

Part I

Background Topics

1 Linear Algebra

1.1 Preface

THE GOAL OF THIS CHAPTER is not to teach you, the reader, linear algebra from scratch - nor to be a thorough source of information on the topic. Rather, my aim is to overview the topic in such a way that new-comers, as well as those who studied linear algebra in an undergraduate university course, will gain the important insights of the topic needed for understanding the rest of the background material and spinors as well.

Instead of teaching the topic from the ground-up, like mathematicians tend to do¹, I prefer to stick to the geometric interpretation of the vector spaces \mathbb{R}^2 and \mathbb{R}^3 (and to a lesser extent \mathbb{R}^n in general). These interpretations can be visualized relatively easily, and thus help in setting up the needed intuition in the student's mind, which becomes handy when the topic turns to more abstract constructs (such as for example vector spaces of matrices or functions).

In my personal experiences, when I was studying the topic I completely failed to understand it (and indeed, failed the courses I took) until it "clicked" for me in regards to 2- and 3-dimensional real spaces, i.e. - visible geometry. Then I didn't even have to study for exams anymore, as everything became clear enough to grasp and develop on the spot even during an exam (except for later, more advanced concepts). That is why, for example, I absolutely adore courses and study materials of the topic² which use animation, such as *3Blue1Brown* great video essay series [Essence of linear algebra](#)³.

There are very few proofs in this chapter, and those that are shown are not completely rigorous. For more in-depth materials, see the last section (further read). With that out of the way - let's begin!

¹ In my view, courses that "build" linear algebra step-by-step give the students good knowledge of the structures of vector spaces, but tend to miss the intuitive view of what these structures can *do*. This is exactly the difference between the "pure" mathematics of the mathematician and the mathematics as a tool of the scientist.

² And other mathematical topics as well.

³ Temporary sidenote which should become a citation for the mentioned *3B1B* video series

1.2 Vectors and Vector Spaces

VECTORS ARE the most basic and important element in linear algebra. There are several different approaches to defining a vector: physicists like to talk about vectors as objects with magnitude and direction. Computer scientists and programmers tend to view vectors as lists of numbers (and sometimes other types of objects as well). To the mathematician, vectors are elements of *vector sets*, which are defined rigorously and precisely.

The existence of different definitions leads to much confusion⁴: for example, we usually think of matrices as *acting* on vectors, so how can matrices be vectors themselves? Also - are vectors just list of numbers, or are there some such lists that aren't vectors? Can we always reduce any vector to a list of numbers? And what about the case of functions as vectors? Etc., etc.

Therefore, for now I choose to limit the definition of vectors to the so-called "physicist's definition": a vector is an object which has both a magnitude (also *length* and *norm*), and a direction. These can be easily visualized in 2- and 3-dimensional real spaces as arrows. We will later see how this translates into lists of numbers (and later still how we can define more abstract and inclusive vectors). Since vectors don't have positions, we can freely move them around in space, and normally present them as originating from the same point in space (??).

Let's now overview the basic operations that can be done with vectors. Vectors can be scaled by any real number: scaling a vector means that we're changing the length of the vector without changing its direction (??). Note that direction here means the line from the vector's origin to its head: when we scale a vector by a negative number $\alpha \in \mathbb{R}$ we flip the vector's orientation and scale its length by the absolute value of α - the vector is then considered to stay in the same direction in space (??, again).

Vectors can also be added together using the *parallelogram rule*: given two vectors a and b we first place b such that its origin lies on the head of a . The vector c from the origin of a to the head of b is the result of the addition.

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⁴ I admit that this is a classic case of "citation needed", but it is something I come across often.

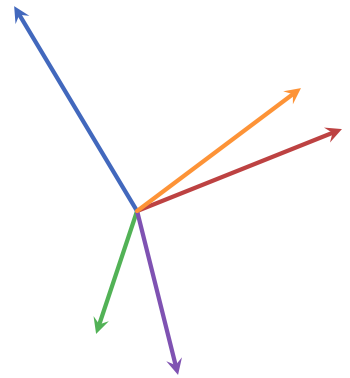


Figure 1.1: Some vectors placed in 2-dimensional space such that they all originate from the same point.

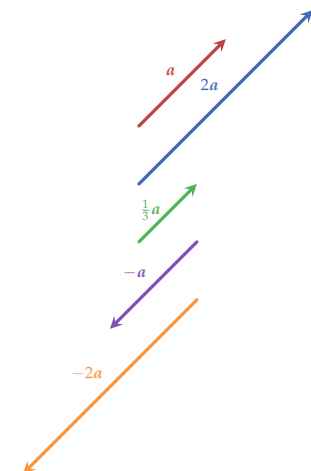
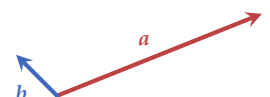


Figure 1.2: Some vectors placed in the origin of a 2-dimensional Cartesian coordinate system.



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1.3 *Linear Transformations*

1.4 *Matrices*

1.5 *Eigenvectors and Eigenvalues*

1.6 *Quaternions and Octanions*

1.7 *Advanced Notation and Einstein's Summation*

1.8 *Further Reading*

2 *Geometric Algebra*

2.1 *Preface*

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3 *Abstract Algebra*

3.1 *Preface*

This is a temp text.

4 Lie Groups and Algebras

4.1 Preface

This is a temp text.

Part II

Spinors

