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ORBITAL MANEUVER OPTIMIZATION

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Curso de Engenharia Aeroespacial

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ORBITAL MANEUVER OPTIMIZATION

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dedicar…

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"Pointy end up, flamey end down."
— Tim Dodd, Everyday Astronaut

Resumo

RESUMO

Abstract

This work presents the development and characterization process of a cold gas thruster vectorization system. The motor is required to have a thrust of 2 N and a chamber pressure of 5 bar. The chosen vectorization method for testing was the jet vane. The constructed motor had slight deviations from the requirements, with a specific impulse of 46.6 s. This motor was mounted on a control mechanism of the deflecting blade, and this assembly was coupled to a three-component scale for force and moment characterization. As a final result, the control derivatives for lateral force and moment were obtained. Finally, the methodological issues encountered and engineering trade-offs identified for the system were presented.

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F	Empuxo propulsivo
$\dot{m}$	Vazão mássica
v_e	Velocidade de exaustão média
p_c	Pressão de câmara
p_e	Pressão de saída média
p_{amb}	Pressão ambiente
A_c	Área da seção transversal da câmara
A_e	Área da seção transversal da saída da tubeira
A_t	Área da seção transversal da garganta
ε	Razão de expansão
I_{sp}	Impulso específico
C_F	Coeficiente de empuxo
C^*	Velocidade característica
F_x	Força horizontal, transversal ao motor foguete
F_y	Força vertical, na direção do empuxo propulsivo
M	Torque resultante
δ	Deflexão da lâmina (*jet vane*)
$F_{x\delta}$	Derivada da força lateral em relação à deflexão da lâmina
M_δ	Derivada de momento em relação à deflexão da lâmina

Sumário

1 Introduction

1.1 Context

primeiro satelite manobravel

discutir manobra interplanetária vs órbita terrestre

GOCE

daedalus

Space exploration relies on clever resource management, since satellites have a finite amount of resources (propellant and other consumables) to fulfill their mission. Up to this date, all space hardware is expendable, that is, when the consumables required for mission maintenance are finished, the mission ends, marking the end of the exploration of a very expensive engineered system. Thus the need for optimization arises in this domain.

Contrary to science fiction, where spaceships seem to be constantly propelled by their thrusters, real life satellites change their courses in discrete moments of maximum thrust application, surrounded by (usually long) coasting periods. This is due to the relatively high power delivered by traditional rocket engines, which can, in the matter of seconds or minutes, greatly alter a satelite's orbit. Certain more modern propulsion systems, such as electric rocket engines, are somewhat of an exception; this technicality will be discussed in further sections.

Orbital maneuvers are necessary in all stages of a satellite's lifecycle. In the beginning of a mission, the satellite is released by the launch vehicle in an orbit that is usually not the mission's orbit. Therefore, an *injection maneuver* is necessary to bring the satellite into an operational orbit. This is usually the biggest maneuver a satellite must execute during its lifecycle, consuming a high fraction of its propellant storage CITE.

During a mission, the satellite must perform sporadic *maintenance maneuvers*, which are small course correction maneuvers to mitigate external perturbations such as atmospheric drag, oblateness effects (if undesired), influence of celestial bodies, and solar radiation pressure. Their frequency and magnitude vary depending on mission requirements, and in industrial applications, must not conflict with other, possibly simultaneous events (such as observation of a ground target), as well as taking into account pointing constraints, since a spacecraft might have sensitive sensors that must not be pointed at the sun, or have solar panels that need uninterrupted illumination. Those are by far the most common type of maneuver, and a loose, non-exhaustive classification arises naturally.

The simplest type of maneuver is that of *orbit raising*, which consists in bringing the satellite from a (often circular) orbit and increasing its semimajor-axis (and thus, its period) until a desired value is reached. This maneuver is commonly found in LEO applications, due to the presence of atmospheric drag; notably, it is performed by the ISS about once a month CITE. From a theoretical standpoint, it presents a simple, introductory case, often restricted to two dimensions instead of three. There are plenty of theoretical results about it, most notably the Hohmann transfer, a two-impulse maneuver which is known to be the two-impulse optimal from a plethora of theoretical tools. Other more elaborate results include the bielliptic transfer, which can be shown to surpass Hohmann's performance in certain conditions by allowing a third impulse. Another scenario that falls under this category is that of high orbit injections, such as LEO to GEO or LEO to MEO.

A second type of maneuver is a *plane change* maneuver. Satellites move (approximately) in a plane which contains its position and velocity vectors and the center of Earth. By changing the direction of the velocity, this plane may be change. Common cases include an inclination change during orbital insertion, which may be required if the inclination of the target orbit is different to the latitude of the launch center CITE. Another plane change instance is that of a change in the right ascension of the ascending node (RAAN), which is especially useful for SSOs. SSO injection requires that the orbit be placed approximately perpendicular to the Sun; this requires careful positioning of the ascending node. Another interesting case is that of a combined plane change and orbit raising maneuver, such as that starting from an inclined LEO orbit targeting a GEO (thus, equatorial) orbit. A clever combination of both requirements can allow for great performance gains as compared to sequential maneuvers.

A final type of maneuver is the *phasing* maneuver. This maneuver consists in changing the position occupied by the satellite within the same orbit at a certain time. This maneuver is very important for *orbital rendez-vous*, where not only it is required that two vessels share the same orbit, but also they must have the same position and velocity at the same time. Another application for this type of maneuver is that of *rideshare injection*, where a swarm of satellites is carried by a launcher hub and they must be distributed around a shared orbit, with certain angular intervals in between. The execution of such a maneuver usually involves placing the satellite in an intermediate orbit with slightly different period than the initial one, and waiting multiple revolutions for the convergence of the satellite and the (mobile) target. A notable, recurring example of this is the rendez-vous of the Soyuz capsule with the ISS, which can take up to 3 days CITE.

Finally, at the end-of-life, there are legal constraints on where a satelite may be disposed of. LEO missions have a deadline for deorbiting into Earth's atmosphere, while GEO satellites are usually placed into a cemetery orbit which does not intersect the highly prized GEO region. As an end-of-life procedure, feasibility is of utmost importance, while ensuring optimality increases the lifespan of the mission.

1.2 Problem statement

This work aims to develop modern numerical methods for orbital maneuver optimization in Earth orbit. Combinations of propulsive and orbital models are to be paired with adequate numerical schemes and theoretical tools to produce feasible maneuvers that also satisfy certain optimality conditions. The main deliverable shall be a code package capable of generating and optimizing maneuvers between an initial and final orbital state, for an allowed time of flight in between, as well as the mathematical formulation and derivation of such a problem.

The main models to be studied are those of two body Keplerian dynamics and impulsive maneuvers, as they offer the most opportunities for validation with analytical results. Further models to be studied, if time allows it, are continuous thrust propulsion models and two body dynamics with oblateness perturbations (J2 effects).

It is desired to validate the numerical algorithms with certain known analytical results, such as the Hohmann transfer, reproduce certain methods from the literature, and apply some of the formalism of optimal control (in the form of primer vector theory) to the solutions obtained.

It is not in the scope of this work to compare different numerical schemes; a sufficient one shall be found and exploited throughout. However, a novel, experimental method for optimal control synthesis based on polynomial optimization may be attempted if time allows it CITE.

The problem of orbital maneuvers is very general and it is possible to abstract it from the specifics of a particular satellite's hardware by reasoning with position and (changes in) velocity. Therefore, application cases shall be representative of classes of maneuvers, instead of restricting their application to the specifics of one mission. This works focuses on Earth exploration activities, thus excluding lunar and interplanetary transfers.

1.3 Hypotheses

All hardware restrictions such as need for contact with a ground station, pointing constraints, mission objectives (such as observation of a ground target), and possibility of hardware failure and imprecisions are neglected. Attitude dynamics are assumed to be as fast as needed and always precise. Further assumptions depend on the propulsion and orbital model chosen and shall be discussed in future sections.

1.4 Objectives

The objective of this work is

1.5 Justificativa

brasil começa a ter satélites em LEO

1.6 Organização do trabalho

2 Theory and fundamentals

2.1 Optimal Control

Optimal control is the area of control theory which tries to find the best control action to satisfy some requirements, such as altering a system's state in some way desired way. Here, "best"is defined as maximizing or minimizing some performance metric. In practice, and in particular in the scope of this work, this can be interpreted as attaining a target orbit in a certain amount of time, while minimizing fuel consumption.

The mathematical nature of an optimal control problem varies greatly depending on the nature of the system, the requirements, and the objective. Here, a selected subset of this vast theory shall be presented. Suppose a continuous time dynamical system operating on times $t \in [0, t_f]$, where $t_f \in \mathbb{R}$, given by

$$\dot{X}(t) = f(X(t), u(t)) \tag{2.1}$$

where $X(t) : \mathbb{R} \to \mathbb{R}^n$ is the state vector trajectory describing the system state, $u(t) : \mathbb{R} \to \mathbb{R}^m$ is the control vector trajectory and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is the function describing its temporal dynamics. At the initial time, the system is supposed to be in a given state X_i such that

$$X(0) = X_i \tag{2.2}$$

and at the final time t_f, it is desired that the system have a state X_f such that

$$X(t_f) = X_f. \tag{2.3}$$

To complete the optimal control problem, a performance metric needs to be introduced. In general, any functional of the form $J[X(t), u(t)]$ may be taken as this performance metric; however, a common form with desireable properties, which shall be adopted in this work, is given by

$$J[X(t), u(t)] = h(X(t_f)) + \int_0^{t_f} L(X(t), u(t))dt \tag{2.4}$$

where the functions $h(X)$ and $L(X, u)$ are respectively called the *terminal cost* and the *temporal cost* functions.

The optimal control problem is then that of finding a control trajectory $u(t)$ that minimizes (or maximizes) the performance metric. Here the problem shall be presented as a minimization problem; but the formulation is perfectly analogous for a maximization problem. That said, the complete optimal control problem may be stated as finding $u(t)$ such that

$$u(t) \in \arg \min_{u(t), X(t)} J[X(t), u(t)] \tag{2.5}$$

subject to

$$\dot{X}(t) = f(X(t), u(t)) \tag{2.6}$$

$$X(0) = X_i \tag{2.7}$$

$$X(t_f) = X_f \tag{2.8}$$

In general, this is a very hard problem. Add introduction on Hamiltonian.

2.2 Orbital Mechanics

Orbital mechanics concerns itself with the motion of bodies in space subject to gravitational and disturbance forces. A variety of models exist, differing in precision and availability of analytical tools. The simpler the model, the more analytical tools are available, and the smaller the precision. The simplest model of all, and the basis for all others, is the two body problem, where a central massive body is supposed to be stationary while a moving satellite is subject to its gravitational attraction, also known as Keplerian motion.

2.2.1 Two Body Motion

Let r be the 3-dimensional position of a satellite, and μ the gravitational parameter of the central body. The dynamics of the satellite's position are given by

$$\ddot{r} = -\frac{\mu}{\|r\|^3} r, \tag{2.9}$$

thus configuring a 6-dimensional state vector $X = \begin{bmatrix} r^T & v^T \end{bmatrix}^T$, where v is the satellite's velocity. It is proven that no analytical solution exists for this differential equation;

however, much is known about its solutions.

In this model, the possible trajectories are known to be conics, and therefore restricted to a plane. For bound satellites, that is, those in orbit around the central body, this trajectory is an ellipse where the central body lies on one of its foci. Mathematically, a "bound" satellite is one whose specific energy (mechanical energy over mass of the satellite), given by

$$\epsilon = -\frac{\mu}{\|r\|} + \frac{v^2}{2}, \tag{2.10}$$

is negative. The trajectory is closed, and the movement is periodic with period

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{2.11}$$

where a is the semi-major axis of the ellipse.

In this case, an alternative state vector may be introduced in the form of the Keplerian elements. These are:

- a: semi-major axis of the ellipse;
- e: excentricity of the ellipse;
- i: inclination of the orbit's plane with respect to the Equatorial plane;
- Ω: right ascension of the ascending node, that is, angle between FIND REFERENCE DIRECTION and the direction where the satellite crosses the Equatorial plane from South to North;
- ω: argument of perigee, or angle, in the plane of the orbit, between the ascending node and the perigee (point of smallest distance to the central body);
- θ: true anomaly, or angle between the perigee and the current position of the satellite.

These elements are related to the Cartesian state vector through ADD PERIFOCAL EQUATIONS.

In this formulation, all elements but the true anomaly are constant in time. The true anomaly can be related to time implictly through two other quantities, the mean anomaly M and the excentric anomaly E:

$$M = 2\pi \frac{t - t_p}{T} \tag{2.12}$$

$$E - e \sin E = M \tag{2.13}$$

$$\tan \frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \tag{2.14}$$

where t_p is the time of the last perigee passage. By computing the mean anomalies in an initial and a final time, and solving the notorious Kepler's equation (2.13), and finally finding a suitable true anomaly with (2.14), a semi-analytical temporal solution can be found. The process of finding the position of a satellite in the future is called *orbit propagation*.

2.2.2 Lambert's Problem

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Conway

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4 METODOLOGIA

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algo usado

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6 CONCLUSÃO

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