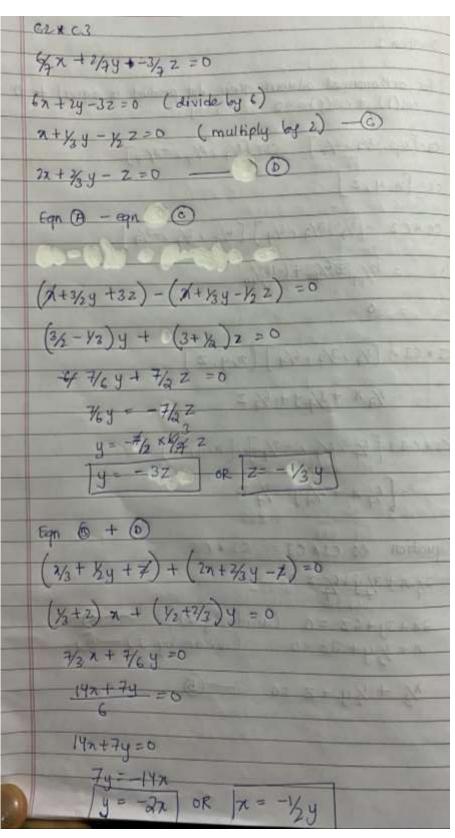
DIMENSIONALITY REDUCTION

QUESTION 1

Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7,2/7,-3/7]. Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that $x^2+y^2+z^2=1$. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

	Assignment \$8
	Question 1
	For arthonormal column, their dot product is squal to 0 col(i) + col(j) == 0 for i = j
	C1-[2/2, 3/2, 4/2] C2=[5/2, 2/2, -3/2]
	c3= [n, y, z]
	C1 * C2 - [4, 13/7, 4/4] [4/4, 12/4, 1-3/4]
	= 12/49 + 6/49 + (-18)/49
	C1 * C3 = [4,13/4,4/4] [n,4,2]
	= 1/2 x + 3/4 y + 1/4 z
	C2*C3=[4412/41-3/4][n14,2]
	= {4n+2/4y-1/4z
	Equation & CIKC3 == CIKC3
	247+344+420
	2x+3y+6z =0 (divide by 2) x+3/y+3z=0 (divide by 3) -(4)
1	1/3 + 1/4 + Z = 0 B
1	



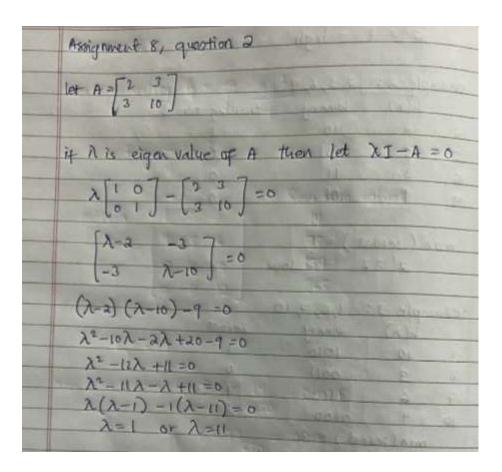
x: y: z = -2: 1: -3

QUESTION 2

Find the eigenvalues and eigenvectors of the following matrix:



You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.



When
$$\lambda = 11$$

$$\begin{bmatrix}
1 & -2 & -3 \\
-3 & 11 - 10
\end{bmatrix} = \begin{bmatrix}
9 & -3 \\
-3 & 1
\end{bmatrix} = B$$

B. $\bar{n} = 0$ - ... \bar{n} is eigen vector

$$\begin{bmatrix}
9 & -3 \\
-3 & 1
\end{bmatrix} \begin{bmatrix}
n_1 \\
n_2
\end{bmatrix} = 0$$

$$\begin{bmatrix}
9 & -3 \\
-3 & 1
\end{bmatrix} \delta$$

Osing row reduction

$$R_2 = 3R_2 + R_1$$

$$\begin{bmatrix}
9 & -3 \\
-3 & 1
\end{bmatrix} \delta$$

$$\begin{bmatrix}
9 & -3 \\
-3 & 1
\end{bmatrix} \delta$$

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\end{bmatrix} \delta$$

Osing row reduction

$$R_2 = 3R_2 + R_1$$

Osing row reduction

$$R_2 = 3R_2 + R_1$$

Osing row reduction

if
$$n_1 = 1$$
 $n_2 = 3$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \text{eigen vectors}.$$
When $\lambda = 1$

$$\begin{bmatrix} \lambda - 2 & -3 \\ -3 & \lambda - 10 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -3 \\ -3 & -9 \end{bmatrix} = 0 \quad \text{(B)}$$

$$\begin{bmatrix} -1 & -3 \\ -3 & -9 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -3 \\ -3 & -9 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = 0$$

using row reduction
$$R_2 = 3R_{11} + R_2$$

$$\begin{bmatrix} -1 & -3 \\ -6 & 0 \end{bmatrix}$$

$$-n_1 - 3n_2 = 0$$

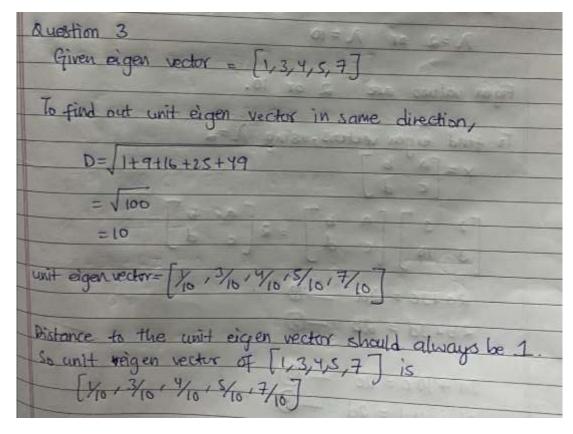
$$-n_1 = 3n_2$$
if $n_1 = 1$ $n_2 = -\frac{1}{3}$

$$\vdots$$

$$\begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \end{bmatrix} = eigen vectors$$

QUESTION 3

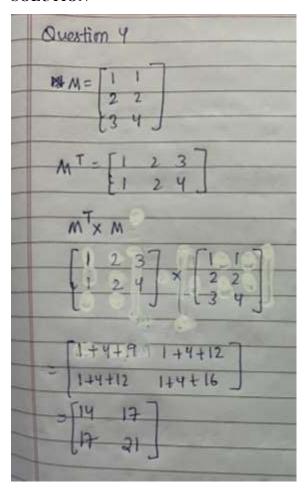
Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.



QUESTION 4

Suppose we have three points in a two-dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

SOLUTION



QUESTION 5

Consider the diagonal matrix M =



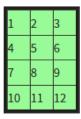
Compute its Moore-Penrose pseudoinverse.

SOLUTION

	0 = hE	5.007
Given dia	gonal matrix = 1	M= 0
	Dui on the	[0 0 0]
	e the Moore pour	ose psoudo-invene of above
	10-6	(D- A)
Manuel O	MASTE DIGITAL - IMMENSE	of a placemal matrix 5 is
Moore p	urose pseudo-inverse th diagonal elemen	t of 2 is 6 \$0, then
Moore p if the replace	invose pseudo-invense ith diagonal element it by 1/8	e of a diagonal matrix & ie to g & is 6 \$ 0, then
replace	invose pseudo-inverse it diagonal elemen it by 1/6 nverse matrix = [1]	0 0 7
replace	it by 1/6	e of a diagonal matrix S is 0 of S is 0 .

QUESTION 6

When we perform a CUR decomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:



Calculate the probability distribution for the rows.

Question 6		40.00	N. S. L.
row probabilis	y P; = Z	Milia	1
f=	≤ M _{ij} ²	0.10	1-6-6
		Row sum	values
matrix =	1 2 3	14	
	4 5 6	77	1
		194	3 6
	7 8 9	365	
	10 11 12	000	o sail
		Toron I	T KALL
£= (So	25	P-C C-
row probabilit	ies are > Pj=	650	No sec
	P2 = 7	7/650 = 0.118	1000
	P3 = 19	74/650 = 0.298	RES IN-
	$P_{q} = 3$	365/650 = 0.562	12,00