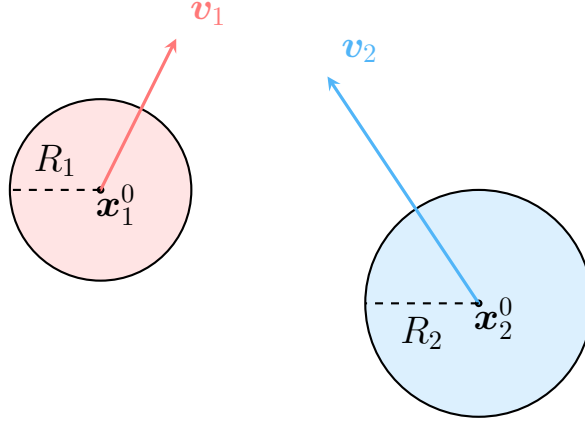


Calculations

May 30, 2020

1 Collision of Two Spherical Particles Moving at Constant Velocities

Two spherical particles with radii R_1 and R_2 , have initial positions \mathbf{x}_1^0 and \mathbf{x}_2^0 and move with constant velocities \mathbf{v}_1 and \mathbf{v}_2 , respectively. At what time, if at all, will they collide?



The position of particle 1 as a function of time is

$$\mathbf{x}_1(t) = \mathbf{x}_1^0 + \mathbf{v}_1 t, \quad (1)$$

The position of particle 2 as a function of time is identical, i.e.

$$\mathbf{x}_2(t) = \mathbf{x}_2^0 + \mathbf{v}_2 t. \quad (2)$$

Therefore, the distance $\mathbf{r}(t)$ between the particles is

$$\mathbf{r}(t) = \mathbf{x}_1(t) - \mathbf{x}_2(t) = \mathbf{x}_1^0 + \mathbf{v}_1 t - \mathbf{x}_2^0 - \mathbf{v}_2 t, \quad (3)$$

or in explicit vector form

$$\begin{aligned} \mathbf{r}(t) &= \begin{pmatrix} x_1^0 + v_1^x t - x_2^0 + v_2^x t \\ y_1^0 + v_1^y t - y_2^0 + v_2^y t \\ z_1^0 + v_1^z t - z_2^0 + v_2^z t \end{pmatrix} \\ &= \begin{pmatrix} (x_1^0 - x_2^0) - (v_1^x - v_2^x) t \\ (y_1^0 - y_2^0) - (v_1^y - v_2^y) t \\ (z_1^0 - z_2^0) - (v_1^z - v_2^z) t \end{pmatrix} \\ &= \begin{pmatrix} \Delta x_0 - \Delta v^x t \\ \Delta y_0 - \Delta v^y t \\ \Delta z_0 - \Delta v^z t \end{pmatrix}. \end{aligned} \quad (4)$$

The square of the norm of $\mathbf{r}(t)$ is thus

$$\begin{aligned}
\|\mathbf{r}\|^2 &= (\Delta x_0 - \Delta v^x t)^2 + (\Delta x_0 - \Delta v^x t)^2 + (\Delta x_0 - \Delta v^x t)^2 \\
&= (\Delta x_0)^2 - 2\Delta x_0 \Delta v^x t + (\Delta v^x)^2 t^2 + \dots \\
&= (\Delta x_0)^2 + (\Delta y_0)^2 + (\Delta z_0)^2 - 2(\Delta x_0 \Delta v^x + \Delta y_0 \Delta v^y + \Delta z_0 \Delta v^z) t + \left((\Delta v^x)^2 + (\Delta v^y)^2 + (\Delta v^z)^2 \right) t^2 \\
&= \|\Delta \mathbf{x}_0\|^2 - 2\langle \Delta \mathbf{x}_0, \Delta \mathbf{v} \rangle t + \|\Delta \mathbf{v}\|^2 t^2.
\end{aligned} \tag{5}$$

The times $t_{1,2}$ for which the particles collide can be calculated by solving equation 5 for the case

$$\|\mathbf{r}\|^2 = (R_1 + R_2)^2 = R^2, \tag{6}$$

i.e. when the distance between the particles is the sum of their radii.

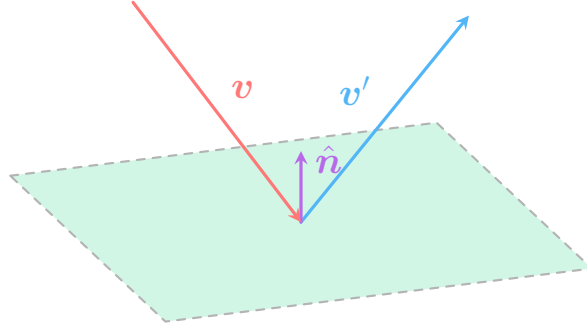
Using the quadratic formula,

$$\begin{aligned}
t_{1,2} &= \frac{2\langle \Delta \mathbf{x}_0, \Delta \mathbf{v} \rangle \pm \sqrt{4\langle \Delta \mathbf{x}_0, \Delta \mathbf{v} \rangle^2 - 4\|\Delta \mathbf{v}\|^2 (\|\Delta \mathbf{x}_0\|^2 - R^2)}}{2\|\Delta \mathbf{v}\|^2} \\
&= \frac{\langle \Delta \mathbf{x}_0, \Delta \mathbf{v} \rangle \pm \sqrt{\langle \Delta \mathbf{x}_0, \Delta \mathbf{v} \rangle^2 - \|\Delta \mathbf{v}\|^2 (\|\Delta \mathbf{x}_0\|^2 - R^2)}}{\|\Delta \mathbf{v}\|^2}.
\end{aligned} \tag{7}$$

The condition for which a collision will happen is therefore

$$\langle \Delta \mathbf{x}_0, \Delta \mathbf{v} \rangle^2 \geq \|\Delta \mathbf{v}\|^2 (\|\Delta \mathbf{x}_0\|^2 - R^2). \tag{8}$$

2 Collision of a Spherical Particle and a Planar Wall



2.1 Time to Collision

Using point-plane distance.

2.2 Velocity Change

The projection of \mathbf{v} onto $\hat{\mathbf{n}}$ is

$$\text{proj}_{\hat{\mathbf{n}}} \mathbf{v} = \langle \mathbf{v}, \hat{\mathbf{n}} \rangle \hat{\mathbf{n}}, \tag{9}$$

and on a direction orthogonal to $\hat{\mathbf{n}}$ is

$$\mathbf{v} - \langle \mathbf{v}, \hat{\mathbf{n}} \rangle \hat{\mathbf{n}}. \tag{10}$$

\mathbf{v} can be reconstructed from these two components,

$$\mathbf{v} = \langle \mathbf{v}, \hat{\mathbf{n}} \rangle \hat{\mathbf{n}} + \mathbf{v} - \langle \mathbf{v}, \hat{\mathbf{n}} \rangle \hat{\mathbf{n}}. \tag{11}$$

Similarly, \boldsymbol{v}' has the $\hat{\boldsymbol{n}}$ opposite and equal in magnitude to that of \boldsymbol{v} , and the orthogonal component equal to that of \boldsymbol{v} , yielding

$$\begin{aligned}\boldsymbol{v}' &= -\langle \boldsymbol{v}, \hat{\boldsymbol{n}} \rangle \hat{\boldsymbol{n}} + \boldsymbol{v} - \langle \boldsymbol{v}, \hat{\boldsymbol{n}} \rangle \hat{\boldsymbol{n}} \\ &= \boldsymbol{v} - 2\langle \boldsymbol{v}, \hat{\boldsymbol{n}} \rangle \hat{\boldsymbol{n}}.\end{aligned}\tag{12}$$