On immediate extensions of valued fields

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joint work with Franz-Viktor Kuhlmann

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If $(L|K,v)$ is a finite extension of valued fields, such that the extension of v from K to L is unique, then

$$[L:K] = p^n (vL : vK)[Lv : Kv]$$

where $n \geq 0$, $p = \mathrm{char} Kv$ if it is positive and $p = 1$ otherwise.

If $p^n > 1$, then $(L|K,v)$ is called a **defect extension**.

An extension $(F|K,v)$ of valued fields is called **immediate** if

$$(vF : vK) = [Fv : Kv] = 1.$$

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$K(x)$ - a rational function field over a field K

$(K(x), v) \qquad v\left(x^m \frac{f}{g}\right) = m, \quad \text{for } f, g \in K[x] \setminus \{0\}, \quad x \nmid f, g.$

$(K((x)), v) \qquad v\left(\sum_{i=m}^{\infty} a_i x^i\right) = m, \ \ a_m \neq 0.$

$vK(x) = \mathbb{Z} = vK((x)),$
$K(x)v = K = K((x))v$

- $(K((x))|K(x), v)$ is an immediate extension,
- $(K((x)), v)$ is (the) maximal immediate extension of $(K(x), v)$.

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Fact: Every valued field admits a maximal immediate extension.

Goals:

- Describe the structure of maximal immediate extensions of certain classes of valued fields.
- Describe the structure of maximal immediate **algebraic** extensions of certain classes of valued fields.
- Determine the classes of valued fields which admit unique (up to isomorphism) maximal immediate extensions.

Problem: Describing all possible extensions of a valuation from a given field (K, v) to a rational function field $L|K$.

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Do the maximal immediate extensions of a given valued field have finite or infinite transcendence degree?

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Do the maximal immediate extensions of a given valued field have finite or infinite transcendence degree?

Theorem 1

Take a henselian field (K, v) and an extension $(L|K, v)$ of finite transcendence degree. Assume that v is nontrivial on L and at least one of the following cases holds:

1) vL/vK is not a torsion group, or $Lv|Kv$ is transcendental;

2) vL/vK contains elements of arbitrarily high order,

3) Lv contains elements of arbitrarily high degree over Kv;

4) L contains an infinite separable-algebraic extension of K.

Then each maximal immediate extension of (L, v) has infinite transcendence degree over L. If in addition the cofinality of vL is countable, then already $(L, v)^c$ has infinite transcendence degree over L.

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A valued field is called **maximal** if it admits no proper immediate extensions.

Every maximal field (M, v) is:

- henselian,
- complete,
- defectless, i.e., $[L : M] = (vL : vM)[Lv : Mv]$ for every finite extension $L|M$,

A finite extension of maximal field is again a maximal field.

Theorem 2

Take a maximal field (K, v) of characteristic 0 or of positive characteristic p and finite p-degree. If $(L|K, v)$ is an algebraic extension, then the field (L, v) is maximal if and only if $L|K$ is finite.

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Uniqueness of maximal immediate extensions

A valued field (K, v) of residue characteristic p is called a Kaplansky field if it satisfies the following conditions:

(K1) if $p > 0$ then vK is p-divisible $\quad (pvK = vK)$,

(K2) the residue field Kv is perfect,

(K3) the residue field Kv admits no finite separable extension of degree divisible by p.

Theorem 3 (I. Kaplansky)

The maximal immediate extension of a Kaplansky field (K, v) is unique up to valuation preserving isomorphism over K.

- There are valued fields admitting non-isomorphic maximal immediate extensions.

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General assumption:
(K, v) is a henselian field of residue characteristic p such that:
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Theorem 4

Assume that (K, v) admits a maximal immediate extension of finite transcendence degree. Then

- *(K, v) admits no immediate separable-algebraic extensions,*
- *(K, v) admits no separable-algebraic defect extensions,*
- *the perfect hull of K is contained in the completion of K,*
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Assume that Γ/vK is a torsion group and $k|Kv$ is an algebraic extension, both countably generated.

When do we have an extension of v to the rational function field $K(x_1, \ldots, x_n)$ such that

$$vK(x_1, \ldots, x_n) = \Gamma \text{ and } K(x_1, \ldots, x_n)v = k\,? \tag{1}$$

Theorem 5

Assume in addition that (K1) and (K2) hold. Then there is an extension of v from K to $K(x_1, \ldots, x_n)$ such that (1) holds if and only if

- *at least one of the two extensions $\Gamma|vK$ and $k|Kv$ is infinite*

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Assume that $(vK : pvK)[Kv : (Kv)^p] = \infty$

Then in particular $[K : K^p] = \infty$.

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There is a class of valued fields which admit an algebraic maximal immediate extension as well as one of infinite transcendence degree.

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