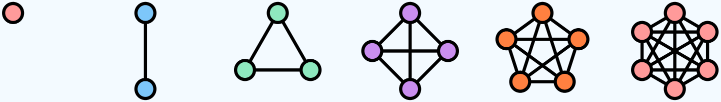
Graphs

Definition

The **complete graph** K_n is the graph with n vertices where every pair of different vertices is connected by an edge (Also called a **clique**).

Example

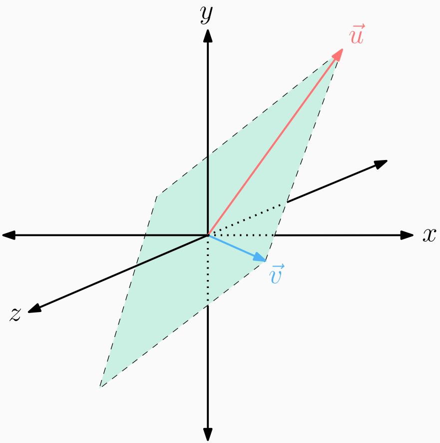
The cliques $K_1, \ldots, K_6$:



K_1 K_2 K_3 K_4 K_5 K_6

Subspaces

Similarily, any non-zero vector in $\mathbb{R}^3$ also spans a line going through the origin. In addition, any two linearly independent vectors span a **plane** going through the origin.

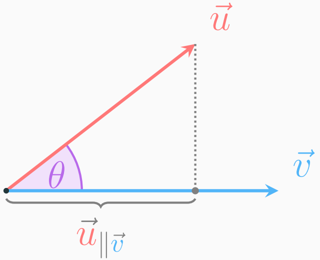


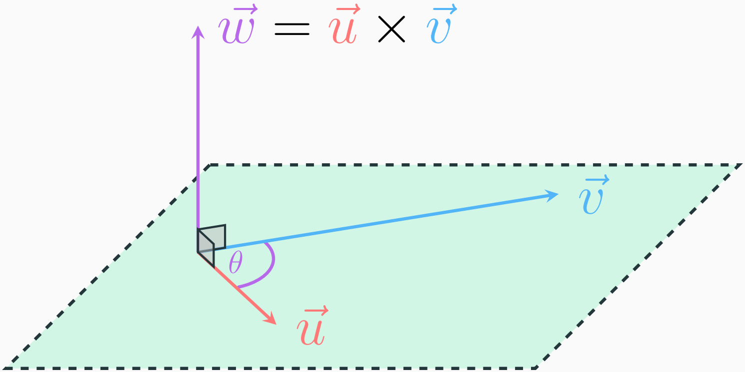
The Dot Product

If we rotate the two vectors such that one of them lies on the horizontal direction, we can draw a perpendicular line from $\vec{u}$ to $\vec{v}$. Using trigonometry we get

$$\cos(\theta) = \frac{\vec{u}_{\parallel \vec{v}}}{\|\vec{u}\|},$$

where $\vec{u}_{\parallel \vec{v}}$ is the length of the projection of $\vec{u}$ on $\vec{v}$.





$\vec{w} = \vec{u} \times \vec{v}$
$\vec{v}$
θ
$\vec{u}$

Matrices from Linear Transformations

Of course, this can be generalized to any transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ as

$$M = \begin{array}{c} \begin{array}{cccc} T(\hat{e}_1) & T(\hat{e}_2) & \quad & T(\hat{e}_n) \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \end{array}.$$

The numbers a_{ij} are called the **elements** of the Matrix, where i is the **row** of the element, and j is the **column** of the element.

In addition, each column of the matrix tells us how the respective standard basis vector is transformed.

The Definite Integral

Of course, we can refine the approximation by increasing the number of rectangles (which is equivalent to reducing Δx, since $\Delta x = \frac{b-a}{N}$):