

Exercise 14: Exam Preparation (Taken From Summer Semester 2016)

Linear Algebra (Vectors)

The following three vectors in \mathbb{R}^3 are given:

$$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \vec{b} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}, \vec{c} = \begin{pmatrix} -16 \\ 0 \\ -10 \end{pmatrix}$$

1. Calculate the vector $2\vec{a} + \vec{b}$.
2. Calculate the inner product $\vec{a} \cdot \vec{b}$.
3. What is the angle between \vec{a} and \vec{b} ?
4. Are the three vectors $\vec{a}, \vec{b}, \vec{c}$ linearly independent? Prove your answer.
5. Give a geometrical description of the shape of the set of points in space described by $\{\vec{x} \in \mathbb{R}^3 \mid \vec{c} \cdot \vec{x} = 0\}$.
(Note: $\vec{c} \cdot \vec{x}$ denotes the inner (scalar) product of \vec{c} and \vec{x})

Linear Algebra (2×2 matrices and linear mapping)

The matrix $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ is given.

1. What is the rank of A ? Give a reason for your answer.
2. Calculate $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Draw the vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ in a Cartesian coordinate system.
3. The linear mapping (i.e. linear transformation) associated with A is $f: \vec{x} \mapsto A \cdot \vec{x}$. Describe (in words) how an arbitrary vector \vec{x} is transformed geometrically by f .
4. Calculate the matrix A^2 .
5. Calculate the determinant of A .
6. Determine the matrix A^{-1} (if it exists).
7. Determine the eigenvalues of A .

Linear Algebra (Larger Matrices and Linear Systems)

The matrix $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 7 \end{pmatrix}$ is given.

1. Calculate the product $A \cdot \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix}$.
2. $A \cdot \vec{x} = \vec{b}$, with $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, is a system of $m = 3$ linear equations for $n = 3$ unknowns. How many unknowns can be chosen arbitrarily? (Give a reason for your answer)

Computer Science (Programming)

The following Java method `f` gets an integer array `x` and a single integer `a` as its arguments:

```

1 public int f(int x[], int a)
2 {
3     int i = 0;
4     boolean b = true;
5     while (b && (i < x.length))
6     {
7         if (x[i] == a)
8             b = false;
9         else
10            i = i+1;
11    }
12    if (b)
13    {
14        println("Error!");
15        return -1;
16    }
17    else
18        return i;
19 }

```

What does the function return?

Computer Science (Representation of Numbers)

1. What is the binary representation of the decimal number 63?
2. What is the decimal representation of the hexadecimal number 2A5?
3. Give the 8-bit two's complement representation of the decimal number -84.
4. What is the exact value of the fraction which is represented in the ternary base (base-3) by $0.22222222 \dots$ (i.e. with infinitely many 2s after the period)?

Computer Science (Rule-Based Simulation)

1. Given the following L-system: $A \rightarrow (RF)FA$,
which string is produced after 3 steps of application of this rule to the start word 'A'?
Draw the graphical structure in the place which is obtained from this string by the following turtle interpretation:
 - R: Rotate 45 degrees clockwise.
 - F: Move forward 1 unit.
 - A: Do nothing.
 - (: Add position and angle to stack.
 -): Pop position and angle from stack.
2. A modified L-system is given: $A \rightarrow [RFA]FA$.
Draw the corresponding graphical structure after 3 steps.
3. How does the number of single lines (obtained from an "F" symbol) grow (quantitatively) with the number of steps for L1? For L2?

Calculus (Univariate Functions, Differentiation)

Given the following function: $f(x) = \frac{1}{3}x^3 - 4x^2 + 7x - 5$,

1. Find all x values where the function f has local extrema and classify them as minima or maxima.
2. Find where the function is increasing/decreasing, and all x values of inflection points.

Calculus (Integration)

Compute the total area between the function $f(x) = 6x^2 + 6x - 12$, the x -axis and the lines $x_1 = 0$ and $x_2 = 2$.