

Matrices from Linear Transformations

Of course, this can be generalized to any transformation

$T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ as

The diagram illustrates the construction of a matrix M from a linear transformation T . Above the matrix, three boxes represent the transformed basis vectors: $T(\hat{e}_1)$ (green), $T(\hat{e}_2)$ (purple), and $T(\hat{e}_n)$ (orange). Arrows point from these boxes to the corresponding columns of the matrix M . The matrix is shown as a large set of parentheses containing three columns. The first column (green) contains elements a_{11} , a_{21} , a vertical ellipsis, and a_{n1} . The second column (purple) contains a_{12} , a_{22} , a vertical ellipsis, and a_{n2} . The third column (orange) contains a_{1n} , a_{2n} , a vertical ellipsis, and a_{nn} . Ellipses between the columns indicate that there are n columns in total. The elements a_{ij} are color-coded: the row index i is red and the column index j is blue.

$$M = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}.$$

The numbers a_{ij} are called the **elements** of the Matrix, where i is the **row** of the element, and j is the **column** of the element.

In addition, each column of the matrix tells us how the respective standard basis vector is transformed.