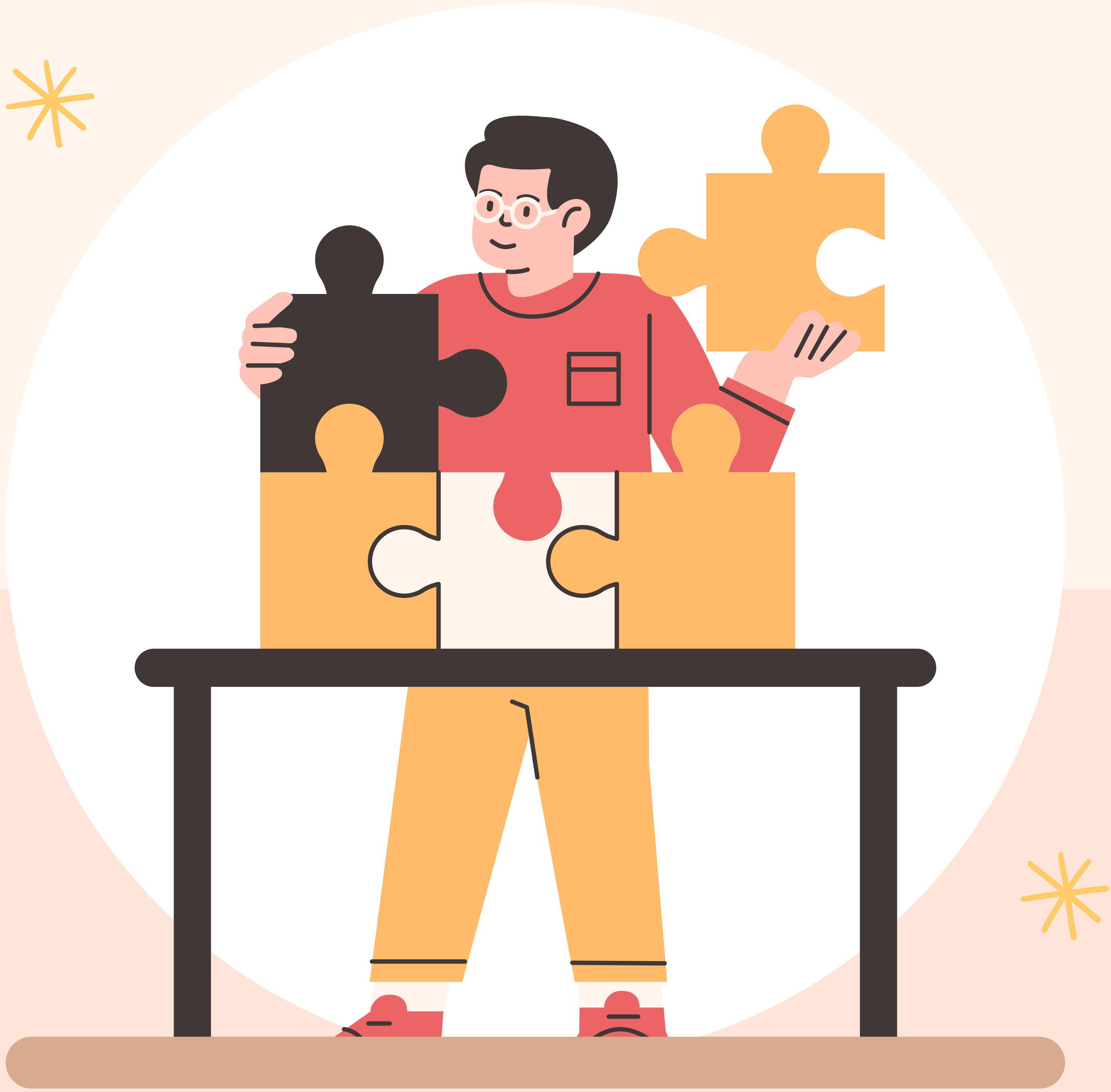
Komputasi Numerik

PERTEMUAN 10

Interpolasi Newton-Gregory, Stirling dan Bessel

2024/2025



Komnum Week 9

Apa Yang Akan Kita Pelajari?

01 Interpolasi Newton-Gregory

02 Interpolasi Stirling

02 Interpolasi Bessel

Tabel Beda Interpolasi Newton-Gregory

s	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\ldots$
0	x_0	f_0					
			Δf_0				
1	x_1	f_1		$\Delta^2 f_0$			
			Δf_1		$\Delta^3 f_0$		
2	x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$	$\ldots$
			Δf_2		$\Delta^3 f_1$		
$\vdots$	$\vdots$	$\vdots$					
$\vdots$	$\vdots$	$\vdots$					
$\vdots$	$\vdots$	$\vdots$					
			Δf_{n-3}		$\Delta^3 f_{n-4}$		
$n-2$	x_{n-2}	f_{n-2}		$\Delta^2 f_{n-3}$		$\Delta^4 f_{n-4}$	$\ldots$
			Δf_{n-2}		$\Delta^3 f_{n-3}$		
$n-1$	x_{n-1}	f_{n-1}		$\Delta^2 f_{n-2}$			
			Δf_{n-1}				
n	x_n	f_n					

Interpolasi Newton-Gregory

Interpolasi Newton-Gregory Forward:

$$f(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \cdots + \frac{s(s-1)(s-2)\cdots(s-n+1)}{n!}\Delta^n f_0$$

Interpolasi Newton-Gregory Backward:

$$f(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)}{2!}\Delta^2 f_{-2} + \frac{s(s+1)(s+2)}{3!}\Delta^3 f_{-3} + \cdots + \frac{s(s+1)(s+2)\cdots(s+n-1)}{n!}\Delta^n f_{-n}$$

$$\boxed{s = \frac{x_s - x_0}{h}, \quad h = \Delta x}$$

Contoh Interpolasi Newton-Gregory

Contoh interpolasi dengan orde-3:

Interpolasi Newton-Gregory Forward:

$$f_3(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0$$

Interpolasi Newton-Gregory Backward:

$$f_3(x_s) = f_0 + s\Delta f_0 + \frac{s(s+1)}{2!}\Delta^2 f_{-1} + \frac{s(s+1)(s+2)}{3!}\Delta^3 f_{-2}$$

$$\boxed{s = \frac{x_s - x_0}{h}, \quad h = \Delta x}$$

Interpolasi Newton-Gregory

Contoh Soal 1

Carilah nilai f(xs) untuk xs = 3, jika diketahui fungsi tersebut menghasilkan nilai tabel sebagai berikut:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
2	31			
		351		
5	382		810	
		1161		486
8	1543		1296	
		2457		
11	4000			

Gunakan Interpolasi Newton-Gregory Forward!

Jawaban Contoh 1

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
2	31			
		351		
5	382		810	
		1161		486
8	1543		1296	
		2457		
11	4000			

$$f_3(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0$$

$$s = \frac{x_s - x_0}{h}, \quad h = \Delta x \to h = 3$$

$$s = \frac{3-2}{3} = \frac{1}{3}$$

-90

117

$$f_3(3) = 31 + \frac{1}{3}(351) + \frac{(\frac{1}{3})(\frac{1}{3}-1)}{2!}(810) +$$

30

$$\frac{(\frac{1}{3})(\frac{1}{3}-1)(\frac{1}{3}-2)}{3!}(486)$$

$$f_3(3) = 88$$

Interpolasi Newton-Gregory

Contoh Soal 2

Diketahui Tabel sebagai berikut:

x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
2	18.00			
		108.53		
2.4	126.53		192.82	
		301.34		196.42
2.8	427.87		389.24	
		690.58		316.11
3.2	1118.45		705.35	
		1395.93		476.34
3.6	2514.38		1181.69	
		2577.62		683.03
4	5092.00		1864.72	
		4442.34		
4.4	9534.34			

- Carilah nilai f(x), ketika x = 3,3 dengan menggunakan Newton-Gregory Forward
- Carilah nilai f(x), ketika x = 3,3 dengan menggunakan Newton-Gregory Backward
- Note:
  - Hitung Error jika diketahui f(3,3) = 1385,77

Jawaban Contoh 2

- Newton-Gregory **Forward**:

x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
2	18.00			
		108.53		
2.4	126.53		192.82	
		301.34		196.42
2.8	427.87		389.24	
		690.58		316.11
3.2	1118.45		705.35	
		1395.93		476.34
3.6	2514.38		1181.69	
		2577.62		683.03
4	5092.00		1864.72	
		4442.34		
4.4	9534.34			

$$f(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \cdots + \frac{s(s-1)(s-2)\cdots(s-n+1)}{n!}\Delta^n f_0$$

$$s = \frac{x_s - x_0}{h}, \quad h = \Delta x$$

$$s = \frac{3.3 - 3.2}{h}, \quad h = 3.2 - 2.8 = 0.4$$
$$= 0.25$$

348.98

$$f_3(3.3) = 1118.45 + (0.25)(1395.93) +$$

-110.78

$$\frac{(0.25)(0.25-1)}{2!}(1181.69) +$$

37.35

$$\frac{(0.25)(0.25-1)(0.25-2)}{3!}(683.03)$$

$$= 1394$$

Jawaban Contoh 2

- Newton-Gregory **backward**:

x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
2	18.00			
		108.53		
2.4	126.53		192.82	
		301.34		196.42
2.8	427.87		389.24	
		690.58		316.11
3.2	1118.45		705.35	
		1395.93		476.34
3.6	2514.38		1181.69	
		2577.62		683.03
4	5092.00		1864.72	
		4442.34		
4.4	9534.34			

$$f(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)}{2!}\Delta^2 f_{-2} + \frac{s(s+1)(s+2)}{3!}\Delta^3 f_{-3} + \cdots + \frac{s(s+1)(s+2)\cdots(s+n-1)}{n!}\Delta^n f_{-n}$$

$$s = \frac{x_s - x_0}{h}, \quad h = \Delta x$$

$$s = \frac{3.3 - 3.2}{h}, \quad h = 3.2 - 2.8 = 0.4$$

$$= 0.25$$

172.64

$$f_3(3.3) = 1118.45 + (0.25)(690.58) +$$

60.82

$$\frac{(0.25)(0.25+1)}{2!}(389.24) +$$

23.02

$$\frac{(0.25)(0.25+1)(0.25+2)}{3!}(196.43)$$

$$= 1374.93$$

Interpolasi Stirling

Interpolasi Stirling:

Rata-Rata

$$f_n(x) = f_0 + s\Delta f +$$

Ambil

Ambil

Rata-Rata

$$\frac{s^2}{2!}\Delta^2 f_{-1} + \frac{s(s^2-1)}{3!}\Delta^3 f +$$

Rata-Rata

$$\frac{s^2(s^2-1)}{4!}\Delta^4 f_{-2} + \frac{s(s^2-1)(s^2-4)}{5!}\Delta^5 f + \ldots$$

Ambil

$$\boxed{\begin{aligned} \Delta f &= \frac{1}{2}(\Delta f_{-1} + \Delta f_0) \\ \Delta^3 f &= \frac{1}{2}(\Delta^3 f_{-2} + \Delta^3 f_{-1}) \\ \Delta^5 f &= \frac{1}{2}(\Delta^5 f_{-3} + \Delta^5 f_{-2}) \end{aligned}}$$

$$\boxed{s = \frac{x_s - x_0}{h}, \quad h = \Delta x}$$

Tentang Interpolasi Stirling:

- Stirling merupakan rerata dari rumusan Gauss Forward dan backward. Karena menggunakan rerata beda gasal di atas dan di bawah garis sentral, serta beda genap pada garis sentral
- Kelebihan dan kekurangan metode Stirling sama dengan metode Newton-Gregory. Perbedaannya, Stirling lebih optimal jika digunakan untuk mencari nilai f(xs) dengan xs di sekitar titik tengah

Tabel Beda Interpolasi Stirling

f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$	$\Delta^7 f$	$\Delta^8 f$
f_{-4}								
	Δf_{-4}							
f_{-3}		$\Delta^2 f_{-4}$						
	Δf_{-3}		$\Delta^3 f_{-4}$					
f_{-2}		$\Delta^2 f_{-3}$		$\Delta^4 f_{-4}$				
	Δf_{-2}		$\Delta^3 f_{-3}$		$\Delta^5 f_{-4}$			
f_{-1}		$\Delta^2 f_{-2}$		$\Delta^4 f_{-3}$		$\Delta^6 f_{-4}$		
	$\boldsymbol{\Delta f_{-1}}$		$\boldsymbol{\Delta^3 f_{-2}}$		$\boldsymbol{\Delta^5 f_{-3}}$		$\boldsymbol{\Delta^7 f_{-4}}$	
$\boldsymbol{f_0}$		$\boldsymbol{\Delta^2 f_{-1}}$		$\boldsymbol{\Delta^4 f_{-2}}$		$\boldsymbol{\Delta^6 f_{-3}}$		$\boldsymbol{\Delta^8 f_{-4}}$
	$\boldsymbol{\Delta f_0}$		$\boldsymbol{\Delta^3 f_{-1}}$		$\boldsymbol{\Delta^5 f_{-2}}$		$\boldsymbol{\Delta^7 f_{-3}}$	
f_1		$\Delta^2 f_0$		$\Delta^4 f_{-1}$		$\Delta^6 f_{-2}$		
	Δf_1		$\Delta^3 f_0$		$\Delta^5 f_{-1}$		$\Delta^7 f_{-2}$	
f_2		$\Delta^2 f_1$		$\Delta^4 f_0$		$\Delta^6 f_{-1}$		
	Δf_2		$\Delta^3 f_1$		$\Delta^5 f_0$			
f_3		$\Delta^2 f_2$		$\Delta^4 f_1$				
	Δf_3		$\Delta^3 f_2$					
f_4		$\Delta^2 f_3$						
	Δf_4							
f_5								

Garis Sentral

Interpolasi Bessel

Interpolasi Bessel:

$$f_n(x) = f + u\Delta f_0 +$$

$$\frac{(u^2-\frac{1}{4})}{2!}\Delta^2 f + \frac{u(u^2-\frac{1}{4})}{3!}\Delta^3 f_{-1} +$$

$$\frac{(u^2-\frac{1}{4})(u^2-\frac{9}{4})}{4!}\Delta^4 f + \frac{u(u^2-\frac{1}{4})(u^2-\frac{9}{4})}{5!}\Delta^5 f_{-2} + \ldots$$

Rata-Rata

Rata-Rata

Ambil

Ambil

Rata-Rata

$$\boxed{s = \frac{x_s - x_0}{h}, \quad h = \Delta x}$$

$$\boxed{u = s - \frac{1}{2}}$$

$$f = \frac{1}{2}(f_0 + f_1)$$

$$\Delta^2 f = \frac{1}{2}\left(\Delta^2 f_{-1} + \Delta^2 f_0\right)$$

$$\Delta^4 f = \frac{1}{2}\left(\Delta^4 f_{-2} + \Delta^4 f_{-1}\right)$$

Tabel Beda Interpolasi Bessel

f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$	$\Delta^7 f$	$\Delta^8 f$
f_{-4}								
	Δf_{-4}							
f_{-3}		$\Delta^2 f_{-4}$						
	Δf_{-3}		$\Delta^3 f_{-4}$					
f_{-2}		$\Delta^2 f_{-3}$		$\Delta^4 f_{-4}$				
	Δf_{-2}		$\Delta^3 f_{-3}$		$\Delta^5 f_{-4}$			
f_{-1}		$\Delta^2 f_{-2}$		$\Delta^4 f_{-3}$		$\Delta^6 f_{-4}$		
	Δf_{-1}		$\Delta^3 f_{-2}$		$\Delta^5 f_{-3}$		$\Delta^7 f_{-4}$	
$\boldsymbol{f_0}$		$\boldsymbol{\Delta^2 f_{-1}}$		$\boldsymbol{\Delta^4 f_{-2}}$		$\boldsymbol{\Delta^6 f_{-3}}$		$\boldsymbol{\Delta^8 f_{-4}}$
	$\boldsymbol{\Delta f_0}$		$\boldsymbol{\Delta^3 f_{-1}}$		$\boldsymbol{\Delta^5 f_{-2}}$		$\boldsymbol{\Delta^7 f_{-3}}$	
$\boldsymbol{f_1}$		$\boldsymbol{\Delta^2 f_0}$		$\boldsymbol{\Delta^4 f_{-1}}$		$\boldsymbol{\Delta^6 f_{-2}}$		$\boldsymbol{\Delta^8 f_{-3}}$
	Δf_1		$\Delta^3 f_0$		$\Delta^5 f_{-1}$		$\Delta^7 f_{-2}$	
f_2		$\Delta^2 f_1$		$\Delta^4 f_0$		$\Delta^6 f_{-1}$		
	Δf_2		$\Delta^3 f_1$		$\Delta^5 f_0$			
f_3		$\Delta^2 f_2$		$\Delta^4 f_1$				
	Δf_3		$\Delta^3 f_2$					
f_4		$\Delta^2 f_3$						
	Δf_4							
f_5								

Garis Sentral

Interpolasi Stirling

Contoh Soal 1

Diketahui Tabel sebagai berikut:

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
3	-639,90			
		-680,95		
3,6	-1320,85		-411,63	
		-1092,58		-104,01
4,2	-2413,43		-515,64	
		-1608,22		-82,11
4,8	-4021,66		-597,76	
		-2205,98		-40,62
5,4	-6227,64		-638,38	
		-2844,36		23,83
6	-9072,00		-614,55	
		-3458,92		
6,6	-12530,92			

Carilah nilai f(x), ketika x = 4.9 dengan menggunakan Interpolasi Stirling Orde 3

Jawaban Contoh 1

- Stirling:

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
3	-639,90			
		-680,95		
3,6	-1320,85		-411,63	
		-1092,58		-104,01
4,2	-2413,43		-515,64	
		-1608,22		-82,11
4,8	-4021,66		-597,76	
		-2205,98		-40,62
5,4	-6227,64		-638,38	
		-2844,36		23,83
6	-9072,00		-614,55	
		-3458,92		
6,6	-12530,92			

$$f_n(x) = f_0 + s\Delta f + \frac{s^2}{2!}\Delta^2 f_{-1} + \frac{s(s^2-1)}{3!}\Delta^3 f$$

$$s = \frac{x_s - x_0}{h}, \quad h = \Delta x$$

$$s = \frac{4.9 - 4.8}{h}, \; h = 4.8 - 4.2 = 0.6$$

$$s = 0.17$$

$$\Delta f = \frac{-2205.98 - 1608.23}{2} = -1907.11$$

$$\Delta^3 f = \frac{-40.63 - 82.1}{2} = -61.36$$

Jawaban Contoh 1

$$f_n(x) = f_0 + s\Delta f + \frac{s^2}{2!}\Delta^2 f_{-1} + \frac{s(s^2-1)}{3!}\Delta^3 f$$

-317.85

$$f_3(4.9) = -4021.66 + (0.17)(-1907.11) +$$

-8.3

$$\frac{(0.17)^2}{2!}(-597.75) +$$

$$\frac{(0.17)((0.17)^2-1)}{3!}(-61.36)$$

1.66

$$= -4346.16$$

Interpolasi Bessel

Contoh Soal 1

Diketahui Tabel sebagai berikut:

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
3	-639,90			
		-680,95		
3,6	-1320,85		-411,63	
		-1092,58		-104,01
4,2	-2413,43		-515,64	
		-1608,22		-82,11
4,8	-4021,66		-597,76	
		-2205,98		-40,62
5,4	-6227,64		-638,38	
		-2844,36		23,83
6	-9072,00		-614,55	
		-3458,92		
6,6	-12530,92			

Carilah nilai f(x), ketika x = 4.9 dengan menggunakan Interpolasi Bessel Orde 3

Jawaban Contoh 1

- Bessel:

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
3	-639,90			
		-680,95		
3,6	-1320,85		-411,63	
		-1092,58		-104,01
4,2	-2413,43		-515,64	
		-1608,22		-82,11
4,8	-4021,66		-597,76	
		-2205,98		-40,62
5,4	-6227,64		-638,38	
		-2844,36		23,83
6	-9072,00		-614,55	
		-3458,92		
6,6	-12530,92			

$$f_n(x) = f + u\Delta f_0 + \frac{(u^2 - \frac{1}{4})}{2!}\Delta^2 f + \frac{u(u^2 - \frac{1}{4})}{3!}\Delta^3 f_{-1}$$

$$s = \frac{x_s - x_0}{h}, \quad h = \Delta x$$

$$s = \frac{4.9 - 4.8}{h}, \; h = 4.8 - 4.2 = 0.6$$

$$s = 0.17$$

$$u = s - \frac{1}{2} = -0.33$$

$$f = \frac{f_0 + f_1}{2} = -5124.65$$

$$\Delta^2 f = \frac{\Delta^2 f_{-1} + \Delta^2 f_0}{2} = -618.06$$

Jawaban Contoh 1

$$f_n(x) = f + u\Delta f_0 +$$

$$\frac{(u^2 - \frac{1}{4})}{2!}\Delta^2 f + \frac{u(u^2 - \frac{1}{4})}{3!}\Delta^3 f_{-1}$$

735.33

$$f_3(4.9) = -5124.65 + (-0.33)(-2205.98) +$$

42.92

$$\frac{(-0.33)^2 - \frac{1}{4}}{2!}(-2205.98) +$$

$$\frac{(-0.33)\left((-0.33)^2 - \frac{1}{4}\right)}{3!}(-40.63)$$

-0.31

$$= -4346.72$$

Interpolasi Hermite

$$
\begin{aligned}
f_n(x) \;=\; & \frac{\sin(x-x_1)\sin(x-x_2)\ \cdots\ \sin(x-x_n)}{\sin(x_0-x_1)\sin(x_0-x_2)\ \cdots\ \sin(x_0-x_n)} + \\
& \frac{\sin(x-x_0)\sin(x-x_2)\ \cdots\ \sin(x-x_n)}{\sin(x_1-x_0)\sin(x_1-x_2)\ \cdots\ \sin(x_1-x_n)} + \\
& \qquad \vdots \\
& \frac{\sin(x-x_0)\sin(x-x_1)\ \cdots\ \sin(x-x_{n-1})}{\sin(x_n-x_0)\sin(x_n-x_1)\ \cdots\ \sin(x_n-x_{n-1})}
\end{aligned}
$$

Tentang Interpolasi Hermite:

- Karena 'diturunkan' dari rumus Interpolasi Lagrange, maka kelebihan dan kekurangan dari interpolasi Hermite secara umum sama dengan Interpolasi Lagrange

Komnum Week 8

TERIMA KASIH

Sampai Bertemu Kembali