Definably Discrete Complete Expansions of Ordered Fields

Jafar S. Eivazloo and **Rahim Moshtaghnazm**

Department of Mathematics, University of Tabriz,

P.O.Box: 51666-17766, Tabriz, Iran.

eivazloo@tabrizu.ac.ir, R_moshtagnazm@tabrizu.ac.ir

Abstract

Let $\mathcal{M} = (M, <, +, \cdot, 0, 1, \dots)$ be a first order expansion of an ordered field. We say that $\mathcal{M}$ is definably discrete complete if every definable discrete subset of M has a least upper bound element in $M \cup \{\infty\}$. These expansions in which every unary definable subset is discrete or has interior, are definably (Dedekind) complete. Type complete (locally o-minimal) structures have been studied in [3], [1], and [2]. If $\mathcal{M}$ is definably complete and type complete, then every definable subset of M is discrete or has interior (see [2]). Here, we prove the converse of this result: If $\mathcal{M}$ is definably discrete complete and every definable subset of M is discrete or has interior, then $\mathcal{M}$ is type complete.

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References

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