APPENDIX 1

CONVERSION FACTORS AND CONSTANTS

Conversion Factors (arranged alphabetically)

Acceleration ($L\ t^{-2}$)*

1 m/sec^2 = 3.2808 ft/sec^2 = 39.3701 in./sec^2
1 ft/sec^2 = 0.3048 m/sec^2 = 12.0 in./sec^2
g_0 = 9.80665 m/sec^2 = 32.174 ft/sec^2 (standard gravity pull at earth's surface)

Area (L^2)

1 ft^2 = 144.0 in.^{2} = 0.092903 m^2
1 m^2 = 1550.0 in.^{2} = 10.7639 ft^2
1 in.^{2} = 6.4516 $\times$ 10^{-4} m^2

Density ($M\ L^3$)

Specific gravity is dimensionless, but has the same numerical value as *density* expressed in g/cm^3 or kg/m^3
1 kg/m^3 = 6.24279 $\times$ 10^{-2} lbm/ft^3 = 3.61273 $\times$ 10^{-5} lbm/in.^{3}
1 lbm/ft^3 = 16.0184 kg/m^3
1 lbm/in.^{3} = 2.76799 $\times$ 10^4 kg/m^3

*The letters in parentheses after each heading indicate the dimensional parameters (L = length, M = mass, t = time, and T = temperature).

Energy, also Work or Heat ($M\ L^2\ t^{-2}$)

1.0 Btu = 1055.056 J (joule)
1.0 kW-hr = 3.60059×10^6 J
1.0 ft-lbf = 1.355817 J
1.0 cal = 4.1868 J
1.0 kcal = 4186.8 J

Force ($M\ L\ t^{-2}$)

1.0 lbf = 4.448221 N
1 dyne = 10^{-5} N
1.0 kg (force) [used in Europe] = 9.80665 N
1.0 ton (force) [used in Europe] = 1000 kg (force)
1.0 N = 0.2248089 lbf
1.0 millinewton (mN) = 10^{-3} N
Weight is the *force* on a *mass* being accelerated by gravity (g_0 applies at the surface of the earth)

Length (L)

1 m = 3.2808 ft = 39.3701 in.
1 ft = 0.3048 m = 12.0 in.
1 in. = 2.540 cm = 0.0254 m
1 mile = 1.609344 km = 1609.344 m = 5280.0 ft
1 nautical mile = 1852.00 m
1 mil = 0.0000254 m = 1.00×10^{-3} in.
1 micron (μm) = 10^{-6} m
1 astronomical unit (au) = 1.49600×10^{11} m

Mass (M)

1 slug = 32.174 lbm
1 kg = 2.205 lbm = 1000 g
1 lbm = 16 ounces = 0.4536 kg

Power ($M\ L^2\ t^{-3}$)

1 Btu/sec = 0.2924 W (watt)
1 J/sec = 1.0 W = 0.001 kW
1 cal/sec = 4.186 W
1 horsepower = 550 ft-lbf/sec = 745.6998 W
1 ft-lbf/sec = 1.35581 W

Pressure ($M\ L^{-1}\ t^{-2}$)

1 bar = 10^5 N/m^2 = 0.10 MPa
1 atm = 0.101325 MPa = 14.696 psia

1 mm of mercury = 13.3322 N/m^2
1 MPa = 10^6 N/m^2
1 psi or lbf/in.^{2} = 6894.757 N/m^2

Speed (or linear velocity) ($L\ t^{-1}$)

1 ft/sec = 0.3048 m/sec = 12.00 in./sec
1 m/sec = 3.2808 ft/sec = 39.3701 in./sec
1 knot = 0.5144 m/sec
1 mile/hr = 0.4770 m/sec

Specific Heat ($L^2\ t^{-2}\ T^{-1}$)

1 g-cal/g-°C = 1 kg-cal/kg-K = 1 Btu/lbm-°F = 4.186 J/g-°C = 1.163×10^{-3} kW-hr/kg-K

Temperature (T)

1 K = 9/5 R = 1.80 R
0°C = 273.15 K
0°F = 459.67 R
$C = (5/9)(F - 32) \qquad F = (9/5)C + 32$

Time (t)

1 mean solar day = 24 hr = 1440 min = 86,400 sec
1 calendar year = 365 days = 3.1536×10^7 sec

Viscosity ($M\ L^{-1}\ t^{-1}$)

1 centistoke = 1.00×10^{-6} m^2/sec
1 centipoise = 1.00×10^{-3} kg/m sec
1 lbf-sec/ft^2 = 47.88025 kg/m sec

Constants

J	Mechanical equivalent of heat = 4.186 joule/cal = 777.9 ft-lbf/Btu = 1055 joule/Btu
R'	Universal gas constant = 8314.3 J/kg-mole-K = 1545 ft-lbf/lbm-mole-R
V_{mole}	Molecular volume of an ideal gas = 22.41 liter/kg-mole at standard conditions
e	Electron charge = $1.6021176 \times 10^{-19}$ coulomb
ε_0	Permittivity of vacuum = 8.854187×10^{-12} farad/m
	Gravitational constant = 6.673×10^{-11} m^3/kg-sec
	Boltzmann's constant $1.38065003 \times 10^{-23}$ J/°K
	Electron mass 9.109381×10^{-31} kg
	Avogadro's number 6.022142×10^{26}/kg-mol
σ	Stefan–Boltzman constant 5.6696×10^{-8} $W/m^2\text{-}K^{-4}$

APPENDIX 2

PROPERTIES OF THE EARTH'S STANDARD ATMOSPHERE

Sea level pressure is 0.101325 MPa (or 14.696 psia or 1.000 atm).

Altitude (m)	Temperature (K)	Pressure Ratio	Density (kg/m^3)
0 (sea level)	288.150	1.0000	1.2250
1,000	281.651	8.8700×10^{-1}	1.1117
3,000	268.650	6.6919×10^{-1}	9.0912×10^{-1}
5,000	255.650	5.3313×10^{-1}	7.6312×10^{-1}
10,000	223.252	2.6151×10^{-1}	4.1351×10^{-1}
25,000	221.552	2.5158×10^{-2}	4.0084×10^{-2}
50,000	270.650	7.8735×10^{-4}	1.0269×10^{-3}
75,000	206.650	2.0408×10^{-5}	3.4861×10^{-5}
100,000	195.08	3.1593×10^{-7}	5.604×10^{-7}
130,000	469.27	1.2341×10^{-8}	8.152×10^{-9}
160,000	696.29	2.9997×10^{-9}	1.233×10^{-9}
200,000	845.56	8.3628×10^{-10}	2.541×10^{-10}
300,000	976.01	8.6557×10^{-11}	1.916×10^{-11}
400,000	995.83	1.4328×10^{-11}	2.803×10^{-12}
600,000	999.85	8.1056×10^{-13}	2.137×10^{-13}
1,000,000	1000.00	7.4155×10^{-14}	3.561×10^{-15}

Source: U.S. Standard Atmosphere, National Oceanic and Atmospheric Administration, National Aeronautics and Space Administration, and U.S. Air Force, Washington, DC, 1976 (NOAA-S/T-1562).

APPENDIX 3

SUMMARY OF KEY EQUATIONS FOR IDEAL CHEMICAL ROCKETS

Parameter	Equations	Equation Numbers
Average exhaust velocity, v_2 (m/sec or ft/sec) (assume that $v_1 = 0$)	$v_2 = c - (p_2 - p_3)A_2/\dot{m}$	2–16
	When $p_2 = p_3$, $v_2 = c$	
	$v_2 = \sqrt{[2k/(k-1)]RT_1[1 - (p_2/p_1)^{(k-1)/k}]}$	3–16
	$= \sqrt{2(h_1 - h_2)}$	3–15
Effective exhaust velocity, c (m/sec or ft/sec)	$c = c^* C_F = F/\dot{m} = I_s g_0$	3–32
	$c = v_2 + (p_2 - p_3)A_2/\dot{m}$	2–16
Thrust, F (N or lbf)	$F = c\dot{m} = cm_p/t_p$	2–17
	$F = C_F p_1 A_t$	3–31
	$F = \dot{m}v_2 + (p_2 - p_3)A_2$	2–14
	$F = \dot{m}I_s g_0 = I_s \dot{w}$	
Characteristic velocity, c^* (m/sec or ft/sec)	$c^* = c/C_F = p_1 A_t/\dot{m}$	3–32
	$c^* = \dfrac{\sqrt{kRT_1}}{k\sqrt{[2/(k+1)]^{(k+1)/(k-1)}}}$	3–32
	$c^* = I_s g_0/C_F = F/(\dot{m}C_F)$	3–32, 3–33
Thrust coefficient, C_F (dimensionless)	$C_F = c/c^* = F/(p_1 A_t)$	3–31, 3–32
	$C_F = \sqrt{\dfrac{2k^2}{k-1}\left(\dfrac{2}{k+1}\right)^{(k+1)/(k-1)}\left[1 - \left(\dfrac{p_2}{p_1}\right)^{(k-1)/k}\right]} + \dfrac{p_2 - p_3}{p_1}\dfrac{A_2}{A_t}$	3–30
Total impulse	$I_t = \int F\, dt = Ft = I_s w$	2–1, 2–2, 2–5
Specific impulse, I_s (sec)	$I_s = c/g_0 = c^* C_F/g_0$	
	$I_s = F/\dot{m}g_0 = F/\dot{w}$	2–5
	$I_s = v_2/g_0 + (p_2 - p_3)A_2/(\dot{m}g_0)$	2–16
	$I_s = I_t/(m_p g_0) = I_t/w$	2–4, 2–5

Parameter	Equations	Equation Numbers
Propellant mass fraction, (dimensionless)	$\zeta = m_p/m_0 = \frac{m_0 - m_f}{m_0}$	2–8, 2-9
	$\zeta = 1 - \mathbf{MR}$	4–4
Mass ratio of vehicle or stage, **MR** (dimensionless)	$\mathbf{MR} = \frac{m_f}{m_0} = \frac{m_0 - m_p}{m_0}$ $= m_f/(m_f + m_p)$	2–7
	$m_0 = m_f + m_p$	2–10
Vehicle velocity increase in gravity-free vacuum, Δv (m/sec or ft/sec) (assume that $v_o = 0$)	$\Delta u = -c \ln \mathbf{MR} = c \ln \frac{m_0}{m_f}$ $= c \ln m_0/(m_0 - m_p)$ $= c \ln(m_p + m_f)/m_f$	4–6 4–5, 4-6
Propellant mass flow rate, $\dot{m}$ (kg/sec or lb/sec)	$\dot{m} = Av/V = A_1 v_1/V_1$ $= A_t v_t/V_t = A_2 v_2/V_2$	3–24
	$\dot{m} = F/c = p_1 A_t/c^*$	2–17, 3–31
	$\dot{m} = p_1 A_t k \frac{\sqrt{[2/(k+1)]^{(k+1)/(k-1)}}}{\sqrt{kRT_1}}$	3–24
	$\dot{m} = m_p/t_p$	
Mach number, M (dimensionless)	$M = v/a$ $= v/\sqrt{kRT}$ At throat, $v = a$ and $M = 1.0$	3–11
Nozzle area rate, ϵ	$\epsilon = A_2/A_t$	3–19
	$\epsilon = \frac{1}{M_2}\sqrt{\left[\frac{1 + \frac{k-1}{2} M_2^2}{1 + \frac{k-1}{2}}\right]^{(k+1)/(k-1)}}$	3–14
Isentropic flow relationships for stagnation and free-stream conditions	$T_0/T = (p_0/p)^{(k-1)/k} = (V/V_0)^{(k-1)}$ $T_x/T_y = (p_x/p_y)^{(k-1)/k} = (V_y/V_x)^{k-1}$	3–7
Satellite velocity, u_s, in circular orbit (m/sec or ft/sec)	$v_s = R_0\sqrt{g_0/(R_0 + h)}$	4–26
Escape velocity, v_e (m/sec or ft/sec)	$v_e = R_0\sqrt{2g_0/(R_0 + h)}$	4–25
Liquid propellant engine mixture ratio r and propellant flow $\dot{m}$	$r = \dot{m}_0/\dot{m}_f$ $\dot{m} = \dot{m}_0 + m_f$ $m_f = \dot{m}/(r + 1)$ $m_0 = r\dot{m}/(r + 1)$	6–1 6–2 6–4 6–3
Average density ρ_{av} for (or average specific gravity)	$\rho_{av} = \frac{\rho_0 \rho_f (r + 1)}{r\rho_f + \rho_0}$	7–2
Characteristic chamber length L*	$L^* = V_c/A_t$	8–9
Solid propellant mass flow rate $\dot{m}$	$\dot{m} = A_b r \rho_0$	11—1
Solid propellant burning rate r	$r = ap_1^n$	11-3
Ratio of burning area A_b to throat area A_t	$K = A_b/A_t$	11–14
Temperature sensitivity of burning rate at constant pressure	$\sigma_p = \frac{1}{r}\left(\frac{\delta r}{\delta T}\right)_p$	11–4
Temperature sensitivity of pressure at constant K	$\pi_K = \frac{1}{p_1}\left(\frac{\delta p}{\delta T}\right)_K$	11-5

APPENDIX 4

DERIVATION OF HYBRID FUEL REGRESSION RATE EQUATION IN CHAPTER 15

Terry A. Boardman

Listed below is an approach for analyzing hybrid fuel regression, based on a simplified model of heat transfer in a turbulent boundary layer. This approach, first developed by Marxman and Gilbert (see Ref. 15–9), assumes that the combustion port boundary layer is divided into two regions separated by a thin flame zone. Above the flame zone the flow is oxidizer rich, while below the flame zone the flow is fuel rich (see Fig. 15–7). An expression is developed to relate fuel regression rate to heat transfer from the flame to the fuel surface. For the definition of the symbols in this appendix, please see the list of symbols in Chapter 15.

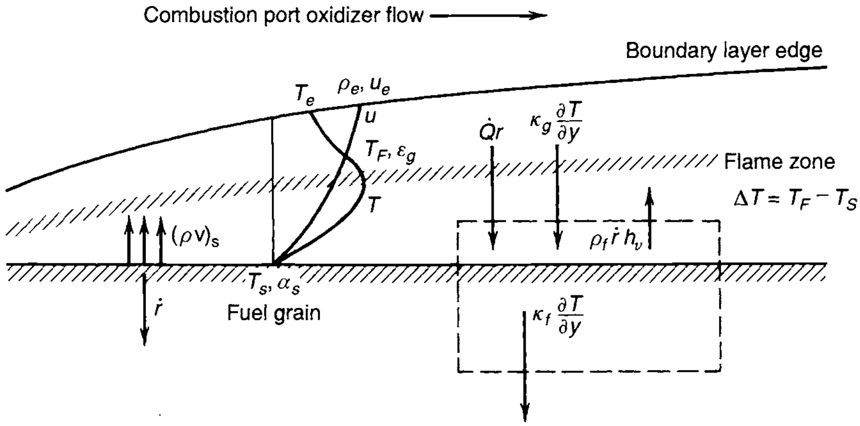
Figure A4–1 illustrates a simplified picture of the energy balance at the fuel grain surface. Neglecting radiation and in-depth conduction in the fuel mass, the steady-sate surface energy balance becomes

$$\dot{Q}_c = \rho_f \dot{r} h_v \tag{A4–1}$$

where $\dot{Q}_c$ is the energy transferred to the fuel surface by convection, and ρ_f, $\dot{r}$, and h_v are respectively the solid fuel density, surface regression rate, and overall fuel heat of vaporization or decomposition. At the fuel surface the heat transferred by convection equals that transferred by conduction, so that

$$\dot{Q}_s = h\Delta T = \kappa_g \left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{A4–2}$$

where h is the convective heat transfer film coefficient, ΔT is the temperature difference between the flame zone and the fuel surface, κ_g is the gas phase conductivity, and $\partial T/\partial y|_{y=0}$ is the local boundary layer temperature gradient evaluated at the fuel surface. The central problem in determining the hybrid fuel regression rate is thereby reduced to determining the basic aerothermal



General steady-state energy balance:

Energy input fuel surface = Energy out of fuel surface

$$Q_{convection} + Q_{radiation\ in} = Q_{conduction\ out} + Q_{phase\ change} + Q_{radiation\ out}$$

$$h\Delta T \text{ or } \kappa_g \frac{\partial T}{\partial y}\bigg|_{y=0} + \alpha\varepsilon_g \sigma T_F^4 = \kappa_f \frac{\partial T}{\partial y} + \rho_f \dot{r} h_v + \varepsilon_s \sigma T_s^4$$

Neglecting radiation and solid phase heat conduction

$$\kappa_g \frac{\partial T}{\partial y}\bigg|_{y=0} = \rho_f \dot{r} h_v$$

FIGURE A4–1. Energy Balance at Fuel Grain Surface.

properties of the boundary layer. Approximate solutions to the flat plate boundary layer problem are well established (Ref. A4–1) and show that the heat transfer coefficient at the wall (in this case, the fuel surface) is related to the skin friction coefficient via the following relationship (called Reynolds' analogy)

$$C_h = \frac{C_f}{2} \mathrm{Pr}^{-2/3} \tag{A4–3}$$

where C_f is the skin friction coefficient with blowing (defined in this case as the evolution of vaporized fuel from the fuel surface and proportional to ρv evaluated at the fuel surface), C_h is the Stanton number, and Pr is the Prandtl number (Stanton, Prandtl, and Reynolds number definitions are summarized in Table A4–1). Furthermore, the Stanton number can be written in terms of the heat flux to the fuel surface as

$$C_h = \frac{\dot{Q}_s}{\Delta h \rho_e u_e} \tag{A4–4}$$

TABLE A4–1. Dimensionless Numbers Used in Hybrid Boundary Layer Analysis

Parameter	Definition	Comment
Stanton number, C_h	$\frac{h}{\rho_e u_e c_p}$	Dimensionless heat transfer coefficient
Prandtl number, Pr	$\frac{c_p \mu_e g_0}{\kappa_g}$	Ratio of momentum transport via molecular diffusion to energy transport by diffusion
Reynolds number, Re_x	$\frac{\rho_e u_e x}{g_0 \mu_e}$	Ratio of gas inertial forces to viscous forces (x is distance from leading edge of fuel grain)

where Δh is the enthalpy difference between the flame zone and the fuel surface, and ρ_e, u_e are the density and velocity of oxidizer at the edge of the boundary layer. Combining Equations A4–1, A4–3, and A4–4, the regression rate of the fuel surface can be written as

$$\dot{r} = \frac{C_f}{2} \frac{\Delta h}{h_v} \frac{\rho_e u_e}{\rho_f} \mathrm{Pr}^{-2/3} \tag{A4–5}$$

From boundary layer theory, one can show that the skin friction coefficient without blowing (C_{f_0}) is related to the local Reynolds number by the relation

$$\frac{C_{f_0}}{2} = 0.0296 \mathrm{Re}_x^{-0.2} \qquad (5 \times 10^5 \le \mathrm{Re}_x \le 1 \times 10^7) \tag{A4–6}$$

Experiments (Ref. A4–2) conducted to determine the effect of blowing on skin friction coefficients have shown that C_f is related to C_{f_0} by the following

$$\frac{C_f}{C_{f_0}} = 1.27 \beta^{-0.77} \qquad (5 \le \beta \le 100) \tag{A4–7}$$

where the blowing coefficient β is defined as

$$\beta = \frac{(\rho v)_s}{\rho_e u_e C_f/2} \tag{A4–8}$$

In a turbulent boundary layer, the Prandtl number is very nearly equal to 1. It can be shown that for $\mathrm{Pr} = 1$, β, as defined in Eq. A4–8, is also equal to $\Delta h/h_v$ (see Appendix 5). Noting that $\rho_e u_e$ is the definition of oxidizer mass velocity (G), Eq. A4–5 can be written in the final form as

$$\dot{r} = 0.036 \frac{G^{0.8}}{\rho_f} \left(\frac{\mu}{x}\right)^{0.2} \beta^{0.23} \tag{A4–9}$$

The coefficient 0.036 applies when the quantities are expressed in the English Engineering system of units as given in the list of symbols at the end of Chapter 15. In some hybrid motors, radiation may be a significant contributor to the total fuel surface heat flux. Such motors include those with metal additives to the fuel grain (such as aluminum) or motors in which soot may be present in significant concentrations in the combustion chamber. In these instances, Eq. A4–1 must be modified to account for heat flux from a radiating particle cloud. The radiative contribution affects surface blowing, and hence the convective heat flux as well, so that one cannot simply add the radiative term to Eq. A4–1. Instead, one can show (Ref. A4–3) that the total heat flux to the fuel surface (and hence the fuel regression rate) is expressed by

$$\dot{Q}_s = \rho_f \dot{r} h_v = \dot{Q}_c e^{-Q_{\text{rad}}/Q_c} + Q_{\text{rad}} \tag{A4–10}$$

which reduces to Eq. A4–1 if $\dot{Q}_{\text{rad}} = 0$. The radiation heat flux has been hypothesized to have the following form

$$\dot{Q}_{\text{rad}} = \sigma \alpha T_F^4 (1 - e^{-ACz}) \tag{A4–11}$$

where the term $1 - e^{-ACz}$ is ε_g, the emissivity of particle-laden gas. Here, σ is the Stefan–Boltzmann constant, α is the fuel surface absorptivity, A is the particle cloud attenuation coefficient, C is the particle cloud concentration (number density), and z is the radiation path length. By assuming that the particle cloud concentration is proportional to chamber pressure and the optical path length is proportional to port diameter, experimenters (see Ref. 15–14) have approximated the functional dependencies of Eq. A4–11 for correlating metallized fuel grain regression rates with expressions of the following form

$$\dot{r} = \dot{r}\left\{G_0, L, \left(1 - e^{-P/P_{\text{ref}}}\right), \left(1 - e^{-D/D_{\text{ref}}}\right)\right\} \tag{A4–12}$$

REFERENCES

A4–1. H. Schlichting, "Boundary Layer Theory," Pergamon Press, Oxford, 1955.

A4–2. L. Lees, "Convective Heat Transfer with Mass Addition and Chemical Reactions," *Combustion and Propulsion, Third AGARD Colloquium*, New York, Pergamon Press, 1958, p. 451.

A4–3. G. A. Marxman, E. E. Woldridge, and R. J. Muzzy, "Fundamentals of Hybrid Boundary Layer Combustion," *AIAA Paper 63-505*, December 1963.

APPENDIX 5

ALTERNATIVE INTERPRETATIONS OF BOUNDARY LAYER BLOWING COEFFICIENT IN CHAPTER 15

Terry A. Boardman

The blowing coefficient β is an important parameter affecting boundary layer heat transfer. It is interesting to note that, although it is defined as the non-dimensional fuel mass flow rate per unit area normal to the fuel surface, it is also a thermochemical parameter equivalent to the nondimensional enthalpy difference between the fuel surface and the flame zone. In terms of the fuel mass flux, β is defined as

$$\beta = \frac{(\rho v)_s}{\rho_e u_e C_f/2} \tag{A5–1}$$

For the definition of the letter symbols please refer to the list of symbols of Chapter 15. Noting that $C_f/2 = C_h \, \mathrm{Pr}^{-2/3}$, Eq. A5–1 can be rewritten as

$$\beta = \frac{(\rho v)_s}{\rho_e u_e} \frac{\mathrm{Pr}^{-2/3}}{C_h} \tag{A5–2}$$

Recalling that the heat flux at the fuel surface is

$$Q_s = h(T_f - T_s) \tag{A5–3}$$

and that the definition of Stanton number is

$$C_h = \frac{h}{\rho_e u_e c_p} \tag{A5–4}$$

Eq. A5–4 can be rewritten as

$$C_h = \frac{\dot{Q}_s}{\Delta h \rho_e u_e} \tag{A5–5}$$

From energy balance considerations, heat flux to the fuel surface in steady state is equivalent to

$$\dot{Q}_s = \rho_f \dot{r} h_v \tag{A5–6}$$

so that Eq. A5–2 becomes

$$\beta = \frac{(\rho v)_s}{\rho_f r} \frac{\Delta h}{h_v} \mathrm{Pr}^{-2/3} \tag{A5–7}$$

Since $(\rho_v)_s = \rho_f \dot{r}$ at the fuel surface, the fuel regression rate, Eq. A5–7, becomes

$$\beta = \frac{\Delta h}{h_v} \mathrm{Pr}^{-2/3}$$

As has been previously stated, the Prandtl number in a turbulent boundary layer is very nearly equal to 1 so that the final form for the blowing coefficient is

$$\beta = \frac{\Delta h}{h_v}$$

Thus, the blowing coefficient is shown to describe the nondimensional enthalpy difference between the fuel surface and flame zone, as well as the nondimensional fuel surface regression rate.