

Definition

The **complete graph** K_n is the graph with n vertices where every pair of different vertices is connected by an edge (Also called a **clique**).

Example

The cliques K_1, \dots, K_6 :



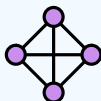
K_1



K_2



K_3



K_4



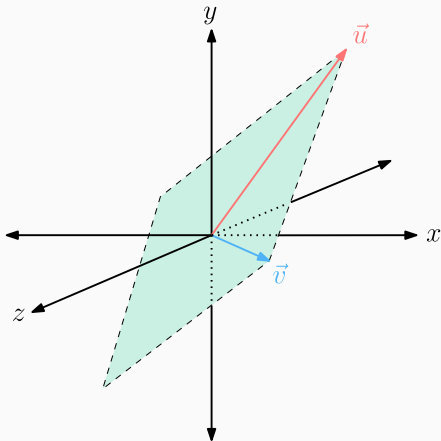
K_5



K_6

Subspaces

Similarly, any non-zero vector in \mathbb{R}^3 also spans a line going through the origin. In addition, any two linearly independent vectors span a **plane** going through the origin.

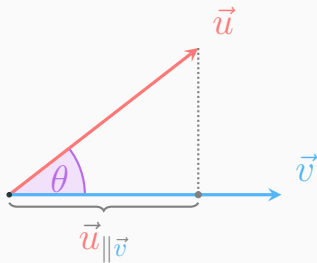


The Dot Product

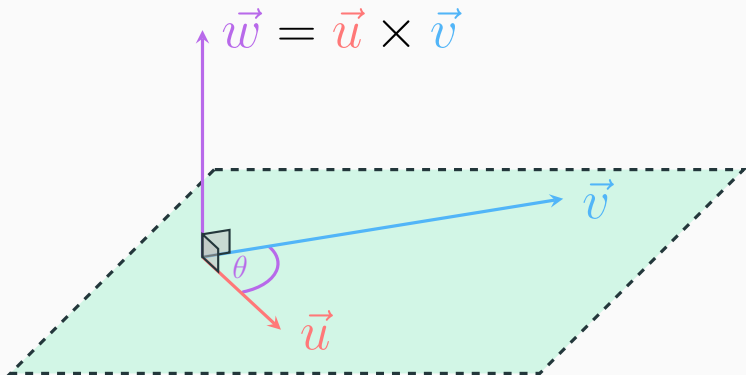
If we rotate the two vectors such that one of them lies on the horizontal direction, we can draw a perpendicular line from \vec{u} to \vec{v} . Using trigonometry we get

$$\cos(\theta) = \frac{\vec{u}_{\parallel\vec{v}}}{\|\vec{u}\|},$$

where $\vec{u}_{\parallel\vec{v}}$ is the length of the projection of \vec{u} on \vec{v} .



The Cross Product



Matrices from Linear Transformations

Of course, this can be generalized to any transformation

$T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ as

The diagram illustrates the construction of the matrix M for a linear transformation T . Above the matrix, three boxes represent the transformed basis vectors: $T(\hat{e}_1)$ (green), $T(\hat{e}_2)$ (purple), and $T(\hat{e}_n)$ (orange). Arrows point from these boxes to the corresponding columns of the matrix M . The matrix is shown as a large set of parentheses containing three columns. The first column (green) contains elements a_{11} , a_{21} , a vertical ellipsis, and a_{n1} . The second column (purple) contains a_{12} , a_{22} , a vertical ellipsis, and a_{n2} . The third column (orange) contains a_{1n} , a_{2n} , a vertical ellipsis, and a_{nn} . Ellipses between the columns indicate that there are n columns in total. The elements a_{ij} are color-coded: the row index i is red and the column index j is blue.

$$M = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}.$$

The numbers a_{ij} are called the **elements** of the Matrix, where i is the **row** of the element, and j is the **column** of the element.

In addition, each column of the matrix tells us how the respective standard basis vector is transformed.

The Definite Integral

Of course, we can refine the approximation by increasing the number of rectangles (which is equivalent to reducing Δx , since $\Delta x = \frac{b-a}{N}$):

