3.

$$g_1(\underline{x}) = -\frac{1}{2}\begin{bmatrix} x_1-3 & x_2-6 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2-6 \end{bmatrix} - \frac{1}{2}\ln\left[\det\begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}\right]$$

$$g_1(\underline{x}) = -\frac{1}{2}\begin{bmatrix} 2x_1-6 & \frac{1}{2}x_2-3 \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2-6 \end{bmatrix} - \cancelto{0}{\frac{1}{2}\ln(1)}$$

$$g_1(\underline{x}) = -\frac{1}{2}\left[(2x_1-6)(x_1-3) + \left(\frac{1}{2}x_2-3\right)(x_2-6)\right]$$

$$g_1(\underline{x}) = -\frac{1}{2}\left[2x_1^2 - 6x_1 - 6x_1 + 18 + \frac{1}{2}x_2^2 - 3x_2 - 3x_2 + 18\right]$$

$$g_1(\underline{x}) = -\frac{1}{2}\left[2x_1^2 - 12x_1 + 18 + \frac{1}{2}x_2^2 - 6x_2 + 18\right]$$

$$g_1(\underline{x}) = -x_1^2 + 6x_1 - 9 - \frac{1}{4}x_2^2 + 3x_2 - 9$$

$$g_1(\underline{x}) = -x_1^2 + 6x_1 - \frac{1}{4}x_2^2 + 3x_2 - 18$$

$$g_2(\underline{x}) = -\frac{1}{2}\begin{bmatrix} x_1-3 & x_2+2 \end{bmatrix}\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2+2 \end{bmatrix} - \frac{1}{2}\ln\left[\det\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right]$$

$$g_2(\underline{x}) = -\frac{1}{2}\begin{bmatrix} \frac{x_1}{2}-\frac{3}{2} & \frac{x_2}{2}+1 \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2+2 \end{bmatrix} - \frac{1}{2}\ln(4)$$

$$g_2(\underline{x}) = -\frac{1}{2}\left[\left(\frac{x_1}{2}-\frac{3}{2}\right)(x_1-3) + \left(\frac{x_2}{2}+1\right)(x_2+2)\right] - \frac{1}{2}\ln(4)$$

$$g_2(\underline{x}) = -\frac{1}{2}\left[\frac{x_1^2}{2} - \frac{3}{2}x_1 - \frac{3}{2}x_1 + \frac{9}{2} + \frac{x_2^2}{2} + x_2 + x_2 + 2\right] - \frac{1}{2}\ln(4)$$

$$g_2(\underline{x}) = -\frac{1}{2}\left[\frac{x_1^2}{2} - 3x_1 + \frac{9}{2} + \frac{x_2^2}{2} + 2x_2 + 2\right] - \frac{1}{2}\ln(4)$$

$$g_2(\underline{x}) = -\frac{x_1^2}{4} + \frac{3}{2}x_1 - \frac{9}{4} - \frac{x_2^2}{4} - x_2 - 1 - \frac{1}{2}\ln(4)$$

$$g_2(\underline{x}) = -\frac{x_1^2}{4} + \frac{3}{2}x_1 - \frac{x_2^2}{4} - x_2 - \frac{13}{4} - \frac{1}{2}\ln(4)$$

(a) $g_1(\underline{x}) - g_2(\underline{x}) = 0 \Rightarrow$

$$-x_1^2+6x_1-\frac{1}{4}x_2^2+3x_2-18-\left(-\frac{x_1^2}{4}+\frac{3}{2}x_1-\frac{x_2^2}{4}-x_2-\frac{13}{4}-\frac{1}{2}\ln(4)\right)=0$$

$$-x_1^2+\frac{x_1^2}{4}+6x_1-\frac{3}{2}x_1-\cancel{\frac{1}{4}x_2^2}+\cancel{\frac{1}{4}x_2^2}+3x_2+x_2\overbrace{-18+\frac{13}{4}}^{-14,75}+\frac{1}{2}\ln(4)=0$$

$$-\frac{3x_1^2}{4}+\frac{(12-3)}{2}x_1+4x_2-14,056=0$$

$$4x_2=14,056-\frac{9}{2}x_1+\frac{3}{4}x_1^2$$

$$\therefore\ x_2=3,514-1,125x_1+0,1875x_1^2$$

(b) $x_2=3,514-1,125x_1+0,1875x_1^2$ é uma parábola

Interseção com o eixo x_1:

$$x_2=0 \Rightarrow 0,1875x_1^2-1,125x_1+3,514=0$$

$$x_1=\frac{1,125\pm\sqrt{\boxed{1,125^2-4\cdot 0,1875\cdot 3,514}}}{2\cdot 0,1875}$$ $<0 \rightarrow \nexists$ interseção com o eixo x_1.

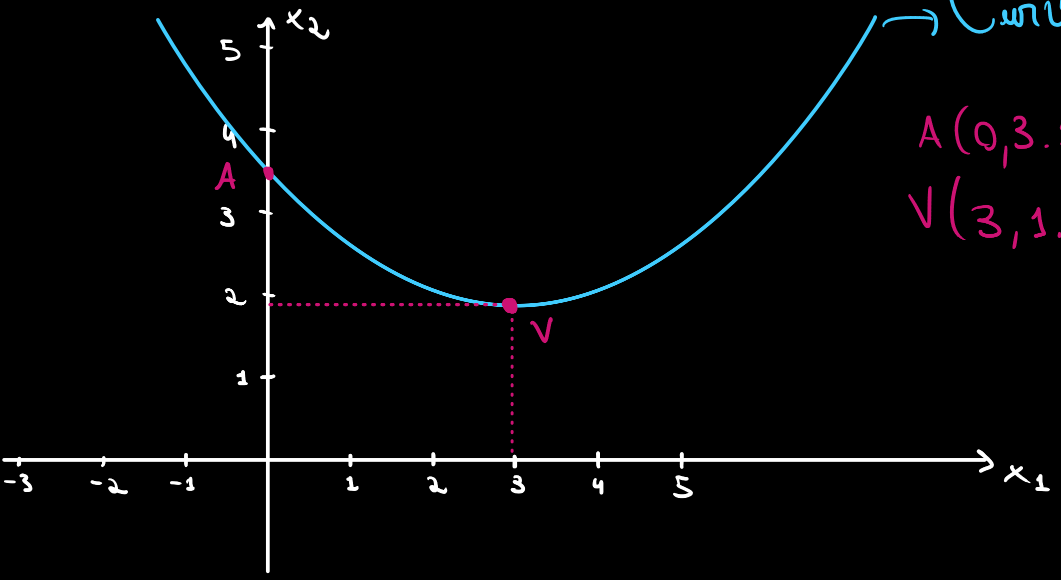
Interseção com o eixo x_2: $x_1=0 \Rightarrow x_2=3,514 \rightarrow A(0,3.154)$

Vértice: $\frac{\partial x_2}{\partial x_1}=0 \Rightarrow \frac{\partial}{\partial x_1}\left[3,514-1,125x_1+0,1875x_1^2\right]=0$

$$-1,125+2\cdot 0,1875x_1=0 \Rightarrow x_1=\frac{1,125}{2\cdot 0,1875}=3$$

$$x_2(3)=3,514-1,125\cdot 3+0,1875\cdot 3^2=1,826$$

$\Big\}\ V(3,1.826)$

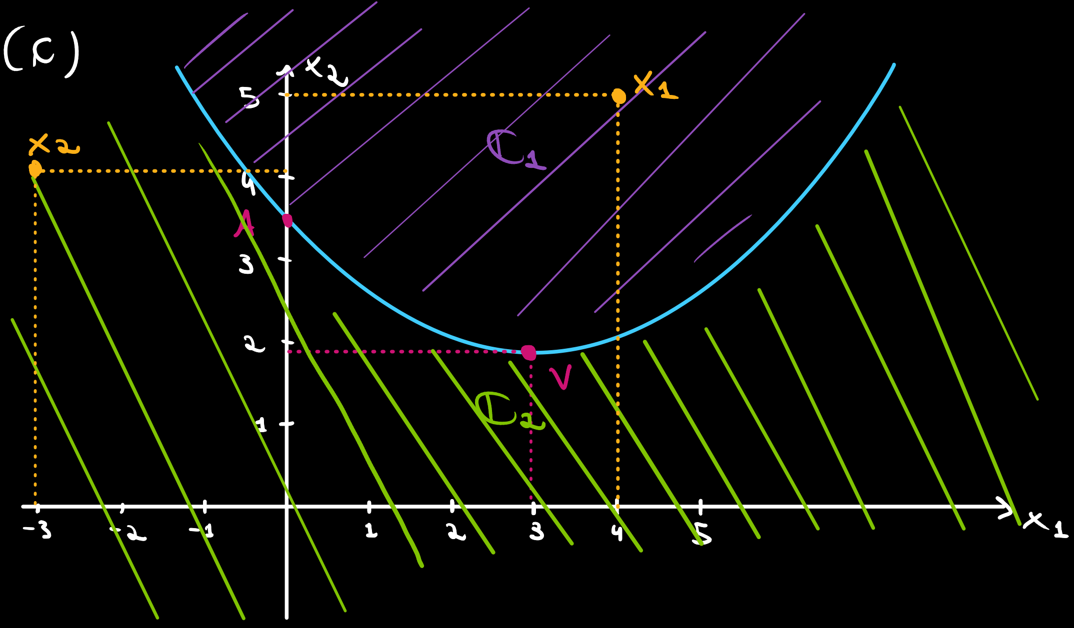


Curva de decisão

A(0,3.154)

V(3,1.826)

(c)



A(0 3.154)

V(3, 1.826)

$x_1 = (4, 5)$

$x_2 = (-3, 4)$

Supondo que todo ponto que esteja acima da parábola pertence à classe C_1 e que todo ponto abaixo da parábola pertence à classe C_2 então, visualmente: $x_1 \in C_1$ e $x_2 \in C_2$. Algebricamente:

P/ x_1: $x_1 = (4,5)$ e $x_2 = 3,514 - 1,125x_1 + 0,1875x_1^2$

fazendo $x_2(4) = 3,514 - 1,125.4 + 0,1875.4^2 = 2,014$

Como a coordenada x_2 de x_1, ou seja 5 é maior que $x_2(4)$ então x_1 é um ponto acima da parábola $\therefore$ $x_1 \in C_1$

P/ x_1: $x_1 = (-3,4)$ e $x_2 = 3,514 - 1,125x_1 + 0,1875x_1^2$

fazendo $x_2(-3) = 3,514 - 1,125.(-3) + 0,1875.(-3)^2 = 8,5765$

Como a coordenada x_2 de x_2, ou seja 4 é maior que $x_2(-3)$ então x_1 é um ponto abaixo da parábola $\therefore$ $x_2 \in C_2$.