Optimal Impulsive Orbital Maneuver Synthesis Through Direct Optimization

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Plan

1. Introduction

2. Theory

Context

Problem Statement

Central Question

What is the most efficient sequence of maneuvers that takes a spacecraft from an initial state to a final state in a given time?

- Efficient: least propellant usage
- General case in mind (no particular analytical solutions)
- How much time? Feasibility, trade-offs?
- How many impulses?
- Is it optimal?

Hypotheses

- Choice for *impulsive propulsion* $\rightarrow$ reducible to parameter optimization
- Good numerical solvers: Ipopt[6]
- Many local optima (non-convex problem)
- Expect *Primer vector* theory provides (some) solutions

Objectives

- Apply primer vector theory;
- Study how much time fo transfer, and how to find it;
- Compare numerical and analytical results;
- Discuss applications in common scenarios;

Justification

Some institutions already know how to optimize orbital maneuvers. Why study it again?

(a) Orekit library provides maneuver analysis, and indirect optimization (outdated).

(b) CNES Patrius library provides analysis, not Synthesis.

(c) NASA's Mystic software (Dawn Discovery mission) is not available for download.

No widely available orbital maneuver optimization software.

Plan

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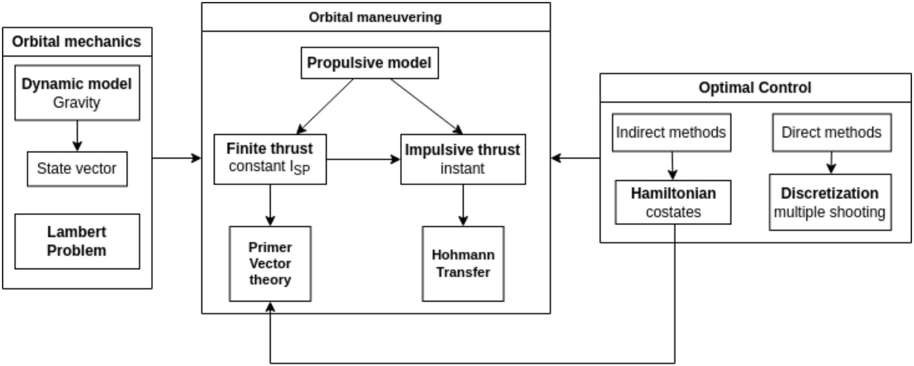
Theory components



Two-Body Motion

Keplerian dynamics:

$$\ddot{\mathbf{r}} = -\frac{\mu}{\|\mathbf{r}\|^3}\mathbf{r} \tag{1}$$

Closed, periodic elliptical trajectory for negative energy (bound satellite).

State vector choice:

- **Cartesian**: $\mathbf{x} = \begin{bmatrix} \mathbf{r} \ \textit{(position)} \\ \mathbf{v} \ \textit{(velocity)} \end{bmatrix}$
- **Keplerian** (elliptical orbit only):

$$\mathbf{x} = \begin{bmatrix} a \ \textit{(semi-major axis)} \\ e \ \textit{(eccentricity)} \\ i \ \textit{(inclination)} \\ \Omega \ \textit{(RAAN)} \\ \omega \ \textit{(argument of perigee)} \\ \theta \ \textit{(true anomaly)} \end{bmatrix}$$

COLOCAR FIGURA

Lambert Problem

Statement

What is the orbit of a satellite that passes by position $\mathbf{r}_2$ at a time Δt after being in position $\mathbf{r}_1$?

- **Importance**: auxiliary role in orbital maneuvering (feasible transfer);
- If $\mathbf{v_1}$ is found, problem is solved;
- Issue with $\mathbf{r}_1 \parallel \mathbf{r}_2$ (eg, perigee & apogee): orbit plane unknown
- Universal variable formulations: simple, cannot handle indetermination [3][5]
- Cartesian formulation [4]

Generic optimal control problem

Given a dynamical system $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$, a fixed initial condition $\mathbf{x}(0) = \mathbf{x}_i$, a total time t_f and a final condition $\mathbf{x}(t_f) = \mathbf{x}_f$, find control trajectory $\mathbf{u}(t)$ minimizing (max.) objective $J[\mathbf{x}(t), \mathbf{u}(t)] = h(\mathbf{x}(t_f)) + \int_0^{t_f} L(x(t), u(t))dt$.

Indirect method

- Hamiltonian [1]: $H = L(\mathbf{x}, \mathbf{u}) + \lambda^T f(\mathbf{x}, \mathbf{u})$
- costate λ
- Pontryagin's Minimum Principle:

$$\mathbf{u} = \arg\min_{\mathbf{u} \in \mathcal{U}} H[\mathbf{x}, \mathbf{u}, \lambda]. \qquad (2)$$

- Solve for $\mathbf{x}$, λ, $\mathbf{u}$

Direct method

- Discretize time, diff. eq.
- Numerical integration: Euler, RK4, RK8
- Trajectories $\rightarrow \mathbf{x}_k, \mathbf{u}_k$
- Parameter optimization
- Solve for $\mathbf{x}$, $\mathbf{u}$

Extended dynamics

$$\ddot{\mathbf{r}} = -\frac{\mu}{\|\mathbf{r}\|^3}\mathbf{r} + \frac{\mathbf{F}}{m}. \tag{3}$$

Thrust **F** related to mass m through *propulsion model*.

Finite Thrust

- $F = -\dot{m}v_e$
- constant exhaust velocity v_e and specific impulse $v_e = I_{sp}g_0$ (CSI)
- $F \leq F_{\max}$
- Extra state: mass

Impulsive thrust $(F_{\max} \to \infty)$

- Discrete impulses, *coasting arcs*
- Tsiolkovsky's Equation [3]:

$$\Delta v = v_e \ln\left(\frac{m_i}{m_f}\right) \tag{4}$$

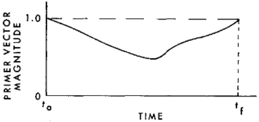
- $\mathbf{v}(t_i^+) = \mathbf{v}(t_i^-) + \Delta\mathbf{v}$
- $\min \int -\dot{m}dt \leftrightarrow \min \sum \Delta v_i$

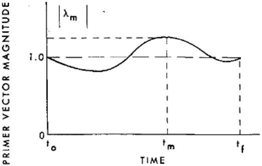
Primer vector theory

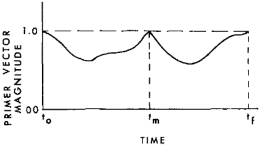
- Apply Hamiltonian to finite thrust CSI case [2] $(\int \dot{m} dt \ll m(0))$
- *bang-bang* control
- *primer vector* $\mathbf{p} = -\lambda_v$ (velocity costate)
- Optimal thrust statisfies

$$\mathbf{F} = \begin{cases} F_{\max} \frac{\mathbf{p}}{p} & , p > 1 \\ 0 & , p < 1 \end{cases} \tag{5}$$

- Extension to impulsive case
  1. $\mathbf{p}(t)$ and $\dot{\mathbf{p}}(t)$ continuous;
  2. $\|\mathbf{p}\| \leq 1$, impulses happen when $\|\mathbf{p}\| = 1$;
  3. $\mathbf{p}$ has the direction of impulse at the impulse instants;
  4. $\frac{d\|\mathbf{p}\|}{dt} = 0$ at impulses at $t \in (0, t_f)$.







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