Basic Maths for Non-mathematicians

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$$\int_a^b f(x)\,\mathrm{d}x = \lim_{\Delta x\to 0}\sum_{k=1}^{N} f(x_k)\Delta x$$

$$(AB)^\top = B^\top A^\top$$

$$\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m$$

$$\vec{v} = \sum_{i=1}^{n} \alpha_i \hat{e}_i$$

$$A = Q\Lambda Q^{-1}$$

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

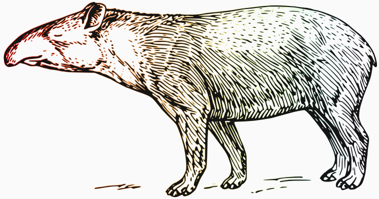
$$A\vec{v} = \lambda\vec{v}$$

$$\mathrm{Rot}(\theta) = \begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}$$

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

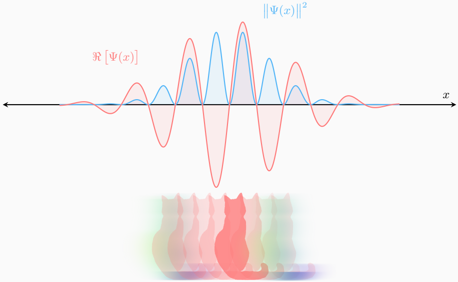
$$\langle \hat{e}_i, \hat{e}_j\rangle = \delta_{ij}$$

$$T\left(\alpha\vec{u} + \beta\vec{v}\right) = \alpha T\left(\vec{u}\right) + \beta T\left(\vec{v}\right)$$



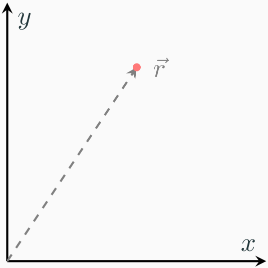
Chapter 10: Linear Algebra, Calculus and (a bit of) Quatum Physics

$$i\hbar\frac{\partial}{\partial t}\left|\Psi(t)\right\rangle = \hat{H}\left|\Psi(t)\right\rangle$$

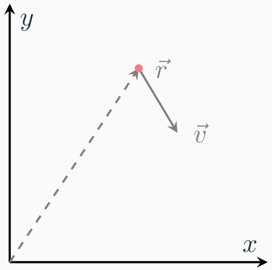


Classical Mechanics

In **classical mechanics** we charecterize a point-like particle as having a well-defined **position** represented as an $\mathbb{R}^3$ vector:



A particle also has a **velocity** (change of position over time), which is also represented by an $\mathbb{R}^3$ vector:



The position and velocity of a particle are time-dependent, i.e. they are functions of the time t:

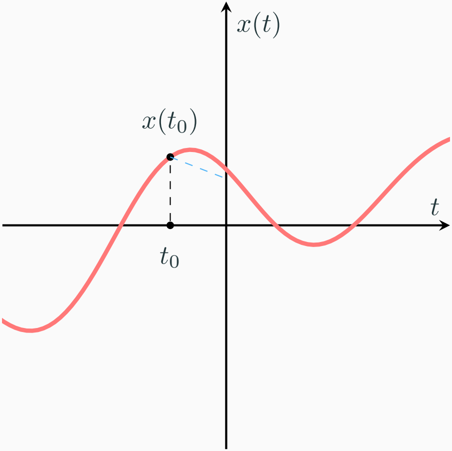
$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix},$$

$$\vec{v}(t) = \begin{pmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{pmatrix}.$$

Each component of $\vec{v}$ is the time derivative of the respective component of $\vec{r}$:

$$v_x(t) = \frac{\mathrm{d}x(t)}{\mathrm{d}t},$$
$$v_y(t) = \frac{\mathrm{d}y(t)}{\mathrm{d}t},$$
$$v_z(t) = \frac{\mathrm{d}z(t)}{\mathrm{d}t}.$$

For example, if we plot $x(t)$ over t (i.e. the x-position of a particle over time), the derivative at some time t_0 is the x-component of the velocity of the partile at that time:



MORE STUFF

Quantum Mechanics

Quantum mechanics, on the other hand, handles physical systems differently: a particle has no specific place nor velocity, but instead it has a function $\Psi : \mathbb{R}^3 \to \mathbb{C}$ which completely encodes all the information about the particle at any given time.

This kind of function is called a **wave function**.

The wave function of a particle by itself has no physical meaning.

However, the product $\Psi\Psi^* = |\Psi|^2$ (i.e. the product of the function with its complex-conjugate) **does** have a physical interpretation: at any given point $\vec{r}$, the value $|\Psi(\vec{r})|^2$ is the **probability density** to find the