Part I

Background Topics

1 *Linear Algebra*

1.1 *Preface*

THE GOAL OF THIS CHAPTER is not to teach you, the reader, linear algebra from scratch - nor to be a thorough source of information on the topic. Rather, my aim is to introduce important "advanced" concepts for those who took a basic linear algebra course as part of an undergraduate university program. These concpets should help you gain a basic knowledge of the topics needed for understanding the rest of the background material, as well as the topic of spinors itself.

My approach to teching topics in linear algebra - and in mathematics as a whole - is to first build an intuition and only then formalize and generalize the ideas as needed. In my personal experiences, when I was studying linear algebra I completely failed to understand it (and indeed, failed the course) until it "clicked" for me in regards to 2- and 3-dimensional real spaces, i.e. - visible geometry. After that I didn't even have to study for exams anymore, as everything became clear enough to grasp and develop on the spot even during an exam (except for later, more advances concepts). That is why, for example, I absolutely adore courses and study materials of the topic[1] which use animation, such as *3Blue1Brown* great video essay series Essence of linear algebra[2].

There are very few proofs in this chapter, and those that are shown are not completely rigorous. For more in-depth materials, see the last section (further read). With that out of the way - let's begin!

[1] And other mathematical topics as well.
[2] Temporary sidenote which should become a citation for the mentioned 3B1B video series

1.2 *Dual Vectors and Dual Spaces*

1.2.1 *Linear measurements and rulers (or: why should I care about dual vectors?)*

Frequently in linear algebra we want to measure vectors. A measure in this context is a way to assign each vector a real number which somehow reflects its properties (i.e. direction and/or magnitude). Mathematically, we are looking for a function $\phi : \mathbb{R}^n \to \mathbb{R}$ - we feed it a vector, and it returns some measurement.

One common way to measure vectors is using the norm: the norm of a vector in $\mathbb{R}^n$ using the standard basis set $\boldsymbol{e}_1, \boldsymbol{e}_2, \dots, \boldsymbol{e}_n$ is given by

$$\|\boldsymbol{v}\| = \sqrt{v_1^2 + v_2 + \cdots + v_n^2}, \tag{1.1}$$

where the numbers $v_1, v_2, \dots, v_n$ are the components in each of the respective basis vectors.

While the norm does tell us something useful about a vector, it has two drawbacks: it doesn't tell us anything about the vector's orientation in space, and even worse - it is not linear. We can try and simplify the calculation of the norm by dropping the square root and calculating the *square norm*, but that too isn't linear, nor does it tell us anything about the vector's orientation.

Another way of measuring vectors is by using *rulers*. Rulers are nothing more than a set of graduation lines with an orientation in space (Figure 1.2). We can therefore represent a ruler using a vector: the magnitude of the vector is the frequency of the graduation lines, while its direction is the direction of the ruler (which is orthogonal to the graduation lines).

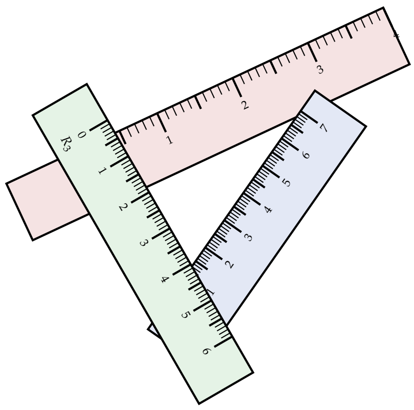


Figure 1.1: Three rulers, R_1, R_2, R_3, each with its own orientation and frequency of graduation lines.

Now, we usually rotate rulers to align with the orientation of the magnitude we wish to measure - however here we want our rulers to also measure orientation. Therefore, instead of rotating the ruler to align with a vector we wish to measure, we *project* the vector on the ruler and then take our measurement.

1.2.2 *Introducing some formalism*

1. Dual vectors form a vector space V^*.
2. Formal definition of dual spaces.
3. Examples of dual vectors of functions?..

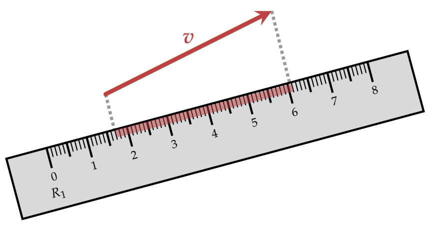
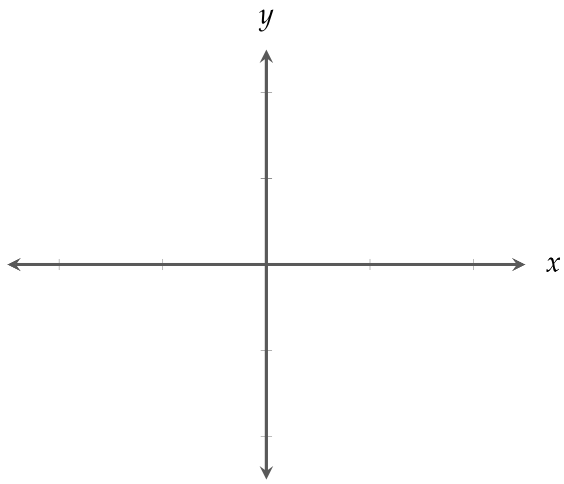


Figure 1.2: Measuring a vector using a ruler by projecting the vector onto the ruler. The result of the projection is drawn as a red line on the ruler.



1.2.3 *Basis sets and coordinate transformations*

1. Dual basis: converting from a basis set in V to its dual in V^*.
2. Covariance of dual vectors basis change vs. contra-varience of vectors.

1.3 *Further Reading*

2 Geometric Algebra

2.1 Preface

This is a temp text.

3 Abstract Algebra

3.1 Preface

This is a temp text.

4 Lie Groups and Algebras

4.1 Preface

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Part II

Spinors