root Theorem

The Rational Root Theorem stipulates that :

1. If f is a polynomial with integer coefficients, with rational zeroes ($n \in \mathbb{Q}$), then they must be of the form $\frac{p}{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient. Should various values of p multiply, at least one combination of two values must equal the integer/coefficient in question. The same stands for q.

An example : Given the equation $f(x) = 8x^4 + 2x^3 + 5x^2 - 4x - 3$, find its rational roots.

1. First, identify which integers/coefficients p and q are factors of. In this case, q is a factor of 8, and p is a factor of -3.
2. Next, list out all the factors, or possible values of p and q. Possible values of p are ± 1 and ± 3. The possible values of q are ± 1, ± 2, ± 4, and ± 8.
3. Thus, from the factors we can extrapolate possible rational zeroes. Possible zeroes are : ± 1, $\pm\frac{1}{2}$, $\pm\frac{1}{4}$, $\pm\frac{1}{8}$, ± 3, $\pm\frac{3}{2}$, $\pm\frac{3}{4}$, and $\pm\frac{3}{8}$.

2.5 Laws of Trigonometric Functions

1. **Law of Cosines**

   The law of cosines allows one to find the side of any triangle, so long as it is a real triangle. The law of cosines is as follows :

$$a^2 = b^2 + c^2 - 2bc * \cos(A) \tag{8}$$

2. **Law of Sines**

   The law of sines allows one to find the ratios between sides very easly.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \tag{9}$$

2.6 Double Angle Identities

The Double Angle Identity is a useful identity to know when dealing with unknown angles. There is one identity for each of the three basic trigonometric functions, but only the sine and cosine identities are relevant in this context. The DAI for sine and cosine is as follows :

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \tag{10}$$

$$\begin{aligned} \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 1 - 2\sin^2(\theta) \\ &= 2\cos^2(\theta) - 1 \end{aligned} \tag{11}$$

2.7 Dot Products

1. Given two lines of the following equations, find the angle between these two lines using the dot product method.

$$y = \frac{3}{2}x + 2 \tag{12}$$

$$y = \frac{5}{3}x + 4 \tag{13}$$

2. Vectors take the form of ⟨ x,y ⟩, where x is the direction and y is the magnitude. We can convert the slopes of these equations above into vectors which we can then extract the angle from using the dot product of these two vectors.

3. Slopes are represented as $\frac{\Delta y}{\Delta x}$, or the change in (delta of) y over the change in x.

4. We can convert these slopes ($\frac{3}{2}$ and $\frac{5}{3}$, respectively) into vectors simply by putting in the Δx as x in the vector and Δy as y in the vector. Doing this, we get the vectors :

$$\langle 2, 3 \rangle \tag{14}$$

$$\langle 3, 5 \rangle \tag{15}$$

5. With these vectors, we can calculate their dot product. To manually calculate the dot product, you must perform the following :

$$x_1 x_2 + y_1 y_2 = s \tag{16}$$

   Plugging in our vector values, it looks like this :

$$2(3) + 3(5) = s = 21 \tag{17}$$

6. This scalar, denoted as s, is the dot product, but it is not the resulting angle yet. We must perform a few more operations. The resultant angle is equal to :

$$s = \sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}\cos(\theta) \tag{18}$$

   Plugging in our values, we get this :

$$21 = \sqrt{2^2 + 3^2}\sqrt{3^2 + 5^2}\cos(\theta) \tag{19}$$

7. After evaluating for θ, we get

$$\arccos \frac{21}{\sqrt{2^2 + 3^2}\sqrt{3^2 + 5^2}} = \theta \tag{20}$$

$$\theta \approx 2.72^\circ \tag{21}$$

2.8 Properties of Logarithms

Logarithms have 7 basic properties. All of these properties exist within the realm of assumption that $b \in \mathbb{R}$, or that b exists within (is an element of) the set of all real numbers.

1. **One-to-one**

$$\log_b(y) = \log_b(x) \Leftrightarrow x = y : b > 1 \tag{22}$$

2. **Property of One**

$$\log_b(1) = 0 \tag{23}$$

This means that for any base b raised to the 0th power, the result will always equal one.

3. **Multiplication Property**

$$\log_b(xy) = \log_b(x) + \log_b(y) \tag{24}$$

4. **Division Property**

$$\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y) \tag{25}$$

5. **Inverse Property**

$$b^{\log_b(x)} = x \Leftrightarrow \log_b(b^x) = x \therefore \log_b(b) = 1, \ln(e) = 1, \log(10) = 1. \tag{26}$$

This first part is true because this is simply b^a, where a is the original exponent. Logarithms evaluate for the exponent (a) that when base b is raised to a, it will result in the value represented as x.

The second is true because $\log_b(b) = 1$, meaning that they cancel out. That simplifies to $1x = x$.

6. **Change of Base (incl. formula)**

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)} \tag{27}$$

Here, a is our new desired base. One should ideally make it 10.