Exercise 11: Derivatives of Real Functions

1. Draw the following functions on a grid: $x^2,\ -x^2,\ x^2+3,\ x^2-5,\ x^2-2x,\ x^2-3x+5$.
2. Calculate the following derivatives:
   i. $\frac{\mathrm{d}}{\mathrm{d}x}\left(5x^4-3x^2+5\right)$
   ii. $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^3-6x+5}{x-7}\right)$
   iii. $\frac{\mathrm{d}}{\mathrm{d}x}P_n(x)$, where $P_n(x)$ is a real polynomial of order n.
   iv. $\frac{\mathrm{d}^n}{\mathrm{d}x^n}P_n(x)$
   v. $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x},\ \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{2\sqrt{x}}$
   vi. $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x^3-2x},\ \frac{\mathrm{d}}{\mathrm{d}x}e^{-2\sqrt{x}}$
   vii. $\frac{\mathrm{d}^7}{\mathrm{d}x^7}e^{-x}$
   viii. $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x-\sin(x)\right),\ \frac{\mathrm{d}}{\mathrm{d}x}\sin\left(x^2\right)$
   ix. $\frac{\mathrm{d}^8}{\mathrm{d}x^8}\cos(x)$
3. Analyze the following functions (i.e. find points where the function intersects the axes, find all extrema and their type - including inflection points, and where the function is increasing or decreasing):
   (a) $f(x)=x^3-x^2-x+1$.
   (b) $f(x)=e^{-\frac{1}{2}x^2}$.
4. **Extra Question** (if time permits)
   i. Using matrix multiplication, show that if a line has slope m, a perpendicular line would have a slope $-\frac{1}{m}$.
   ii. Find the slope of the function $f(x)=\sqrt{1-x^2}$ at $x=\frac{1}{\sqrt{2}}$ without using derivates.
   iii. Find the derivative of $f(x)$ at $x=\frac{1}{\sqrt{2}}$ with derivation and compare the result to the one obtained in the previous section.