Optimal Impulsive Orbital Maneuver Synthesis Through Direct Optimization

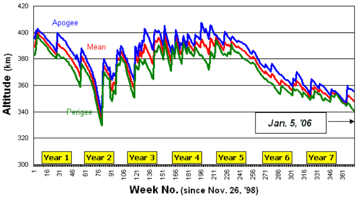
Pedro Kuntz Puglia[1] Willer Santos[2] Emilien Flayac[3]

[1]ITA, Student
[2]ITA, Professor (AESP)
[3]ISAE-SUPAERO, Professor (DISC)

Plan

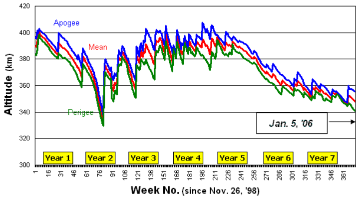
1. Introduction
2. Theory
3. Methodology
4. Results

Context

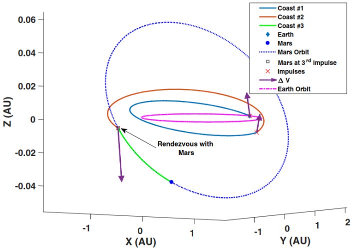


(a) ISS reboost [11]

Context

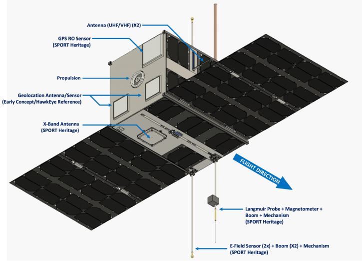


(a) ISS reboost [11]



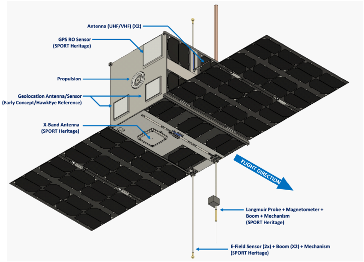
(b) Impulsive transfer [13]

Context

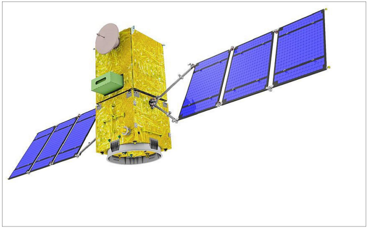


(a) ITASAT2 [9]

Context



(a) ITASAT2 [9]



(b) Amazonia 1 PMM [8]

Problem Statement

Central Question

What is the most efficient sequence of impulses that takes a spacecraft from an initial state to a final state in a given time?

Problem Statement

Central Question

What is the most efficient sequence of impulses that takes a spacecraft from an initial state to a final state in a given time?

- LEO environment: applicability & perturbations
- General case in mind (no particular analytical solutions)
- Compare maneuvers with different perturbation models
- Local solutions backed by optimal control theory: **primer vector**
    - How many impulses?
    - Develop theory for perturbations

Plan

1 Introduction

2 Theory

3 Methodology

4 Results

Two-Body Motion

Keplerian dynamics:

$$\ddot{\mathbf{r}} = -\frac{\mu}{\|\mathbf{r}\|^3}\mathbf{r} \tag{1}$$

Closed, planar, periodic elliptical trajectory for negative energy (bound satellite).

J2 perturbation

- Largest LEO perturbation [7]
- Earth's oblateness
- Conservative force
  - $\ddot{\mathbf{r}} = g(\mathbf{r})$
- Nonplanar trajectory

Drag perturbation

- Second largest LEO perturbation
- Needs atmospheric model
- Dissipative force
  - $\ddot{\mathbf{r}} = g(\mathbf{r}) + d(\mathbf{r}, \mathbf{v})$

Optimal control

Optimal control

Generic optimal control problem

Given a dynamical system $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$, a fixed initial condition $\mathbf{x}(0) = \mathbf{x}_i$, a total time t_f and a final condition $\mathbf{x}(t_f) = \mathbf{x}_f$, find control trajectory $\mathbf{u}(t)$ minimizing objective $J[\mathbf{x}(t), \mathbf{u}(t)] = h(\mathbf{x}(t_f)) + \int_0^{t_f} L(x(t), u(t))dt$.

Generic optimal control problem

Given a dynamical system $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$, a fixed initial condition $\mathbf{x}(0) = \mathbf{x}_i$, a total time t_f and a final condition $\mathbf{x}(t_f) = \mathbf{x}_f$, find control trajectory $\mathbf{u}(t)$ minimizing objective $J[\mathbf{x}(t), \mathbf{u}(t)] = h(\mathbf{x}(t_f)) + \int_0^{t_f} L(x(t), u(t))dt$.

Indirect method

- Hamiltonian with Pontryagin's Minimum Principle [2]:

$$H = L(\mathbf{x}, \mathbf{u}) + \boldsymbol{\lambda}^T f(\mathbf{x}, \mathbf{u}) \tag{2}$$

$$\dot{\boldsymbol{\lambda}} = -\left(\frac{\partial H}{\partial \mathbf{x}}\right)^T \tag{3}$$

$$\mathbf{u}^\star(t) \in \arg\min_{\mathbf{u}\in\mathcal{U}} H[\mathbf{x}(t), \mathbf{u}, \boldsymbol{\lambda}(t)] \tag{4}$$

Generic optimal control problem

Given a dynamical system $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$, a fixed initial condition $\mathbf{x}(0) = \mathbf{x}_i$, a total time t_f and a final condition $\mathbf{x}(t_f) = \mathbf{x}_f$, find control trajectory $\mathbf{u}(t)$ minimizing objective $J[\mathbf{x}(t), \mathbf{u}(t)] = h(\mathbf{x}(t_f)) + \int_0^{t_f} L(x(t), u(t))dt$.

Indirect method

- Hamiltonian with Pontryagin's Minimum Principle [2]:

$$H = L(\mathbf{x}, \mathbf{u}) + \boldsymbol{\lambda}^T f(\mathbf{x}, \mathbf{u}) \tag{2}$$

$$\dot{\boldsymbol{\lambda}} = -\left(\frac{\partial H}{\partial \mathbf{x}}\right)^T \tag{3}$$

$$\mathbf{u}^\star(t) \in \arg\min_{\mathbf{u} \in \mathcal{U}} H[\mathbf{x}(t), \mathbf{u}, \boldsymbol{\lambda}(t)] \tag{4}$$

Direct method

- Multiple shooting
- Numerical integration
- Trajectories $\rightarrow \mathbf{x}_k, \mathbf{u}_k$
- Solve for $\mathbf{x}$, $\mathbf{u}$

OC Solution Hierarchy

TODO

Orbital Maneuvering

Extended dynamics

$$\ddot{\mathbf{r}} = g(\mathbf{r}) + \boldsymbol{\Gamma}. \tag{5}$$

Extended dynamics

$$\ddot{\mathbf{r}} = g(\mathbf{r}) + \boldsymbol{\Gamma}. \tag{5}$$

Control input is acceleration $\boldsymbol{\Gamma} = \Gamma\hat{\mathbf{u}}$. Objective: $\min \int_0^{t_f} \Gamma(t)dt$

Extended dynamics

$$\ddot{\mathbf{r}} = g(\mathbf{r}) + \mathbf{\Gamma}. \tag{5}$$

Control input is acceleration $\mathbf{\Gamma} = \Gamma \hat{\mathbf{u}}$. Objective: $\min \int_0^{t_f} \Gamma(t) dt$

- Impulsive thrust
  - Engineering: independent of propulsive plant
  - Physics: high thrust $\approx$ impulse
  - Mathematics: minimum is unattainable with bounded acceleration
- Velocity discontinuities, *coasting arcs*

$$\mathbf{v}(t_i^+) = \mathbf{v}(t_i^-) + \Delta\mathbf{v} \tag{6}$$

- Hamiltonian results: *primer vector* theory

- Hamiltonian [6]

$$H = (1 + \boldsymbol{\lambda}_v^T \hat{\mathbf{u}})\Gamma + \dots \quad (7)$$

- $(1 + \boldsymbol{\lambda}_v^T \hat{\mathbf{u}})$ as small as possible when $\Gamma > 0$
- $\therefore \hat{\mathbf{u}} \parallel -\boldsymbol{\lambda}_v$
- Define primer vector $\mathbf{p} = -\boldsymbol{\lambda}_v$: optimal direction of impulse
- Necessary conditions for optimal $\boldsymbol{\Gamma}^\star$:
  - $\mathbf{p}(t)$ and $\dot{\mathbf{p}}(t)$ are continuous;
  - $\|\mathbf{p}\| \leq 1$;
  - $\|\mathbf{p}\| = 1$ at the impulse instants ($\mathbf{p} = \hat{\mathbf{u}}$);
- Suboptimal trajectory:
  - $(\|\mathbf{p}(0)\| = 1)$ and $\partial_t \|\mathbf{p}\| \,|_{t=0} > 0$, adding an initial coast will lower the cost;
  - $\|\mathbf{p}(t)\| > 1$ for some $t \in [0, t_f]$, adding an impulse will lower the cost.

- Conservative model

$$\begin{bmatrix} \dot{\mathbf{p}} \\ \ddot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_3 & \mathbf{I}_3 \\ \left[\frac{\partial g}{\partial \mathbf{r}}\right]^T & \mathbf{0}_3 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \dot{\mathbf{p}} \end{bmatrix} = \mathbf{A}_p(\mathbf{r}) \begin{bmatrix} \mathbf{p} \\ \dot{\mathbf{p}} \end{bmatrix} \quad (8)$$

  - **ODE method**: numerical integration
  - Linear ODE: Primer vector transition matrix (PVTM) $\Phi_p(t)$
  - Same as ODE for variational perturbations: use state transition matrix (STM) $\Phi_\delta(t)$
  - **STM method**: $\Phi_p(t) = \Phi_\delta(t) = \frac{\partial \mathbf{x}(t)}{\partial \mathbf{x}_0}$
  - Keplerian model:
    - **Glandorf method**: analytical form of $\Phi_p(t) = \Phi_\delta(t)$
  - **STM**, **Glandorf** widely used in literature [4][12][6]

Primer Vector Calculation

- Non-conservative model

$$\begin{bmatrix} \dot{\mathbf{p}} \\ \ddot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_3 & \mathbf{I}_3 \\ \left[\frac{\partial}{\partial \mathbf{r}}\left(g(\mathbf{r}) + d(\mathbf{r},\mathbf{v})\right)\right]^T & -\left(\frac{\partial d(\mathbf{r},\mathbf{v})}{\partial \mathbf{v}}\right)^T \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \dot{\mathbf{p}} \end{bmatrix} = \mathbf{A}_p(\mathbf{r},\mathbf{v}) \begin{bmatrix} \mathbf{p} \\ \dot{\mathbf{p}} \end{bmatrix}. \quad (9)$$

  - Not found in the literature
  - **Not** the same as variational perturbation ODE
  - Only **ODE method** available

Model and primer vector summary

Model	Class	Primer vector method Glandorf	STM	ODE
Two-Body	Keplerian	✓	✓	✓
J2	Conservative	×	✓	✓
J2+Drag	Non-conservative	×	×	✓

Plan

1. Introduction
2. Theory
3. Methodology
4. Results

Numerical Tools

Numerical Tools

- Julia [3] language

Numerical Tools

- Julia [3] language
- `SatelliteToolbox` [5]: auxiliary orbit propagation

Numerical Tools

- Julia [3] language
- `SatelliteToolbox` [5]: auxiliary orbit propagation
- CasADi [1]: optimization interface

Numerical Tools

- Julia [3] language
- `SatelliteToolbox` [5]: auxiliary orbit propagation
- CasADi [1]: optimization interface
- Ipopt [14]: robust nonlinear optimizer. Local, deterministic, gradient-based

Algorithms

Algorithms

- Maneuver representation
  - `C`: coasting arc
  - `I`: impulse
  - `ICI`, `CICIC`, `CICICIC`, etc
- Impulse: boundary condition between coasting arcs
- Coasting arc
  - Optimization: RK8 Cartesian multiple shooting
  - Propagation: integration or `SatelliteToolbox`
- Primer vector calculation
  - Solve two point boundary value problem between consecutive impulses with PVTM
- Search for **number of impulses** [10]
  1. Solve `ICI`
  2. Solve `CICIC`
  3. Primer vector calculation
  4. Add impulses when $\max \|\mathbf{p}\| > 1$
  5. Repeat 3-5 until all necessary conditions satisfied

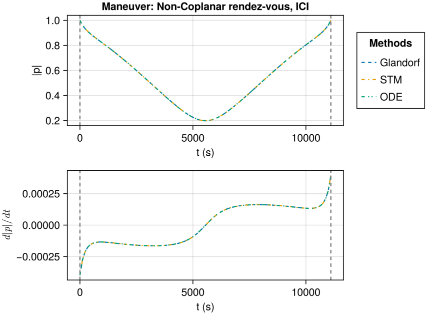
Plan

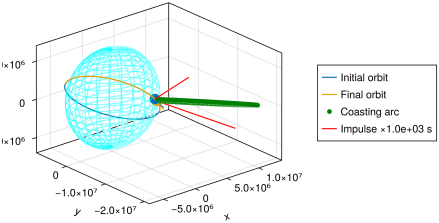
1. Introduction
2. Theory
3. Methodology
4. Results

Maneuver Scenarios

	Circle to Circle		Noncoplanar rendez-vous	
Element	Initial	Final	Initial	Final
a	7000.0 km	9000.0 km	6748.1 km	6778.1 km
e	0.0	0.0	0.0	0.0
i	51.0°	51.0°	42.1°	42.0°
Ω	0.0°	0.0°	120.2°	120.0°
ω	0.0°	0.0°	0.0°	0.0°
θ	0.0°	180.0°	175.0°	180.0°
Maneuver time	3560.541		11107.158	

- Circle to circle
  - Keplerian model: Hohmann transfer
- Noncoplanar rendez-vous
  - Keplerian model: results in the literature [10]

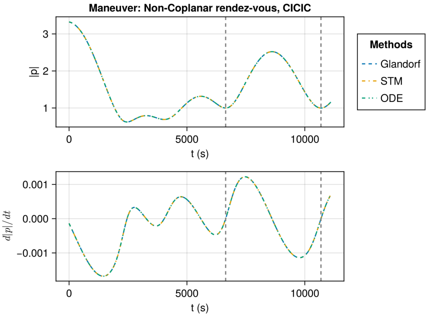


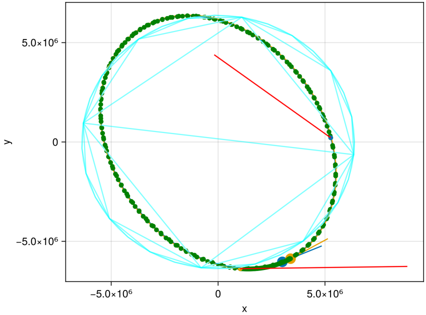


Local Extremum

Impulse	1	2	**Total**
t (s)	0.0	11107.1576	11107.1576
Δv (m/s)	11740.94035	11708.69678	23449.63713

Keplerian Noncoplanar Rendez-vous

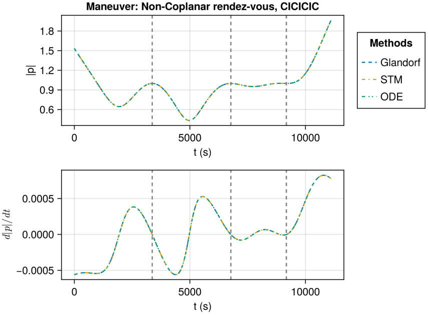


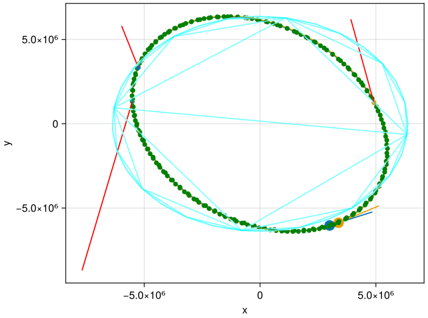


Addition of impulse

Impulse	1	2	**Total**
t (s)	6644.30733	10689.86179	11107.1576
Δv (m/s)	37.29252	16.20984	53.50237

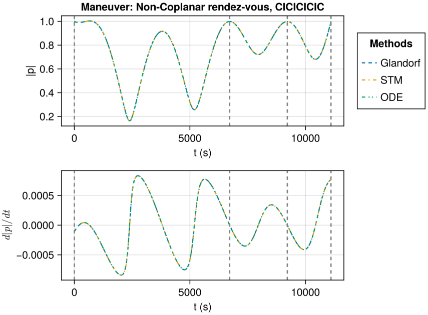
Keplerian Noncoplanar Rendez-vous

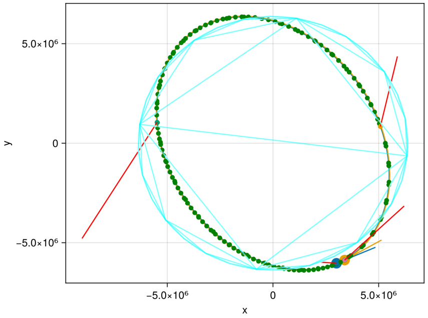




Addition of impulse

Impulse	1	2	3	**Total**
t (s)	3370.1071	6774.61652	9176.42663	11107.1576
Δv (m/s)	11.76294	10.56494	20.74554	43.07342





Local extremum

Impulse	1	2	3	4	**Total**
t (s)	0.00394	6724.60052	9217.50949	11107.15747	11107.1576
Δv (m/s)	3.58823	9.34687	15.01486	8.19599	36.14596

J2+Drag circle to circle

Model Comparison

Scenario	Keplerian		J2		J2+Drag	
	n_i	Δv (m/s)	n_i	Δv (m/s)	n_i	Δv (m/s)
Circle to circle	2	887.56199	3	893.05336	3	893.05339
Noncoplanar rendez-vous	4	36.14596	3	56.00653	3	56.01418

- Perturbations alter cost, and **number of impulses** as well
  - Planar Keplerian $\rightarrow$ nonplanar J2(+Drag)
- Primer vector algorithm can offer >100x increase in performance
- Short time frame: drag has little effect
- Conjecture:
  J2+Drag maneuver $\neq$ J2 maneuver $\iff$ STM $\neq$ ODE method?
  - Long timespans
  - High drag: aerobraking, Very Low Earth Orbit

Conclusions

References I

Joel A E Andersson, Joris Gillis, Greg Horn, James B Rawlings, and Moritz Diehl.
CasADi – A software framework for nonlinear optimization and optimal control.
Mathematical Programming Computation, 11(1):1–36, 2019.

Dimitri P. Bertsekas.
Dynamic Programming and Optimal Control Vol. I.
Athena Scientific, 1st edition, 1995.

Jeff Bezanson, Alan Edelman, Stefan Karpinski, and Viral B Shah.
Julia: A fresh approach to numerical computing.
SIAM Review, 59(1):65–98, 2017.

Kevin Bokelmann and Ryan Russell.
Optimization of impulsive europa capture trajectories using primer vector theory.
The Journal of the Astronautical Sciences, 67, 05 2019.

Ronan Arraes Jardim Chagas, Thatcher Chamberlin, Helge Eichhorn, Yakir Luc Gagnon, Alberto Mengali, Chris Binz, Yasushi SHOJI, Federico Stra, Forrest Gasdia, Hiroaki Yamazoe, Julia TagBot, Srikumar, chenyongzhi, and justbyoo.
Juliaspace/satellitetoolbox.jl: v1.0.0, January 2025.

B. Conway.
Spacecraft Trajectory Optimization.
Cambridge Aerospace Series. Cambridge University Press, 2010.

References III

H.D. Curtis.
Orbital Mechanics: For Engineering Students.
Aerospace Engineering. Butterworth-Heinemann, 2020.

INPE.
Multi Mission Platform (MMP).
https://www.inpe.br/amazonia1/en/about_satellite/pmm.php.
[Accessed 06-11-2025].

Erik Kulu.
ITASAT-2 - Nanosats Database — nanosats.eu.
https://www.nanosats.eu/sat/itasat-2.
[Accessed 06-11-2025].

Ya-Zhong Luo, Jin Zhang, Hai yang Li, and Guo-Jin Tang.
Interactive optimization approach for optimal impulsive rendezvous using primer vector and evolutionary algorithms.
Acta Astronautica, 67(3):396–405, 2010.

NASA.
Iss environment.
Available at
https://web.archive.org/web/20080213164432/http://pdlprod3.hosc.msf
aboutiss/D6.html., 2009.

Bruno Victorino Sarli and Yasuhiro Kawakatsu.
Orbit transfer optimization for asteroid missions using weighted cost function.
Journal of Guidance, Control, and Dynamics, 38(7):1241–1250, 2015.

Ehsan Taheri and John Junkins.
How many impulses redux.
The Journal of the Astronautical Sciences, 67, 12 2019.

Andreas Wächter and Lorenz Biegler.
On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming.
Mathematical programming, 106:25–57, 03 2006.