MAPREDUCE AND PAGERANK

QUESTION 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer i and produces the list of pairs (p,i) such that p is a prime divisor of i. For example, map(12) = [(2,12),(3,12)].

The reduce function is addition. That is, reduce(p,[i_1,i_2,...,i_k]) is (p,i_1+i_2+...+i_k).

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

SOLUTION

prime number : 2,3,5,7,11....

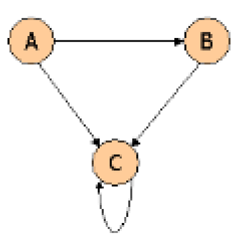
map (15) : [3,15], [5,15]
map (21) : [3,21], [7,21]
map (24) : [2,24], [3,24]
map (30) : [2,30], [3,30], [5,30]
map (49) : [7,49]

by combining all common elements parts i.e compare left element and add right most to get the solution

reduce (2,54)
reduce (3,90)
reduce (5,45)
reduce (7,70)

QUESTION 2:

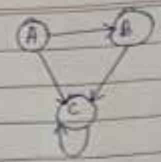
Consider three Web pages with the following links:



Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRanks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRanks a, b, and c of the three pages A, B, and C, respectively.

SOLUTION

②



$$M = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 0 & 0 \\ B & 1/2 & 0 & 0 \\ C & 1/2 & 1 & 1 \end{array}$$

Page rank equation:

$$r = \beta M \cdot r^{0} + (1-\beta)\left[\tfrac{1}{N}\right]_{N}$$

$\beta = 0.7 = 7/10$

$$r^{0} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\beta M = \tfrac{7}{10}\begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 7/20 & 0 & 0 \\ 7/20 & 7/10 & 7/10 \end{bmatrix}$$

$$\beta M \cdot r^{0} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 7/10 & 0 & 0 \\ 7/20 & 7/10 & 7/10 \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}}_{3\times1}$$

$$= \begin{bmatrix} 0 \\ 0.1155 \\ 0.35 \times 0.33 + 0.7 \times 0.33 + 0.7 \times 0.33 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.1155 \\ 0.5775 \end{bmatrix}$$

$$(1-\beta)\left[\tfrac{1}{N}\right]_{N}$$

$$= (1-0.7)\begin{bmatrix} 1/3 \\ 1/3 \\ 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$r^{1} = \begin{bmatrix} 0 \\ 0.1155 \\ 0.5795 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.2155 \\ 0.6795 \end{bmatrix}$$

$$r^{2} = \beta M r^{1} + (1-\beta)\left[\tfrac{1}{n}\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.2155 \\ 0.6795 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.035 \\ 0.6601 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.125 \\ 0.7601 \end{bmatrix}$$

$$r^{3} = \beta M r^{2} + (1-\beta)\left[\tfrac{1}{n}\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.125 \\ 0.7601 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.035 \\ 0.66157 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.135 \\ 0.76157 \end{bmatrix}$$

$$r^{4} = \beta M r^{3} + (1-\beta)\left[\tfrac{1}{n}\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.135 \\ 0.76157 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.035 \\ 0.6626 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.135 \\ 0.7626 \end{bmatrix}$$

$$r^{5} = \beta M r^{4} + (1-\beta)\left[\tfrac{1}{n}\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.135 \\ 0.7626 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.035 \\ 0.6633 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.135 \\ 0.7633 \end{bmatrix}$$

After 5th iteration.

Page rank

$$= \begin{bmatrix} 0.1 \\ 0.135 \\ 0.7633 \end{bmatrix} \times 3$$

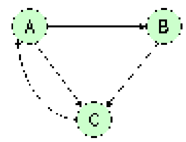
$$= \begin{bmatrix} 0.3 \\ 0.405 \\ 2.289 \end{bmatrix}$$

$a = 0.3$

$b = 0.405$

$c = 2.289$

QUESTION 3:



Suppose we compute PageRank with β=0.85. Write the equations for the PageRanks *a*, *b*, and *c* of the three pages A, B, and C, respectively.

SOLUTION

classmate
Date
Page

Assignment 1

Question 3

Formula

$A = \beta \cdot C + (1-\beta)\frac{1}{3}$

$B = \beta * \frac{A}{2} + (1-\beta)\frac{1}{3}$

$C = \beta * \left(\frac{A}{2} + B\right) + (1-\beta)\frac{1}{3}$

Since $\beta = 0.85$

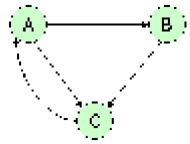
$A = 0.85C + (1-0.85)\frac{1}{3}$ $\quad B = 0.85 \times 0.5A + (1-0.85)\frac{1}{3}$

$\boxed{A = 0.85C + 0.05}$ $\quad \boxed{B = 0.425A + 0.05}$

$C = 0.85\,[0.5A + 0.85] + 0.05$

$\boxed{C = 0.425A + 0.85B + 0.05}$

QUESTION 4:



Assuming no "taxation," compute the PageRanks a, b, and c of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank $a = b = c = 1$. Compute as far as the 5th iteration, and also determine what the PageRanks are in the limit.

SOLUTION

Question 4

Formula = $A = C$

$B = A/2$

$C = A/2 + B$

At 0^{th} iteration

$A = 1$ $B = 1$ $C = 1$

1^{st} iteration

$A = 1$ $B = 1/2$ $C = 3/2$

2^{nd} iteration

$A = 3/2$ $B = 1/2$ $C = 1/2 + 1/2 = 1$

3^{rd} iteration

$A = 1$ $B = 3/2 \times 1/2 = 3/4$ $C = 3/4 + 1/2$

$C = 5/4$

4th iteration

$A = 5/4 \quad B = 1/2 \quad C = 1/2 + 3/4$

$C = 5/4$

At 5th iteration

$A = 5/4 \quad B = 5/8 \quad C = 5/8 + 1/2$

$C = 9/8$

$\therefore$ Page rank at 5th iteration are

$A = 5/4$

$B = 5/8$

$C = 9/8$