Numerical Computing

MEETING 2

Graph, Tabulation, Bisection, Regula Falsi

2024/2025

Comnum Week 2

What Will We Discover?

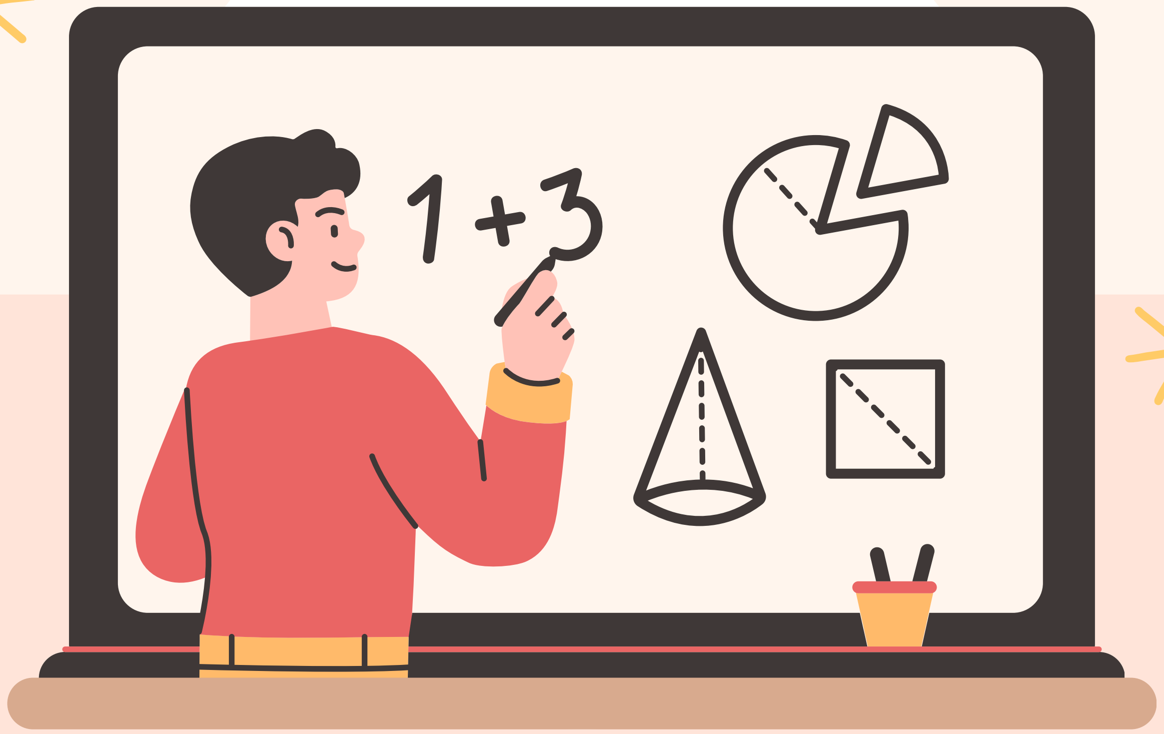
01 Comprehending the Foundations of Equations

02 Accolade Method; Graph, Tabulation, Bolzano (Bisection), Regula Falsi

03 Task 1

Objective

- Search:
  - solution of the equation
  - Determine the value of X for f(X) = 0.
  - The function f(x) intersects the x-axis.



find the root

$f(x) = x^2 - x - 6$

$(x - 3)(x + 2) = 0$

$x = 3 \rightarrow f(x) = 0$

$x = -2 \rightarrow f(x) = 0$

In a polynomial equation of degree two, for instance

$f(x) = x^2 + x - 2$

To determine x1 and x2, we can employ the ABC formula.

$$X_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Methods for determining the roots of an equation:

$f(x) = x^4 - 3x - 2 = 0$

$f(x) = e^{-x} - x = 0$

$f(x) = x^3 + x^2 - 3x - 3 = 0$

Methods for resolution

experimenting, inputting the value of x, so that f(x) equals 0

The results are sluggish and not guaranteed to be located. 😊

Comnum Week 2

In the upcoming two meetings, we will explore various methods for determining the roots of equations.

Comprehending the Foundations of Equations

For polynomials of degree 2, there exists a remarkable formula known as "ABC," which can analytically assist in determining the roots of the equation.

Meanwhile, for polynomials of degree 3 or 4, the existing formulas are rather intricate. We must utter “gladium laviosa” numerous times before we can apply them. Nevertheless, these formulas can still be utilized analytically.

polynomial degree > 4?

All we can do is attempt to resolve it through a series of numerical approaches. For this purpose, there are various methods available for selection.

Comprehending the Foundations of Equations

The most straightforward method for identifying the roots of a high-degree polynomial equation is to graph the function in Cartesian coordinates. Subsequently, determine the points of intersection of the function with the X-axis.

Another simple method?!...
Yes, but it requires patience. This involves trial and error. Select any value of x and determine if you can achieve $f(x) = 0$.

If it fails, experiment with alternative values of x until you are fortunate enough to find $f(x) = 0$.

Both of these methods can indeed be classified as an approach effort, albeit not systematic. Conversely, numerous approach techniques can be broadly categorized into two major groups, namely:

- Akolade Method Group (this week)
- Open Method Group (upcoming meeting)

Welcome to Week Two.

1. Graphical Method

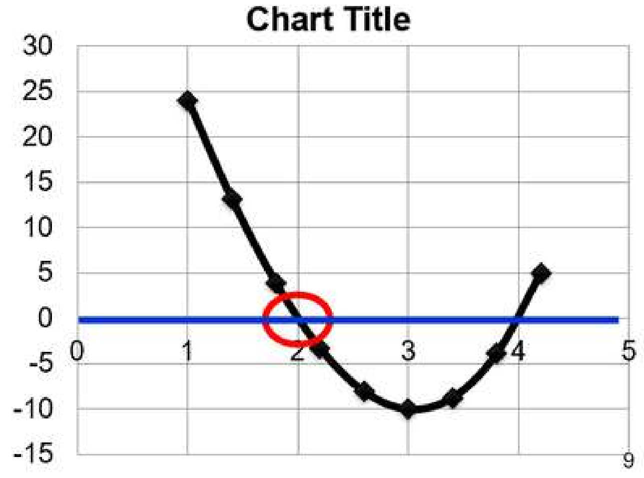
- Preliminary assessment
- Unable to compute Ea (approximate error); only Et (true error) can be calculated.
- First, create a table to construct the graph.
- The graph illustrates the points at which the function f(x) intersects the x-axis.
- This point signifies the value of x for which f(x) equals zero.

$$\% \text{ error} = \left| \frac{\#\text{experimental} - \#\text{actual}}{\#\text{actual}} \right| \times 100$$

determine the approximate roots of the equation $f(x) = x^3 + x^2 - 34x + 56$

01 First, establish a table.

x	f(x)
1	24
1,4	13,104
1,8	3,872
2,2	-3,312
2,6	-8,064
3	-10
3,4	-8,736
3,8	-3,888
4,2	4,928



02 The current price is established as x = 2.

Thus, we can compute Et, specifically:

$$Et = \left| \frac{2 - (2{,}2)}{2} \right| * 100\ \% = 10\ \%$$

Et = error concerning the actual value

Welcome to Week Two

1. Graphical Method

- Preliminary assessment
- Unable to compute Ea (approximate error); only Et (true error) can be calculated.
- First, establish a table to construct the graph.
- The graph illustrates the points at which the function f(x) intersects the x-axis.
- This point signifies the value of x for which f(x) equals zero.

$$\% \text{ error} = \left|\frac{\#\,\text{experimental} - \#\,\text{actual}}{\#\,\text{actual}}\right| \times 100$$

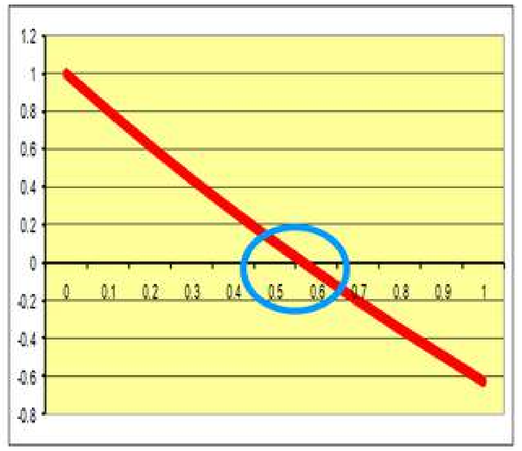
determine the approximate roots of the equation

$$f(x) = e^{-x} - x$$

01 First, create a table using an Excel formula.

=EXP(-A1)-A1

x	f(x)
0.5	0.106531
0.51	0.090496
0.52	0.074521
0.53	0.058605
0.54	0.042748
0.55	0.02695
0.56	0.011209
0.57	-0.00447
0.58	-0.0201
0.59	-0.03567
0.6	-0.05119



02 The precise value is established as x = 0.56714329.

Thus, we can compute Et, specifically:

$$Et = \left|\frac{0{,}56714329 - 0{,}57}{0{,}56714329}\right| * 100\ \% = 0{,}5\ \%$$

Et = error concerning the actual price

Welcome to Week Two.

1.2. Tabulation Technique

- This tabulation method serves as an extension of the graphic method. While the graphic method yields only a rough approximation, the tabulation method allows for more precise results.

Example 1:

Obtain the approximate roots of the equation provided below.

$f(x) = x^3 + x^2 - 34x + 56$

x	f(x)
1	24
1,4	13,104
1,8	**3,872**
2,2	**-3,312**
2,6	-8,064
3	-10
3,4	-8,736
3,8	-3,888
4,2	4,928

x	f(x)
1,8	3,872
1,82	3,460968
1,84	3,055104
1,86	2,654456
1,88	2,259072
1,9	1,869
1,92	1,484288
1,94	1,104984
1,96	0,731136
1,98	0,362792
2	**0**
2,02	-0,35719
2,04	-0,70874

Tabulation Technique

Example 2:

Obtain the approximate roots of the equation. $f(x) = e^{-x} - x$

x	f(x)
0	1
0.1	0.804837
0.2	0.618731
0.3	0.440818
0.4	0.27032
0.5	**0.106531**
0.6	**-0.05119**
0.7	-0.20341
0.8	-0.35067
0.9	-0.49343
1	-0.63212

x	f(x)
0.5	0.106531
0.51	0.090496
0.52	0.074521
0.53	0.058605
0.54	0.042748
0.55	0.02695
0.56	**0.011209**
0.57	**-0.00447**
0.58	-0.0201
0.59	-0.03567
0.6	-0.05119

x	f(x)
0.56	0.011209
0.561	0.009638
0.562	0.008068
0.563	0.006498
0.564	0.004929
0.565	0.00336
0.566	0.001792
0.567	**0.000225**
0.568	**-0.00134**
0.569	-0.00291
0.57	-0.00447

Tabulation Technique

- It is established that the actual price is x = 0.56714329.

Thus, we can compute Et, specifically:

- Et = error concerning the actual price

$$Et = \left| \frac{0,56714329 - 0,567}{0,56714329} \right| * 100\ \% = 0,025\ \%$$

Welcome to Week Two.

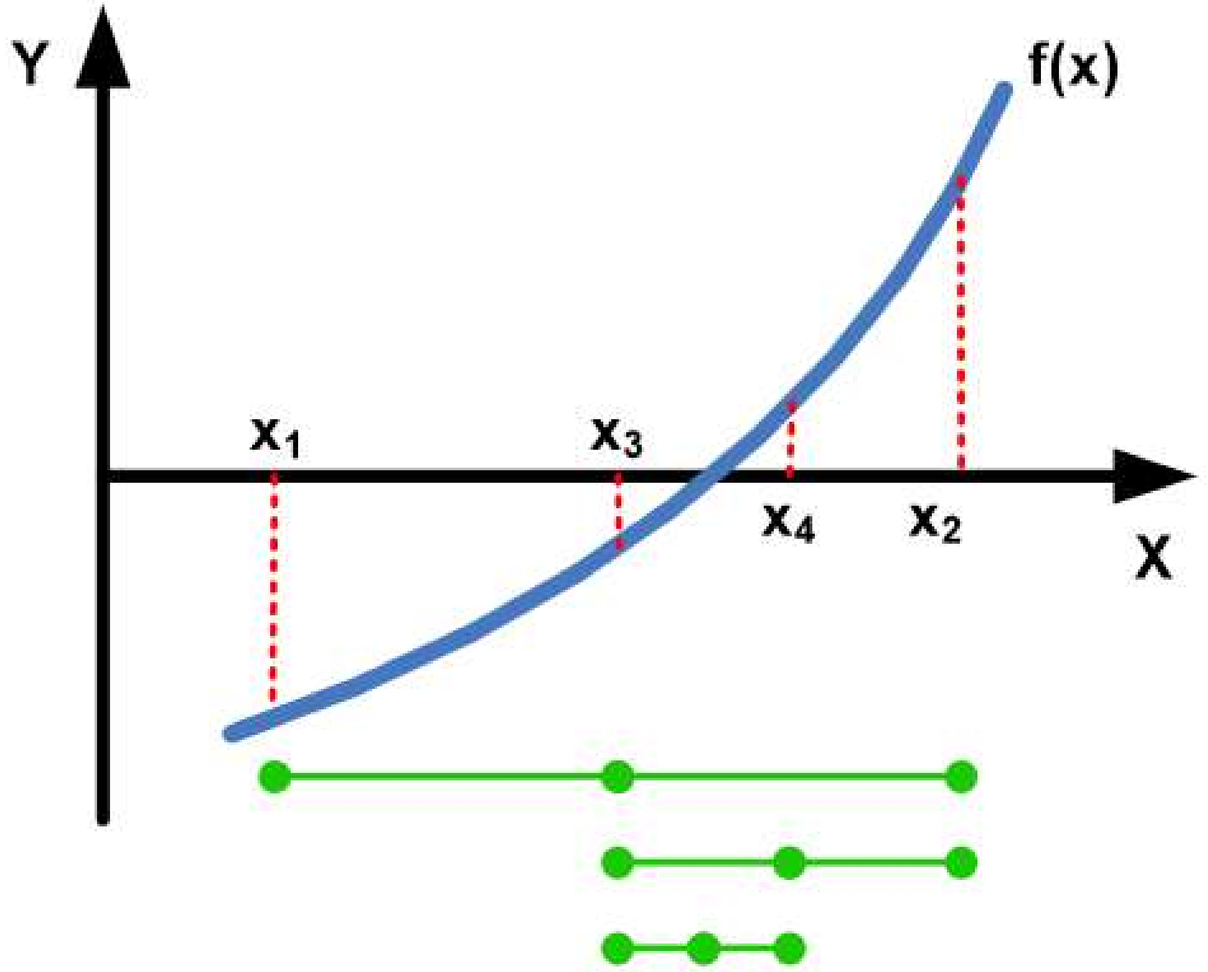
1.3. The Bisection Method

Bolzano/Biseksi

- Refined estimates derived from graphs
- Also referred to as the half-interval method (interval halving), Bolzano's method, or bisection.
- Ea and Et can be computed.
- Algorithm:
- Select the initial estimates XL (Xlower) and Xu (Xupper), ensuring that f(XL) * f(Xu) < 0, indicating a sign change between f(XL) and f(Xu).

Bolzano Techniques

- The phrase "sign change" in this method carries significant implications. Given the continuous nature of the function, the presence of two function values, f(xi) and f(xi+n), with differing signs indicates that the function intersects the coordinate axis (at least once) between xi and xi+n.
- Remember, we are seeking the value of x for which f(x) equals zero.



The Bisection Technique

2. First estimation of roots : $Xr = \frac{Xl + Xu}{2}$

3. To find the next sub-interval
   - If $f(Xl).\, f(Xr) < 0$
     - The root lies in the first subinterval,
       - Then -> Xu = Xr, continue to number 4
   - If $f(Xl).\, f(Xr) > 0$
     - The roots lies in the second subinterval,
       - Then -> XL = Xr, continue to number 4
   - If $f(Xl).\, f(Xr) = 0$ akar = Xr, stop

4. Compute the new estimated root: $Xr = \frac{Xl + Xu}{2}$

5. Decide: - If the root is quite accurate -> stop
   - If the root is inaccurate -> step 3

Example Question 1

Known,

$$f(x) = x^3 + 10x^2 - 7x - 196$$

Lower Bound (XL) = -5

Upper Bound (Xu) = 8

The actual value of X = 4

Find the root of x using the bisection method (value 24)

- notes:
  - each iteration look for Et and Ea
  - 2 digits after comma
  - look for iteration 1 to iteration 3
  - write the formula first

The Bisection Technique

Example

Known,

$$f(x) = x^3 + 10x^2 - 7x - 196$$

Lower Bound (XL) = -5
Upper Bound (Xu) = 8
Actual value of X = 4

Formula

$$Xr = \frac{Xl + Xu}{2}$$

Answer

iterasi 1

Xl = -5
Xu = 8

$$Xr = \frac{-5 + 8}{2}$$

Xr = 1.5

$$Et = \frac{4 - 1.5}{4}$$

Et = 62.5
Ea = belum bisa

f(Xl) = -36
f(Xr) = -180.625

f(xl) x f(xu) = -36 x -180.63
>0
maka ==> interval ke 2 (kanan)

iterasi 2

Xl = 1.5
Xu = 8

$$Xr = \frac{1.5 + 8}{2}$$

Xr = 4.75

$$Et = \frac{4 - 4.75}{4}$$

Et = 18.75

$$Ea = \frac{4.75 - 1.5}{4.75}$$

Ea = 68.42

f(Xl) = -180.625
f(Xr) = 103.54688

f(xl) x f(xu) = -180.63 x 103.55
< 0
maka ==> interval ke 1 (kiri)

The Bisection Method

Sample Inquiries

known,

$$f(x) = x^3 + 10x^2 - 7x - 196$$

Lower Bound(XL) = -5
Upper Bound (Xu) = 8
Actual value of X = 4

Formula

Xr =	Xl	+	Xu
		2	

Answer

iterasi 3			
Xl =	1.5		
Xu =	4.75		
Xr =	1.5	+	4.75
		2	
Xr =	3.125		
Et =	4	-	3.125
		4	
Et =	21.875		
Ea =	3.125	-	4.75
		3.125	
Ea =	52.00		

Bisection Method

Example Problem 2

Known,

$$f(x) = x^3 + x^2 - 34x - 56$$

Lower Bound (XL) = -2
Upper Bound (Xu) = 3
Actual value of X = 2

Find the root of x using the bisection method

- notes:
  - each iteration look for Et and Ea
  - 2 digits after comma
  - look for iteration 1 to iteration 3
  - write the formula first

Formula

$$Xr = \frac{Xl + Xu}{2}$$

Bisection Method

Example Problem 2

Known,

$$f(x) = x^3 + x^2 - 34x - 56$$

Lower Bound (XL) = -2
Upper Bound (Xu) = 3
Actual value of X = 2

Find the root of x using the bisection method

- notes:
  - each iteration look for Et and Ea
  - 2 digits after comma
  - look for iteration 1 to iteration 3
  - write the formula first

Formula

Xr =	Xl	+	Xu
		2	

Answer

iterasi 1

Xl =	-2				f(Xl) =	120
Xu =	3				f(Xu) =	-10
Xr =	-2	+	3			
		2				
Xr =	0,5					
Et =	2	-	0,5			
		2				
Et =	75					
Ea =	belum bisa					
f(Xl) = f (-2) =	120					
f(Xr) = f (0,5) =	39,38					
f(xl) x f(xu) =		120	x	39,38		
	> 0					2
	maka ==> interval ke 2 (kanan)					

Bisection Method

Example Problem 2

Known,

$$f(x) = x^3 + x^2 - 34x - 56$$

Lower Bound (XL) = -2

Upper Bound (Xu) = 3

Actual value of X = 2

Find the root of x using the bisection method

- notes:
  - each iteration look for Et and Ea
  - 2 digits after comma
  - look for iteration 1 to iteration 3
  - write the formula first

Formula

$$Xr = \frac{Xl + Xu}{2}$$

Answer

iterasi 2

Xl =	0,5		
Xu =	3		
Xr =	0,5	+	3
		2	
Xr =	1,75		
Et =	2	-	1,75
		2	
Et =	12,5		
Ea =	1,75	-	0,5
		1,75	
Ea =	71,43		
f(Xl) = f (0,5) =	39,38		
f(Xr) = f (1,75) =	4,92		

f(xl) x f(xu) =		39,38	x	4,92
	> 0			

maka ==> interval ke 2 (kanan)

The Bisection Method

Example Problem 2

Known:

$$f(x) = x^3 + x^2 - 34x - 56$$

Upper Bound (XL) = -2
Lower Bound (Xu) = 3
Actual Value of X = 2

Find the root of x using the bisection method (value 24)

- Note:
  - Calculate Et and Ea in each iteration
  - Precision to 2 decimal places
  - Find 1st to 3rd iteration
  - Note the formula

Formula

Xr =	Xl	+	Xu
		2	

Answer

iterasi 3

Xl =	1,75		
Xu =	3		
Xr =	1,75	+	3
		2	
Xr =	2,375		
Et =	2	-	2,375
		2	
Et =	18,75		
Ea =	2,375	-	1,75
		2,375	
Ea =	26,32		
f(Xl) = f (1,75) =	4,92		
f(Xr) = f (2,375) =	-5,71		
f(xl) x f(xu) =		4,92	x	-5,71
	< 0		

maka ==> interval ke 1 (kiri)

Komnum Week 2

Regula Falsi (False Position) Method

- taksiran lebih halus dr m. bagidua
- m. regula falsi or m. interpolasi Linier
- algoritma :
  1. pilih taksiran interval x_L & x_u
  2. taksiran awal ⇒ $$x_r = x_u - \frac{f(x_u)(x_L - x_u)}{f(x_L) - f(x_u)}$$
  3. u mencari sub interval berikutnya :
     → idem dgn m. bagi dua

Regula Falsi (False Position) Method

Example Problem

Known;

$f(x) = x^3 + 10x^2 - 7x - 196$

Lower bound (Xl) = – 5
Upper Bound (Xu) = 8
Actual value of X = 4

Find the root of x using the False Position Method (with a value of 24)

- Note :
  - Calculate Et and Ea in each iteration
  - Precision to 2 decimal places
  - Find 1st to 3rd iteration
  - Note the formula

Regula Falsi (False Position) Method

Jawab :

$$X_r = X_u - \frac{f(X_u)\,(X_L - X_u)}{f(X_L) - f(X_u)}$$

$$f(x) = x^3 + 10x^2 - 7x - 196$$

Upper Bound (Xl) = – 5
Lower Bound (Xu) = 8
Actual value of X = 4

iterasi 1

Xl =	-5	f(Xl) =	-36
Xu =	8	f(Xu) =	900

$$X_r = 8 - \frac{900\,(-5 - 8)}{-36 - 900}$$

Xr = -4.50

$$E_t = \frac{4 - -4.50}{4}$$

Et = 212.50
Ea = belum bisa

f(Xl) = -36
f(Xr) = -53.13

f(xl) x f(xu) = -36 x -53.13

>0

maka ==> interval ke 2 (kanan)

False Position Method

iterasi 2

Xl = -4.50   f(Xl) = -53.13
Xu = 8.00   f(Xu) = 900

$$Xr = 8 - \frac{900 \; (-4.5 - 8)}{-53.13 - 900}$$

Xr = -3.80

$$Et = \frac{4 - -3.80}{4}$$

Et = 195.08

$$Ea = \frac{-3.80 - -4.50}{-3.80}$$

Ea = 18.32

f(Xl) = -53.13
f(Xr) = -79.74

f(xl) x f(xu) = -53.13 x -79.74

>0

maka ==> interval ke 2 (kanan)

iterasi 3

Xl = -3.80   f(Xl) = -79.74
Xu = 8.00   f(Xu) = 900

$$Xr = 8 - \frac{900 \; (-3.80 - 8)}{-79.74 - 900}$$

Xr = -2.84

$$Et = \frac{4 - -2.84}{4}$$

Et = 171.07

$$Ea = \frac{-2.84 - -3.80}{-2.84}$$

Ea = 33.80

https://its.id/m/komnum25

Komnum Week 2

Group Task

1. Create an example problem, you can make your own or search from the internet:
   a. Graph = 2 groups
   b. Tabulation = 2 groups
   c. Bisection = 4 groups
   d. Regula Falsi = 4 groups
2. PPT File + Group name and members
3. Provide examples of real life implementations of the methods used

Ditanya :

- Search for Et and Ea in every iteration
- Precision to 2 decimal places
- Find 1st to 3rd iteration
- Note the formula

Komnum Week 2

THANK YOU

See you next week!