Exercise 5: Eigenvectors and Eigenvalues

Problem 1: Geometric Interpretation

What are the eigenvectors and corresponding eigenvalues for the following transformations (answer without direct calculations)?

1. $T(\vec{v}) = -3\vec{v}$.
2. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$.
3. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$.

Problem 2: Calculating Eigenvectors and Eigenvalues

Calculate all eigenvectors and corresponding eigenvalues for the following transformations:

1. $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$
2. $\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
4. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
5. $\begin{pmatrix} 5 & 4 \\ 2 & -2 \end{pmatrix}$

Problem 3: Challange

What do you expect would are the eigenvectors and eigenvalues of the 3-dimensional rotation matrices by φ, ψ around the y- and z-axes, respectively? Explain and then calculate them directly. The two matrices are:

$$R^y_\varphi = \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) \\ 0 & 1 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) \end{pmatrix}, \quad R^z_\psi = \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$