Exercise 10: Limits of Real Functions

Calculate the following limits:

1. $\lim_{x\to\pm\infty} x^5 - 3$, $\lim_{x\to\pm\infty} 4x^3 - 2x^7 + 103x^5$.
2. $\lim_{x\to\pm\infty} \frac{1}{x}$, $\lim_{x\to 0} \frac{1}{x}$, $\lim_{x\to\pm\infty} \frac{1}{x^2}$, $\lim_{x\to 0} \frac{1}{x^2}$
3. $\lim_{x\to\infty} \frac{x^4-3x^2+10x}{-2x^2-5}$, $\lim_{x\to -1} \frac{2x^2+x-1}{x+1}$
4. $\lim_{x\to\pm\infty} \frac{P_n(x)}{P_m(x)}$, where $P_k(x)$ is a real polynomial of order k, n is even, m is odd and $n > m$.

   Note: A real polynomial $P_k(x)$ is defined as $P_k(x) = \sum_{i=0}^{k} a_i x^i$, with $a_i \in \mathbb{R}$ and $a_k \neq 0$.
5. $\lim_{x\to\pm\infty} \sin(x)$, $\lim_{x\to\pm\infty} \tan(x)$
6. $\lim_{x\to 0} \frac{\sin(x)}{x}$, $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$
7. $\lim_{x\to\pm\infty} e^x$, $\lim_{x\to\pm\infty} e^{-x}$, $\lim_{x\to 0^+} \log(x)$, $\lim_{x\to\pm\infty} \log(x)$