HMM for DNA Sequence

In this problem, you will use HMM to decode a simple DNA sequence. It is well known that a DNA sequence is a series of components from $\{A, C, G, T\}$. Now let's assume there is one hidden variable S that controls the generation of DNA sequence. S takes 2 possible states $\{S_1, S_2\}$. Assume the following transition probabilities for HMM M

$$P(S_1|S_1) = 0.8$$
, $P(S_2|S_1) = 0.2$, $P(S_1|S_2) = 0.2$, $P(S_2|S_2) = 0.8$

emission probabilities as following

$$P(A|S_1) = 0.4$$
, $P(C|S_1) = 0.1$, $P(G|S_1) = 0.4$, $P(T|S_1) = 0.1$
 $P(A|S_2) = 0.1$, $P(C|S_2) = 0.4$, $P(G|S_2) = 0.1$, $P(T|S_2) = 0.4$

and start probabilities as following

$$P(S_1) = 0.5, P(S_2) = 0.5$$

Assume the observed sequence is x = CGTCAG, calculate:

 \bullet P(x|M)