0.1

For Naive Bayes, $v_{NB} = \underset{v_j \in V}{\operatorname{argmax}} P(v_j) \prod_i P(a_i | v_j)$,

let H denote class healthy and V denote class virulent

$P(+1)\,P(\text{single}|H)\,P(\text{light}|H)\,P(\text{one}|H) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{27}$

$P(V)\,P(\text{single}|V)\,P(\text{light}|V)\,P(\text{one}|V) = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{54}$

$\frac{4}{27} > \frac{1}{54}$, thus $v_{NB} = +1$. Its class is healthy

0.2

for text classification,

$$v_{NB} = \underset{v_j \in V}{\operatorname{argmax}} P(v_j) \prod_{i \in \text{Positions}} P(a_i | v_j) \qquad P(a_i | v_j) = \frac{n_k + 1}{n + |\text{Voc}|}$$

$P(A)\,P(\text{cats}|A)\,P(\text{eat}|A) \cdots P(\text{bones}|A)$

L1 $= \frac{1}{2} \frac{0+1}{8+6} \cdot \frac{0+1}{8+6} \cdots \frac{0+1}{8+6} = \frac{1}{2 \times 14^7}$ $n = 8$ in A and no word in target sentence is in A

$P(B)\,P(\text{cats}|B)\,P(\text{eat}|B) \cdots P(\text{bones}|B)$

L2 $= \frac{1}{2} \frac{1+1}{5+6} \cdot \frac{0+1}{5+6} \cdot \frac{0+1}{5+6} \cdot \frac{0+1}{5+6} \cdot \frac{1+1}{5+6} \cdot \frac{0+1}{5+6} \cdot \frac{0+1}{5+6} = \frac{2^2}{11^7}$

$\frac{2^2}{11^7} > \frac{1}{2 \times 14^7}$, so this sentence belongs to class B

0.3

Let the symbols below denote each feature and class.

Shape: circle = C, diamond = D, hexagon = H

Nose: big = B, small = S

smile = $\mathcal{A}$, not smile = $\neg\mathcal{A}$

1. $P(\mathcal{A})P(D|\mathcal{A})P(S|\mathcal{A}) = \frac{4}{8} \cdot \frac{1}{4} \cdot \frac{2}{4} = \frac{1}{16}$

$P(\neg\mathcal{A})P(D|\neg\mathcal{A})P(S|\neg\mathcal{A}) = \frac{4}{8} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{32}$

So the last face is smiling

2. Shape → Smile ← Nose

$P(C) = 3/8$ $P(B) = 5/8$

$P(D) = 2/8$ $P(S) = 3/8$

$P(H) = 3/8$

$\Rightarrow P(\mathcal{A}|C,B) = \frac{P(\mathcal{A},C,B)}{P(C,B)} = \frac{1/8}{2/8} = \frac{1}{2}$

$P(\mathcal{A}|D,B) = \frac{1/8}{1/8} = 1$

$P(\mathcal{A}|H,B) = \frac{0/8}{2/8} = 0$

$P(\mathcal{A}|C,S) = \frac{1/8}{1/8} = 1$

$P(\mathcal{A}|D,S) = \frac{0/8}{1/8} = 0$

$P(\mathcal{A}|H,S) = \frac{1/8}{1/8} = 1$

For $P(\neg\mathcal{A}|\ )$, we just need to use 1 − the above probabilities

Shape → Nose

↓ Smile

$P(C) = 3/8$ $P(B|C) = \frac{P(C,B)}{P(C)} = \frac{2/8}{3/8} = \frac{2}{3}$

$P(D) = 2/8$ $P(B|D) = \frac{1/8}{2/8} = \frac{1}{2}$

$P(H) = 3/8$ $P(B|H) = \frac{2/8}{3/8} = \frac{2}{3}$

$P(\mathcal{A}|B) = \frac{2/8}{5/8} = \frac{2}{5}$

$P(\mathcal{A}|S) = \frac{2/8}{3/8} = \frac{2}{3}$

For $P(S|C)$, $P(S|D)$, $P(S|H)$ and $P(\neg\mathcal{A}|B)$, $P(\neg\mathcal{A}|S)$ we just need to calculate the difference between 1 and the above probabilities

0. 3. Smile

↙ ↘

Shape Nose