

Example: polynomial evaluation.

input: polynomial f & evaluation point x .

output: $f(x)$.

$$f(x) = \sum_{i=0}^d a_i x^i.$$

How to store in C++?
Save list of coefficients
 a_0, a_1, \dots, a_d .

```
int polyEval(int* a, int d, int x)
{
    // Note: a must point to an array
    // of d+1 elements!
    // if  $f = \sum_{i=0}^d a_i x^i$ ,  $a[i] = a_i$ 
    int sum = 0;
    for (int i = 0; i <= d; i++)
        sum += a[i] * pow(x, i);
    return sum;
}
```

$f(x) = 3 + 7x + 2x^2$

How to call? `int a[3] = {3, 7, 2};` \equiv
`int result = polyEval(a, 2, 97);`
// computes $f(97)$

How expensive is it to call `polyEval`?

Let's count the # of multiplications
as a function of d .

```
for (int i = 0; i <= d; i++)
    sum += a[i] * pow(x, i);
```

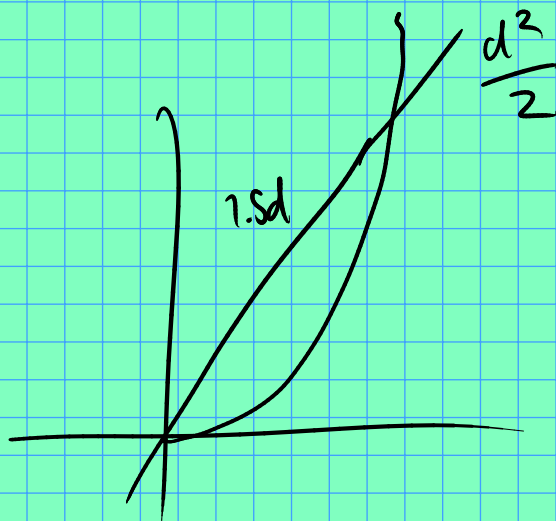
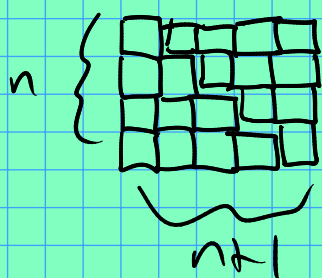
↑
d+1

costs i mults.

Total # mults: $1 + 2 + 3 + 4 + \dots + (d+1)$

$$= \frac{(d+1)(d+2)}{2} \approx \frac{d^2}{2}$$

$$1 + 2 + \dots + n = \frac{n(n+1)}{2}$$



Idea for saving on multiplications:

$\text{pow}(x, i+1)$ is easy given $\text{pow}(x, i)$!
just mult. by x .

```
int polyEvalBetter(int* a, int d, int x)
{
    int sum = 0;
    int powXi = 1; // stores x^i
    for (int i = 0; i <= d; i++) {
```

```
sum += a[i] * powxi;  
powxi *= x;
```

```
}
```

```
return sum;
```

```
}
```

Now how many multiplications?

$2(d+1)$. Much better!

Challenge problem: can you do this with only $d+1$ or fewer multiplications?

(And a linear # of additions, of course.)