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Interactive optimization approach for optimal impulsive rendezvous using primer vector and evolutionary algorithms ☆

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ABSTRACT

In this paper, a new optimization approach combining primer vector theory and evolutionary algorithms for fuel-optimal non-linear impulsive rendezvous is proposed. The optimization approach is designed to seek the optimal number of impulses as well as the optimal impulse vectors. In this optimization approach, adding a midcourse impulse is determined by an interactive method, i.e. observing the primer-magnitude time history. An improved version of simulated annealing is employed to optimize the rendezvous trajectory with the fixed-number of impulses. This interactive approach is evaluated by three test cases: coplanar circle-to-circle rendezvous, same-circle rendezvous and non-coplanar rendezvous. The results show that the interactive approach is effective and efficient in fuel-optimal non-linear rendezvous design. It can guarantee solutions, which satisfy the Lawden's necessary optimality conditions.

1. Introduction

The fuel-optimal rendezvous problem has been extensively studied, and a lot of algorithms have been proposed [1–8]. The existing algorithms can guarantee to obtain a global solution for linearized impulsive rendezvous, but not so efficient and effective in solving non-linear rendezvous problems. The widely used tools for optimizing non-linear impulsive rendezvous are the primer vector method and the Lambert algorithm [3,4]. By using these tools and classical gradient-based optimization algorithms, Jezewski and Rozendaal [4], Gross and Prussing [5], Prussing and Chiu [6] and Hughes et al. [8] had successfully solved different types of unperturbed two-body multiple-impulse rendezvous problems. However, as the limitations of the gradient-based optimization methods, these optimization approaches cannot guarantee the global solution, which was concluded by Hughes et al. [8] and was also observed in our experiments. In this paper, by combing primer vector theory and the global convergence ability of evolutionary algorithms, we propose a new interactive optimization approach for fuel-optimal non-linear rendezvous problem.

The paper is organized as follows: in Section 2, the primer vector theory is simply described. The proposed interactive optimization approach is provided in Section 3. Section 4 is devoted to three examples. Several issues about the proposed approach and its application are discussed in Section 5. Finally, conclusions are made.

2. Primer vector theory

Primer vector theory provides a set of first order necessary conditions that a trajectory must meet to be locally optimal. The necessary conditions, first derived by Lawden, are expressed in terms of the primer vector, which is defined as the adjoint to the velocity vector in the variational Hamiltonian formulation [8,9]. If any of Lawden's conditions are violated, the rendezvous trajectory is not optimal and we can use the primer vector history to obtain information on how to improve its

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Δv cost. In his initial work, Lawden solved a fixed-time rendezvous problem. His theory was extended by Lion and Handelsman [3] and later, by Jezewski and Rozendaal [4], to solve the *n*-impulse optimal rendezvous problem. For the rendezvous problem considered in this paper, we can express the primer vector equations using calculus of variations theory. The initial trajectory or first-guess is labeled as the reference trajectory. Since the primer vector is a first order theory based on local variations, it will converge to locally optimal neighboring trajectory of the reference trajectory. For the general dynamic equations

$$\begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\mu \dfrac{\mathbf{r}}{r^3} + \mathbf{\Gamma} \end{cases} \tag{1}$$

where the thrust acceleration is approximately by impulses, i.e. $\mathbf{\Gamma} = \sum_{i=1}^{n} \Delta \mathbf{v}_i \delta(t - t_i)$. The primer vector $\boldsymbol{\lambda}_v$ obeys the following equation,

$$\ddot{\boldsymbol{\lambda}}_v = \mathbf{G}(\mathbf{r})\boldsymbol{\lambda}_v \tag{2}$$

where $\mathbf{G}(\mathbf{r})$ is the gravity gradient matrix

$$\mathbf{G}(\mathbf{r}) = \frac{\partial - \mu \frac{\mathbf{r}}{r^3}}{\partial \mathbf{r}} = \frac{\mu}{r^3} \left\{ \frac{3}{r^2} \begin{bmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & yz & z^2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} \tag{3}$$

where $\mathbf{r} = (x, y, z)^T$ is the spacecraft position vector on the reference trajectory and μ is the earth's gravitational constant.

The spacecraft position is found using Eqs. (1). As evident in Eqs. (2), the primer vector state cannot be derived simultaneously with the spacecraft state. The spacecraft trajectory must be propagated first for the primer vector history to be computed. Using calculus of variations, Lawden derived four necessary conditions expressed in terms of the primer vector defined by Eqs. (2), for an optimal rendezvous trajectory:

(1) The primer vector $\boldsymbol{\lambda}_v$ and its first derivative $\dot{\boldsymbol{\lambda}}_v$ are continuous for $t \in [t_0, t_f]$.
(2) $t \in [t_0, t_f], |\boldsymbol{\lambda}_v| \le 1$, impulses occur when $|\boldsymbol{\lambda}_v| = 1$.
(3) At an impulse, the optimal impulse direction is $\boldsymbol{\lambda}_v / |\boldsymbol{\lambda}_v|$
(4) At an interior impulses (not at the initial or final times) $\dot{\lambda}_v = \dot{\boldsymbol{\lambda}}_v^T \boldsymbol{\lambda}_v = 0$.

These conditions are necessary (but not sufficient) for an optimal trajectory. When solving a rendezvous problem using primer vector theory, we first need to evaluate the primer vector history along the reference trajectory. Solving for the primer vector history is equivalent to solving a two point boundary value problem. For an *n*-impulse rendezvous trajectory with its known solution $(t_1, \Delta\mathbf{v}_1; t_2, \Delta\mathbf{v}_2; \ldots t_n, \Delta\mathbf{v}_n)$, we can obtain the primer vector history using the following shooting method.

(1) *Set boundary conditions*

Choose two impulses from the obtained impulses: $\Delta\mathbf{v}_i$, $\Delta\mathbf{v}_j$, $i \neq j$. We know from Lawden's conditions that, at an impulse, the primer vector is in the direction of the thrust vector, thus we can let

$$\boldsymbol{\lambda}_v^*(t_i) = \Delta\mathbf{v}_i / \Delta v_i$$
$$\boldsymbol{\lambda}_v^*(t_j) = \Delta\mathbf{v}_j / \Delta v_j \tag{4}$$

(2) *Determine* $\boldsymbol{\lambda}_r^*(t_0)$, $\boldsymbol{\lambda}_v^*(t_0)$

Letting $\boldsymbol{\lambda}_r(t_0)$, $\boldsymbol{\lambda}_v(t_0)$ and $\mathbf{r}(t_0)$, $\mathbf{v}(t_0)$ as initial conditions, integrate Eqs. (1) and (2), respectively and then obtain $\boldsymbol{\lambda}_v(t_i)$, $\boldsymbol{\lambda}_v(t_j)$;

use Newton method to solve the following six-degree non-linear equations

$$\boldsymbol{\lambda}_v(t_i) - \boldsymbol{\lambda}_v^*(t_i) = 0$$
$$\boldsymbol{\lambda}_v(t_j) - \boldsymbol{\lambda}_v^*(t_j) = 0 \tag{5}$$

and obtain the solution $\boldsymbol{\lambda}_r^*(t_0)$, $\boldsymbol{\lambda}_v^*(t_0)$.

(3) *Determine* $\boldsymbol{\lambda}_v(t)(t \in [t_0, t_f])$

with $\boldsymbol{\lambda}_r^*(t_0)$, $\boldsymbol{\lambda}_v^*(t_0)$ as initial state, compute the primer vector history by integrating Eqs. (2).

Once the primer vector history is computed, we can determine the optimality of the trajectory using Lawden's necessary conditions. If any of Lawden's conditions are violated, the rendezvous trajectory is not optimal and we can use the primer vector history to obtain information on how to improve its Δv cost. Improving non-optimal trajectories using primer vector theory is the main contribution of Lion and Handelsman [3]. For a non-optimal primer vector history, two types of actions are possible to lower Δv cost: (1) moving the time of the initial or final impulse and (2) adding and/or moving an interior impulse.

The general expression for time variations at both endpoints is given as

$$\delta J = -\dot{\lambda}_v(t_0)\Delta v_0 dt_0 - \dot{\lambda}_v(t_f)\Delta v_f dt_f \tag{6}$$

where the initial and final time variations are given by dt_0 and dt_f, respectively. To have a lower cost, δJ must be less than zero. If the following terminologies are defined: (1) initial coast: $dt_0 > 0$; (2) early departure: $dt_0 < 0$; (3) final coast: $dt_f < 0$; and (4) late arrival :$dt_f > 0$; then the following four cases cover all the non-zero primer slope combinations.

(1) If $\dot{\lambda}_v(t_0) > 0$ and $\dot{\lambda}_v(t_f) < 0 \Rightarrow$ apply initial coast and final coast.

(2) If $\dot{\lambda}_v(t_0) > 0$ and $\dot{\lambda}_v(t_f) > 0 \Rightarrow$ apply initial coast and late arrival.

(3) If $\dot{\lambda}_v(t_0) < 0$ and $\dot{\lambda}_v(t_f) < 0 \Rightarrow$ apply early departure and final coast.

(4) If $\dot{\lambda}_v(t_0) < 0$ and $\dot{\lambda}_v(t_f) > 0 \Rightarrow$ apply early departure and late arrive.

When no further improvement is achieved by varying the endpoints times, an additional impulse should be added. Assume a midcourse impulse is added at time t_m, the objective function variation is obtained considering the first order terms only as

$$\delta J = c(1 - \dot{\boldsymbol{\lambda}}_v(t_m)^T \boldsymbol{\eta}) \tag{7}$$

where the following are mid-impulse parameters: c is the magnitude of the impulse, and $\boldsymbol{\eta}$ is a unit vector in the direction of the impulse. Then, for an improvement in cost, $\delta J < 0$. The greatest decrease in the cost function will be achieved if the impulse is applied at the maximum of λ_v at time t_m and in the direction of $\boldsymbol{\lambda}_v(t_m)/\lambda_v(t_m)$. The mid-impulse should decrease the cost but might not produce an optimal trajectory in the sense of Lawden's conditions. Therefore, it is necessary to optimize the mid-impulse.

According to above descriptions, the improvement methods from a non-optimal rendezvous trajectory to an optimal rendezvous trajectory are diverse, besides all methods are dependent on time histories. Up to date, there is no strict mathematical theory to support how to choose impulse task series. Generally speaking, different task series will result in different optimization results. The widely employed methods are to use gradient-based algorithm to optimize simultaneously the initial impulse time, the final impulse time and the interior impulses' location and time [3–8], in which the gradient is calculated using the Lawden theory. As the local convergence of the gradient-based optimization algorithms, these methods always encounter converge problem and cannot guarantee the global solution. In this paper, we propose an interactive approach by employing the global convergence ability of evolutionary algorithms and directly observing the primer vector history.

3. Interactive optimization approach

3.1. Outline

The proposed interactive optimization approach outline is described as follows:

Step 1 Two-impulse reference solution

1.1 Let n=2, and assume the initial and final impulses are applied at t_0 and t_f, respectively. Obtain the two-impulse reference solution by using the Lambert algorithm.
1.2 Obtain the primer vector history using the shooting algorithm described in Section 2 and determine whether it satisfies Lawden's conditions.
1.3 If satisfied, go to Step 1.4; otherwise, go to Step 2.1.
1.4 Optimize the initial and final impulse times. If a smaller Δv is obtained, go to Step 2.3; otherwise, the two-impulse reference solution is the optimal solution, end.

Step 2. Add initial coast or final coast.

2.1 Determine whether to add initial and final coast by observing the primer-magnitude figure. For a two-impulse reference trajectory, Fig. 1 shows four

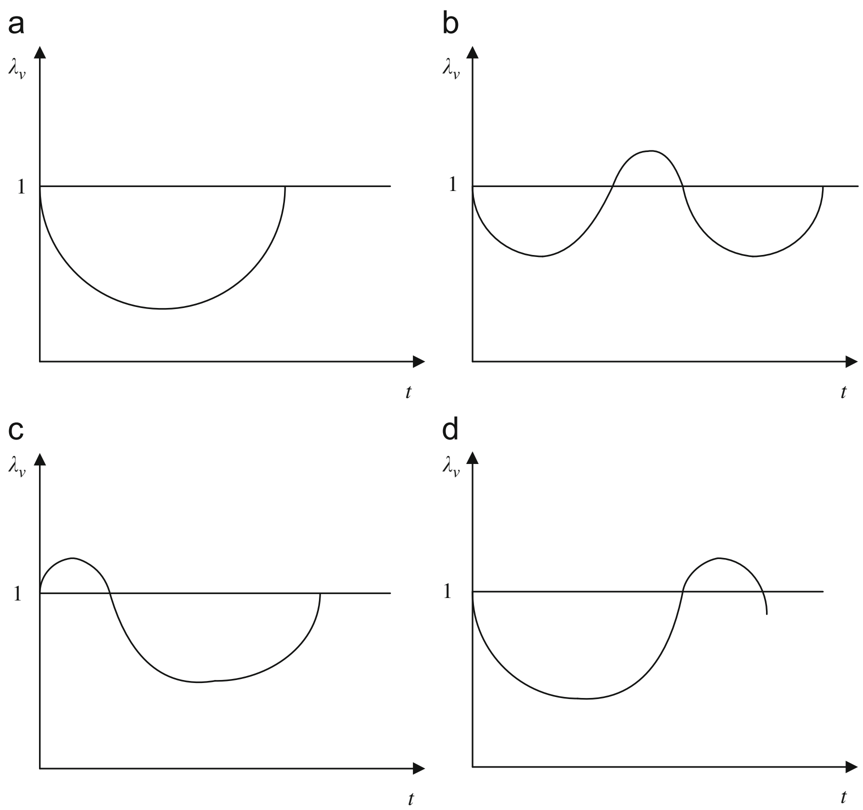

Fig. 1. Four representative primer magnitude categories. (a) Optimal, (b) non-optimal (additional impulse required), (c) non-optimal (initial coast required) and (d) non-optimal (final coast required).

representative primer vector histories corresponding to different improvement methods.

2.2 If the primer-magnitude figure matches Fig. 1c or d, the initial or final coast should be added. Let the initial and final impulse times as optimization variables, and obtain the improved two-impulse solution.

2.3 Determine whether this improved solution satisfies Lawden's conditions. If satisfied, end; otherwise, go to Step 3.

Step 3. Search mid-impulse.

3.1 Add a mid-impulse: n=n+1.

3.2 Execute the following optimization iteration: the evolutionary algorithms are employed to obtain the n-impulse optimal solution using the multi-impulse Lambert optimization programming models as described in Section 3.2.

Step 4. Re-compute n-impulse solution.

4.1 Obtain the primer-magnitude history corresponding to this n-impulse and determine whether this n-impulse solution satisfies Lawden's conditions.

4.2 If satisfied, end. Otherwise, considering the stochastic character of the evolutionary algorithms, re-execute Step 3.2 by ten times and determine whether the best solution of these ten runs satisfies Lawden's conditions.

4.3 If satisfied, end. Otherwise, go to Step 3.1.

In this optimization approach, the improvements including adding a mid-impulse are determined by observing the primer-magnitude histories. Thus, this optimization approach needs the designers' participation and it is interactive. Fig. 1 only provides four representative primer-magnitude histories. In fact, primer vector histories in practical application are not limited to these ones. Practical design would need more designers' experience.

3.2. Multi-impulse optimization model using the Lambert algorithm

The initial conditions for rendezvous are $\mathbf{r}_0, \mathbf{v}_0, t_0$ and the terminal conditions are $\mathbf{r}_f, \mathbf{v}_f, t_f$.

Assuming an impulsive $\Delta\mathbf{v}_i$ is applied, the superscript '$-$' indicates the state before an impulse, and '+' indicates the state after an impulse. We get

$$\begin{cases} \mathbf{r}_i^+ = \mathbf{r}_i^- \\ t_i^+ = t_i^- \\ \Delta\mathbf{v}_i = \mathbf{v}_i^+ - \mathbf{v}_i^- \end{cases} \tag{8}$$

without loss of generality, let

$$\begin{aligned} \mathbf{r}_i &= \mathbf{r}_i^+ = \mathbf{r}_i^- \\ t_i &= t_i^+ = t_i^- \end{aligned} \tag{9}$$

and assume that $\mathbf{r}(t+\Delta t) = \mathbf{f}(\mathbf{r}(t), \mathbf{v}(t), t, t+\Delta t)$ and $\mathbf{v}(t+\Delta t) = \mathbf{g}(\mathbf{r}(t), \mathbf{v}(t), t, t+\Delta t)$ are the solutions to Eqs. (1).

For an intermediate impulse $i \neq 1$, $i \neq n$, where $n(\geq 2)$ is the number of impulses, the following conditions must be satisfied

$$\begin{aligned} \mathbf{r}_i &= \mathbf{f}(\mathbf{r}_{i-1}, \mathbf{v}_{i-1}^+, t_{i-1}, t_i) \\ \mathbf{v}_i^- &= \mathbf{g}(\mathbf{r}_{i-1}, \mathbf{v}_{i-1}^+, t_{i-1}, t_i) \end{aligned} \tag{10}$$

The chosen independent variables, i.e. the optimization variables are impulse times and the first $n-2$ impulse vectors:

$$\begin{array}{ll} t_i & i = 1, 2, \ldots, n \\ \Delta\mathbf{v}_j & j = 1, 2, \ldots, n-2 \end{array} \tag{11}$$

and the objective function is

$$J = \Delta v = \sum_{i=1}^{n} |\Delta\mathbf{v}_i| \tag{12}$$

Calculating $\mathbf{r}_1$ and $\mathbf{v}_1^-$, and $\mathbf{r}_n$ and $\mathbf{v}_n{}^+$ using Eqs. (10), we have

$$\begin{cases} \mathbf{v}_i^+ = \mathbf{v}_i^- + \Delta\mathbf{v}_i \\ \mathbf{r}_{i+1} = \mathbf{f}(\mathbf{r}_i, \mathbf{v}_i^+, t_i, t_{i+1}) \\ \mathbf{v}_{i+1}^- = \mathbf{g}(\mathbf{r}_i, \mathbf{v}_i^+, t_i, t_{i+1}) \end{cases} \quad (i = 1, 2, \ldots, n-2) \tag{13}$$

The rendezvous conditions are satisfied by solving a Lambert's problem "Lambert $(\mathbf{r}_{n-1}, \mathbf{r}_n, t_n - t_{n-1})$". With the solution for the Lambert's problem, we can solve for $\Delta\mathbf{v}_{n-1}$ and $\Delta\mathbf{v}_n$ and then obtain the total Δv.

3.3. Evolutionary algorithms

Evolutionary algorithms (EAs) utilize principles of natural selection and are robust adaptive search schemes suitable for searching non-linear, discontinuous, and high-dimensional spaces. This class of algorithms is being increasingly applied to obtain optimal or near-optimal solutions to many complex real-world optimization problems. It has therefore been tempting for the EA research community at large to categorize these algorithms as universal problem solvers. The EA has many different approaches, for example, genetic algorithms, evolution strategies, evolutionary programming, genetic programming, and other improved editions. The parallel simulated annealing using the simplex method (PSASM) is a recently developed new global optimization algorithm

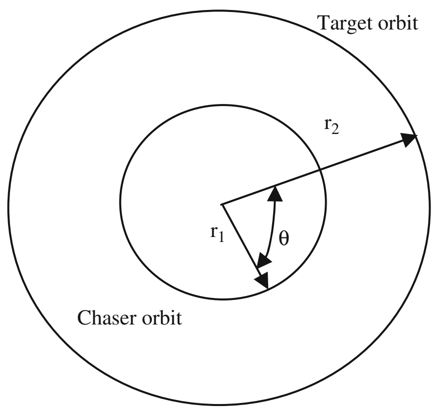

Fig. 2. Circle-to-circle rendezvous.

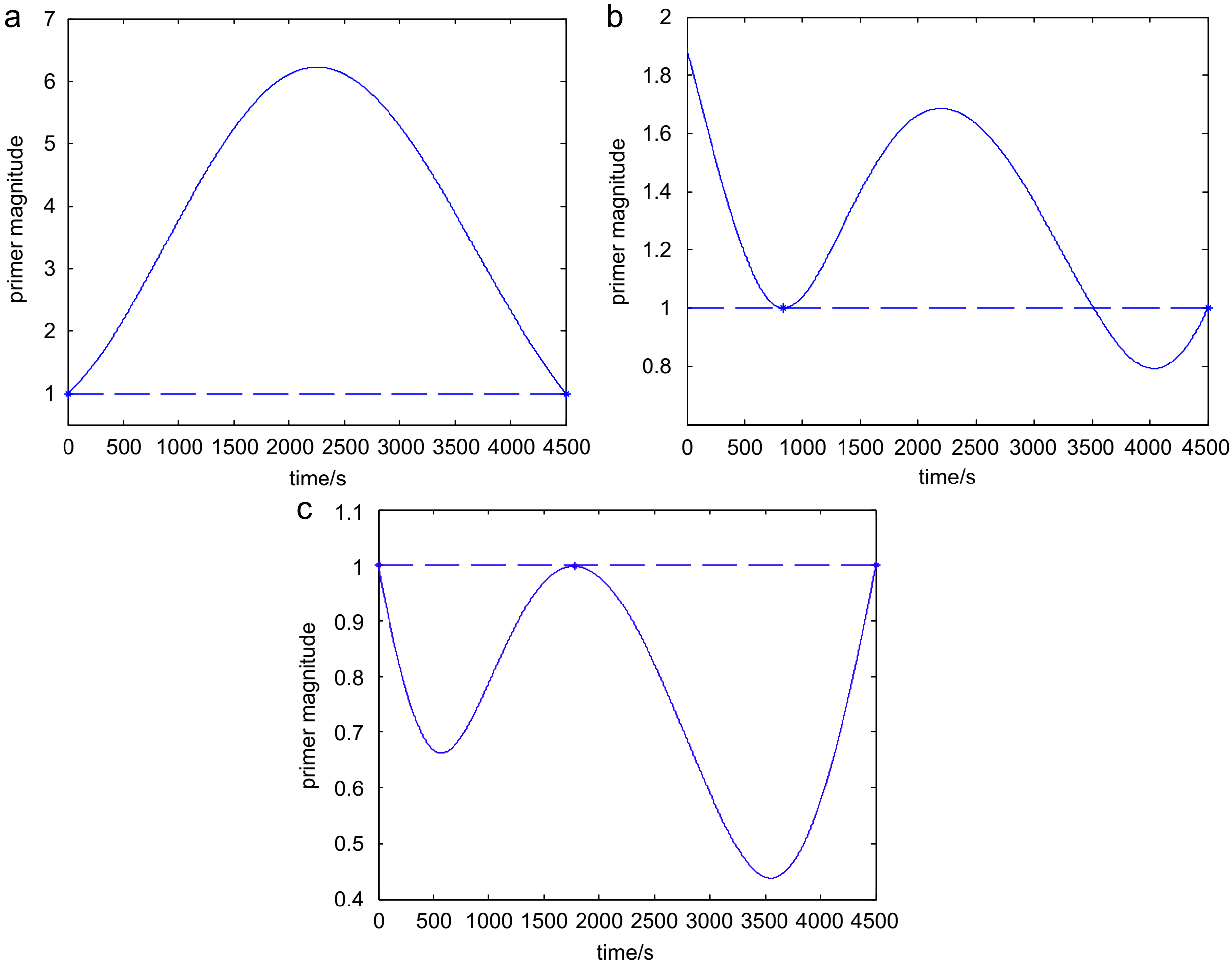

Fig. 3. The primer magnitude history of the coplanar circle-to-circle rendezvous (* denotes the impulse position): (a) two-impulse reference solution, (b) two-impulse best solution, (c) three-impulse optimal solution.

Table 1
Solutions for the coplanar circle-to-circle rendezvous problem.

Index	Impulses (t_i (s), Δv_i (m/s))			Δv (m/s)
	i=1	i=2	i=3	
Two-impulse reference	(0, 34.9)	(4500, 23.0)		57.9
Two-impulse best	(837.3, 25.1)	(4500, 13.9)		39.0
Three-impulse optimal	(0, 6.1)	(1779.8, 8.7)	(4500, 14.4)	29.3

[10]. The PSASM combines the advantages of simulated annealing with that of the simplex method. We have tested genetic algorithms, simulated annealing, hybrid genetic algorithm, and the PSASM in this interactive optimization approach. Among these evolutionary algorithms, the PSASM demonstrated the best performance. The results provide here are the results of the PSASM. The details of the PSASM can be found in [10] and in the test cases of this paper it is terminated after a total number of objective function calculations of 50,000.

4. Examples

The proposed approach is tested by three cases: circle-to-circle rendezvous, same-circle rendezvous and non-coplanar rendezvous.

4.1. Coplanar circle-to-circle rendezvous

We first test our approach by the case of coplanar circle-to-circle rendezvous as described in Fig. 2. Prussing and Chiu [6] solved this type of problems using primer vector method and gradient-based optimization techniques. In our test case: r_1 = 6748 km, r_2 = 6778 km, θ = 2°, t_f = 4500 s.

Fig. 3a plots the primer-magnitude time history of the two-impulse reference solution. Apparently, it does not satisfy the Lawden's conditions. First, the initial and final impulse times are optimized. The obtained two-impulse best solution's primer magnitude is plotted in Fig. 3b, which is similar as Fig. 1b. Therefore, a mid-impulse should be added. The three-impulse solution's primer-magnitude time history is plotted in Fig. 3c, which satisfies the Lawden's conditions. These three solutions for this circle-to-circle rendezvous problem are listed in Table 1.

4.2. Same-circle rendezvous

The proposed approach is further tested by a representative problem, i.e. the same-circle rendezvous problem of a chaser in a circular orbit with a target in that same circular orbit [6,7]. A chaser and its target are shown

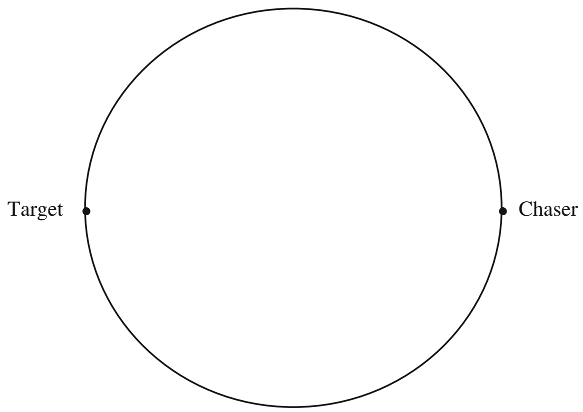

Fig. 4. Chaser and target in same circular orbit.

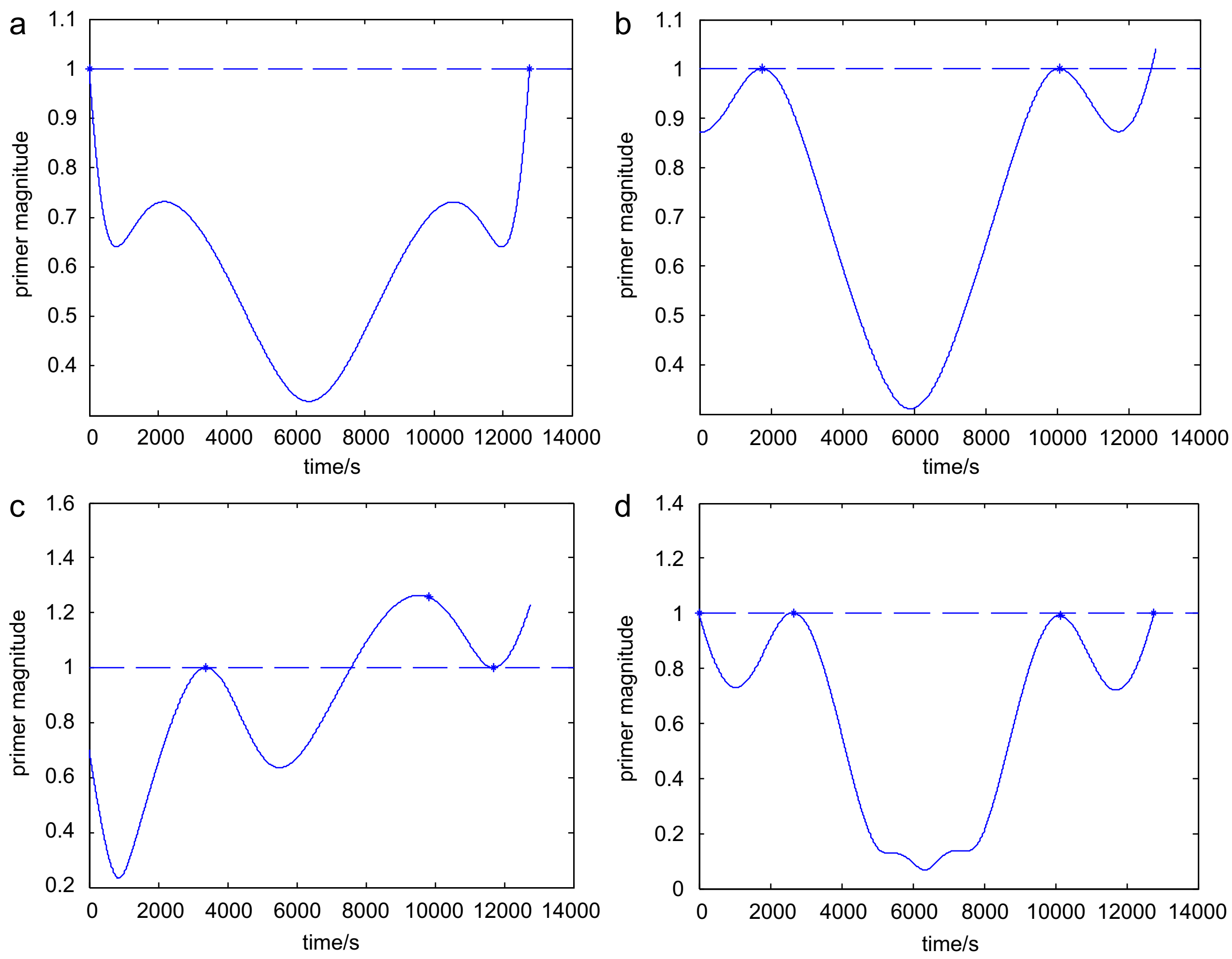

Fig. 5. The primer-magnitude time history of the same circle-to-circle rendezvous (* denotes the impulse position): (a) two-impulse reference solution, (b) two-impulse best solution, (c) three-impulse best solution, (d) four-impulse optimal solution.

Table 2
Solutions for the 400 km same-circle rendezvous problem.

Index	Impulses (t_i (s), Δv_i (m/s))				Δv (m/s)
	i=1	i=2	i=3	i=4	
Two-impulse reference	(0, 2309.7)	(12773.3, 2309.7)			4619.4
Two-impulse best	(1763.6, 859.7)	(10070.2, 859.7)			1719.4
Three-impulse best	(3375.9, 859.8)	(9825.1, 0.1)	(11685.1, 859.7)		1719.5
Four-impulse optimal	(0, 391.6)	(2648.9, 236.5)	(10130.1, 235.8)	(12773.3, 392.5)	1256.3

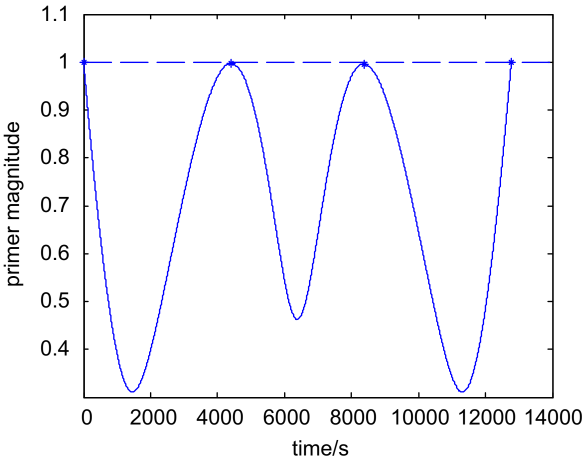

Fig. 6. The primer-magnitude time history of the other four-impulse solution for the same-circle rendezvous problem (Δv=1450.4 m/s).

in Fig. 4 for an initial separation angle of 180°. When the rendezvous transfer time is 2.3 times of the orbital period, Prussing and Chiu [6] and Prussing [7] reported that a four-impulse optimal solution existed for this transfer time. This same-circle rendezvous has a very high Δv (larger than 1200 m/s), mainly because that the phasing angle is too large (180°) while the transfer time is too short (2.3 orbits), therefore the chaser vehicle needs large velocity change to catch up with the target vehicle. If the transfer time increases to 30 times of the orbital period, the Δv reduces to less than 100 m/s. In this paper, we will show how the four-impulse solution is obtained by the interactive approach. Herein, a 400 km circular orbit case is tested by the proposed approach.

Fig. 5a plots the primer-magnitude time history of the two-impulse reference solution. It seems to satisfy the Lawden's conditions and Δv is 4619.4 m/s. However, we optimize the initial and final impulse times and obtain an improved solution with a smaller Δv of 1719.4 m/s. Apparently, the two-impulse reference solution is not the fuel-optimal solution even though its primer-magnitude time history seems to satisfy the Lawden's conditions. The two-impulse best solution's primer-magnitude time history is plotted in Fig. 5b. It does not satisfy the Lawden's conditions. Then, a mid-impulse is added and the three-impulse solution's primer-magnitude time history is plotted in Fig. 5c. It still does not satisfy Lawden's conditions. Another mid-impulse is added, and the four-impulse solution is obtained and its primer-magnitude time history is illustrated in Fig. 5d, which satisfies the Lawden's conditions. These four solutions for this same-circle rendezvous problem are listed in Table 2.

As demonstrated by Colasurdo and Pastrone [11], Sandrik [12] and Luo et al. [13], this same-circle rendezvous problem has an interesting peculiarity with two four-impulse solutions, both satisfying the Lawden's conditions. The other solution's primer-magnitude time history is plotted in Fig. 6. Its Δv is 1450.4 m/s and larger than the solution in Fig. 5d and Table 2.

4.3. Non-coplanar rendezvous

The third test case is a more practical rendezvous problem, a non-coplanar rendezvous problem. We describe the rendezvous problem using the classical orbit elements $\mathbf{E}=(a,\ i,\ e,\ \Omega,\ \omega,\ v)$, where a is the semimajor axis, i is the inclination, e is the eccentricity, Ω is the right ascension of the ascending node (RAAN), ω is the argument of perigee, and $v$is the true anomaly. The rendezvous initial conditions are

$$\mathbf{E}_{tar}=(6771.1\,\text{km}, 42^{\circ}, 0, 120^{\circ}, 0, 180^{\circ})$$
$$\mathbf{E}_{cha}=(6741.1\,\text{km}, 42.1^{\circ}, 0, 120.2^{\circ}, 0, 175^{\circ})$$

The whole transfer time equals two target orbital periods, t_f=11107.2 s.

Fig. 7a plots the primer-magnitude time history of the two-impulse reference solution. It seems to satisfy the Lawden's conditions. However, as Δv is 23449.5 m/s, it should not be the fuel-optimal solution. Δv is greatly reduced to 53.5 m/s by optimizing the initial and final impulse times. The obtained two-impulse best solution's primer-magnitude time history is plotted in Fig. 7b, the latter part is similar as Fig. 1b. Therefore, a mid-impulse should be added. The three-impulse solution's primer-magnitude time history is plotted in Fig. 7c. It still does not satisfy the Lawden's conditions. Another mid-impulse is added, and the four-impulse solution is obtained and its primer-magnitude time history is plotted in Fig. 7d, which satisfies the Lawden's conditions. These four solutions for this non-coplanar rendezvous problem are listed in Table 3.

5. Discussions

In these existing optimization approaches using primer vector theory for optimal rendezvous design [4–6,8], the

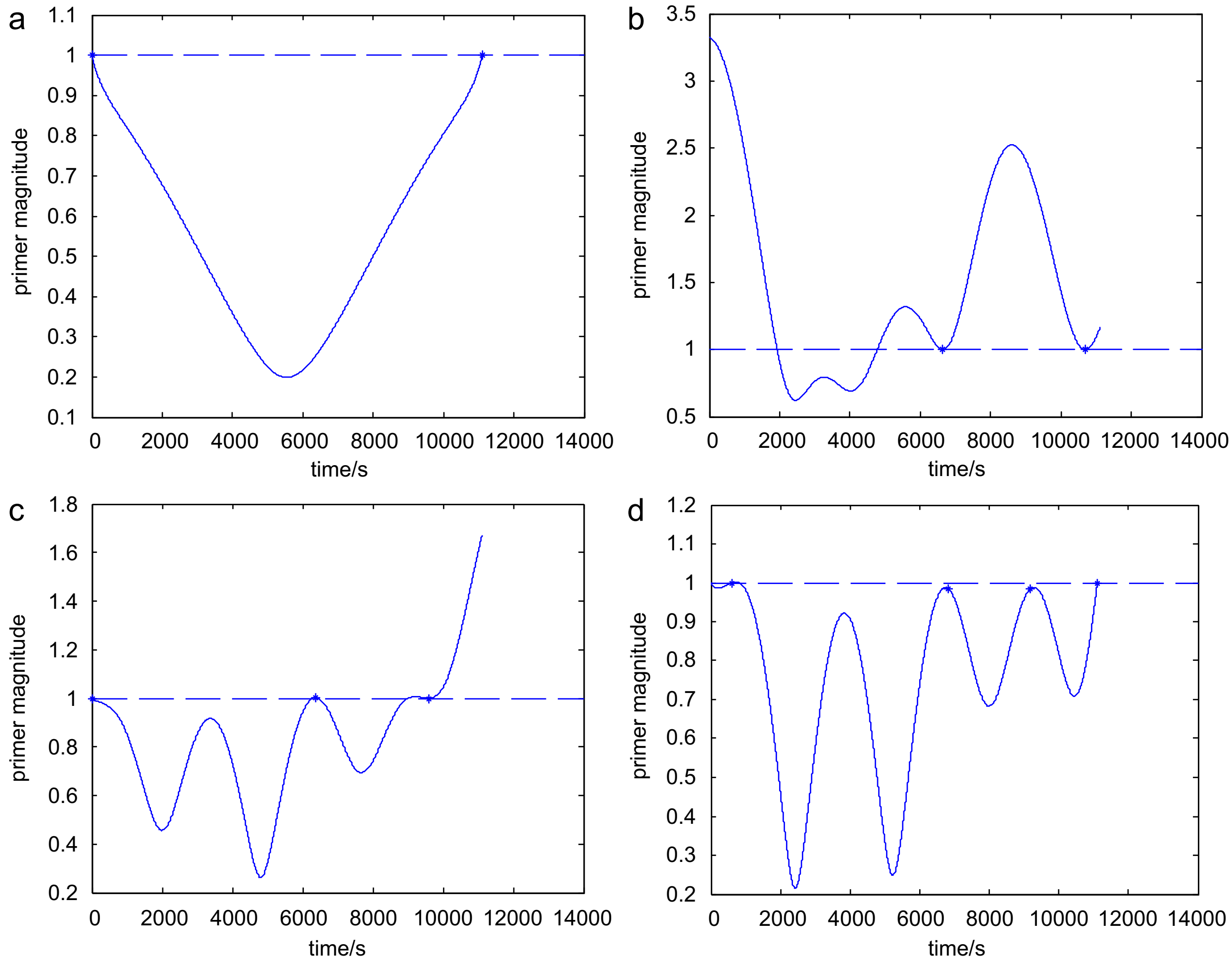

Fig. 7. The primer-magnitude time history of the non-coplanar rendezvous (* denotes the impulse position): (a) two-impulse reference solution, (b) two-impulse best solution, (c) three-impulse best solution and (d) four-impulse optimal solution.

Table 3
Solutions for the non-coplanar rendezvous problem.

Index	Impulses (t_i (s), Δv_i (m/s))				Δv (m/s)
	i=1	i=2	i=3	i=4	
Two-impulse reference	(0, 11740.9)	(11107.2, 11708.6)			23449.5
Two-impulse best	(6643.7, 37.3)	(10689.4, 16.2)			53.5
Three-impulse best	(0, 7.9)	(6373.4, 18.0)	(9603.9, 17.3)		40.2
Four-impulse optimal	(593, 5.6)	(6809.0, 10.2)	(9183.6, 11.9)	(11107.2, 8.5)	36.2

improvements including adding an impulse are determined by using the primer-magnitude time history. It is found from our experiment that we can easily determine which improvements should be made from observing primer-magnitude time history for those simple rendezvous problems as the test case of the coplanar circle-to-circle rendezvous where the transfer time is less than one orbital period. However, for these more practical and complex problems as the test cases of the same-circle rendezvous and the non-coplanar rendezvous where the transfer time is more than two orbital periods, the primer-magnitude time histories are not limited to these ones as in Fig. 1. They are more complex and various, as demonstrated in Figs. 5 and 7. Most of the time multiple actions for improvements are possible.

Besides, we observe that the fuel-optimal rendezvous with fixed-number of impulses has many local minims and even if two solutions have equal Δv cost, their

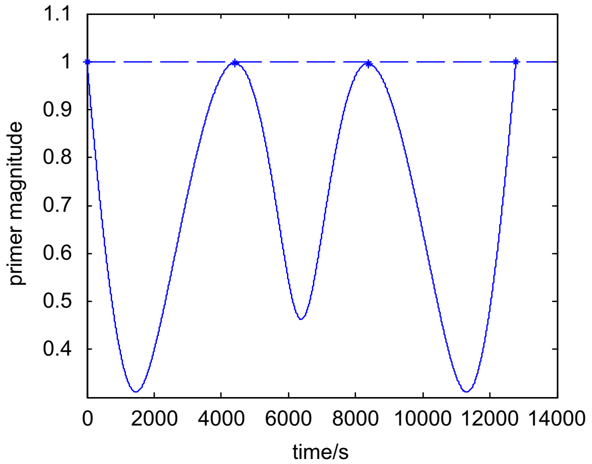

Fig. 8. The primer-magnitude time history of another three-impulse solution for the same-circle rendezvous problem (Δv=1720.8 m/s).

Table 4
Statistical results of the PSASM for the non-coplanar rendezvous (50 independent runs for each configuration).

Index	Δv (m/s)			
	Best	Worst	Mean	Std
Two-impulse best	53.5140	53.5140	53.5140	0
Three-impulse best	40.1948	40.2000	40.1954	8.6543e−004
Four-impulse optimal	36.1974	36.2478	36.2122	0.0124
Five-impulse best	36.2251	38.7801	36.7402	0.6409

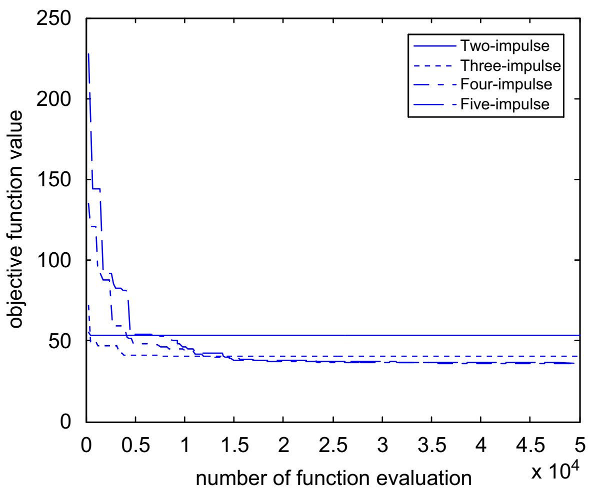

Fig. 9. The representative convergence time histories of the PSASM in obtaining the best solution for the no coplanar rendezvous (Δv=36.28 m/s).

primer-magnitude time history may be very different. Fig. 8 plots the primer-magnitude time history of another three-impulse solution for the same-circle rendezvous. It has only an error of 0.8% of Δv compared with the solution

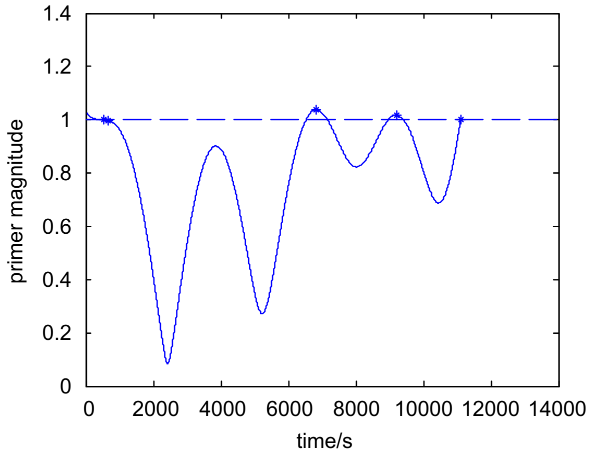

Fig. 10. The primer-magnitude time history of the five-impulse best solution for the non-coplanar rendezvous (Δv=36.22 m/s).

plotted in Fig. 5, but its primer vector history is evidently different from the latter.

Furthermore, as the Lawden's conditions are only necessary conditions for a fuel-optimal solution. That means one solution that satisfies the Lawden's conditions is not certain to be the optimal solution, which has been testified by the two-impulse reference solution of two latter test cases, and also by four-impulse solutions of the same-circle rendezvous. These two four-impulse solutions both satisfy the Lawden's conditions, but the second solution is not the fuel-optimal solution as its Δv is evidently larger than the first one.

In the gradient-based optimization approach in [4–6,8], it combines Eqs. (6) and (7) to form a unique cost function. This gradient is then used in the optimization algorithm to simultaneously move the endpoints times and the midcourse impulses positions. The optimization process is greatly dependent on the primer vector. However, as the above mentioned three peculiarities related to the primer vector, this type of approach is subjected to convergence problems in obtaining global solutions for complex rendezvous problems.

In our proposed interactive approach, these problems can be overcome. Only the primer-magnitude history is observed to define whether to add a mid-impulse, and no further information related to the primer vector is employed. The mid-impulses are directly searched in a large-scale space by solving an n-impulse problem using the PSASM. As the PSASM has good global convergence ability, it can easily determine the best n-impulse solution. Table 4 provides the statistical results of the PSASM in 50 independent runs for each configuration of the non-coplanar rendezvous problem. Fig. 9 plots the PSASM convergence time histories. From Table 4 and Fig. 9, it can be concluded that the PSASM is efficient and effective in obtaining the best solution for the n-impulse problem. If n-impulse is found better than $(n-1)$-impulse solution and it also satisfies the Lawden's condition, we would be confident that the n-impulse is the final global solution. For the non-coplanar rendezvous problem, the four-impulse solution is better than three-impulse, and also satisfies the Lawden's conditions. In order to testify the optimality of the four-impulse solution, we further test the five-impulse case. Fig. 10 plots the primer magnitude of the five-impulse best solution and Table 4 also reports the statistical results. From Table 4, Δv of the five-impulse is larger than the four-impulse. Besides, as demonstrated in Fig. 10, the second-impulse position always converges to overlap the first-impulse position. This also testifies that the four-impulse solution is the optimal solution.

6. Conclusions

An interactive optimization approach for fuel-optimal non-linear rendezvous problem is proposed by combing primer vector theory and evolutionary algorithms. Adding impulses by directly observing the primer-magnitude time history makes this approach intuitive and simple. Solution to n-impulse rendezvous optimization problem using the parallel simulated annealing using simplex method can guarantee the global solution. The proposed approach has been successfully applied to obtain the fuel-optimal solution with the optimal number of impulses for three test cases. The approach is especially useful for preliminary design for rendezvous mission with unknown optimal number of impulses. The disturbances such as J2, and atmospheric drag are not considered in this study. Future work will develop interactive global optimization approach for perturbed non-linear rendezvous.

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