Exercises

- Problem 1: Draw a perceptron that does A AND B; is it possible? (If possible, then specify the weights)

- Problem 2: Draw a perceptron that does A OR B; is it possible? (If possible, then specify the weights)

- Problem 3: Draw a perceptron that does A XOR B; is it possible? (If possible, then specify the weights)

- Problem 4: Write down the update rule for Stochastic Gradient Descent.

  Let $w \in R^n$ be the weight vector, $x^{(i)}$ be the ith sample from the training data, and $L(w,x)$ be a loss function (assume that it is continuous and differentiable). We observe example $x(i)$ and predict using the current hypothesis w. We then update the weight vector. Express the new weight vector as a function of w, $L(w,x^{(i)})$, and the learning rate R.

  $$w \leftarrow ...$$

- Problem 5: Consider two perceptrons defined by the threshold expression $w_0 + w_1x_1 + w_2x_2 > 0$.

  Perceptron A has weight values

  $$w_0 = 1, w_1 = 2, w_2 = 1$$

  and perceptron B has the weight values

  $$w_0 = 0, w_1 = 2, w_2 = 1$$

  True or false? Perceptron A is *more general than* perceptron B. (*more general than* is defined in Chapter 2.)

Problem 6: Design a two-input perceptron that implements the boolean function $A \wedge \neg B$, and a 2-layer network of perceptrons that implements A XOR B.

Problem 7: Consider a two-dimensional space XY; X and Y are inputs of a perceptron. We have seen that the decision surface of a perceptron is always a straight line. We also know that perceptrons can apply OR and AND operations to boolean values.

Now consider a neural network with two layers of perceptrons (a hidden layer with n perceptrons and an output layer of 1 perceptron). Based on the above observations, what kind of decision surface do you think such a network can at least form?