USAD Mathematics Reference

Created for the Lane Tech ACADEC Team

2025-2026

1 Introduction

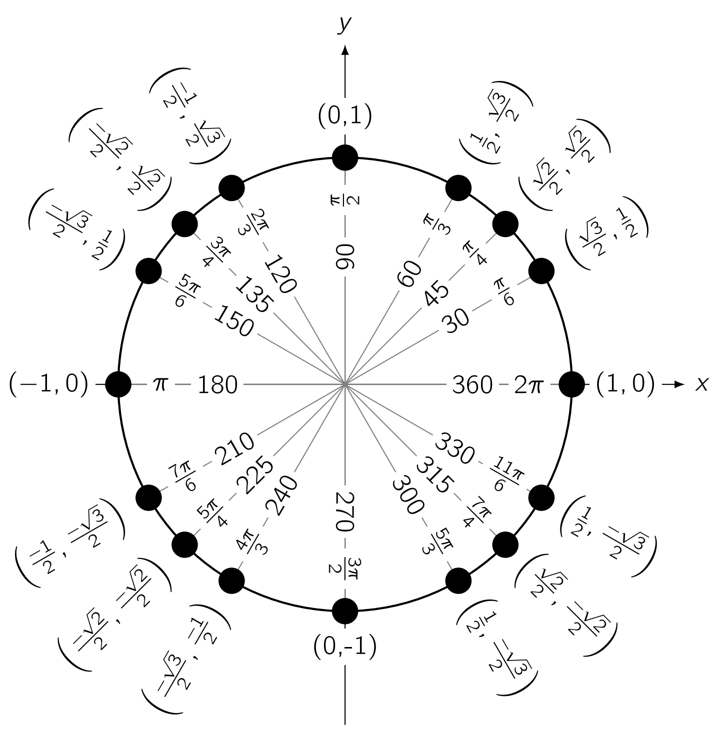
This Reference will cover a variety of topics and mathematical rules that are often need of additional practice to better succeed at USAD Competitions. The first part of this Reference will elaborate on various basic mathematics concepts, ideas, complete with supplementary visuals.

The second part of this Reference will go over, solve, and explain questions commonly found on USAD Mathematics exams, approaches to these questions, and a variety of methods and explanations as to how these solutions work. Figures and supplementary text, along with step-by-step solutions to practice problems, will be provided. Any concepts at greater than an Advanced Algebra level will be explained.

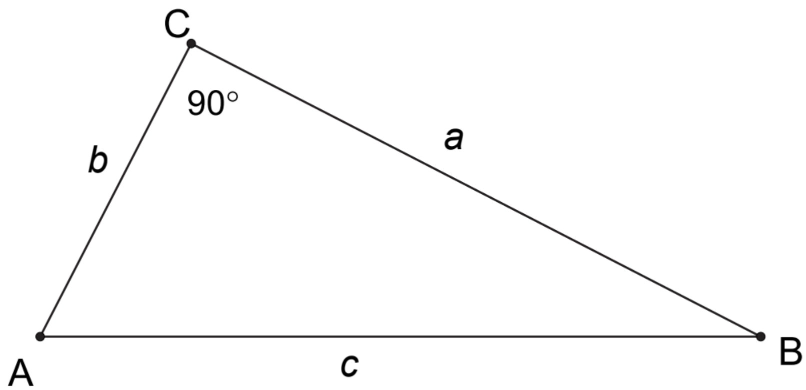
2 Useful Mathematics Concepts, Rules, and Identities

2.1 Radians, Degrees and the Unit Circle

1. Two mathematical systems are used to describe a given angle : degrees and radians.
   (a) The degree system divides one full rotation into 360 degrees.
   (b) The radian system divides one full rotation into 2π.
   (c) Both systems employ a 'standard angle reference' on the unit circle, which is the positive X axis.
2. Below is the unit circle. The first value is the angle from the 'standard reference angle' in degrees, radians and then the coordinate location of that angle's intersection with the circumference of the circle.



2.2 Right Triangle Relationships



1. In the right triangle above, the capital letters represent the angle which they are adjacent, and the lowercase letters sides. Your knowledge of *"SOH CAH TOA"* is assumed.

   In a right triangle, such as shown above, the following identities hold true :

$$\sin(A) = \cos(B) = \frac{a}{c} \tag{1}$$

$$\cos(A) = \cos(B) = \frac{b}{c} \tag{2}$$

$$\tan(A) = \cot(B) = \frac{a}{b} \tag{3}$$

$$\cot(A) = \tan(B) = \frac{b}{a} \tag{4}$$

2.3 Useful Equations

1. Circle Equation

   The equation for a circle is as follows, where r is the radius of the circle, a is the horizontal offset and b is the vertical offset from (0,0):

$$r^2 = (x + a)^2 + (y + b)^2 \tag{5}$$

2.4 Rational Root Theorem

2.5 Dot Products

1. Given two lines of the following equations, find the angle between these two lines using the dot product method.

$$y = \frac{3}{2}x + 2 \tag{6}$$

$$y = \frac{5}{3}x + 4 \tag{7}$$

2. Vectors take the form of $\langle$ x,y $\rangle$, where x is the direction and y is the magnitude. We can convert the slopes of these equations above into vectors which we can then extract the angle from using the dot product of these two vectors.

3. Slopes are represented as $\frac{\Delta y}{\Delta x}$, or the change in (delta of) x over the change in y.

4. We can convert these slopes ($\frac{3}{2}$ and $\frac{5}{3}$, respectively) into vectors simply by putting in the Δx as x in the vector and Δy as y in the vector. Doing this, we get the vectors :

$$\langle 2, 3 \rangle \tag{8}$$

$$\langle 3, 5 \rangle \tag{9}$$

5. With these vectors, we can calculate their dot product. To manually calculate the dot product, you must perform the following :

$$x_1 x_2 + y_1 y_2 = s \tag{10}$$

   Plugging in our vector values, it looks like this :

$$2(3) + 3(5) = s = 21 \tag{11}$$

6. This scalar, denoted as s, is the dot product, but it is not the resulting angle yet. We must perform a few more operations. The resultant angle is equal to :

$$s = \sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}\cos\theta \tag{12}$$

   Plugging in our values, we get this :

$$21 = \sqrt{2^2 + 3^2}\sqrt{3^2 + 5^2}\cos\theta \tag{13}$$

7. After evaluating for θ, we get

$$\arccos\frac{21}{\sqrt{2^2 + 3^2}\sqrt{3^2 + 5^2}} = \theta \tag{14}$$

$$\theta \approx 2.72^\circ \tag{15}$$

2.6 Properties of Logarithms