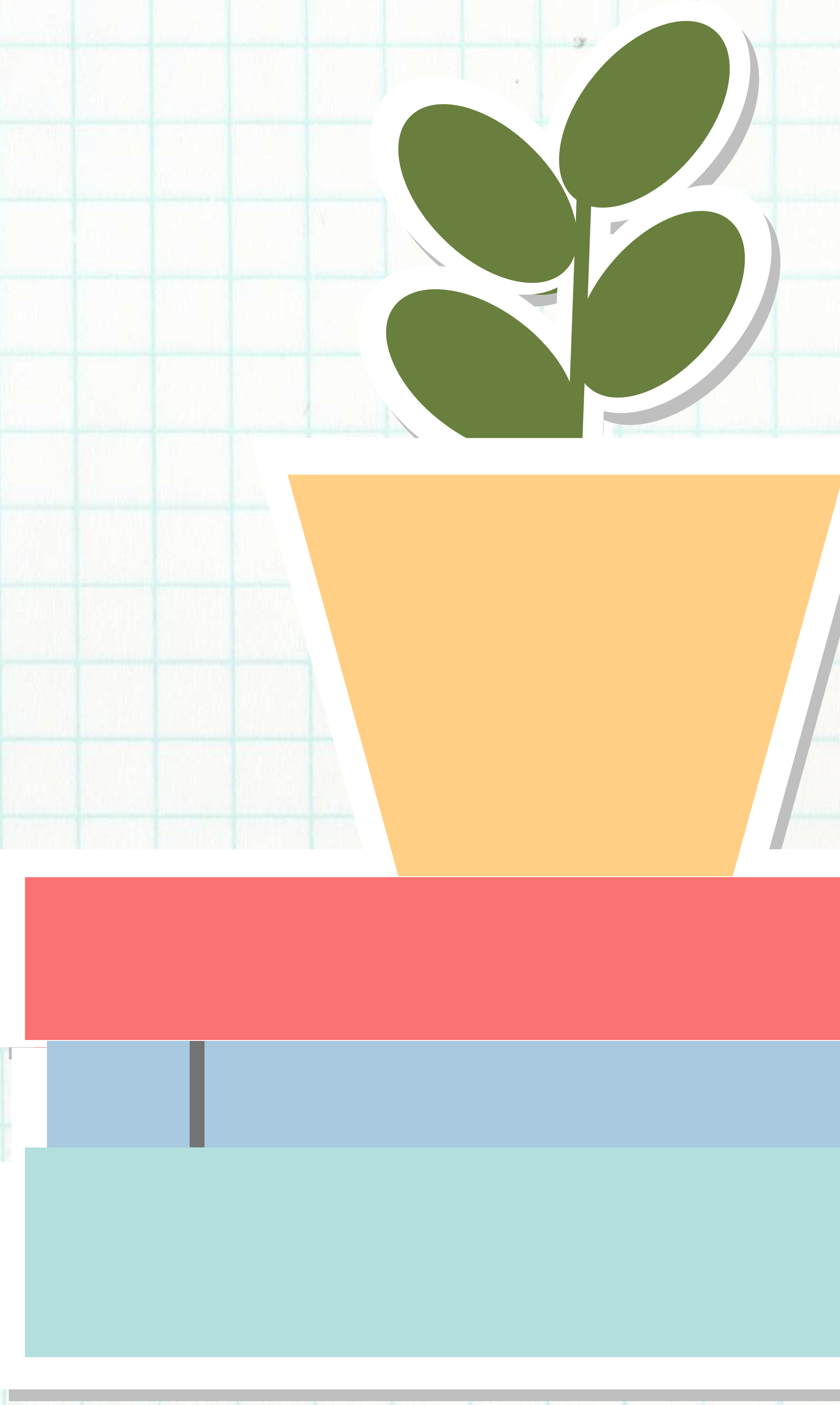
EIGEN VALUE & EIGEN VEKTOR

OBJECTIVES

1. Eigen Value
2. Vektor Eigen
3. Power Matrix



NILAI EIGEN & VEKTOR EIGEN

$$\boxed{A_{n \times n} \; X_{n \times 1} = \lambda \, . \, X_{n \times 1}}$$

vektor eigen ↲ ↳ nilai eigen / sebenarnya

Diket $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ adalah vektor eigen dari $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$

? ⇒ berapakah nilai eigen ? ?

jwb: $\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} . \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \lambda . \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\therefore \lambda = 3$$

CONTOH SOAL

pers karakteristik A :

$$\boxed{\det(\lambda . I - A) = 0}$$

polinom karakteristik A : ~> menghasilkan persamaan

$\det(\lambda . I - A) = \lambda^n + c_1 \lambda^{n-1} + c_2 \lambda^{n-2} + \dots + c_n$

variabel ↵   ↳ konstanta

x:

Cari nilai eigen dari $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$

jawab:

- polinom karakteristik A :

$\det(\lambda . I - A) = \det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}\right)$

$= \det \begin{pmatrix} \lambda - 3 & -2 \\ 1 & \lambda \end{pmatrix}$

$= \lambda^2 - 3\lambda + 2$

- pers. karakteristik A :

$\det(\lambda . I - A) = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$(\lambda - 1)(\lambda - 2) = 0$

- $\therefore \lambda = 1$ dan $\lambda = 2$ adalah nilai-nilai eigen dari A

CONTOH SOAL

6.2

3 : Carilah nilai eigen dari $A = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$

Jawab :

- polinom karakteristik A :

$$\det(\lambda . I - A) = \det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}\right)$$

$$= \det\begin{pmatrix} \lambda + 2 & 1 \\ -5 & \lambda - 2 \end{pmatrix}$$

$$= \lambda^2 - 4 + 5$$

$$= \lambda^2 + 1$$

- pers. karakteristik A

$$\det(\lambda . I - A) = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \sqrt{-1}$$

- $\therefore$ krn $\lambda = \sqrt{-1}$, mk tidak ada nilai eigen untuk A

CONTOH SOAL

: 4 : Carilah nilai eigen dari : $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$

jawab :

pers. karakteristik A : $\det(\lambda . I - A) = 0$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}\right) = 0$$

$$\det \begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -4 & 17 & \lambda - 8 \end{pmatrix} = 0 \quad \Rightarrow \text{using kofaktor}$$

$$\lambda \underbrace{\begin{bmatrix} \lambda & -1 \\ 17 & \lambda - 8 \end{bmatrix}}_{\det} - 4 \underbrace{\begin{bmatrix} -1 & 0 \\ \lambda & -1 \end{bmatrix}}_{\det} = 0$$

$$\lambda(\lambda^2 - 8\lambda + 17) - 4(1) = 0$$

$$\lambda^3 - 8\lambda^2 + 17\lambda - 4 = 0$$

↳ kemungkinan $\boxed{-4}$ < ± 1, ± 2, ± 4 } coba satu² mana yg benar

mis ⇒ $\lambda = 4$

↳ $\lambda^3 - 8\lambda^2 + 17\lambda - 4 = (\lambda - 4)(\lambda^2 - 4\lambda + 1)$

$= (\lambda - 4)$ ↓ pakai rumus ABC

kmd ketemu nilai² eigennya

adalah ⇒ $\lambda = 4$

$\lambda = 2 + \sqrt{3}$

$\lambda = 2 - \sqrt{3}$

TEOREMA

6.3

teorema 1

jk $A_{n \times n}$ mk pernyataan berikut ekivalen satu sama lain:

→ a) $\lambda \rightarrow$ nilai eigen dari A

→ b) $(\lambda . I - A) . x = 0$ memp. pemecahan yang tak trivial (banyak pemecahan)

→ c) ada vektor tak nol x shingga $A . x = \lambda . x$

→ d) λ adalah pemecahan riil dari pers karakte-ristik $\det(\lambda . I - A) = 0$

CONTOH SOAL

: 5 :

Cari basis & ruang eigen $A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

↳ cari vektor eigen

Jawab:

1) Cari λ → pers karakteristik $A \Rightarrow (\lambda - 1)(\lambda - 5)^2 = 0$

→ ∴ $\lambda = 1$, $\lambda = 5$ } buktikan

→ ∴

2) Cari $X \Rightarrow (\lambda I - A) . X = 0$

$$\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}\right) \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

CONTOH SOAL

$$\begin{pmatrix} \lambda-3 & 2 & 0 \\ 2 & \lambda-3 & 0 \\ 0 & 0 & \lambda-5 \end{pmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ jk $\lambda = 5$ mk menj :

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

maka $\Rightarrow$ $\left.\begin{aligned} x_1 &= -s \\ x_2 &= s \\ x_3 &= t \end{aligned}\right\}$

catt : - index besar terletak di kiri =
- index besar mrp komb lin dr kecil

$$x = \begin{bmatrix} -s \\ s \\ t \end{bmatrix} = \begin{bmatrix} -s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix}$$

$$= s\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

vektor eigen $\underline{u}$ $\lambda = 5$ $\longrightarrow$ $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ dan $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\Rightarrow$ jk $\lambda = 1$ mk menj :

$$\begin{bmatrix} -2 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

mk $\Rightarrow$ $x_1 = t$

$x_2 = t$

$x_3 = 0$

$$x = \begin{bmatrix} t \\ t \\ 0 \end{bmatrix} = t\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ vektor eigen $\underline{u}$ $\lambda = 1$ $\longrightarrow$ $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$$A = \begin{vmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{vmatrix}$$

CONTOH SOAL

Carilah nilai eigen dan vector eigen dari matrix A

nilai eigen

$\det(\lambda . I - A) = 0$

$$\det \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} \lambda & 1 & 3 \\ -2 & \lambda - 3 & -3 \\ 2 & -1 & \lambda - 1 \end{vmatrix} = 0$$

baris pertama => kofaktor

$\lambda * ((\lambda-3)*(\lambda-1)-3) +$

$- 1 * (- 2 * (\lambda - 1) + 6) +$

$3 * (2 - (\lambda - 3)*2) =$

$\lambda =$	-2
$\lambda =$	2
$\lambda =$	4

vektor eigen

$(\lambda . I - A) . X = 0$

$$\begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{vmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \end{vmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$\begin{vmatrix} \lambda & 1 & 3 \\ -2 & \lambda \ \ 3 & -3 \\ 2 & -1 & \lambda - 1 \end{vmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \end{vmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Jika λ = -2

$$\begin{bmatrix} -2 & 1 & 3 \\ -2 & -5 & -3 \\ 2 & -1 & -3 \end{bmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \end{vmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Carilah nilai X1, X2 dan X3 dengan Gauss

1	-0.5	-1.5	0
-2	-5	-3	0
2	-1	-3	0

1	-0.5	-1.5	0
0	-6	-6	0
2	-1	-3	0

1	-0.5	-1.5	0
0	-6	-6	0
0	0	0	0

1	-0.5	-1.5	0
0	1	1	0
0	0	0	0

$$\begin{matrix} x1 = & t \\ x2 = & -t \\ x3 = & t \end{matrix} \quad = t \begin{matrix} 1 \\ -1 \\ 1 \end{matrix}$$

Jadi vektor eigen untuk λ = - 2 adalah

$$x = \begin{vmatrix} 1 \\ -1 \\ 1 \end{vmatrix}$$

Jika λ = 2

$$\begin{bmatrix} 2 & 1 & 3 \\ -2 & -1 & -3 \\ 2 & -1 & 1 \end{bmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \end{vmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Carilah nilai X1, X2 dan X3 dengan Gauss

1	0.5	1.5	0
-2	-1	-3	0
2	-1	1	0

1	0.5	1.5	0
0	0	0	0
2	-1	1	0

1	0.5	1.5	0
0	0	0	0
0	-2	-2	0

1	0.5	1.5	0
0	0	0	0
0	1	1	0

$$\begin{matrix} x1 = & -t \\ x2 = & -t \\ x3 = & t \end{matrix} \quad = t \begin{matrix} -1 \\ -1 \\ 1 \end{matrix}$$

Jadi vektor eigen untuk λ = 2 adalah

$$x = \begin{vmatrix} -1 \\ -1 \\ 1 \end{vmatrix}$$

Jika λ = 4

$$\begin{bmatrix} 4 & 1 & 3 \\ -2 & 1 & -3 \\ 2 & -1 & 3 \end{bmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \end{vmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Carilah nilai X1, X2 dan X3 dengan Gauss

1	0.25	0.75	0
-2	1	-3	0
2	-1	3	0

1	0.25	0.75	0
0	1.5	-1.5	0
2	-1	3	0

1	0.25	0.75	0
0	1.5	-1.5	0
0	-1.5	1.5	0

1	0.25	0.75	0
0	1	-1	0
0	-1.5	1.5	0

1	0.25	0.75	0
0	1	-1	0
0	0	0	0

$$\begin{matrix} x1 = & -t \\ x2 = & t \\ x3 = & t \end{matrix} \quad = t \begin{matrix} -1 \\ 1 \\ 1 \end{matrix}$$

Jadi vektor eigen untuk λ = 4 adalah

$$x = \begin{vmatrix} -1 \\ 1 \\ 1 \end{vmatrix}$$

TEOREMA

Theorem 7.1.1. *If A is an $n \times n$ triangular matrix (upper triangular, lower triangular, or diagonal), then the eigenvalues of A are the entries on the main diagonal of A.*

Example 4 By inspection, the eigenvalues of the lower triangular matrix

$$A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & \frac{2}{3} & 0 \\ 5 & -8 & -\frac{1}{4} \end{bmatrix}$$

are $\lambda = \frac{1}{2}$, $\lambda = \frac{2}{3}$, and $\lambda = -\frac{1}{4}$.

CONTOH SOAL

Example 5 Find bases for the eigenspaces of

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Solution. The characteristic equation of A is $\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$, or in factored form, $(\lambda - 1)(\lambda - 2)^2 = 0$ (verify); thus, the eigenvalues of A are $\lambda = 1$ and $\lambda = 2$, so there are two eigenspaces of A.

By definition,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

is an eigenvector of A corresponding to λ if and only if $\mathbf{x}$ is a nontrivial solution of $(\lambda I - A)\mathbf{x} = \mathbf{0}$, that is, of

$$\begin{bmatrix} \lambda & 0 & 2 \\ -1 & \lambda - 2 & -1 \\ -1 & 0 & \lambda - 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

If $\lambda = 2$, then (3) becomes

$$\begin{bmatrix} 2 & 0 & 2 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this system yields (verify)

$$x_1 = -s, \qquad x_2 = t, \qquad x_3 = s$$

Thus, the eigenvectors of A corresponding to $\lambda = 2$ are the nonzero vectors of the form

$$\mathbf{x} = \begin{bmatrix} -s \\ t \\ s \end{bmatrix} = \begin{bmatrix} -s \\ 0 \\ s \end{bmatrix} + \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix} = s\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Since

$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

are linearly independent, these vectors form a basis for the eigenspace corresponding to $\lambda = 2$.

If $\lambda = 1$, then (3) becomes

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & -1 & -1 \\ -1 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this system yields (verify)

$$x_1 = -2s, \qquad x_2 = s, \qquad x_3 = s$$

Thus, the eigenvectors corresponding to $\lambda = 1$ are the nonzero vectors of the form

$$\begin{bmatrix} -2s \\ s \\ s \end{bmatrix} = s\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

so that

$$\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

is a basis for the eigenspace corresponding to $\lambda = 1$.

POWER MATRIX

Teori 7.1.3

jika $k \to$ int $(+)$

$\lambda \to$ eigen value matrix A

$x \to$ eigen vektor $\to A$

maka $\lambda^k \to$ eigen value $\to A^k$.

$x \to$ eigen vektor $\to A$

misal : $A^{13} = A \cdot A \dots A$

matrix $A \to \lambda$, x

$A^k \to \lambda^k$, x

$\to$ tetap

CONTOH SOAL

contoh :

jika diketahui $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

mempunyai $\to \lambda = 2 \longrightarrow$ eigen vektor $\to \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = 1 \longrightarrow \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

berapakah $\to$ eigen value, eigen vektor $\}$ dari matrix A^k

jika $k = 7 \longrightarrow A^7$

CONTOH SOAL

jawab:

eigen value $\rightarrow \lambda^k \Rightarrow 2^k \Rightarrow 2^7 = 128$

$\rightarrow$ tapi eigen vektor tetap $\rightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\rightarrow \lambda^k = 1^7 = 1$

$\rightarrow$ eigen vektor tetap $\rightarrow \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

POWER MATRIX

•) mencari A^k dpt dgn cara $\rightsquigarrow$ biasa $\rightsquigarrow$ $A \cdot A \cdot A \cdot A$

atau dgn rumus : $\boxed{A^k = P \cdot D^k \cdot P^{-1}}$

P $\rightsquigarrow$ matrix diagonalizes A

$\llcorner$ kumpulan dr eigen vektor A

$D = P^{-1} \cdot A \cdot P$ $\rightsquigarrow$ menghasilkan matrix diagonal

D^k $\Rightarrow$ matrix D dimana, tiap item pd diagonalnya dipangkat k

CONTOH SOAL

contoh :

Temukan A^{13} u matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

P dari matrix A adalah

$$P = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$D = P^{-1} . A . P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

maka

$$A^{13} = P . D^{13} . P^{-1}$$

$$= \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2^{13} & 0 & 0 \\ 0 & 2^{13} & 0 \\ 0 & 0 & 1^{13} \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -8190 & 0 & -16382 \\ 8191 & 8192 & 8191 \\ 8191 & 0 & 16383 \end{bmatrix}$$

CONTOH SOAL

2. (Nilai 72) Diketahui matrix A sebagai berikut

$$A = \begin{vmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{vmatrix}$$

Ditanya :

1. Nilai eigen. Catt : carilah determinan dengan menggunakan kofaktor kolom ke-2
2. Vektor Eigen
3. Carilah A^3 dengan menggunakan vektor eigen

   Catt :

   a. Urutan matrix P adalah :
      1. Kolom 1 → nilai eigen terkecil
      2. Kolom 2 → nilai eigen terkecil berikutnya
      3. Kolom 3 → nilai eigen terbesar

   b. Cari P^{-1} dengan menggunakan OBE

   c. $A^k = P \cdot (D^k \cdot P^{-1})$

      Hitung dulu $D^k \cdot P^{-1}$, setelah itu baru dikalikan dengan P

   d. $D = P^{-1} \cdot (A \cdot P)$

      Hitung dulu A . P, setelah itu baru di kalikan P^{-1}

$$A = \begin{bmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

1) Nilai eigen $\Rightarrow \lambda$

$$\det(\lambda . I - A) = \varnothing$$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}\right) = \varnothing$$

$$\det\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}\right) = \varnothing$$

3

$$\det . \begin{bmatrix} \lambda-3 & -8 & 0 \\ -3 & \lambda-1 & 0 \\ 0 & 0 & \lambda-8 \end{bmatrix} = \varnothing$$

Cari det dgn menggo. kofaktor kolom ke 2 (2)

$$8.(-3(\lambda-8)) + (\lambda-1)(\lambda-3)(\lambda-8) = \varnothing$$

3

$$8(-3\lambda+24) + (\lambda-1)(\lambda^2-11\lambda+24) = \varnothing$$

$$-24\lambda + 192 + (\lambda-1)(\lambda^2-11\lambda+24) = \varnothing$$

$$-24\lambda + 192 + \lambda^3 - 11\lambda^2 + 24\lambda - \lambda^2 + 11\lambda - 24 = \varnothing$$

$$\lambda^3 - 12\lambda^2 + 11\lambda + 168 = \varnothing$$

3

$$(\lambda+3)(\lambda-7)(\lambda-8) = \varnothing$$

3

$\therefore$ nilai eigen $\longrightarrow$ $\lambda = -3$

$\lambda = 7$

$\lambda = 8$.

2) Vektor eigen

$\Rightarrow$ Cari $x \Rightarrow (\lambda . I - A) . \bar{x} = \bar{0}$

$$\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}\right) . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

3

$$\begin{bmatrix} \lambda - 3 & -8 & 0 \\ -3 & \lambda - 1 & 0 \\ 0 & 0 & \lambda - 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ jk $\lambda = -3$

$$\begin{bmatrix} -6 & -8 & 0 \\ -3 & -4 & 0 \\ 0 & 0 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

mk $\Rightarrow$ $-6x_1 - 8x_2 = \emptyset$

$\boxed{x_2 = t}$

$-6x_1 - 8t = \emptyset$

$-6x_1 = 8t$

$x_1 = -\frac{8t}{6}$

$\boxed{x_1 = -\frac{4}{3}t}$

$-11 x_3 = \emptyset$

$\boxed{x_3 = \emptyset}$

$$x = \begin{bmatrix} -\frac{4}{3}t \\ t \\ \emptyset \end{bmatrix} = t \begin{bmatrix} -\frac{4}{3} \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ vektor eigen u $\lambda = -3$ adalah $\begin{bmatrix} -\frac{4}{3} \\ 1 \\ 0 \end{bmatrix}$

$\rightarrow$ jk $\lambda = 7$

$$\begin{bmatrix} 4 & -8 & 0 \\ -3 & 6 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

mk $\Rightarrow$ $4x_1 - 8x_2 = \emptyset$

$\boxed{x_2 = t}$

$4x_1 - 8t = \emptyset$

$4x_1 = 8t$

$\boxed{x_1 = 2t}$

$-x_3 = \emptyset$

$\boxed{x_3 = \emptyset}$

$$x = \begin{bmatrix} 2t \\ t \\ \emptyset \end{bmatrix} = t \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ vektor eigen u $\lambda = 7$ adalah $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

$\rightarrow$ jk $\lambda = 8$

$$\begin{bmatrix} 5 & -8 & 0 \\ -3 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

mk $\Rightarrow$ $5x_1 - 8x_2 = \emptyset \quad | \times 3$

$-3x_1 + 7x_2 = \emptyset \quad | \times 5$

$15x_1 - 24x_2 = \emptyset$

$-15x_1 + 35x_2 = \emptyset$ +

$11x_2 = \emptyset$

$\boxed{x_2 = \emptyset}$

$-3x_1 + 7x_2 = \emptyset$

$-3x_1 + 7.0 = \emptyset$

$-3x_1 = \emptyset$

$\boxed{x_1 = 0}$

$0x_3 = \emptyset$

$\boxed{x_3 = t}$

$$x = \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} = t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ vektor eigen u $\lambda = 8$ adalah $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

③ A^3 dgn mengg vektor eigen ⑥

$$A^3 = P \cdot D^k \cdot P^{-1}$$

$P \rightarrow$ kumpulan vektor eigen

$D \rightarrow P^{-1} \cdot A \cdot P$

$\rightarrow$ matrix diagonal

$\rightarrow$ kumpulan nilai eigen

D^k = matrix D yg tiap item pd diagonalnya dipangkatkan k

P^{-1} dicari dgn OBE

$$P = \begin{bmatrix} -4/3 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3

	-1,3333	2	0			
P =	1	1	0			
	0	0	1			
	-1,3333	2	0	1	0	0
	1	1	0	0	1	0
	0	0	1	0	0	1
	1,00	-1,50	0,00	-0,75	0,00	0,00
	1,00	1,00	0,00	0,00	1,00	0,00
	0,00	0,00	1,00	0,00	0,00	1,00
	1,00	-1,50	0,00	-0,75	0,00	0,00
	0,00	2,50	0,00	0,75	1,00	0,00
	0,00	0,00	1,00	0,00	0,00	1,00
	1,00	-1,50	0,00	-0,75	0,00	0,00
	0,00	1,00	0,00	0,30	0,40	0,00
	0,00	0,00	1,00	0,00	0,00	1,00
	1,00	0,00	0,00	-0,30	0,60	0,00
	0,00	1,00	0,00	0,30	0,40	0,00
	0,00	0,00	1,00	0,00	0,00	1,00

THANK YOU