# Exercise 2: Vectors

## Problem 1: General Vectors Operations

The following column vectors are defined:

$$\vec{u} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$
$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 0 \\ 5 \end{pmatrix}, \quad \vec{c} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

1. Calculate $\vec{u} + \vec{v}$, $\vec{u} - \vec{w}$, $\vec{u} \cdot \vec{v}$, $\vec{u} \cdot \vec{w}$.
   What does the result for $\vec{u} \cdot \vec{v}$ mean for these two vectors?
2. Draw $\vec{u}$, $\vec{v}$, $\vec{u} + \vec{v}$, $-\vec{v}$ on a cartesian coordinate system.
3. Calculate $5\vec{a} - 3\vec{b}$.
4. Calculate $\vec{a} + \vec{w}$, $\vec{a} + \vec{b}$, $\vec{b} \cdot \vec{w}$, $\vec{a} \cdot \vec{c}$.
5. What are the lengths of $\vec{u}$, $\vec{v}$, $\vec{a}$ and $\vec{c}$?
6. What is the angle between $\vec{v}$ and the $x$-axis?
7. What would be the cartesian coordinates of the vector $\vec{v}$ rotated by $42°$ counter clockwise?
8. What is the angle between $\vec{a}$ and $\vec{b}$?
9. Calculate $\vec{c} = \vec{a} \times \vec{b}$. What is the general formula for all the vectors that are orthogonal to $\vec{c}$?

## Problem 2: Linear Combinations of Vectors

Write the vector $\vec{v} = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 7 \end{pmatrix}$ as a linear combination of the following vectors:

$$\vec{u}_1 = \begin{pmatrix} -2 \\ 5 \\ 0 \\ 5 \end{pmatrix}, \ \vec{u}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \ \vec{u}_3 = \begin{pmatrix} -4 \\ 4 \\ -8 \\ -2 \end{pmatrix}.$$

## Problem 3: Linear Independence of Vectors

Which of the following sets of vectors are linearly independent?

1. $\vec{a} = \begin{pmatrix} 1 \\ 0 \\ 3 \\ -2 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 2 \\ 6 \\ 0 \\ 1 \end{pmatrix}$
2. $\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} -2 \\ 4 \\ -10 \end{pmatrix}$
3. $\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}, \ \vec{c} = \begin{pmatrix} -9 \\ 1 \\ 6 \end{pmatrix}$
4. $\vec{a} = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \ \vec{c} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}, \ \vec{d} = \begin{pmatrix} -1 \\ -7 \\ 7 \end{pmatrix}$