Pertemuan 10

Interpolasi Newton-Gregory
Interpolasi Stirling
Interpolasi Bessel

Bilqis

INTERPOLASI

Jika pada materi pencocokan kurva sebelumnya anda diminta menaksir bentuk fungsi melalui sederetan data, maka sekarang kita diminta untuk mengestimasi nilai fungsi $f(x)$ di antara beberapa nilai fungsi yang diketahui (tanpa mengetahui bentuk fungsi yang menghasilkannya).

INTERPOLASI

Untuk menaksir harga tengahan diantara titik-titik data yang telah ada :

- Polinomial Newton
- Polinomial Lagrange
- Interpolasi Newton-Gregory
- Interpolasi Stirling
- Interpolasi Bessel

Interpolasi Newton-Gregory

Newton-Gregory Forward (NGF) → dari atas ke bawah kanan

Newton-Gregory Backward (NGB)→ dari bawah ke atas kanan

Beda Hingga

Tabel Beda Diagonal Maju

s	x	f(x)	Δf(x)	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	…
0	x_0	f_0					
			Δf_0				
1	x_1	f_1		$\Delta^2 f_0$			
			Δf_1		$\Delta^3 f_0$		
2	x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$	
			Δf_2		$\Delta^3 f_1$		
.	.	.					
.	.	.					
.	.	.					
					$\Delta^3 f_{n-4}$		
n-2	x_{n-2}	f_{n-2}		$\Delta^2 f_{n-3}$		$\Delta^4 f_{n-4}$	
			Δf_{n-2}		$\Delta^3 f_{n-3}$		
n-1	x_{n-1}	f_{n-1}		$\Delta^2 f_{n-2}$			
			Δf_{n-1}				
n	x_n	f_n					

Interpolasi Newton-Gregory

Newton-Gregory Forward (NGF)

$$f(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \ldots + \frac{s(s-1)(s-2)\ldots(s-n+1)}{n!}\Delta^n f_0$$

$$s = \frac{x_s - x_0}{h} \qquad \text{dengan } h = \Delta x$$

Newton-Gregory Backward (NGB)

$$f(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)}{2!}\Delta^2 f_{-2} + \frac{s(s+1)(s+2)}{3!}\Delta^3 f_{-3} + \ldots + \frac{s(s+1)(s+2)\ldots(s+n-1)}{n!}\Delta^n f_{-n}$$

$$s = \frac{x_s - x_0}{h} \qquad \text{dengan } h = \Delta x$$

Interpolasi Newton-Gregory

Newton-Gregory Forward (NGF)

$$f(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \dots + \frac{s(s-1)(s-2)(s-3)}{4!}\Delta^4 f_0$$

$$s = \frac{x_s - x_0}{h} \qquad \text{dengan } h = \Delta x$$

Newton-Gregory Backward (NGB)

$$f(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)}{2!}\Delta^2 f_{-2} + \frac{s(s+1)(s+2)}{3!}\Delta^3 f_{-3} + \dots + \frac{s(s+1)(s+2)(s+3)}{4!}\Delta^4 f_{-n}$$

$$s = \frac{x_s - x_0}{h} \qquad \text{dengan } h = \Delta x$$

Interpolasi Newton-Gregory

contoh :

carilah nilai $f(x_s)$ untuk x_s = 1,03, jika diketahui fungsi tsb menghasilkan nilai² sbb :

x	1,0	1,3	1,6	1,9	2,2	2,5	2,8
f(x)	1,449	2,060	2,645	3,216	3,779	4,338	4,898

Interpolasi Newton-Gregory

contoh : carilah nilai $f(x_s)$ untuk x_s = 1,03, jika diketahui fungsi tsb menghasilkan nilai² sbb :

x	1,0	1,3	1,6	1,9	2,2	2,5	2,8
f(x)	1,449	2,060	2,645	3,216	3,779	4,338	4,898

Langkah 1 → mencari nilai beda

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$	$\Delta^6 f(x)$
1,0	1,449						
		0,611					
1,3	2,060		-0,026				
		0,585		0,012			
1,6	2,645		-0,014		-0,006		
		0,571		0,006		0,004	
1,9	3,216		-0,008		-0,002		-0,001
		0,563		0,004		0,003	
2,2	3,779		-0,004		0,001		
		0,559		0,005			
2,5	4,338		0,001				
		0,560					
2,8	4,898						

1,03 ada di dekat titik awal. shg NGF lebih cocok digunakan.

Interpolasi Newton-Gregory

$$f(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \ldots +$$

$$s = \frac{x_s - x_0}{h} \qquad \text{dengan } h = \Delta x$$

Langkah 2 → mencari nilai s (lebar interval)

$$s = (1{,}03 - 1) / (1{,}3 - 1) = 0{,}1$$

Langkah 3 → mencari nilai $f(x_s)$

$$f(1{,}03) = 1{,}449 + 0{,}1(0{,}611) + \frac{0{,}1\,(0{,}1-1)}{2!} \,.\, -0{,}026 + \frac{0{,}1\,(0{,}1-1)(0{,}1-2)}{3!} \,.\, 0{,}012$$

$$+ \frac{0{,}1\,(0{,}1-1)(0{,}1-2)(0{,}1-3)}{4!} \,.\, -0{,}006 + \frac{0{,}1\,(0{,}1-1)(0{,}1-2)(0{,}1-3)(0{,}1-4)}{5!} \,.\, 0{,}004$$

$$+ \frac{0{,}1\,(0{,}1-1)(0{,}1-2)(0{,}1-3)(0{,}1-4)(0{,}1-5)}{6!} \,.\, -0{,}001$$

$$= 1{,}5118136$$

(4) Carilah nilai $f(x_s)$ untuk $x_s = 3$, jk diketahui fungsi tsb menghasilkan nilai sbb : ~ cari hingga $\Delta^3 f(x)$ dgn menggunakan : Newton Gregory Forward

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
2	31			
		351 (2)		
5	382		810 (2)	
		1161 (2)		486 (2)
8	1543		1296 (2)	
		2457 (2)		
11	4000			

$$f(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \ldots +$$

$$s = \frac{x_s - x_0}{h} \qquad \text{dengan } h = \Delta x$$

jawab:

$$f(x_s) = f_0 + s \cdot \Delta f_0 + \frac{s(s-1)}{2!} \Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!} \Delta^3 f_0$$

$$s = \frac{x_s - x_0}{h} \qquad h = \Delta x \Rightarrow h = 3 \quad 2$$

$$x_s = 3$$

$$s = \frac{3-2}{3} = \frac{1}{3} \quad 2$$

$$f(3) = \underset{2}{31} + \frac{1}{3} \cdot 351 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2} \cdot 810 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3.2} \cdot 486$$

$$f(3) = 88 \quad 2$$

Interpolasi Newton-Gregory

contoh : carilah nilai $f(x_s)$ untuk x_s = 2,67, jika diketahui fungsi tsb menghasilkan nilai² sbb :

x	1,0	1,3	1,6	1,9	2,2	2,5	2,8
f(x)	1,449	2,060	2,645	3,216	3,779	4,338	4,898

Langkah 1 → mencari nilai beda

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$	$\Delta^6 f(x)$
1,0	1,449						
		0,611					
1,3	2,060		-0,026				
		0,585		0,012			
1,6	2,645		-0,014		-0,006		
		0,571		0,006		0,004	
1,9	3,216		-0,008		-0,002		<u>-0,001</u>
		0,563		0,004		<u>0,003</u>	
2,2	3,779		-0,004		<u>0,001</u>		
		0,559		<u>0,005</u>			
2,5	4,338		<u>0,001</u>				
		<u>0,560</u>					
2,8	<u>4,898</u>						

2,67 ada di dekat titik akhir. Jadi NGB adalah pilihan terbaik.

Interpolasi Newton-Gregory

Langkah 2 → mencari nilai s (lebar interval)

$$s = (2{,}67 - 2{,}8) \,/\, (1{,}3 - 1) = -0{,}43333$$

$$f(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)}{2!}\Delta^2 f_{-2} + \frac{s(s+1)(s+2)}{3!}\Delta^3 f_{-3} + \ldots +$$

Langkah 3 → mencari nilai $f(x_s)$

$$f(2{,}67) = 4{,}898 + -0{,}433(0{,}560) + \frac{-0{,}433\,(-0{,}433+1)}{2!}\,.\,0{,}001 + \frac{-0{,}433\,(-0{,}433+1)(-0{,}433+2)}{3!}\,.\,0{,}005$$

$$+ \frac{0{,}433\,(0{,}433+1)(0{,}433+2)(0{,}433+3)}{4!}\,.\,0{,}001$$

$$+ \frac{0{,}433\,(0{,}433+1)(0{,}433+2)(0{,}433+3)(0{,}433+4)}{5!}\,.\,0{,}003$$

$$+ \frac{0{,}433\,(0{,}433+1)(0{,}433+2)(0{,}433+3)(0{,}433+4)(0{,}433+5)}{6!}\,.\,-0{,}001$$

$$= \;4{,}654783$$

Contoh Soal :

Diketahui Tabel sebagai berikut :

x	y	Δ f(x)	Δ 2 f(x)	Δ 3 f(x)
2	18,00			
		108,53		
2,4	126,53		192,82	
		301,34		196,42
2,8	427,87		389,24	
		690,58		316,11
3,2	1118,45		705,35	
		1395,93		476,34
3,6	2514,38		1181,69	
		2577,62		683,03
4	5092,00		1864,72	
		4442,34		
4,4	9534,34			

Ditanya

- Carilah nilai f(x) , ketika x = 3,3 dengan menggunakan Newton Gregory **forward,** xo=3,2
- Carilah nilai f(x) , ketika x = 3,3 dengan menggunakan Newton Gregory **Backward,** xo=3,2
- Catt :

  penilaian ... k pada perhitungan detail,

  Misal $\frac{s(s-1)}{2!}\Delta^2 f_0$ = 324.

  Jadi hitung setiap data yang dibatasi dengan tanda “+”.
- Hitung error jika diketahui f(3,3) = 1385,77

Newton Gregory **forward**

$$f(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \dots + \frac{s(s-1)(s-2)\dots(s-n+1)}{n!}\Delta^n f_0$$

$$s = \frac{x_s - x_0}{h} \qquad \text{dengan } h = \Delta x$$

X = 3.3

Xo = 3.2

$$s = \frac{3.3 - 3.2}{0.4}$$

s = 0.250

fo = 1118.45

f(3,3) = **1118.45** + **0.250 * 1395.93**

348.982

-110.784 + $\frac{\mathbf{0.250 * (0.250 - 1)}}{\mathbf{1 * 2}}$ *** 1181.69**

37.353 + $\frac{\mathbf{0.250 * (0.250 - 1) * (0.250 - 2)}}{\mathbf{1 * 2 * 3}}$ *** 683.03**

f(3,3) = **1394.002**

Newton Gregory Backward

Newton-Gregory Backward (NGB)

$$f(x_s) = f_0 + s\Delta f_{-1} + \frac{s(s+1)}{2!}\Delta^2 f_{-2} + \frac{s(s+1)(s+2)}{3!}\Delta^3 f_{-3} + \dots + \frac{s(s+1)(s+2)\dots(s+n-1)}{n!}\Delta^n f_{-n}$$

$$s = \frac{x_s - x_0}{h} \qquad \text{dengan } h = \Delta x$$

X = 3.3
Xo = 3.2

s = (3.3 - 3.2) / 0.4

s = 0.250

fo = 1118.45

f(3,3) = **1118.45** + **0.250 * 690.58**

172.645

60.819 + **0.250 * (0.250 + 1) * 389.24 / (1 * 2)**

23.018 + **0.250 * (0.250 + 1) * (0.250 + 2) * 196.42 / (1 * 2 * 3)**

f(3,3) = **1374.933**

Tabel Quiz 3 (A)

X	Y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
3	-741				
		555			
6	-186		31752		
		32307		88776	
9	32121		120528		87480
		152835		176256	
12	184956		296784		116640
		449619		292896	
15	634575		589680		145800

X	Y	$\Delta f(x)$	$\Delta\text{^}2f(x)$	$\Delta\text{^}3f(x)$	$\Delta\text{^}4f(x)$
		1039299		438696	
18	1673874		1028376		174960
		2067675		613656	
21	3741549		1642032		204120
		3709707		817776	
24	7451256		2459808		
		6169515			
27	1362077				

Soal Jawaban 2

Diketahui:

- X = 16
- Xo = 15
- Y asli = 897104

Ditanya:

- Nilai f(x) dan Et dengan cara **Newton Gregory Forward**

Jawaban Soal 2

Langkah 1 ⇒ mencari nilai beda

a. $\triangle f_0 = 1039299$

b. $\triangle^2 f_0 = 1028376$

c. $\triangle^3 f_0 = 613656$

d. $\triangle^4 f_0 = 204120$

Langkah 2 ⇒ mencari nilai S

$$s = \frac{X - X_o}{h} = \frac{16 - 15}{3} = \frac{1}{3} = 0{,}33$$

Soal Jawaban 2

Langkah 3 ⇒ mencari nilai persamaan

a. $s \triangle f_0 = 0{,}33 \times 449619 = 346433$

b. $\frac{s(s-1)}{2!} \triangle^2 f_0 = \frac{0{,}33(0{,}33-1)}{2} \times 296784 = -114264$

c. $\frac{s(s-1)(s-2)}{3!} \triangle^3 f_0 = \frac{0{,}33(0{,}33-1)(0{,}33-2)}{3 \times 2 \times 1} \times 176256 = 37880$

d. $\frac{s(s-1)(s-2)(s-3)}{4!} \triangle^4 f_0 = \frac{0{,}33(0{,}33-1)(0{,}33-2)(0{,}33-3)}{4 \times 3 \times 2 \times 1} \times 87480 = -8400$

Soal Jawaban 2

Mencari hasil $f(16)$

a. $f(x) = f_0 + s \bigtriangleup f_0 + \frac{s(s-1)}{2!} \bigtriangleup^2 f_0 + \frac{s(s-1)(s-2)}{3!} \bigtriangleup^3 f_0 + \frac{s(s-1)(s-2)(s-3)}{4!} \bigtriangleup^4 f_0$

b. $f(16) = 634575 + 346433 - 114264 + 37880 - 8400 = 896224$

$$Et = \frac{897104-896224}{897104} \; x \; 100 = 0{,}1$$

Soal Jawaban 3

Diketahui:

a) X = 16
b) Xo = 15
c) Y asli = 897104

Ditanya:

a) Carilah hasil dari persamaan berikut:
   i. $s \triangle f_{-1}$
   ii. $\frac{s(s+1)}{2!} \triangle^2 f_{-2}$
   iii. $\frac{s(s+1)(s+2)}{3!} \triangle^3 f_{-3}$
   iv. $\frac{s(s+1)(s+2)(s+3)}{4!} \triangle^4 f_{-4}$
b) Carilah nilai f(x) dan Et dengan **Newton Gregory Backward**!

Soal Jawaban 3a

Langkah 1 ⇒ mencari nilai beda

a. $\triangle f_{-1} = 449619$

b. $\triangle^2 f_{-2} = 296784$

c. $\triangle^3 f_{-3} = 176256$

d. $\triangle^4 f_{-4} = 87480$

Langkah 2 ⇒ mencari nilai S

$$s = \frac{X - X_o}{h} = \frac{16 - 15}{3} = \frac{1}{3} = 0{,}33$$

Soal Jawaban 3a

Langkah 3 ⇒ mencari nilai persamaan

a. $s \bigtriangleup f_{-1} = 0{,}33 \times 449619 = 149873$

b. $\frac{s(s+1)}{2!} \bigtriangleup^2 f_{-2} = \frac{0{,}33(0{,}33+1)}{2} \times 296784 = 65952$

c. $\frac{s(s+1)(s+2)}{3!} \bigtriangleup^3 f_{-3} = \frac{0{,}33(0{,}33+1)(0{,}33+2)}{3 \times 2 \times 1} \times$
$176256 = 30464$

d. $\frac{s(s+1)(s+2)(s+3)}{4!} \bigtriangleup^4 f_{-4} =$
$\frac{0{,}33(0{,}33+1)(0{,}33+2)(0{,}33+3)}{4 \times 3 \times 2 \times 1} \times 87480 = 12600$

Soal Jawaban 3b

Mencari hasil $f(11)$

a. $f(x) = f_0 + s \triangle f_{-1} + \frac{s(s+1)}{2!} \triangle^2 f_{-2} + \frac{s(s+1)(s+2)}{3!} \triangle^3 f_{-3} + \frac{s(s+1)(s+2)(s+3)}{4!} \triangle^4 f_{-4}$

b. $f(16) = 634575 + 149873 + 65952 + 30464 + 12600 = 893464$

$Et = \frac{897104 - 89364}{897104} x\ 100 = 0{,}41$

Interpolasi Stirling & Bessel (1)

Rumus Interpolasi Stirling :

$$f(x_s) = f_0 + s\underset{\text{Rata-2}}{\overset{\text{Ambil}}{\Delta f}} + \frac{s(s-1)}{2!}\overset{\text{Ambil}}{\Delta^2 f_{-1}} + \frac{s(s-1)(s-2)}{3!}\underset{\text{Rata-2}}{\Delta^3 f} + \frac{x}{4}\,\frac{s(s-1)(s-2)}{3!}\Delta^4 f_{-2}$$

$$+ \frac{s(s-1)(s-2)(s-3)(s-4)}{5!}\Delta^5 f + \frac{x}{6}\,\frac{s(s-1)(s-2)(s-3)(s-4)}{5!}\Delta^6 f_{-3} + \ldots$$

$$\Delta f = \tfrac{1}{2}(\Delta f_{-1} + \Delta f_0)$$
$$\Delta^3 f = \tfrac{1}{2}(\Delta^3 f_{-1} + \Delta^3 f_{-2})$$
$$\Delta^5 f = \tfrac{1}{2}(\Delta^5 f_{-3} + \Delta^5 f_{-2})$$

Interpolasi Stirling : *facts and figures*

1. Stirling merupakan rerata dari rumusan Gauss. Karena menggunakan rerata beda gasal di atas dan di bawah garis sentral, serta beda genap pada garis sentral (*menggunakan tabel beda hal 15 paparan ini*);
2. Kelebihan dan kekurangan metode Stirling sama dengan metode Newton-Gregory. Bedanya, Stirling lebih optimal jika digunakan untuk mencari nilai $f(x_s)$ dengan x_s di sekitar titik tengah;

Interpolasi Stirling & Bessel (2)

Jika dapat diasumsikan :

$\Delta^3 f_{-1} = \Delta^2 f_0 - \Delta^2 f_{-1}$
$\Delta^5 f_{-2} = \Delta^4 f_{-1} - \Delta^4 f_{-2}$
$\Delta^7 f_{-3} = \Delta^6 f_{-2} - \Delta^6 f_{-3}$

Untuk kemudian disubstitusikan pada persamaan Interpolasi Gauss Forward, maka akan diperoleh persamaan **Interpolasi Bessel**, seperti berikut :

$$f(x_s) = \overset{\text{Ambil}}{f_0} + \overset{\text{Ambil}}{s\Delta f_0} + \frac{s(s-1)}{2!}\underset{\text{Rata-2}}{\Delta^2 f} + \frac{1}{3}\,\frac{s(s-1)}{2!}\,(x - \tfrac{1}{2})\,.\,\overset{\text{Ambil}}{\Delta^3 f_{-1}}$$

$$+ \frac{s(s-1)(s-2)(s-3)}{4!}\underset{\text{Rata-2}}{\Delta^4 f} + \frac{1}{5}\,\frac{s(s-1)(s-2)(s-3)}{4!}\,(x - \tfrac{1}{2})\,.\,\Delta^5 f_{-2} + \ldots$$

dengan :

$\Delta^2 f = \tfrac{1}{2}(\Delta^2 f_{-1} + \Delta^2 f_0)$
$\Delta^4 f = \tfrac{1}{2}(\Delta^4 f_{-2} + \Delta^4 f_{-1})$

Contoh Stirling

- Carilah nilai f(x) ketika x = 4,9 dengan menggunakan Stirling

x	y	Δ f(x)	Δ 2 f(x)	Δ 3 f(x)	Δ 4 f(x)	Δ 5 f(x)	Δ 6 f(x)
3	-639,90						
		-680,95					
3,6	-1320,85		-411,63				
		-1092,58		-104,01			
4,2	-2413,43		-515,64		21,90		
		-1608,22		-82,11		19,60	
4,8	-4021,66		-597,76		41,49		3,36
		-2205,98		-40,62		22,95	
5,4	-6227,64		-638,38		64,45		
		-2844,36		23,83			
6	-9072,00		-614,55				
		-3458,92					
6,6	- 12530,92						

Xs = 4,9

Xo = 4,8

$$\mathbf{s} = \frac{\mathbf{4{,}9 \quad - \quad 4{,}8}}{\mathbf{0{,}6}}$$

s = 0,1666667

fo = -4021,66

Δ 2 f(x)= -597,76
Δ 4 f(x)= 41,49
Δ 6 f(x)= 3,36

$$\Delta f = \frac{-1608{,}22 \quad + \quad -2205{,}98}{2}$$

Δ f = -1907,103

$$\Delta 3 f = \frac{-82{,}11 \quad + \quad -40{,}62}{2}$$

Δ 3 f = -61,36819

$$\Delta 5 f = \frac{19{,}60 \quad + \quad 22{,}95}{2}$$

Δ 5 f = 21,275136

f(4,9) = $\boxed{-4021,66}$ + $\boxed{0,167 * -1907,10}$

-317,851

41,511 + $\frac{0,167 * (0,167 - 1)}{1 * 2} * -597,76$

-2,604 + $\frac{0,167 * (0,167 - 1) * (0,167 - 2)}{1 * 2 * 3} * -61,37$

2,157 + $\frac{4,9}{4} * \frac{0,167 * (0,167 - 1) * (0,167 - 2)}{1 * 2 * 3} * 41,49$

0,490 + $\frac{0,167 * (0,167 - 1) * (0,167 - 2) * (0,167 - 3)}{1 * 2 * 3 * 4 * 5} * \underline{(0,167 - 4)}\ 21,28$

0,063 + $\frac{4,9}{6} * \frac{0,167 * (0,167 - 1) * (0,167 - 2) * (0,167 - 3) * (0,167 - 4)}{1 * 2 * 3 * 4 * 5} * 3,36$

f(4,9) = $\boxed{-4297,889}$ -4297,89 -4297,89

Contoh Bessel

- Carilah nilai f(x) ketika x = 4,9 dengan menggunakan Bessel

x	y	Δ f(x)	Δ 2 f(x)	Δ 3 f(x)	Δ 4 f(x)	Δ 5 f(x)	Δ 6 f(x)
3	-639,90						
		-680,95					
3,6	-1320,85		-411,63				
		-1092,58		-104,01			
4,2	-2413,43		-515,64		21,90		
		-1608,22		-82,11		19,60	
4,8	-4021,66		-597,76		41,49		3,36
		-2205,98		-40,62		22,95	
5,4	-6227,64		-638,38		64,45		
		-2844,36		23,83			
6	-9072,00		-614,55				
		-3458,92					
6,6	-12530,92						

$Xs = 4{,}9$

$Xo = 4{,}8$

$$s = \frac{4{,}9 - 4{,}8}{0{,}6}$$

s = 0,167

$fo = -4021{,}66$

Δ fo =	-2205,98
Δ 3 f(x)=	-40,62
Δ 5 f(x)=	22,95

$$\Delta 2 f = \frac{-597{,}76 + -638{,}38}{2}$$

$\Delta 2 f = -618{,}069$

$$\Delta 4 f = \frac{41{,}49 + 64{,}45}{2}$$

$\Delta 4 f = 52{,}970$

f(4.9) = -4021,66 + 0,167 * -2205,98

-367,664

42,921 + 0,167 * (0,167 - 1) * -618,07
1 * 2

4,137 + 1 * 0,167 (0,167 - 1) * (4,9 - 0,5) * -40,62
3 2

-1,592 + 0,167 * (0,167 - 1) * (0,167 - 2) * (0,167 - 3) * 52,970
1 * 2 * 3 * 4

-0,589 + 1 * 0,167 * (0,167 - 1) * (0,167 - 2) * (0,167 - 3) * (4,9 - 0,5) * 22,95
5 1 * 2 * 3 * 4

f(4.3) = -4344,443 -4344,44 -4344,460

Tabel Quiz 3 (A)

X	Y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
3	−741				
		555			
6	−186		31752		
		32307		88776	
9	32121		120528		87480
		152835		176256	
12	184956		296784		116640
		449619		292896	
15	634575		589680		145800

X	Y	$\Delta f(x)$	$\Delta^{2}f(x)$	$\Delta^{3}f(x)$	$\Delta^{4}f(x)$
		1039299		438696	
18	1673874		1028376		174960
		2067675		613656	
21	3741549		1642032		204120
		3709707		817776	
24	7451256		2459808		
		6169515			
27	1362077				

Soal Jawaban 4

Diketahui:

- Xo = 15
- X = 14
- Y sebenarnya = 436366

Ditanya:

a. Selesaikan persamaan berikut!
   - $s\Delta f$
   - $\frac{s(s-1)}{2!}\Delta^2 f_{-1}$
   - $\frac{s(s-1)(s-2)}{3!}\Delta^3 f$
   - $\frac{x}{4}\ \frac{s(s-1)(s-2)}{3!}\Delta^4 f_{-2}$

b. Carilah nilai f(14) dan Et menggunakan **Stirling**!

Soal Jawaban 4a

Langkah 1 => Cari nilai beda!

- $\Delta f =$ 744459
- $\Delta^2 f_{-1} = 589680$
- $\Delta^3 f =$ 365796
- $\Delta^4 f_{-2} = 145800$

Langkah 2 => Cari nilai S

- $s = \dfrac{X - Xo}{h} = \dfrac{14 - 15}{3} = -\dfrac{1}{3}$

Soal Jawaban 4a

Langkah 3 => Masukan variabel dan selesaikan persamaan!

- $s\Delta f = \frac{1}{3} \times 744459 = -248153$
- $\frac{s(s-1)}{2!}\Delta^2 f_{-1} = \frac{-\frac{1}{3}\left(-\frac{1}{3} - 1\right)}{2} \times 589680 = 131040$
- $\frac{s(s-1)(s-2)}{3!}\Delta^3 f = \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3\times2\times1} \times 365796 = -63224$
- $\frac{x}{4}\ \frac{s(s-1)(s-2)}{3!}\Delta^4 f_{-2} = \frac{14}{4}\ \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3\times2\times1} \times 145800 = -88200$

Soal Jawaban 4b

=> Masukan variabel dan selesaikan persamaan f(16)!

- $f(x) = f_0 + s\Delta f + \frac{s(s-1)}{2!}\Delta^2 f_{-1} + \frac{s(s-1)(s-2)}{3!}\Delta^3 f + \frac{x}{4}\ \frac{s(s-1)(s-2)}{3!}\Delta^4 f_{-2}$
- $f(14) = \mathbf{634575 - 248153 + 131040 - 63224 - 88200 = 366038}$

$\Rightarrow$ Cari Et!

- $Et = \frac{436366-366038}{436366} \times 100\% = 16{,}12$

Jadi hasil dari f(16) dan Et adalah 366038

Tabel Quiz 3 (B)

X	Y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
3	-741				
		555			
6	-186		31752		
		32307		88776	
9	32121		120528		87480
		152835		176256	
12	184956		296784		116640
		449619		292896	
15	634575		589680		145800

X	Y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
		1039299		438696	
18	1673874		1028376		174960
		2067675		613656	
21	3741549		1642032		204120
		3709707		817776	
24	7451256		2459808		
		6169515			
27	13620771				

Soal 1

Diketahui:

- Xo = 15
- X = 16
- Y sebenarnya = 897104

Ditanya:

a. Selesaikan persamaan berikut!
   - $s\Delta f$
   - $\frac{s(s-1)}{2!}\Delta^2 f_{-1}$
   - $\frac{s(s-1)(s-2)}{3!}\Delta^3 f$
   - $\frac{x}{4}\ \frac{s(s-1)(s-2)}{3!}\Delta^4 f_{-2}$

b. Carilah nilai f(16) dan Et menggunakan **Stirling**!

Jawaban Soal 1a

Langkah 1 => Cari nilai beda!

- $\Delta f = 744459$
- $\Delta^2 f_{-1} = 589680$
- $\Delta^3 f = 365796$
- $\Delta^4 f_{-2} = 145800$

Langkah 2 => Cari nilai S

- $s = \dfrac{X - Xo}{h} = \dfrac{16 - 15}{3} = \dfrac{1}{3}$

Jawaban Soal 1a

Langkah 3 => Masukan variabel dan selesaikan persamaan!

- $s\Delta f = \frac{1}{3} \times 744459 = 248153$
- $\frac{s(s-1)}{2!}\Delta^2 f_{-1} = \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2} \times 589680 = -65520$
- $\frac{s(s-1)(s-2)}{3!}\Delta^3 f = \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3\times2\times1} \times 365796 = 22580$
- $\frac{x}{4}\,\frac{s(s-1)(s-2)}{3!}\Delta^4 f_{-2} = \frac{16}{4}\,\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3\times2\times1} \times 145800 = 36000$

Jawaban Soal 1b

=> Masukan variabel dan selesaikan persamaan f(16)!

- $f(x) = f_0 + s\Delta f + \frac{s(s-1)}{2!}\Delta^2 f_{-1} + \frac{s(s-1)(s-2)}{3!}\Delta^3 f + \frac{x}{4}\ \frac{s(s-1)(s-2)}{3!}\Delta^4 f_{-2}$
- $f(16) = 634575 + 248153 - 65520 + 22580 + 36000 = 875788$

$\Rightarrow$ Cari Et!

- $Et = \frac{897104 - 875788}{897104} \times 100\% = 2{,}38$

Jadi hasil dari f(16) dan Et adalah 875788 dan 2,38

Soal 2

Diketahui:

- Xo = 15
- X = 16
- Y sebenarnya = 897104

Ditanya:

a. Selesaikan persamaan berikut!

- $s\Delta f_0$
- $\frac{s(s-1)}{2!}\Delta^2 f$
- $\frac{1}{3}\frac{s(s-1)}{2!}\left(x-\frac{1}{2}\right)\Delta^3 f_{-1}$
- $\frac{s(s-1)(s-2)(s-3)}{4!}\Delta^4 f$

b. Carilah nilai f(16) dan Et menggunakan **Bessel**!

Jawaban Soal 2a

Langkah 1 => Cari nilai beda!

- $\Delta f_0 = 1039299$
- $\Delta^2 f = 809.028$
- $\Delta^3 f_{-1} = 438696$
- $\Delta^4 f = 160380$

Langkah 2 => Cari nilai S

- $s = \dfrac{X - Xo}{h} = \dfrac{16 - 15}{3} = \dfrac{1}{3}$

Jawaban Soal 2a

Langkah 3 => Masukan variabel dan selesaikan persamaan!

- $s\Delta f_0 = \frac{1}{3} \times 1039299 = 346433$
- $\frac{s(s-1)}{2!}\Delta^2 f = \frac{\frac{1}{3}\left(\frac{1}{3} - 1\right)}{2} \times 809.028 = -89892$
- $\frac{1}{3}\frac{s(s-1)}{2!}\left(x - \frac{1}{2}\right)\Delta^3 f_{-1} = \frac{1}{3}\frac{\frac{1}{3}\left(\frac{1}{3} - 1\right)}{2}\left(16 - \frac{1}{2}\right) \times 438696 = -251844$
- $\frac{s(s-1)(s-2)(s-3)}{4!}\Delta^4 f = \frac{\frac{1}{3}\left(\frac{1}{3} - 1\right)\left(\frac{1}{3} - 2\right)\left(\frac{1}{3} - 3\right)}{4} \times 160380 = -6600$

Jawaban Soal 2b

=> Masukan variabel dan selesaikan persamaan f(16)!

- $f(x) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f + \frac{1}{3}\frac{s(s-1)}{2!}\left(x - \frac{1}{2}\right)\Delta^3 f_{-1} + \frac{s(s-1)(s-2)(s-3)}{4!}\Delta^4 f$
- $f(16) = 634575 + 346433 - 89892 - 251844 - 6600 = 632672$

$\Rightarrow$ Cari Et!

- $Et = \frac{897104 - 632672}{897104} \times 100\% = 29{,}48$

Jadi hasil dari f(16) dan Et adalah 632672 dan 29,48

Tabel Quiz 3 (C)

X	Y	$\triangle f(x)$	$\triangle 2f(x)$	$\triangle 3f(x)$	$\triangle 4f(x)$
2	-940				
		-5608			
4	-6008		-576		
		-5644		18912	
6	-11652		18336		23040
		12692		41952	
8	1040		60288		30720
		72980		72672	
10	74020		132960		38400

X	Y	$\triangle f(x)$	$\triangle 2f(x)$	$\triangle 3f(x)$	$\triangle 4f(x)$
		205940		111072	
12	279960		244032		46080
		449972		157152	
14	729932		401184		53760
		851156		210912	
16	1581088		612096		
		1463252			
18	3044340				

Soal 3

Diketahui:

a) X = 11
b) Xo = 10

Ditanya:

a) Carilah hasil dari persamaan berikut:
   i. $s \bigtriangleup f_0$
   ii. $\frac{s(s-1)}{2!} \bigtriangleup^2 f$
   iii. $\frac{1}{3} \frac{s(s-1)}{2!} \left(x - \frac{1}{2}\right) \bigtriangleup^3 f_{-1}$
   iv. $\frac{s(s-1)(s-2)(s-3)}{4!} \bigtriangleup^4 f$
b) Carilah nilai f(x) dan Et menggunakan **Bessel**!

Jawaban Soal 3.a

Langkah 1 ⇒ mencari nilai beda

a. $\triangle f_0 = 205940$

b. $\triangle^2 f = 188496$

c. $\triangle^3 f_{-1} = 11072$

d. $\triangle^4 f = 42240$

Langkah 2 ⇒ mencari nilai S

$$s = \frac{X - X_o}{h} = \frac{11 - 10}{2} = \frac{1}{2} = 0{,}5$$

Jawaban Soal 3.a

Langkah 3 ⇒ mencari nilai persamaan

a. $s \triangle f_0 = 0{,}5 \times 205940 = 1029720$

b. $\frac{s(s-1)}{2!} \triangle^2 f = \frac{0{,}5(0{,}5-1)}{2} \times 188496 = -23562$

c. $\frac{1}{3}\,\frac{s(s-1)}{2!}\left(x-\frac{1}{2}\right) \triangle^3 f_{-1} = \frac{1}{3} \times \frac{0{,}5(0{,}5-1)}{2} \times \left(11-\frac{1}{2}\right) \times 11072 = -48594$

d. $\frac{s(s-1)(s-2)(s-3)}{4!} \triangle^4 f = \frac{0{,}5(0{,}5-1)(0{,}5-2)(0{,}5-3)}{4 \times 3 \times 2 \times 1} \times 42240 = -1650$

Jawaban Soal 3.b

⇒ Mencari hasil $f(11)$

a. $f'(x) = f_0 + s \triangle f_0 + \frac{s(s-1)}{2!} \triangle^2 f + \frac{1}{3} \frac{s(s-1)}{2!} \left(x - \frac{1}{2}\right) \triangle^3 f_{-1} + \frac{s(s-1)(s-2)(s-3)}{4!} \triangle^4 f$

b. $f'(11) = 74020 + 102970 - 23562 - 48594 - 1650 = 103184$

⇒ Mencari hasil Et

$a.\ Et = \frac{154418 - 103184}{154418} \times 100\% = 33{,}18$

Jadi hasil dari f(11) *dan* Et *adalah* 103184 dan 33,18

Interpolasi Lagrange (1)

$$f(x_s) = \frac{(x - x_1)(x - x_2)(x - x_3)\ \dots\ (x - x_n)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)\ \dots\ (x_0 - x_n)} \cdot f_0$$

$$+ \frac{(x - x_0)(x - x_2)(x - x_3)\ \dots\ (x - x_n)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)\ \dots\ (x_1 - x_n)} \cdot f_1$$

$$+ \dots$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_2)\ \dots\ (x - x_{n-1})}{(x_n - x_1)(x_n - x_2)(x_n - x_3)\ \dots\ (x_n - x_{n-1})} \cdot f_n$$

Interpolasi Lagrange : *facts and figures*

1. Lagrange tidak memerlukan tabel beda;
2. Aplikatif untuk kasus *equispaced* (h konstan) maupun *non-equispaced* (h tidak konstan);
3. Aplikatif untuk kasus interpolasi dan *invers interpolation*;
4. Efisien untuk mencari nilai fungsi di dekat titik awal, tengah, maupun akhir;

Interpolasi Lagrange (2)

contoh : carilah nilai log 656, jika diketahui nilai² log 654 = 2,8156, log 658 = 2,8182, log 659 = 2,8189, log 661 = 2,8202

n	Log	Nilai
0	654	2,8156
1	658	2,8182
3	659	2,8189
3	661	2,8202

$$\text{Log } 656 = \frac{(656-658)(656-659)(656-661)}{(654-658)(654-659)(654-661)} \,.\, (2{,}8156)$$

$$+ \frac{(656-654)(656-659)(656-661)}{(658-654)(658-659)(658-661)} \,.\, (2{,}8182)$$

$$+ \frac{(656-654)(656-658)(656-661)}{(659-654)(659-658)(659-661)} \,.\, (2{,}8189)$$

$$+ \frac{(656-654)(656-658)(656-659)}{(661-654)(661-654)(661-654)} \,.\, (2{,}8202)$$

$$= 2{,}8168$$

Interpolasi Hermite (1)

$$f(x_s) = \frac{\sin(x - x_1)\,\sin(x - x_2)\,\sin(x - x_3)\,\ldots\,\sin(x - x_n)}{\sin(x_0 - x_1)\,\sin(x_0 - x_2)\,\sin(x_0 - x_3)\,\ldots\,\sin(x_0 - x_n)} \,.\, f_0$$

$$+ \frac{\sin(x - x_0)\,\sin(x - x_2)\,\sin(x - x_3)\,\ldots\,\sin(x - x_n)}{\sin(x_1 - x_0)\,\sin(x_1 - x_2)\,\sin(x_1 - x_3)\,\ldots\,\sin(x_1 - x_n)} \,.\, f_1$$

$$+ \ldots$$

$$+ \frac{\sin(x - x_0)\,\sin(x - x_1)\,\sin(x - x_2)\,\ldots\,\sin(x - x_{n-1})}{\sin(x_n - x_1)\,\sin(x_n - x_2)\,\sin(x_n - x_3)\,\ldots\,\sin(x_n - x_{n-1})} \,.\, f_n$$

Interpolasi Hermite : *facts and figures*

1. *Hermite is truly dedicated for periodic function's problems* (makanya disebut juga interpolasi trigonometrik);
2. Karena 'diturunkan' dari rumus Interpolasi Lagrange, maka kelebihan & kekurangannya secara umum sama dengan Interpolasi Lagrange;

Interpolasi Hermite (2)

contoh : carilah nilai f(x) untuk x = 0,6 radian, jika diketahui tabel berikut :

x	0,4	0,5	0,7	0,8
f(x)	0,0977	0,0088	-0,1577	-0,2192

$$f(0,6) = \frac{\sin(0,6-0,5)\sin(0,6-0,7)\sin(0,6-0,8)}{\sin(0,4-0,5)\sin(0,4-0,7)\sin(0,4-0,8)}\,.\,(0,0977)$$

$$+\frac{\sin(0,6-0,4)\sin(0,6-0,7)\sin(0,6-0,8)}{\sin(0,5-0,4)\sin(0,5-0,7)\sin(0,5-0,8)}\,.\,(0,0088)$$

$$+\frac{\sin(0,6-0,4)\sin(0,6-0,5)\sin(0,6-0,8)}{\sin(0,7-0,4)\sin(0,7-0,5)\sin(0,7-0,8)}\,.\,(-0,1577)$$

$$+\frac{\sin(0,6-0,4)\sin(0,6-0,5)\sin(0,6-0,7)}{\sin(0,8-0,4)\sin(0,8-0,5)\sin(0,8-0,7)}\,.\,(-0,2192)$$

$$= -\,0,07915$$

Pertemuan 11

Diferensiasi

Bilqis

Diferensiasi Numerik

Metode Newton-Gregory (F/B)
Metode Lagrange

Differensiasi / Turunan

- Manfaat :
  1. ketika kita melihat siaran di televisi tentang pemilihan presiden, terdapat *Quick Count* untuk memperkirakan sementara siapa calon yang akan terpilih
  2. turunan fungsi aljabar ini juga berguna dalam pencampuran bahan-bahan bangunan untuk membuat tiang, langit-langit, dan lain sebagainya sehingga bangunan dapat terlihat cantik dan kokoh.

- definisi turunan fungsi (diferensial) adalah fungsi lain dari suatu fungsi sebelumnya,
- contohnya fungsi f dijadikan f'
- secara umum suatu besaran yang berubah seiring perubahan besaran lainnya

penerapan turunan

- Turunan dapat diterapkan untuk menghitung gradien dari garis singgung suatu kurva.
- Turunan dapat digunakan untuk menentukan interval dimana suatu fungsi naik atau turun.
- Turunan dapat diterapkan untuk menentukan nilai stasioner suatu fungsi.
- Turunan dapat diterapkan dalam menyelesaikan permasalahan yang berkaitan dengan persamaaan gerak.
- Turunan dapat digunakan untuk menyelesaikan permasalahan maksimum-minimum.

Diferensiasi Numerik (1)

Ke depan anda akan sering menjumpai 2 jenis operasi matematis dalam kehidupan ilmiah anda, yaitu Diferensiasi Numerik dan Integrasi Numerik.

Untuk diferensiasi numeris, konsepnya hampir sama dengan regresi dan interpolasi. Yaitu mencari nilai turunan sebuah fungsi hanya dengan menggunakan himpunan nilai dari fungsi tersebut.

Permasalahan diferensiasi numeris ini diselesaikan dengan menyatakan fungsi yang dimaksud melalui rumusan interpolasi yang telah di-diferensiasi.

Untuk permasalahan yang bersifat *equispaced*, dapat diselesaikan dengan rumus Newton-Gregory, Stirling atau Bessel. Sementara untuk permasalahan *non-equispaced*, digunakan Lagrange atau Hermite (jika periodik).

Metode Newton-Gregory (1)

Newton-Gregory Forward (NGF)

$$f'(x_s) = \frac{1}{h}\left[\Delta f_0 + \frac{2s-1}{2!}\Delta^2 f_0 + \frac{3s^2-6s+2}{3!}\Delta^3 f_0 + \frac{4s^3-18s^2+22s-6}{4!}\Delta^4 f_0 + \frac{5s^4-40s^3+105s^2-100s+24}{5!}\Delta^5 f_0 + \frac{6s^5-75s^4+340s^3-675s^2+548s-120}{6!}\Delta^6 f_0 + \ldots\right]$$

$$s = \frac{x_s - x_0}{h}$$

Newton-Gregory Backward (NGB)

$$f'(x_s) = \frac{1}{h}\left[\Delta f_{-1} + \frac{2s+1}{2!}\Delta^2 f_{-2} + \frac{3s^2+6s+2}{3!}\Delta^3 f_{-3} + \frac{4s^3+18s^2+22s+6}{4!}\Delta^4 f_{-4} + \frac{5s^4+40s^3+105s^2+100s+24}{5!}\Delta^5 f_{-5} + \frac{6s^5+75s^4+340s^3+675s^2+548s+120}{6!}\Delta^6 f_{-6} + \frac{7s^6+126s^5+875s^4+2940s^3+4872s^2+3528s+720}{7!}\Delta^7 f_{-7}\right]$$

Metode Newton-Gregory (2)

contoh : carilah nilai $f'(x_s)$ untuk x_s = 1,03, jika diketahui fungsi tsb menghasilkan nilai² sbb :

x	1,0	1,3	1,6	1,9	2,2	2,5	2,8
f(x)	1,449	2,060	2,645	3,216	3,779	4,338	4,898

Langkah 1 mencari nilai beda

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$	$\Delta^6 f(x)$
1,0	<u>1,449</u>						
		<u>0,611</u>					
1,3	2,060		<u>-0,026</u>				
		0,585		<u>0,012</u>			
1,6	2,645		-0,014		<u>-0,006</u>		
		0,571		0,006		<u>0,004</u>	
1,9	3,216		-0,008		-0,002		<u>-0,001</u>
		0,563		0,004		0,003	
2,2	3,779		-0,004		0,001		
		0,559		0,005			
2,5	4,338		0,001				
		0,560					
2,8	4,898						

1,03 ada di dekat titik awal, shg NGF lebih cocok digunakan.

Metode Newton-Gregory (3)

Langkah 2 mencari nilai s (lebar interval)

$h = 1,3 - 1 = 0,3$

$s = (1,03 - 1) / h = 0,1$

$$f'(x_s) = \frac{1}{h}\left[\Delta f_0 + \frac{2s-1}{2!}\Delta^2 f_0 + \frac{3s^2-6s+2}{3!}\Delta^3 f_0 + \frac{4s^3-18s^2+22s-6}{4!}\Delta^4 f_0\right.$$

Langkah 3 menggunakan rumus NG untuk mencari nilai $f'(x_s)$

$$f'(1{,}03) = \frac{1}{0{,}3}\left[0{,}611 + \frac{(2\ .\ 0{,}1)-1}{2!}\ .\ -0{,}026 + \frac{3(0{,}1)^2-6(0{,}1)+2}{3!}\ .\ 0{,}012\right.$$

$$+ \frac{4(0{,}1)^3-18(0{,}1)^2+22(0{,}1)-6}{4!}\ .\ -0{,}006 + \frac{5(0{,}1)^4-40(0{,}1)^3+105(0{,}1)^2-100(0{,}1)+24}{5!}\ .\ 0{,}004$$

$$\left.+ \frac{6(0{,}1)^5-75(0{,}1)^4+340(0{,}1)^3-675(0{,}1)^2+548(0{,}1)-120}{6!}\ .\ -0{,}001\right]$$

$= 2{,}088647$

Rumusan diferensiasi metode Newton-Gregory memang di-'turun'-kan dari rumus Interpolasi Newton-Gregory. Sehingga karakteristik, kelebihan-kekurangan, serta cara penggunaan rumusnya pun sama.

Carilah nilai f'(x) ketika x = 3,3
dengan menggunakan Newton Gregory Forward

x	y	Δ f(x)	Δ 2 f(x)	Δ 3 f(x)	Δ 4 f(x)	Δ 5 f(x)	Δ 6 f(x)
2	18.00						
		108.53					
2.4	126.53		192.82				
		301.34		196.42			
2.8	427.87		389.24		119.69		
		690.58		316.11		40.55	
3.2	1118.45		705.35		160.24		5.90
		1,395.93		476.34		46.45	
3.6	2514.38		1,181.69		206.68		
		2,577.62		683.03			
4	5092.00		1,864.72				
		4,442.34					
4.4	9534.34						

Carilah nilai f'(x) ketika x = 3,3
dengan menggunakan Newton Gregory Forward

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2	18.00						
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3.6	2514.38		1,181.69		206.68		
		2,577.62		683.03			
4	5092.00		1,864.72				
		4,442.34					
4.4	9534.34						

Xs = 3.3 h = 0.4

Xo = 3.2

$$s = \frac{3.3 - 3.2}{0.4}$$

s = 0.25

x = 3.30

$$f'(x_s) = \frac{1}{h}\left[\Delta f_0 + \frac{2s - 1}{2!}\Delta^2 f_0 + \frac{3s^2 - 6s + 2}{3!}\Delta^3 f_0\right.$$

$$f'(x) = 2.50 \left\{ 1396 + \frac{2.00 \quad 0.25 - 1}{2} * 1182 + \frac{3 \quad 0.25 \ \hat{} \ 2 - 6 \quad 0.25 + 2}{3 * 2} * 683 \right.$$

1100.506

78.26368

f ' (3,3) = **2946.924**

Carilah nilai f'(x) ketika x = 3,3
dengan menggunakan Newton Gregory Backward

x	y	Δ f(x)	Δ 2 f(x)	Δ 3 f(x)	Δ 4 f(x)
2	18.00				
		108.53			
2.4	126.53		192.82		
		301.34		196.42	
2.8	427.87		389.24		119.69
		690.58		316.11	
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		1,395.93		476.34	
3.6	2514.38		1,181.69		206.68
		2,577.62		683.03	
4	5092.00		1,864.72		
		4,442.34			
4.4	9534.34				

Carilah nilai f'(x) ketika x = 3,3 dengan menggunakan Newton Gregory Backward

x	y	Δ f(x)	Δ 2 f(x)	Δ 3 f(x)	Δ 4 f(x)
2	18.00				
		108.53			
2.4	126.53		192.82		
		301.34		196.42	
2.8	427.87		389.24		119.69
		690.58		316.11	
3.2	1118.45		705.35		160.24
		1,395.93		476.34	
3.6	2514.38		1,181.69		206.68
		2,577.62		683.03	
4	5092.00		1,864.72		
		4,442.34			
4.4	9534.34				

Xs = 3.3 h = 0.4
Xo = 3.2

s = (3.3 - 3.2) / 0.4

s = 0.25

x = 3.30

f'(x) = **2.50** { **691** + (**2.00** 0.25 + 1) / 2 * 389 +

982.5102

120.7187 (**3** **0.25** **^** **2** **+** **6** **0.25** **+** **2**) / (3 * 2) * 196

f(3,3) = **2758.072**

$$f'(x_s) = \frac{1}{h}\left[\Delta f_{-1} + \frac{2s+1}{2!}\Delta^2 f_{-2} + \frac{3s^2+6s+2}{3!}\Delta^3 f_{-3}\right.$$

Metode Lagrange (1)

$$f'(x) = \sum_{m=1}^{n+1} \frac{f_m - 1}{\prod\limits_{\substack{k=1 \\ k \neq m}}^{n+1} (x_{m-1} - x_{k-1})} \left(\sum_{\substack{j=1 \\ j \neq m}}^{n+1} \frac{\prod\limits_{\substack{i=1 \\ i \neq 1}}^{n+4} (x - x_{i-1})}{(x - x_{j-1})} \right)$$

Metode Lagrange (2)

contoh : carilah nilai f'(x) pada x = 2,25 berdasarkan tabel berikut, dengan menggunakan metode Lagrange.

n	x	f(x)
0	1,0	0,00000
1	1,2	0,26254
2	1,5	0,91230
3	1,9	2,31709
4	2,1	3,27194
5	2,5	5,72682
6	3,0	9,88751

Metode Lagrange (3)

$$f'(x) = \frac{(x-x_2)(x-x_3)(x-x_4)+(x-x_1)(x-x_3)(x-x_4)+(x-x_1)(x-x_2)(x-x_4)+(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \cdot f_0 +$$

$$\frac{(x-x_2)(x-x_3)(x-x_4)+(x-x_0)(x-x_3)(x-x_4)+(x-x_0)(x-x_2)(x-x_4)+(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \cdot f_1 +$$

$$\frac{(x-x_1)(x-x_3)(x-x_4)+(x-x_0)(x-x_3)(x-x_4)+(x-x_0)(x-x_1)(x-x_4)+(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \cdot f_2 +$$

$$\frac{(x-x_1)(x-x_2)(x-x_4)+(x-x_0)(x-x_2)(x-x_4)+(x-x_0)(x-x_1)(x-x_4)+(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \cdot f_3 +$$

$$\frac{(x-x_1)(x-x_2)(x-x_3)+(x-x_0)(x-x_2)(x-x_3)+(x-x_0)(x-x_1)(x-x_3)+(x-x_0)(x-x_1)(x-x_2)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \cdot f_4 +$$

= *menghitungnya musti sabar & telaten☺…*

PR

**Buatlah Contoh soal sendiri dan jawab dengan menggunakan **

1. **Stirling**
2. **Bessel**
3. **Differensiasi :**
   1. **NGF**
   2. **NGB**
   3. **langrange**

- ** bentuk File PPT**
- ** upload ke elearning**