Exercise 14: Exam Preparation (Taken From Summer Semester 2016) (Solution)

Linear Algebra (Vectors)

The following three vectors in $\mathbb{R}^3$ are given:

$$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}, \ \vec{c} = \begin{pmatrix} -16 \\ 0 \\ -10 \end{pmatrix}$$

1. Calculate the vector $2\vec{a} + \vec{b}$.

Answer:
Multiplying a vector by a scalar is done element-wise. Thus:

$$\begin{aligned} 2\vec{a} &= \begin{pmatrix} 1 \cdot 2 \\ -2 \cdot 2 \\ 2 \cdot 2 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \end{aligned}$$

Similarly, adding to vectors is done element-wise. Thus:

$$\begin{aligned} 2\vec{a} + \vec{b} &= \begin{pmatrix} 2+6 \\ -4+4 \\ 4+1 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ 0 \\ 5 \end{pmatrix} \end{aligned}$$

2. Calculate the inner product $\vec{a} \cdot \vec{b}$.

Answer:
The inner product of two vectors is equal to the sum of the element-wise product. Thus:

$$\begin{aligned} \vec{a} \cdot \vec{b} &= 1 \cdot 6 + (-2) \cdot 4 + 2 \cdot 1 \\ &= 6 - 8 + 2 \\ &= 0 \end{aligned}$$

3. What is the angle between $\vec{a}$ and $\vec{b}$?

Answer:
The angle θ between any two vectors can be derived from their inner product: $\theta = \arcsin\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right)$.In this case, $\vec{a} \cdot \vec{b} = 0$. Since $\arcsin(0) = 90°$, the angle between the two vectors is $90°$ (i.e. they are orthogonal).

4. Are the three vectors $\vec{a}, \vec{b}, \vec{c}$ linearly independent? Prove your answer.

Answer:

We can construct $\vec{c}$ as a linear combination of $\vec{a}$ and $\vec{b}$ using the scalars -4 and -2, respectively:

$$\begin{aligned} -4\vec{a} - 2\vec{b} &= \begin{pmatrix} -4 \cdot 1 \\ -4 \cdot -2 \\ -4 \cdot 2 \end{pmatrix} + \begin{pmatrix} -2 \cdot 6 \\ -2 \cdot 4 \\ -2 \cdot 1 \end{pmatrix} \\ &= \begin{pmatrix} -4 - 12 \\ 8 - 8 \\ -8 - 2 \end{pmatrix} \\ &= \begin{pmatrix} -16 \\ 0 \\ -10 \end{pmatrix} \\ &= \vec{c} \end{aligned}$$

Therefore, the vectors are linearly depended.

Another way to solve this problem is two treat all three vectors togheter as a matrix: $M = \begin{pmatrix} 1 & 6 & -16 \\ -2 & 4 & 0 \\ -16 & 0 & -10 \end{pmatrix}$. We can then calculate the determinant of this matrix:

$$\begin{aligned} \det(M) &= 1 \cdot 4 \cdot (-10) + 0 + (-2) \cdot 1 \cdot (-16) - 2 \cdot 4 \cdot (-16) - 0 - (-10) \cdot (-2) \cdot 6 \\ &= -40 + 32 + 128 - 120 \\ &= 0 \end{aligned}$$

A determinant which equals 0 means that the columns (and rows) of the matrix are linearly depended. Since the columns of M are the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, these vectors are themselves linearly depended.

5. Give a geometrical description of the shpae of the set of points in space described by $\left\{\vec{x} \in \mathbb{R}^3 \mid \vec{c} \cdot \vec{x} = 0\right\}$. (Note: $\vec{c} \cdot \vec{x}$ dontes the inner (scalar) product of $\vec{c}$ and $\vec{x}$)

Answer:
An inner product between two vectors equaling 0 means that the vectors are orthogonal. Therefore, we are looking for all vectors $\vec{x}$ that are orthogonal to $\vec{c}$. These vectors together form a plane that is itself orthogonal to $\vec{c}$.

Linear Algebra (2×2 matrices and linear mapping)

The matrix $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ is given.

1. What is the rank of A? Give a reason for your answer.

Answer:
Since the columns (or rows, for that matter) of the matrix are linearly independent, the rank of the matrix is equal to its dimensionality: rank $= 2$.

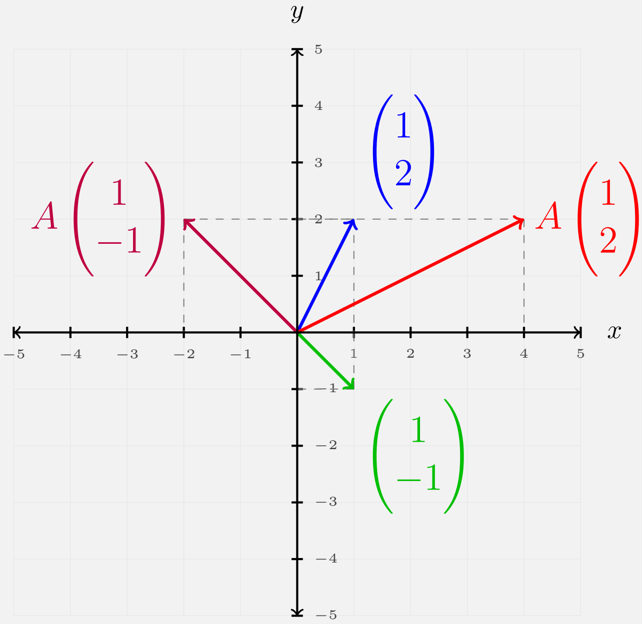
2. Calculate $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Draw the vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ in a Cartesian coordinate system.

Answer:
First, let's calculate $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$:

$$A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \cdot 1 + 2 \cdot 2 \\ 2 \cdot 1 + 0 \cdot 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \cdot 1 + 2 \cdot -1 \\ 2 \cdot 1 + 0 \cdot -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

Drawing all the requested vectors yields the following figure:



3. The linear mapping (i.e. linear transformation) associated with A is $f : \vec{x} \mapsto A \cdot \vec{x}$.
   Describe (in words) how an arbitrary vector $\vec{x}$ is transformed geometrically by f.

Answer:
The first column of the matrix is equal to the result of applying the matrix on the basis vector $\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. We can see that its x- and y components have been switched, and the y-component multiplied by 2. Similarly, The second column of the matrix is the result of applying the matrix on the basis vector $\vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Almost the same exact thing happens to this vector: its x- and y components are switched, but this time its x-component is multiplied by 2. Since any vector in $\mathbb{R}^2$ is a linear combination of these two vectors, the result for such vectors would be the same: their x- and y components would be switched, and both these component multiplied by 2. Switching the two components is equivalent to mirroring the vector by a line going through the origin at a 45° angle.

4. Calculate the matrix A^2.

Answer:

$A^2 = A \cdot A$. Thus:

$$\begin{aligned} A^2 &= \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 \cdot 0 + 2 \cdot 2 & 0 \cdot 2 + 2 \cdot 0 \\ 2 \cdot 0 + 0 \cdot 2 & 2 \cdot 2 + 0 \cdot 0 \end{pmatrix} \\ &= \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \end{aligned}$$

5. Calculate the determinant of A.

Answer:

$$\det(A) = 0 \cdot 0 - 2 \cdot 2 = -4$$

A way to quickly see that the determinant in this case equals -4 is to consider the change in an area by the matrix (since the determinant in $\mathbb{R}^2$ tells us the change in the area due to applying the matrix): flipping two axes means changing the sign of an area, and scaling each axis by 2 means scaling each area by 4. Hence, we get an overall change of -4.

6. Determine the matrix A^{-1} (if it exists).

Answer:
We first notice that the inverse matrix A^{-1} does exist, since the determinant of A is not zero. Now, looking at the solution to question 4: if we multiply the resulting matrix by $\frac{1}{4}$ we get the identity matrix I_2. Thus, the matrix $B = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$, which is simply $B = \frac{1}{4}A$, will achieve this exact solution. Since now $A \cdot B = I_2$, we know that $B = A^{-1}$.

7. Determine the eigenvalues of A.

Answer:
In order to find all eigenvalues of a matrix A, we should solve the equation $|A - \lambda I| = 0$ (with I being the identity matrix):

$$\begin{aligned} |A - \lambda I| &= \left| \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| \\ &= \left| \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| \\ &= \begin{vmatrix} 0 - \lambda & 2 \\ 2 & 0 - \lambda \end{vmatrix} \\ &= (-\lambda)(-\lambda) - 4 \\ &= \lambda^2 - 4 \end{aligned}$$

This means that the eigenvectors are the solution of the equation $\lambda^2 - 4 = 0$, or $\lambda^2 = 4$, i.e. $\lambda = -2, 2$.

Linear Algebra (Larger Matrices and Linear Systems)

The matrix $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 7 \end{pmatrix}$ is given.

1. Calculate the product $A \cdot \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix}$.

Answer:
Using matrix multiplication we get:

$$\text{row}=1,\ \text{col}=1: \quad \overset{\boldsymbol{A}}{\begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 7 \end{pmatrix}} \quad \overset{\boldsymbol{B}}{\begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix}} \Rightarrow c_{11} = [2 \times 2] + [1 \times 0] + [3 \times \text{-}1] = 1$$

$$\text{row}=1,\ \text{col}=2: \quad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 7 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow c_{12} = [2 \times 0] + [1 \times 1] + [3 \times 0] = 1$$

$$\text{row}=2,\ \text{col}=1: \quad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 7 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow c_{21} = [1 \times 2] + [1 \times 0] + [2 \times \text{-}1] = 0$$

$$\text{row}=2,\ \text{col}=2: \quad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 7 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow c_{22} = [1 \times 0] + [1 \times 1] + [2 \times 0] = 1$$

$$\text{row}=3,\ \text{col}=1: \quad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 7 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow c_{31} = [4 \times 2] + [2 \times 0] + [7 \times \text{-}1] = 1$$

$$\text{row}=3,\ \text{col}=2: \quad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 7 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow c_{32} = [4 \times 0] + [2 \times 1] + [7 \times 0] = 2$$

And thus, $A \cdot \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 2 \end{pmatrix}$

2. $A \cdot \vec{x} = \vec{b}$, with $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, is a system of $m = 3$ linear equations for $n = 3$ unknowns. How many unknowns can be chosen arbitrarily? (Give a reason for your answer)

Answer:
We look at the matrix generated by appending $\vec{b}$ to A, and bringing it to its row-echelon form:

$$\begin{pmatrix} 2 & 1 & 3 & 1 \\ 1 & 1 & 2 & 0 \\ 4 & 2 & 7 & 2 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 1 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ 4 & 2 & 7 & 2 \end{pmatrix} \xrightarrow[R_2 \to -R_2]{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & -1 \\ 4 & 2 & 7 & 2 \end{pmatrix} \xrightarrow{R_3 \to R_3 - 4R_1} \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

We can see that the result is a row of zeros. Therefore the rank of the matrix is 2, and thus there is one unknown (as $3 - 2 = 1$) that can be chosen arbitrarily.

Computer Science (Programming)

The following Java method `f` gets an integer array `x` and a single integer `a` as its arguments:

```java
public int f(int x[], int a)
{
    int i = 0;
    boolean b = true;
    while (b && (i < x.length))
    {
        if (x[i] == a)
            b = false
        else
            i = i+1;
    }
    if (b)
    {
```

```
            println("Error!");
            return -1;
        }
        else
            return i;
}
```

What does the function return?

Answer:
The first two lines of the function (except its header) do the following: set new integer variable `i` to `0`, and setting a boolean variable `b` to `true`:

```
    int i = 0;
    boolean b = true;
```

Then, a new loop is run, so long as two conditions are met:

1. `b` is `true` (simply written as `while b`), and
2. `i` is smaller than the length of the array `x`.

If any of these two conditions is not met, the loop stops.
While the loop runs, the function checks whether the `i`-th element in the array (counting the first element as `0`) equals the integer `a` which is an argument passed to the function by the user (see the function's header). This can result in two ways:

1. The `i`-th element indeed equals `a`, in which case `b` is set to `false` (which will cause the loop to finish), or
2. The `i`-th element does not equal `a`, which will result in simply increasing the value of `i` by one.

When the while loop is finished, for whatever reason (either for some `i`, the `i`-th element of the array `x` equals to `a`, or the value of `i` reached the size of the array `x`), the function checks the value of `b`. Once again, there are two options:

1. If `b` is true (which means that no element of the array `x` is equal to `a`), there's a problem: the function went over all the elements of the array `x`, and found no element that is equal to `a`. In this case the function prints `Error!` to screen, and returns `-1`.
2. If `b` is `true` it means that the function found an element of the array `x` that is equal to `a`. It then returns the value of `i`. Since `i` counts the elements of the array `x` until it reaches an element that is equal to `a` (or iterates over the entire array without finding anything), the value of `i` is the position in the array `x` of the element that is equal to `a`.

To summarize: the function checks whether there is an element in the array `x` which equals to the integer `a` (given to it as an argument). If there is such element (or more than one), it returns the index of its first occurrence in the array `x`. If there isn't, it returns `-1` and writes `Error!` to the screen.

Computer Science (Representation of Numbers)

1. What is the binary representation of the decimal number 63?

Answer:
We repeatedly divide 63 by 2 until reaching 1, noting the remainder. Then, we take the resulting remainders from bottom up, and that is the answer:

$63_{10} \rightarrow$ base-2

Result	Quotient	Remainder
63	31	1
31	15	1
15	7	1
7	3	1
3	1	1
1	0	1

Answer: $63_{10} = 111111_2$

There is however a quick answer: since 64 is a power of 2 ($64 = 2^6$), the binary representation of 64 is 1, followed by some number of 0's (in this case 6 zeros). Thus, 63 would be simply represented by six 1's (think for example of any power of 10, say 10000, minus 1: it's the same behavior, except in the binary case the highest digit is 1 instead of 9).

2. What is the decimal representation of the hexadecimal number 2A5?

Answer:
In hexadecimal numbers, each digit stands for a multiplication of 16 to the power of $0, 1, 2, \ldots$, counting from the right:

2	A	5
16^2	16^1	16^0

Thus, the number 2A5 in hexadecimal form is equal to:

$$\begin{aligned} 2 \times 16^2 + A \times 16^1 + 5 \times 16^2 &= 2 \times 256 + 10 \times 16 + 5 \times 1 \\ &= 512 + 160 + 5 \\ &= 677 \end{aligned}$$

(Remember that in base-16, $A = 10$)

3. Give the 8-bit two's complement representation of the decimal number -84.

Answer:
$84 \xrightarrow[\text{8 bit}]{\text{binary}} 01010100 \xrightarrow{\text{invert}} 10101011 \xrightarrow{\text{add 1}} 10101100 = -84_{10}$

4. What is the exact value of the fraction which is represented in the ternary base (base-3) by $0.222222222\ldots$ (i.e. with infinitely many $2s$ after the period)?

Answer:
Setting $x = 0.2222\ldots$, we get $3x = 2.2222\ldots$ (since in base-3 multiplying by 3 is like moving the decimal point one place to the left, same as multiplying by 10 in base-10). But $2.2222\ldots$ is like adding 2 to $0.2222\ldots$, and thus we get $3x = 2 + x$, or simply $x = 1$.
Notice how this is equivalent to the number $0.9999\ldots$ in base-10.

Computer Science (Rule-Based Simulation)

1. Given the following L-system: A $\rightarrow$ (RF)FA,
which string is produced after 3 steps of application of this rule to the start word 'A'?
Draw the graphical structure in the place which is obtained from this string by the following turtle interpretation:

- R: Rotate 45 degrees clockwise.
- F: Move forward 1 unit.

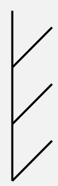
- A: Do nothing.
- (: Add position and angle to stack.
-): Pop position and angle from stack.

Answer:
The rule simply state that that in each step the letter A is replaced with the string '(RF)FA'. Since in the original string there is just one character, and that is 'A' - after the first step we get (RF)FA.
The second step replaces the single A at the end of the string '(RF)FA' with '(RF)FA', and thus after the second step we get '(RF)F(RF)FA'.
Essentially, each step simply concatenates '(RF)FA' to the end of the string (excluding the 'A'). After the third step this will yield '(RF)F(RF)F(RF)FA'.

The string will evolve as following:

0. A
1. (RF)FA
2. (RF)F(RF)FA
3. (RF)F(RF)F(RF)FA

Using the given drawing rules, the total string '(RF)F(RF)F(RF)FA' yields the following drawing:

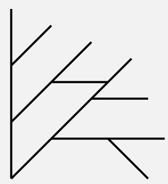


2. A modified L-system is given: A $\rightarrow$ [RFA]FA.
   Draw the corresponding graphical structure after 3 steps.

Answer:
Now there are two 'A's to replace at each step, and each replacement generates two new 'A's. Thus, the system will grow as follows:

0. A
1. (RFA)FA
2. (RF(RFA)FA)F(RFA)FA
3. (RF(RF(RFA)FA)F(RFA)FA)F(RF(RFA)FA)F(RFA)FA

And the drawing after three steps would be:



3. How does the number of single lines (obtained from an "F" symbol) grow (quantitatively) with the number of steps for L1? For L2?

Answer:
In the case of L1, each step N yields two 'F's. Thus, the change is linear: Number of lines $= 2N$. In the case of L2, we get the following number of 'F's in each successive step: $0, 2, 6, 14$. We can notice that these numbers are each a distance of 2 away from 2^{N+1} (i.e. $2, 4, 8, 16$). The growth is exponential, and given by Number of lines $= 2^{N+1} - 2$.

Calculus (Univariate Functions, Differentiation)

Given the following function: $f(x) = \frac{1}{3}x^3 - 4x^2 + 7x - 5$,

1. Find all x values where the function f has local extrema and classify them as minima or maxima.

Answer:
Local extrema are points where the derivative of a function equals 0. Thus, we solve for $f'(x) = 0$:

$$
\begin{aligned}
f'(x) &= \frac{3}{3}x^2 - 2 \cdot 4x + 7 \\
&= x^2 - 8x + 7 \\
&\Downarrow \\
0 &= x^2 - 8x + 7 \\
&\Downarrow \\
x_{1,2} &= \frac{8 \pm \sqrt{64 - 28}}{2} \\
&= \frac{8 \pm \sqrt{36}}{2} \\
&= \frac{8 \pm 6}{2} \\
&= 4 \pm 3 \\
&= 1, 7
\end{aligned}
$$

In order to classify the extrema, we need to use the second derivative of $f(x)$: $f''(x) = 2x - 8$. Substituting point $x = 1,\ x = 7$ in $f''(x)$ yields:

(a) $f''(x = 1) = 2 - 8 = -6 < 0$, and thus at $x = 1$ there is a local maximum.

(b) $f''(x = 7) = 14 - 8 = 6 > 0$, and thus at $x = 7$ there is a local minimum.

2. Find where the function is increasing/decreasing, and all x values of inflection points.

Answer:
There are two extrema (at $x = 1$ and $x = 7$), and thus we need to check three domains:

(a) $x < 1$

(b) $1 < x < 7$

(c) $x > 7$

Since at $x = 1$ the function is at a maximum, we expect that for $x < 1$ the function will increase. Let's verify it by substituting some $x < 1$ into $f'(x)$, say $x = 0$: $f'(x = 0) = \cancel{0^2} - \cancel{8 \cdot 0} + 7 = 7$. Indeed 7 is positive, and thus for $x < 1$ the function increases.

Similarly, we expect the function to decrease for $1 < x < 7$. Checking by substituting $x = 2$ into $f'(x)$: $f'(x = 2) = 2^2 - 8 \cdot 2 + 7 = 4 - 16 + 7 = -5$. Indeed, this shows that the function is decreasing for $1 < x < 7$.

Last but not least, for $x > 7$ we expect the function to increase again (owing to the minimum at $x = 7$). By substituting $x = 10$ we can see that $f'(x = 10) = 10^2 - 8 \cdot 10 + 7 = 100 - 80 + 7 = 27$, i.e. - the function indeed increases for this domain.

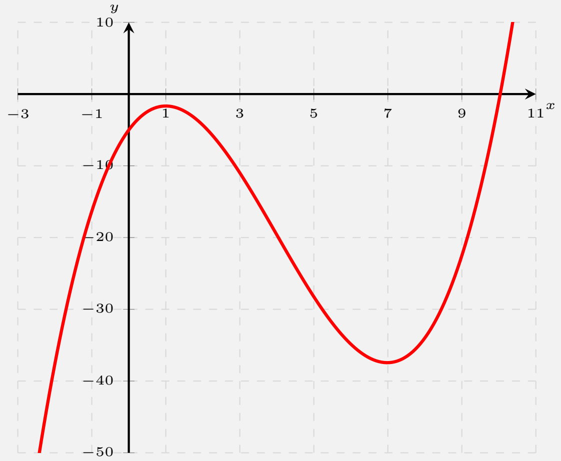
Summarizing:

(a) $x < 1$: $f(x)$ increases.

(b) $1 < x < 7$: $f(x)$ decreases.

(c) $x > 7$: $f(x)$ increases.

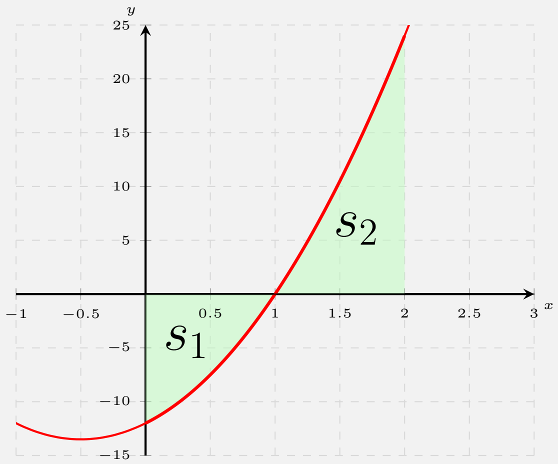
A plot of the function in the relevant domain, showing the extrema:



Calculus (Integration)

Compute the total area between the function $f(x) = 6x^2 + 6x - 12$, the x-axis and the lines $x_1 = 0$ and $x_2 = 2$.

Answer:
The given function is a parabola. It intersects the y-axis at $(0, -12)$, the x-axis at $(1, 0)$ and it has a minimum point at $x = \frac{1}{2}$.
Drawing this information (including the areas we wish to calculate) yields:



It can be seen that this area is divided in two parts (which we'll call s_1 and s_2): the part where $0 \leq x \leq 1$, in which the area is below the x-axis, and the part where $1 \leq x \leq 2$, where the area is above the x-axis. This means we should divide our integration in to two, and take care which function ($y = 6x^2 + 6x - 12$

or $y = 0$) we subtract from which function. On to the calculation:

$$
\begin{aligned}
S &= s_1 + s_2 \\
&= \int_0^1 \Big[0 - (6x^2 + 6x - 12)\Big]\,dx + \int_1^2 \Big[6x^2 + 6x - 12 - 0\Big]\,dx \\
&= \int_0^1 \Big[-6x^2 - 6x + 12\Big]\,dx + \int_1^2 \Big[6x^2 + 6x - 12\Big]\,dx \\
&= \Big[\Big|-2x^3 - 3x^2 + 12x\Big|\Big]_0^1 + \Big[2x^3 + 3x^2 - 12x\Big]_1^2 \\
&= \Big|\cancel{-2(0^3) - 3(0^2) + 12(0)} - (-2 - 3 + 12)\Big| + 2(2^3) + 3(2^2) - 12(2) - (2 + 3 - 12) \\
&= |2 + 3 - 12| + 2(8) + 3(4) - 12(2) - 2 - 3 + 12 \\
&= 7 + 16 + 12 - 24 + 7 \\
&= 18
\end{aligned}
$$