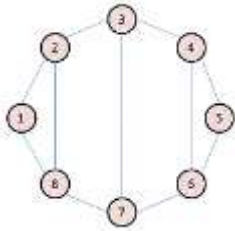


COMMUNITIES

QUESTION 1:

For the following graph:



Write the adjacency matrix A , the degree matrix D , and the Laplacian matrix L . For each, find the sum of all entries and the number of nonzero entries.

SOLUTION

Communities. (Assignment 5)

Question 1

Adjacency matrix, A

	1	2	3	4	5	6	7	8
1	0	1	0	0	0	0	0	1
2	1	0	1	0	0	0	0	1
3	0	1	0	1	0	0	1	0
4	0	0	1	0	1	1	0	0
5	0	0	0	1	0	1	0	0
6	0	0	0	1	1	0	1	0
7	0	0	1	0	0	1	0	1
8	1	1	0	0	0	0	1	0

Degree matrix, D

	1	2	3	4	5	6	7	8
1	2	0	0	0	0	0	0	0
2	0	3	0	0	0	0	0	0
3	0	0	3	0	0	0	0	0
4	0	0	0	3	0	0	0	0
5	0	0	0	0	2	0	0	0
6	0	0	0	0	0	3	0	0
7	0	0	0	0	0	0	3	0
8	0	0	0	0	0	0	0	3

Laplacian Matrix, $L = D - A$

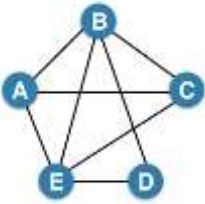
	1	2	3	4	5	6	7	8
1	2	-1	0	0	0	0	0	-1
2	-1	3	-1	0	0	0	0	-1
3	0	-1	3	-1	0	0	-1	0
4	0	0	-1	3	-1	-1	0	0
5	0	0	0	-1	2	-1	0	0
6	0	0	0	-1	-1	3	-1	0
7	0	0	-1	0	0	-1	3	-1
8	-1	-1	0	0	0	0	-1	3

A has 22 non-zero entries
 D has 8 non-zero entries
 L has 30 non-zero entries.

The sum of the entries of A is 22.
 The sum of entries of D is 22.
 The sum of entries of L is 0.

QUESTION 2:

Consider the following undirected graph (i.e., edges may be considered bidirectional):



Run the "trawling" algorithm for finding dense communities on this graph and find all complete bipartite subgraphs of types $K_{3,2}$ and $K_{2,2}$. Note: In the case of $K_{2,2}$, we consider $\{\{W, X\}, \{Y, Z\}\}$ and $\{\{Y, Z\}, \{W, X\}\}$ to be identical.

Question 2

Items:

- $A = \{B, E, C\}$
- $B = \{A, E, D, C\}$
- $C = \{B, A, E\}$
- $D = \{B, E\}$
- $E = \{A, B, C, D\}$

$K_{3,2}$ and $K_{2,2}$

$K_{2,2}$ min support threshold = 3

Item	Support	Item	Support	Item	Support
A	3	B, E	3	B, E	3
B	4	BC	2		
C	3	EC	2		
D	2	A, E	2		
E	4	A, D	2		
		A, C	2		
		E, D	1		
		D, C	2		
		B, A	2		
		B, D	1		
		E, D	2		

1 bipartite graph

$K_{2,2}$ min support = 2

item Support

B, E 3

B, C 2

E, C 2

A, E 2

A, D 2

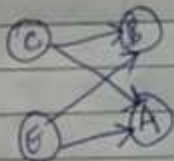
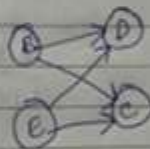
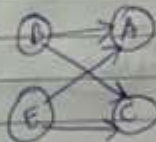
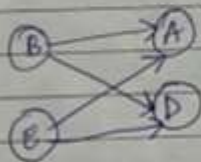
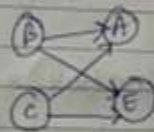
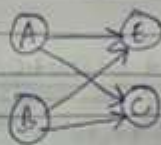
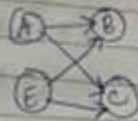
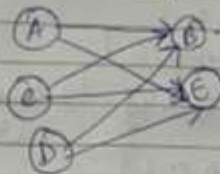
A, C 2

D, C 2

B, A 2

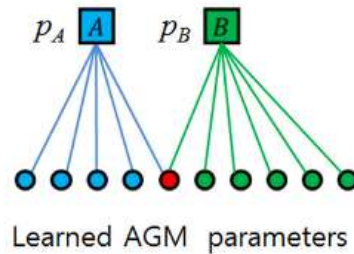
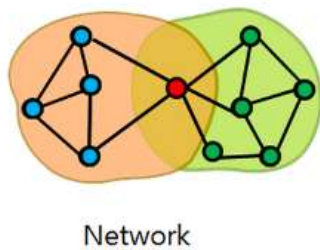
~~E, D~~ 2

7 bipartite graph.



Question 3:

We fit AGM to the network on the left, and found the parameters on the right:



Find the optimal values for p_A and p_B .

SOLUTION

$p_A = \text{Number of edges in the network} / \text{Total possible number of edges} = 7/5 \times 2 = 7/10.$

$p_B = \text{Number of edges in the network} / \text{Total possible number of edges} = 9/6 \times 2 = 9/15.$