

Part I

Background Topics

1 Linear Algebra

1.1 Preface

THE GOAL OF THIS CHAPTER is not to teach you, the reader, linear algebra from scratch - nor to be a thorough source of information on the topic. Rather, my aim is to introduce important “advanced” concepts for those who took a basic linear algebra course as part of an undergraduate university program. These concepts should help you gain a basic knowledge of the topics needed for understanding the rest of the background material, as well as the topic of spinors itself.

My approach to teaching topics in linear algebra - and in mathematics as a whole - is to first build an intuition and only then formalize and generalize the ideas as needed. In my personal experiences, when I was studying linear algebra I completely failed to understand it (and indeed, failed the course) until it “clicked” for me in regards to 2- and 3-dimensional real spaces, i.e. - visible geometry. After that I didn’t even have to study for exams anymore, as everything became clear enough to grasp and develop on the spot even during an exam (except for later, more advanced concepts). That is why, for example, I absolutely adore courses and study materials of the topic¹ which use animation, such as *3Blue1Brown* great video essay series [Essence of linear algebra](#)².

There are very few proofs in this chapter, and those that are shown are not completely rigorous. For more in-depth materials, see the last section (further read). With that out of the way - let’s begin!

¹ And other mathematical topics as well.

² Temporary sidenote which should become a citation for the mentioned *3B1B* video series

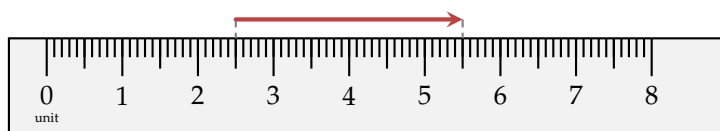
1.2 Dual Vectors and Dual Spaces

1.2.1 Measurements and rulers

USUALLY, DUAL VECTORS ARE TAUGHT by hitting the students with the definition of a dual space and then analyzing its properties. It's all very abstract and often leaves the students with a constant question in mind: "why do we care about dual vectors?"

I would like to take a different approach here: instead of confronting you with the definition and then discuss practical details, I will start with explaining *why* we care about dual vectors in the first place.

Let us begin with discussing rulers³. A ruler is essentially a geometric object which allows one to measure the *lengths* of different objects by counting the number of graduation lines between the beginning and end of an object (section 1.2.1). Of course the geometric objects we use normally are vectors.



³ The idea for this approach comes from a beautiful answer by *Aloizio Macedo* to [a question in the mathematics stack exchange website](#).

Figure 1.1: Measuring a vector using a ruler: the start of the vector sits at 2.5 units, while its head is at 5.5 units. Therefore we say that the vector is $5.5 - 2.5 = 3$ units in length.

Of course, we can have rulers with different distances between consecutive graduation (i.e. they can be more or less "dense"), which would yield different measurements for the same vectors. Another property a ruler has its *orientation*: while it is most common to measure by placing a ruler parallel to the distance we wish to measure, it is not *strictly* necessary. If we imagine that the graduation on a ruler are infinitely long and there are infinitely many of them in both directions, we can easily measure vectors that do not align with the ruler (section 1.2.1).

1.3 Further Reading

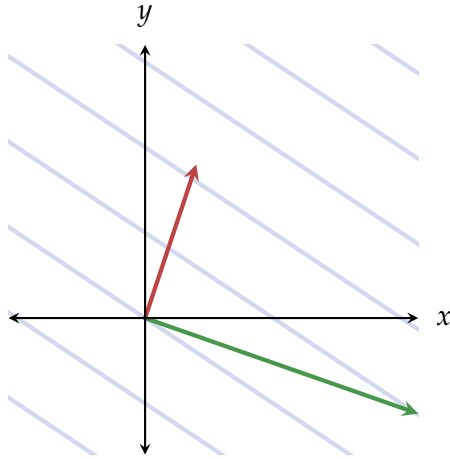


Figure 1.2: Measuring two vectors at an angle to the graduation of some ruler. Here the ruler is represented by infinitely long graduations line in blue. Note that although the green vector appears longer than the red vector, it is measured by the ruler to be about 1 unit long, while the red vector is measured to be a bit more than 2 units in length.

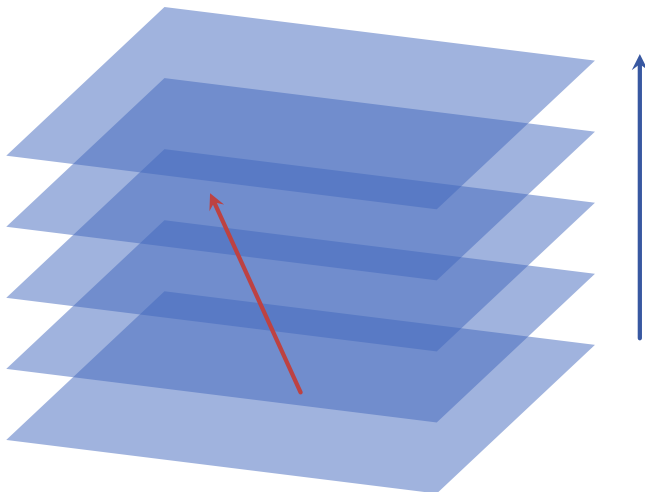


Figure 1.3: Measuring two vectors at an angle to the graduation of some ruler. Here the ruler is represented by infinitely long graduations line in blue. Note that although the green vector appears longer than the red vector, it is measured by the ruler to be about 1 unit long, while the red vector is measured to be a bit more than 2 units in length.

2 *Geometric Algebra*

2.1 *Preface*

This is a temp text.

3 *Abstract Algebra*

3.1 *Preface*

This is a temp text.

4 Lie Groups and Algebras

4.1 Preface

This is a temp text.

Part II

Spinors

