USAD Mathematics Reference

Created for the Lane Tech ACADEC Team

2025-2026

1 Introduction

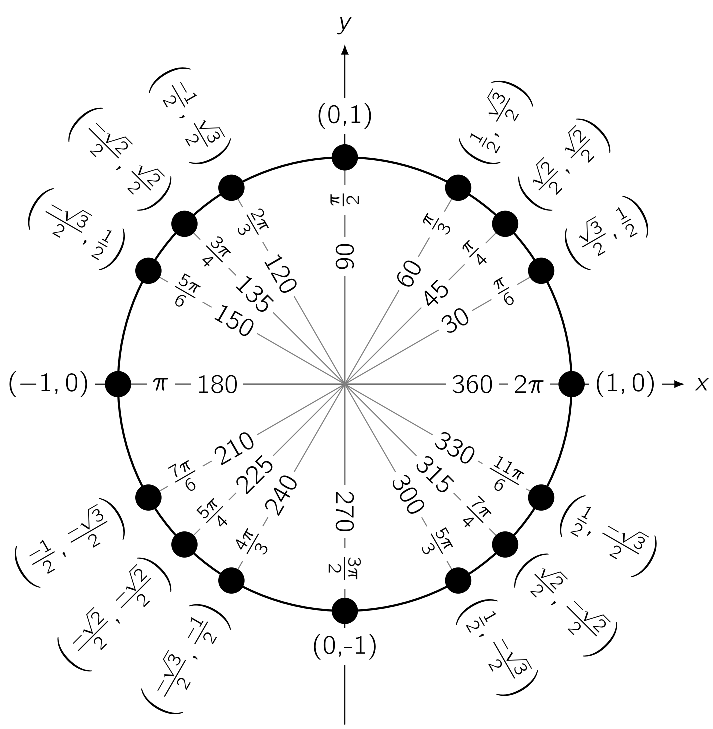
This Reference will cover a variety of topics and mathematical rules that are often need of additional practice to better succeed at USAD Competitions. The first part of this Reference will elaborate on various basic mathematics concepts, ideas, complete with supplementary visuals.

The second part of this Reference will go over, solve, and explain questions commonly found on USAD Mathematics exams, approaches to these questions, and a variety of methods and explanations as to how these solutions work. Figures and supplementary text, along with step-by-step solutions to practice problems, will be provided. Any concepts at greater than an Advanced Algebra level will be explained.

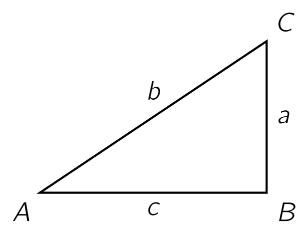
2 Useful Mathematics Concepts, Rules, and Identities

2.1 Radians, Degrees and the Unit Circle

1. Two mathematical systems are used to describe a given angle : degrees and radians.
   (a) The degree system divides one full rotation into 360 degrees.
   (b) The radian system divides one full rotation into 2π.
   (c) Both systems employ a 'standard angle reference' on the unit circle, which is the positive X axis.
2. Below is the unit circle. The first value is the angle from the 'standard reference angle' in degrees, radians and then the coordinate location of that angle's intersection with the circumference of the circle.



2.2 Right Triangle Relationships



1. In the right triangle above, the capital letters represent the angle, and the lowercase letters sides. Your knowledge of *"SOH CAH TOA"* is assumed.

   In a right triangle, such as shown above, the following identities hold true :

$$\sin(A) = \cos(B) = \frac{a}{c} \tag{1}$$

$$\cos(A) = \cos(B) = \frac{b}{c} \tag{2}$$

$$\tan(A) = \cot(B) = \frac{a}{b} \tag{3}$$

$$\cot(A) = \tan(B) = \frac{b}{a} \tag{4}$$

2.3 Useful Equations

1. Circle Equation

   The equation for a circle is as follows, where r is the radius of the circle, a is the horizontal offset and b is the vertical offset from (0,0):

$$r^2 = (x + a)^2 + (y + b)^2 \tag{5}$$

2. Heron's Formula

   Heron's formula allows you to solve for the area of any triangle with 3 given sides, *a, b* and *c*, using the semiperimeter of the triangle, which is defined as half the perimeter.

$$A = \sqrt{s(s - a)(s - b)(s - c)} \tag{6}$$

3. Distance Formula

   The distance formula is a simple formula for getting the distance travelled by an object given a speed r over a given time period, t. Keep in mind, r and t must use the same base units, such as kilometers. The formula is as follows :

$$d = rt \tag{7}$$

   One can additionally use other methods of finding distance travelled, such as through the usage of *integration*. This will be briefly explained later on.

2.4 Rational Root Theorem

2.5 Dot Products

1. Given two lines of the following equations, find the angle between these two lines using the dot product method.

$$y = \frac{3}{2}x + 2 \tag{8}$$

$$y = \frac{5}{3}x + 4 \tag{9}$$

2. Vectors take the form of $\langle$ x,y $\rangle$, where x is the direction and y is the magnitude. We can convert the slopes of these equations above into vectors which we can then extract the angle from using the dot product of these two vectors.

3. Slopes are represented as $\frac{\Delta y}{\Delta x}$, or the change in (delta of) x over the change in y.

4. We can convert these slopes ($\frac{3}{2}$ and $\frac{5}{3}$, respectively) into vectors simply by putting in the Δx as x in the vector and Δy as y in the vector. Doing this, we get the vectors :

$$\langle 2, 3 \rangle \tag{10}$$

$$\langle 3, 5 \rangle \tag{11}$$

5. With these vectors, we can calculate their dot product. To manually calculate the dot product, you must perform the following :

$$x_1 x_2 + y_1 y_2 = s \tag{12}$$

   Plugging in our vector values, it looks like this :

$$2(3) + 3(5) = s = 21 \tag{13}$$

6. This scalar, denoted as s, is the dot product, but it is not the resulting angle yet. We must perform a few more operations. The resultant angle is equal to :

$$s = \sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}\cos\theta \tag{14}$$

   Plugging in our values, we get this :

$$21 = \sqrt{2^2 + 3^2}\sqrt{3^2 + 5^2}\cos\theta \tag{15}$$

7. After evaluating for θ, we get

$$\arccos\frac{21}{\sqrt{2^2 + 3^2}\sqrt{3^2 + 5^2}} = \theta \tag{16}$$

$$\theta \approx 2.72^\circ \tag{17}$$

2.6 Laws of Trigonometric Functions

1. **Law of Cosines**

   The law of cosines allows one to find the side of any triangle, so long as it is a real triangle. The law of cosines is as follows :

$$a^2 = b^2 + c^2 - 2bc * \cos(B) \tag{18}$$

2. **Law of Sines**

   The law of sines allows one to find the ratios between sides very easly.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} * \frac{c}{\sin(C)} \tag{19}$$

2.7 Properties of Logarithms

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