

0.1 VC-Dimension

Consider a 2-dimensional instance space. Take as a hypothesis space, the set of all hypotheses of the form “everything inside rectangle R is positive and everything outside it is negative”, where R can be any axis-parallel rectangle (that is, the sides of the rectangle are parallel to the X and Y axis). This is similar to the illustrations of rule learning in the lectures.

Show that the VC-dimension of this hypothesis space is at least 3.

Show that the VC-dimension of this hypothesis space is at least 4.

Now consider a similar hypothesis space, but for each hypothesis that states “everything inside rectangle R is positive and everything outside it is negative” there is also a hypothesis “everything outside R is positive and everything inside is negative”. Show that the VC-dimension of this hypothesis space is at least 4.

0.2 Sample complexity

Using the concept of VC-dimension, compute an upper bound for the number of examples that may be needed to train a 2-input perceptron such that with 90% certainty it learns a hypothesis with true error $< 5\%$.