Part I

Background Topics

1 *Linear Algebra*

1.1 *Preface*

The goal of this chapter is not to teach you, the reader, linear algebra from scratch - nor to be a thorough source of information on the topic. Rather, my aim is to introduce important “advanced” concepts for those who took a basic linear algebra course as part of an undergraduate university program. These concpets should help you gain a basic knowledge of the topics needed for understanding the rest of the background material, as well as the topic of spinors itself.

My approach to teching topics in linear algebra - and in mathematics as a whole - is to first build an intuition and only then formalize and generalize the ideas as needed. In my personal experiences, when I was studying linear algebra I completely failed to understand it (and indeed, failed the course) until it “clicked” for me in regards to 2- and 3-dimensional real spaces, i.e. - visible geometry. After that I didn’t even have to study for exams anymore, as everything became clear enough to grasp and develop on the spot even during an exam (except for later, more advances concepts). That is why, for example, I absolutely adore courses and study materials of the topic[1] which use animation, such as *3Blue1Brown* great video essay series Essence of linear algebra[2].

There are very few proofs in this chapter, and those that are shown are not completely rigorous. For more in-depth materials, see the last section (further read). With that out of the way - let’s begin!

[1] And other mathematical topics as well.
[2] Temporary sidenote which should become a citation for the mentioned 3B1B video series

1.2 Dual Vectors and Dual Spaces

1.2.1 Measurements and rulers

Usually, dual vectors are taught by hitting the students with the definition of a dual space and then analyzing its properties. It's all very abstract and often leaves the students with a constant question in mind: "why do we care about dual vectors?"

I would like to take a different approach here: instead of confronting you with the definition and then discuss practical details, I will start with explaining *why* we care about dual vectors in the first place.

Let us begin with discussing rulers[3]. A ruler is essentially a geometric measuring tool which allows one to measure the *lengths* of different objects by counting the number of graduation lines between the beginning and end of an object (section 1.2.1). Of course in linear algebra, the geometric objects we measure using rulers are vectors.

[3] The idea for this approach comes from a beautiful answer by *Aloizio Macedo* to a question in the mathematics stack exchange website.

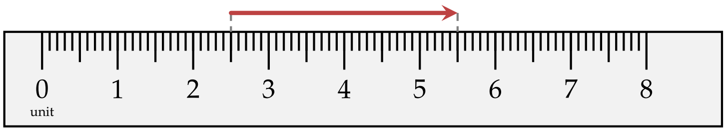

Figure 1.1: Measuring a vector using a ruler: the start of the vector sits at 2.5 units, while its head is at 5.5 units. Therefore we say that the vector is $5.5 - 2.5 = 3$ units in length.

We can represent a ruler in $\mathbb{R}^2$ as follows: the 0-graduation, which I denote as L_0 here, is represented by a line going through the origin with a direction orthogonal to the that of the ruler. The rest of the graduation marks are then drawn as the lines that are parallel to L_0 such that they are spaced equaly apart (see section 1.2.1). To mark the direction of the ruler more clearly, a vector with the same direction of the ruler is added at the origin, such that its length corresponds to the "density" of the graduation marks.

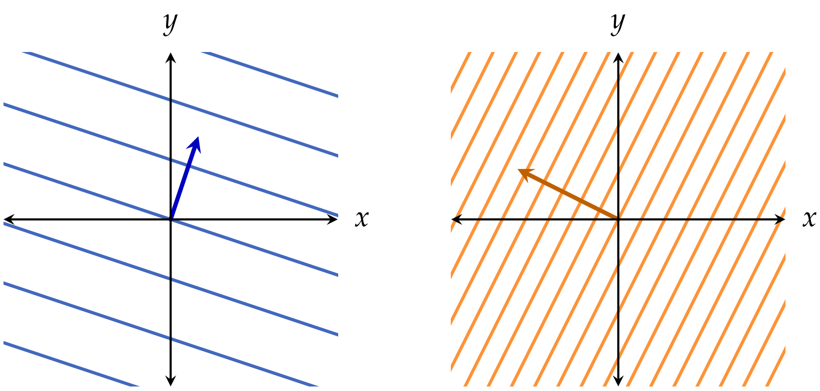

Figure 1.2: Two rulers in $\mathbb{R}^2$, each represented as the infinite set of lines which are parallel to the zero graduation L_0, and are equaly spaced apart. Note the vector drawn at the origin of each figure: it has the same direction as the respective ruler, and its length corresponds to the "density" of the graduation marks - the more dense the graduation marks, the longer the vector.

In $\mathbb{R}^3$ the graduations are represented in the same way, except that *planes* are used instead of line (figure).

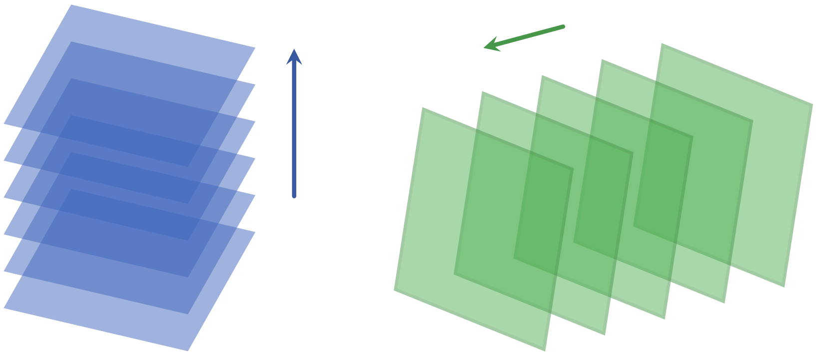

Figure 1.3: Representation of a ruler in $\mathbb{R}^3$.

1.3 *Further Reading*

2 *Geometric Algebra*

2.1 *Preface*

This is a temp text.

3 Abstract Algebra

3.1 Preface

This is a temp text.

4 Lie Groups and Algebras

4.1 Preface

This is a temp text.

Part II

Spinors