INSTITUTO TECNOLÓGICO DE AERONÁUTICA

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OPTIMAL IMPULSIVE ORBITAL MANEUVER SYNTHESIS THROUGH DIRECT OPTIMIZATION AND NECESSARY CONDITIONS VERIFICATION

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Curso de Engenheria Aeroespacial

Pedro Kuntz Puglia

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dedicar…

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"Pointy end up, flamey end down."
— Tim Dodd, Everyday Astronaut

Resumo

RESUMO

Abstract

This work presents the development and characterization process of a cold gas thruster vectorization system. The motor is required to have a thrust of 2 N and a chamber pressure of 5 bar. The chosen vectorization method for testing was the jet vane. The constructed motor had slight deviations from the requirements, with a specific impulse of 46.6 s. This motor was mounted on a control mechanism of the deflecting blade, and this assembly was coupled to a three-component scale for force and moment characterization. As a final result, the control derivatives for lateral force and moment were obtained. Finally, the methodological issues encountered and engineering trade-offs identified for the system were presented.

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F	Empuxo propulsivo
$\dot{m}$	Vazão mássica
v_e	Velocidade de exaustão média
p_c	Pressão de câmara
p_e	Pressão de saída média
p_{amb}	Pressão ambiente
A_c	Área da seção transversal da câmara
A_e	Área da seção transversal da saída da tubeira
A_t	Área da seção transversal da garganta
ε	Razão de expansão
I_{sp}	Impulso específico
C_F	Coeficiente de empuxo
C^*	Velocidade característica
F_x	Força horizontal, transversal ao motor foguete
F_y	Força vertical, na direção do empuxo propulsivo
M	Torque resultante
δ	Deflexão da lâmina (*jet vane*)
$F_{x\delta}$	Derivada da força lateral em relação à deflexão da lâmina
M_δ	Derivada de momento em relação à deflexão da lâmina

Sumário

1 Introduction

1.1 Context

primeiro satelite manobravel

discutir manobra interplanetária vs órbita terrestre

GOCE

daedalus

Space exploration relies on clever resource management, since satellites have a finite amount of resources (propellant and other consumables) to fulfill their mission. Up to this date, all space hardware is expendable, that is, when the consumables required for mission maintenance are finished, the mission ends, marking the end of the exploration of a very expensive engineered system. Thus the need for optimization arises in this domain.

Contrary to science fiction, where spaceships seem to be constantly propelled by their thrusters, real life satellites change their courses in discrete moments of maximum thrust application, surrounded by (usually long) coasting periods. This is due to the relatively high power delivered by traditional rocket engines, which can, in the matter of seconds or minutes, greatly alter a satelite's orbit. Certain more modern propulsion systems, such as electric rocket engines, are somewhat of an exception; this technicality will be discussed in further sections.

Orbital maneuvers are necessary in all stages of a satellite's lifecycle. In the beginning of a mission, the satellite is released by the launch vehicle in an orbit that is usually not the mission's orbit. Therefore, an *injection maneuver* is necessary to bring the satellite into an operational orbit. This is usually the biggest maneuver a satellite must execute during its lifecycle, consuming a high fraction of its propellant storage CITE.

During a mission, the satellite must perform sporadic *maintenance maneuvers*, which are small course correction maneuvers to mitigate external perturbations such as atmospheric drag, oblateness effects (if undesired), influence of celestial bodies, and solar radiation pressure. Their frequency and magnitude vary depending on mission requirements, and in industrial applications, must not conflict with other, possibly simultaneous events (such as observation of a ground target), as well as taking into account pointing constraints, since a spacecraft might have sensitive sensors that must not be pointed at the sun, or have solar panels that need uninterrupted illumination. Those are by far the most common type of maneuver, and a loose, non-exhaustive classification arises naturally.

The simplest type of maneuver is that of *orbit raising*, which consists in bringing the satellite from a (often circular) orbit and increasing its semimajor-axis (and thus, its period) until a desired value is reached. This maneuver is commonly found in LEO applications, due to the presence of atmospheric drag; notably, it is performed by the ISS about once a month CITE. From a theoretical standpoint, it presents a simple, introductory case, often restricted to two dimensions instead of three. There are plenty of theoretical results about it, most notably the Hohmann transfer, a two-impulse maneuver which is known to be the two-impulse optimal from a plethora of theoretical tools. Other more elaborate results include the bielliptic transfer, which can be shown to surpass Hohmann's performance in certain conditions by allowing a third impulse. Another scenario that falls under this category is that of high orbit injections, such as LEO to GEO or LEO to MEO.

A second type of maneuver is a *plane change* maneuver. Satellites move (approximately) in a plane which contains its position and velocity vectors and the center of Earth. By changing the direction of the velocity, this plane may be change. Common cases include an inclination change during orbital insertion, which may be required if the inclination of the target orbit is different to the latitude of the launch center CITE. Another plane change instance is that of a change in the right ascension of the ascending node (RAAN), which is especially useful for SSOs. SSO injection requires that the orbit be placed approximately perpendicular to the Sun; this requires careful positioning of the ascending node. Another interesting case is that of a combined plane change and orbit raising maneuver, such as that starting from an inclined LEO orbit targeting a GEO (thus, equatorial) orbit. A clever combination of both requirements can allow for great performance gains as compared to sequential maneuvers.

A final type of maneuver is the *phasing* maneuver. This maneuver consists in changing the position occupied by the satellite within the same orbit at a certain time. This maneuver is very important for *orbital rendez-vous*, where not only it is required that two vessels share the same orbit, but also they must have the same position and velocity at the same time. Another application for this type of maneuver is that of *rideshare injection*, where a swarm of satellites is carried by a launcher hub and they must be distributed around a shared orbit, with certain angular intervals in between. The execution of such a maneuver usually involves placing the satellite in an intermediate orbit with slightly different period than the initial one, and waiting multiple revolutions for the convergence of the satellite and the (mobile) target. A notable, recurring example of this is the rendez-vous of the Soyuz capsule with the ISS, which can take up to 3 days CITE.

Finally, at the end-of-life, there are legal constraints on where a satelite may be disposed of. LEO missions have a deadline for deorbiting into Earth's atmosphere, while GEO satellites are usually placed into a cemetery orbit which does not intersect the highly prized GEO region. As an end-of-life procedure, feasibility is of utmost importance, while ensuring optimality increases the lifespan of the mission.

1.2 Problem statement

This work aims to develop modern numerical methods for orbital maneuver optimization in Earth orbit. Combinations of propulsive and orbital models are to be paired with adequate numerical schemes and theoretical tools to produce feasible maneuvers that also satisfy certain optimality conditions. The main deliverable shall be a code package capable of generating and optimizing maneuvers between an initial and final orbital state, for an allowed time of flight in between, as well as the mathematical formulation and derivation of such a problem.

The main models to be studied are those of two body Keplerian dynamics and impulsive maneuvers, as they offer the most opportunities for validation with analytical results. Further models to be studied, if time allows it, are continuous thrust propulsion models and two body dynamics with oblateness perturbations (J2 effects).

It is desired to validate the numerical algorithms with certain known analytical results, such as the Hohmann transfer, reproduce certain methods from the literature, and apply some of the formalism of optimal control (in the form of primer vector theory) to the solutions obtained.

It is not in the scope of this work to compare different numerical schemes; a sufficient one shall be found and exploited throughout. However, a novel, experimental method for optimal control synthesis based on polynomial optimization may be attempted if time allows it CITE.

The problem of orbital maneuvers is very general and it is possible to abstract it from the specifics of a particular satellite's hardware by reasoning with position and (changes in) velocity. Therefore, application cases shall be representative of classes of maneuvers, instead of restricting their application to the specifics of one mission. This works focuses on Earth exploration activities, thus excluding lunar and interplanetary transfers.

1.3 Hypotheses

All hardware restrictions such as need for contact with a ground station, pointing constraints, mission objectives (such as observation of a ground target), and possibility of hardware failure and imprecisions are neglected. Attitude dynamics are assumed to be as fast as needed and always precise. Further assumptions depend on the propulsion and orbital model chosen and shall be discussed in future sections.

1.4 Objectives

The objective of this work is to implement an impulsive orbital maneuver optimizer. Given an initial orbital state, a final orbital state, and a transfer time, the goal is to characterize the control history that optimally satisfies theses requirements, spending the least amount of propellant possible. Secondary objectives include:

- Apply primer vector theory to the solutions found. This is a central tool in the field, and provides analytical necessary conditions for verifying optimality;
- Discuss different parameterizations for the problem;
- Compare some of the numerical results with known analytical results, namely the Hohmann transfer;
- Discuss some instances of application of this method to common aerospace scenarios;
- **Optional**: expand the work to continuous thrust propulsive models;
- **Optional**: include orbital perturbations, most notably oblateness effects;

1.5 Justificativa

brasil começa a ter satélites em LEO

1.6 Organização do trabalho

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2 Theory and fundamentals

2.1 Optimal Control

Optimal control is the area of control theory which tries to find the best control action to satisfy some requirements, such as altering a system's state in some way desired way. Here, "best"is defined as maximizing or minimizing some performance metric. In practice, and in particular in the scope of this work, this can be interpreted as attaining a target orbit in a certain amount of time, while minimizing fuel consumption.

The mathematical nature of an optimal control problem varies greatly depending on the nature of the system, the requirements, and the objective. Here, a selected subset of this vast theory shall be presented. Suppose a continuous time dynamical system operating on times $t \in [0, t_f]$, where $t_f \in \mathbb{R}$, given by

$$\dot{X}(t) = f(X(t), u(t)) \tag{2.1}$$

where $X(t) : \mathbb{R} \to \mathcal{X} \subset \mathbb{R}^n$ is the state vector trajectory describing the system state, $u(t) : \mathbb{R} \to \mathcal{U} \subset \mathbb{R}^m$ is the control vector trajectory and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is the function describing its temporal dynamics.

In addition, the control vector might be subject to some some inequality constraints, representing for instance saturation of actuators. Therefore, an admissible control set $\mathcal{U}$ is defined by a vector of constraint functions $g(u)$ as

$$\mathcal{U} = \{u \in \mathbb{R}^m | \; g(u) \leq 0\} \tag{2.2}$$

where the inequality is understood to hold component-wise.

At the initial time, the system is supposed to be in a given state X_i such that

$$X(0) = X_i. \tag{2.3}$$

At the final time t_f, some components of the final state vector are specified, while others are subject to optimization. Let the index set $\mathcal{K}$ be the set of state variables that are fixed at the final time, such that

$$X_k(t_f) = X_{fk}, k \in \mathcal{K} \tag{2.4}$$

for some given values X_{fk}.

To complete the optimal control problem, a performance metric needs to be introduced. In general, any functional of the form $J[X(t), u(t)]$ may be taken as this performance metric; however, a common form with desireable properties, which shall be adopted in this work, is given by

$$J[X(t), u(t)] = h(X(t_f)) + \int_0^{t_f} L(X(t), u(t))dt \tag{2.5}$$

where the functions $h(X)$ and $L(X, u)$ are respectively called the *terminal cost* and the *temporal cost* functions.

The optimal control problem is then that of finding a control trajectory $u(t)$ that minimizes (or maximizes) the performance metric. Here the problem shall be presented as a minimization problem; but the formulation is perfectly analogous for a maximization problem. That said, the complete optimal control problem may be stated as finding the function $u(t)$ such that

$$u(t) = \arg \min_{u(t), X(t)} J[X(t), u(t)] \tag{2.6}$$

subject to

$$\dot{X}(t) = f(X(t), u(t)) \tag{2.7}$$

$$X(0) = X_i \tag{2.8}$$

$$X_k(t_f) = X_{fk}, k \in \mathcal{K} \tag{2.9}$$

In general, this is a very hard problem. The optimization variable $u(t)$ is not merely a vector of parameters but a whole trajectory of them; thus, the search space is enormous. There are techniques to turn this problem into a simple parameter optimization problem, which are known as *direct methods*, which shall be discussed later. There are however tools for extracting necessary conditions for the solution of this problem at all points in time. These are known as *indirect methods*.

One of this tools is the Hamiltonian, a quantity that describes the ensemble of objectives and constraints. It shall be defined for a minimization problem, and maximization problems can be adapted by changing the sign of the performance metric. Given a system of the form in equation (2.1), constraints in the forms of (2.3) and (2.4), and a cost function in the form (2.5), the Hamiltonian H is defined as

$$H(X(t), u(t), \lambda(t)) = L(X, u) + \lambda(t)^T f(X, u) \tag{2.10}$$

for all times t, state and control vectors X and u along a trajectory. $\lambda(t)$ is the costate trajectory, a new set of variables introduced as the continuous-time equivalent of Lagrangian multipliers. These new variables are subject to the differential equation

$$\dot{\lambda} = -\left(\frac{\partial H}{\partial X}\right)^T = -\left(\frac{\partial f}{\partial X}\right)^T \lambda - \left(\frac{\partial L}{\partial X}\right)^T \tag{2.11}$$

and boundary conditions

$$\lambda_k(t_f) = \frac{\partial h(X(t_f))}{\partial X_k}, k \notin \mathcal{K} \tag{2.12}$$

To complete the Hamiltonian approach, Pontryagin's Minimum Principle is introduced. It states that a necessary condition for attaining the minimum in equation (2.6) is that, at all times t, and along the optimal trajectory,

$$u(t) = \arg\min_{u \in \mathcal{U}} H[X(t), u, \lambda(t)]. \tag{2.13}$$

With the control trajectory obtained as a function of $X(t)$ and $\lambda(t)$ from equation (2.13), there are 2n variables, the state and costate trajectories, and 2n boundary conditions, the initial and final states. Thus, the problem is well-posed and configures a Two Point Boundary Value Problem (TPBVP).

2.2 Orbital Mechanics

Orbital mechanics concerns itself with the motion of bodies in space subject to gravitational and disturbance forces. A variety of models exist, differing in precision and availability of analytical tools. The simpler the model, the more analytical tools are available, and the smaller the precision. The simplest model of all, and the basis for all others, is the two body problem, where a central massive body is supposed to be stationary while a moving satellite is subject to its gravitational attraction, also known as Keplerian motion.

2.2.1 Two Body Motion

Let r be the 3-dimensional position of a satellite, and μ the gravitational parameter of the central body. The dynamics of the satellite's position are given by

$$\ddot{r} = -\frac{\mu}{\|r\|^3} r, \tag{2.14}$$

thus configuring a 6-dimensional state vector $X = \begin{bmatrix} r^T & v^T \end{bmatrix}^T$, where v is the satellite's velocity. The system contains a singularity at the states with $\|r\| = 0$, which configures a non-convex domain. In practice, this point is rarely encountered as it lies inside of the central body, thus far from the regions of interest. It is proven that no analytical solution exists for this differential equation; however, much is known about its solutions.

In this model, the possible trajectories are known to be conics, and therefore restricted to a plane. For bound satellites, that is, those in orbit around the central body, this trajectory is an ellipse where the central body lies on one of its foci. Mathematically, a "bound" satellite is one whose specific energy (mechanical energy over mass of the satellite), given by

$$\epsilon = -\frac{\mu}{\|r\|} + \frac{v^2}{2}, \tag{2.15}$$

is negative. The trajectory is closed, and the movement is periodic with period

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{2.16}$$

where a is the semi-major axis of the ellipse.

In this case, an alternative state vector may be introduced in the form of the Keplerian elements. These are:

- a: semi-major axis of the ellipse;
- e: excentricity of the ellipse;
- i: inclination of the orbit's plane with respect to the Equatorial plane;
- Ω: right ascension of the ascending node, that is, angle between FIND REFERENCE DIRECTION and the direction where the satellite crosses the Equatorial plane from South to North;
- ω: argument of perigee, or angle, in the plane of the orbit, between the ascending node and the perigee (point of smallest distance to the central body);

- θ: true anomaly, or angle between the perigee and the current position of the satellite.

These elements are related to the Cartesian state vector through ADD PERIFOCAL EQUATIONS.

In this formulation, all elements but the true anomaly are constant in time. The true anomaly can be related to time implictly through two other quantities, the mean anomaly M and the excentric anomaly E:

$$M = 2\pi \frac{t - t_p}{T} \tag{2.17}$$

$$E - e \sin E = M \tag{2.18}$$

$$\tan \frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \tag{2.19}$$

where t_p is the time of the last perigee passage. By computing the mean anomalies in an initial and a final time, and solving the notorious Kepler's equation (2.18), and finally finding a suitable true anomaly with (2.19), a semi-analytical temporal solution can be found. The process of finding the position of a satellite in the future is called *orbit propagation*. Define an orbit propagator as a funcion $p_o(X_i, t)$ such that

$$X_f = p_o(X_i, t) \tag{2.20}$$

where X_f is the satellite's final state after a time t, with initial state X_i.

2.2.2 Lambert's Problem

An important problem in orbital mechanics is that of the determination of the initial and final velocities of a satellite that passes through two points in space r_1 and r_2 with a time interval Δt in between. This problem first arose in the field of orbit determination but also finds application in the context of orbital maneuvers. Namely, Lambert's Problem seeks to find a feasible solution to a TPBVP, which is of interest to the optimal control TPBVP.

In general, this problem can have multiple solutions, corresponding to prograde and retrograde trajectories, with less than one or multiple revolutions. The resulting orbit can, in general, be elliptic, parabolic or hyperbolic.

Many formulations exist, including CURTIS, SUKHANOV, and numerical integration. They all suffer from a physical indetermination in the case of collinear r_1 and r_2: the plane of the orbit is indeterminate. In this case, one can find many feasible solutions but determining exact velocities requires extra information about the plane of the orbit.

2.3 Orbital Maneuvers

When a satellite is able to maneuver, the Keplerian dynamics of equation (2.14) need to be augmented with the thrust control vector F, which applies a propulsion force on the satellite. Supposing that m is the total current mass of the spacecraft, the dynamics are given by

$$\ddot{r} = -\frac{\mu}{\|r\|^3} r + \frac{F}{m}. \tag{2.21}$$

The generation of thrust F is tied to the consumption of propellant, according to some propulsion model. Three main models exist. The first is the continuous specific impulse continuous thrust model, adequate for chemical engines. The second is the impulsive thrust model, the limiting case of the previous model where a burn is considered to happen instantly. And the last one is the variable specific impulse model, which models electric rocket engines and shall not be explored in this work.

The application of optimal control to the field of orbital maneuvering is mainly concerned with the preservation of propellant. Suppose a satellite has an orbital state X_i and is required to maneuver to an orbital state X_f in a time t_f, and it is desired to minimize the amount of propellant used. A convenient way of expressing this is that it is desired to maximize the final mass of the spacecraft, with constraints:

$$\max_{F(t)} \quad m(t_f) \tag{2.22}$$

$$X(0) = X_i \tag{2.23}$$

$$X(t_f) = X_f \tag{2.24}$$

2.3.1 Constant specific impulse model

Chemical and cold gas thrusters are characterized by an exhaust velocity v_e at which the propellant flow is ejected from the spacecraft. With a propellant flow rate $\dot{m}_p$, the thrust F is given by

$$\|F\| = v_e \dot{m}_p \tag{2.25}$$

The propellant flow is deducted from the spacecraft's mass; therefore it can be stated that $\dot{m} = -\dot{m}_p$. Thus, in this model, the spacecraft's mass is a seventh state variable. A new state vector $X_m = \begin{bmatrix} r^T & v^T & m \end{bmatrix}^T$ is defined and subject to the dynamics

$$\begin{bmatrix} \dot{r} \\ \dot{v} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} v \\ -\frac{\mu}{\|r\|^3} r + \frac{F}{m} \\ -\frac{\|F\|}{v_e} \end{bmatrix} \tag{2.26}$$

In addition, thrusters have limited flow rates, which imposes a maximum thrust magnitude $F_{\max}$:

$$\|F\| \leq F_{\max} \tag{2.27}$$

The objective (2.22) can be developed for this model by integrating $\dot{m}$ as

$$m(t_f) = m(0) - \int_0^{t_f} \frac{\|F\|}{v_e} dt \tag{2.28}$$

Let Γ be the acceleration due to thrust such that $\Gamma = \frac{F}{m}$. The literature CITE CONWAY then suggests considering that the propellant consumption is small compared to the total mass of the satellite, such that it can be stated that

$$m(t_f) \approx m(0) - \frac{m(0)}{v_e} \int_0^{t_f} \|\Gamma\| dt \tag{2.29}$$

Therefore the objective can be restated as

$$\min_{\Gamma(t)} \int_0^{t_f} \|\Gamma\| dt. \tag{2.30}$$

If the control variable is set to be Γ instead of F, this leads to a mass-independent problem. This approximation leads to primer vector theory, and is therefore important. However, the assumption of constant mass should be challenged.

2.3.2 Impulsive propulsion model

A very simple propulsion model that allows for easier solution of the orbital maneuvering problem supposes that the propulsive forces are much greater and operate much faster than the gravitational force, introducing discontinuities in velocity. This is called *impulsive thrust*. The propulsion model relies on Tsiolkovsky's equation,

$$\Delta v = v_e \ln \left(\frac{m_i}{m_f} \right), \tag{2.31}$$

where Δv is the magnitude of an instantaneous change in velocity, v_e is the engine's exhaust velocity (which is treated as a known parameter), m_i is the initial spacecraft amss and m_f, the final mass. Supposing a burn happens at time t_b, the propulsion model can then be expressed through a Dirac delta as

$$\left.\frac{F}{m}\right|_{t=t_b} = \delta(t-t_b)V_e \ln\left(\frac{m(t_b^-)}{m(t_b^+)}\right) = \delta(t-t_b)\Delta v, \tag{2.32}$$

which yields a velocity discontinuity

$$\|v(t_b^+) - v(t_b^-)\| = V_e \ln\left(\frac{m(t_b^-)}{m(t_b^+)}\right) = \Delta v. \tag{2.33}$$

Now, considering a generic maneuver with n burns, there are $n+1$ coasting segments related by the change in velocity $\Delta\vec{v}_j$ associated with the j-th burn. Considering burn times t_j, with $t_f \geq t_{j+1} \geq t_j \geq 0$, and the initial and final times 0 and t_f, the system is subject to boundary conditions

$$X(0) = X_i \tag{2.34}$$

$$r(t_j^+) = r(t_j^-), \qquad \forall j = 1, \ldots, n \tag{2.35}$$

$$v(t_j^+) = v(t_j^-) + \Delta\vec{v}_j, \qquad \forall j = 1, \ldots, n \tag{2.36}$$

$$X(t_f) = X_f \tag{2.37}$$

and to dynamical equation (2.14) in the intermediate times. Using the concept of orbit propagator, this adds the constraints

$$X(t_1^-) = p_o(X(0), t_1) \tag{2.38}$$

$$X(t_{j+1}^-) = p_o(X(t_j^+, t_{j+1} - t_j)), \forall j = 1, \ldots, n-1 \tag{2.39}$$

$$X(t_f) = p_o(X(t_n^+), t_f - t_n) \tag{2.40}$$

to the previous list. Each impulse is described by its time t_j and its velocity change vector Δv_j. Accounting for $X(0)$, all of the intermediate $r(t_j^-)$, $r(t_j^+)$, $v(t_j^-)$ and $v(t_j^+)$, and also the final state $X(t_f)$, plus the impulse parameters, there are $6+12n+6+4n = 16n+12$ unknowns. At the same time, there are $6+(3+3)n+6+6+6(n-1)+6 = 12n+18$ constraints. Therefore, for general initial and final conditions, the *minimal number of impulses* is 2.

Tsiolkovsky's equation can also be applied between the initial time and the final time, thus relating mass at time t_f with the total velocity change, which is the sum of all n

burns executed during the transfer:

$$m(t_f) = m(0) \exp\left(-\frac{\sum_{i=1}^{n} \Delta v_i}{v_e}\right) \tag{2.41}$$

Since v_e and $m(0)$ are not subject to optimization, the objective (2.22) is equivalent to minimizing the sum of magnitudes of impulses used during the transfer:

$$\min \sum_{i=1}^{n} \Delta v_i. \tag{2.42}$$

Thus, in the impulsive case, the problem is *independent of spacecraft mass*, and it can be eliminated from the state vector. However, the introduction of a discrete parameter, the number of burns n, is worth discussing. The optimization of non linear problems with mixed continuous and discrete variables is called Mixed Integer Non-Linear Programming (MINLP), and is much more complicated than regular non-linear programming. Although the implementation of such a solver might be of interest, some simple reasoning and the theory of primer vectors (exposed later) can help determine the number of impulses needed CITE INTERACTIVE PRIMER VECTOR.

2.3.3 Primer vector theory

The application of optimal control theory to orbital maneuvers dates back to Lawden CITE, who introduced the concept of the *primer vector*, which is closely tied to the costate.

3 Bibliographic survey

4 METODOLOGIA

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4.3 Lambert problem formulations

4.4 Optimal impulsive maneuver problem statement

algo usado

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6 CONCLUSÃO

Referências

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FOLHA DE REGISTRO DO DOCUMENTO

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7. INSTITUIÇÃO(ÕES)/ÓRGÃO(S) INTERNO(S)/DIVISÃO(ÕES):
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8. PALAVRAS-CHAVE SUGERIDAS PELO AUTOR:
Cupim; Cimento; Estruturas

9. PALAVRAS-CHAVE RESULTANTES DE INDEXAÇÃO:
Propulsão; Gás Frio; Vetorização de empuxo;

10. APRESENTAÇÃO: (**X**) **Nacional** ( ) **Internacional**
ITA, São José dos Campos. Curso de Mestrado. Programa de Pós-Graduação em Engenharia Aeronáutica e Mecânica. Área de Sistemas Aeroespaciais e Mecatrônica. Orientador: Prof. Dr. Adalberto Santos Dupont. Coorientadora: Profª. Drª. Doralice Serra. Defesa em 05/03/2015. Publicada em 25/03/2015.

11. RESUMO:

RESUMO

12. GRAU DE SIGILO:
(**X**) **OSTENSIVO** ( ) **RESERVADO** ( ) **SECRETO**