Séries
Lista 3

Ex. 4.1

$$Y_t = 5 + e_t - \frac{1}{2} e_{t-1} + \frac{1}{4} e_{t-2}$$

A função de autocorrelação será:

$$Var(Y_t) = Var\left(5 + e_t - \frac{1}{2} e_{t-1} + \frac{1}{4} e_{t-2}\right)$$

$$(*) = \underbrace{Var(5)}_{\text{Var. constante} = 0} + Var(e_t) + \frac{1}{4} Var(e_{t-1}) + \frac{1}{16} Var(*)$$

$* : e_{t-2}$

Prop. variância: constante sai ao quadrado

Prop.: $Var(e_t) = Var(e_{t-1}) = Var(e_{t-2})$

$$\therefore (*) = Var(e_t) + \frac{1}{4} Var(e_t) + \frac{1}{16} Var(e_t)$$

$$= \frac{21}{16} \underbrace{Var(e_t)}_{= \sigma^2 \Rightarrow \text{Var do processo}}$$

$$= \frac{21}{16} \sigma^2$$

Já a autocovariância; (P/ lag = 1),

$$\text{Cov}(Y_t, Y_{t-1}) = \text{Cov}\Big(5 + e_t \underbrace{- \tfrac{1}{2} e_{t-1}} \underbrace{+ \tfrac{1}{4} e_{t-2}},$$
$$5 + \underbrace{e_{t-1}} \underbrace{- \tfrac{1}{2} e_{t-2}} + \tfrac{1}{4} e_{t-3}\Big)$$

$$= \text{Cov}\left(-\tfrac{1}{2} e_{t-1}, e_{t-1}\right) + \text{Cov}\left(\tfrac{1}{4} e_{t-2}, -\tfrac{1}{2} e_{t-2}\right)$$

$$= -\tfrac{1}{2} \underbrace{\text{Cov}(e_{t-1}, e_{t-1})}_{\text{Var}} - \tfrac{1}{8} \underbrace{\text{Cov}(e_{t-2}, e_{t-2})}_{\text{Var}}$$

$$= -\tfrac{1}{2} \text{Var} \underbrace{(e_{t-1})}_{\sigma^2} - \tfrac{1}{8} \underbrace{\text{Var}(e_{t-2})}_{\sigma^2}$$

$$= -5/8\, \sigma^2$$

P/ lag = 2;

$$\text{Cov}(Y_t, Y_{t-2}) = \text{Cov}\Big(5 + e_t - \tfrac{1}{2} e_{t-1} + \tfrac{1}{4} e_{t-2},$$
$$5 + e_{t-2} - \tfrac{1}{2} e_{t-3} + \tfrac{1}{4} e_{t-4}\Big)$$

$$= \text{Cov}\left(e_t + \tfrac{1}{4} e_{t-2}, e_{t-2}\right)$$

$$= \tfrac{1}{4} \text{Cov}(e_{t-2}, e_{t-2}) = \tfrac{1}{4} \text{Var}(e_{t-2})$$

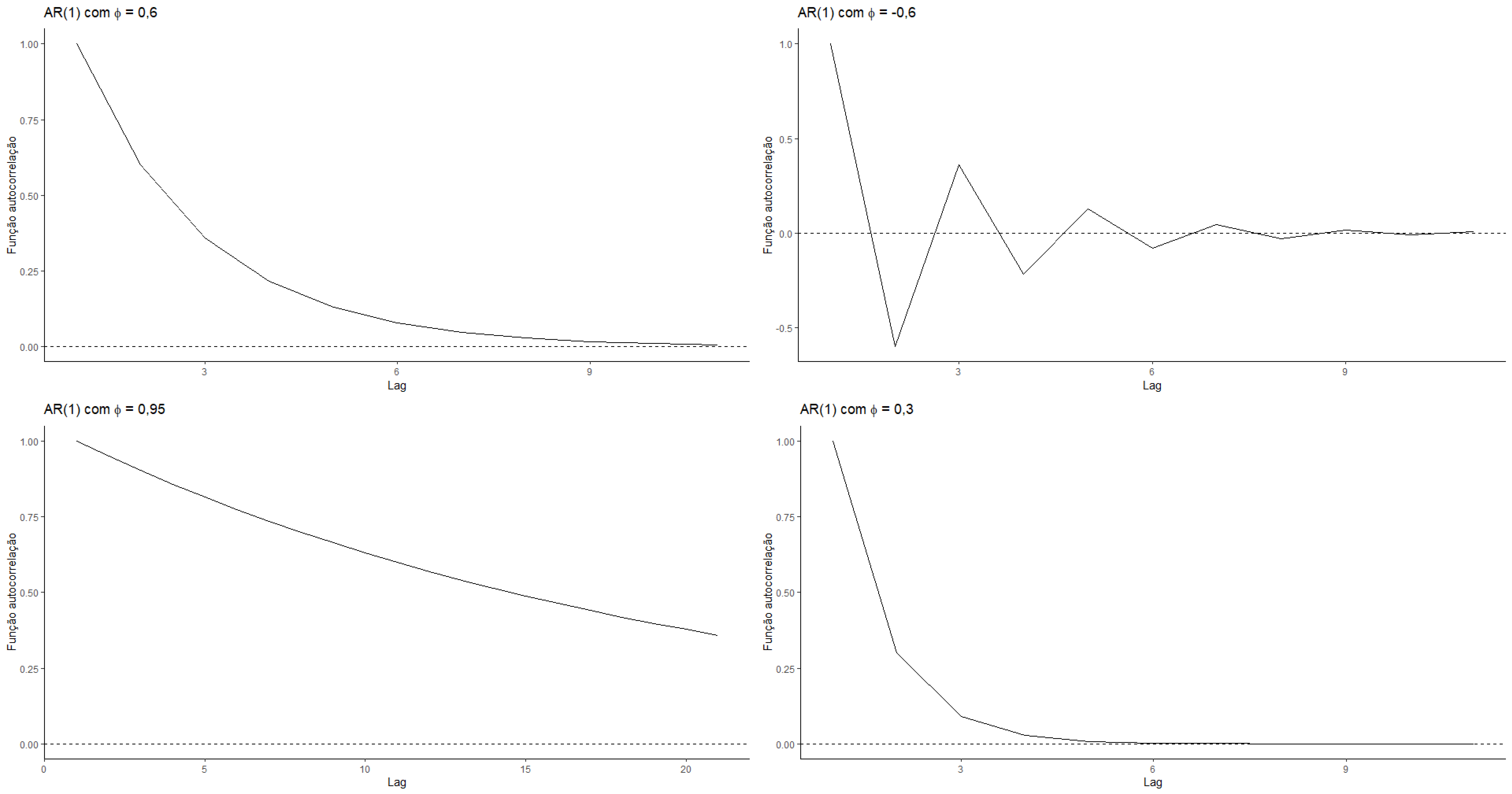
$$= 1/4\, \sigma^2$$

P/ lag = 3,

$$\text{Cov}(Y_t, Y_{t-3}) = \text{Cov}\Big(5 + e_t - \tfrac{1}{2} e_{t-1} + \tfrac{1}{4} e_{t-2}, 5 + e_{t-3} - \tfrac{1}{2} e_{t-4}$$
$$+ 1/4\, e_{t-5}\Big) = 0.$$

Logo, a Autocorrelação será dada por:

$$\rho_s = \begin{cases} 1 & , \ p/\ s=0 \\ \dfrac{-\frac{5}{8}\alpha^2}{\frac{21}{16}\alpha^2} = -\dfrac{10}{21} & , \ p/\ s=1 \\ \dfrac{1/4\,\alpha^2}{21/16\,\alpha^2} = 4/21 & , \ p/\ s=2 \\ 0 & , \ p/\ s=3 \end{cases}$$


AR(1) com ϕ = 0,6
AR(1) com ϕ = -0,6
AR(1) com ϕ = 0,95
AR(1) com ϕ = 0,3
Função autocorrelação
Lag
1.00
0.75
0.50
0.25
0.00
1.0
0.5
0.0
-0.5
3
6
9
0
5
10
15
20

4.15) Modelo AR 1 $Y_t = \phi Y_{t-1} + e_t$

Se $|\phi| = 1$, então processo não estacionário.

Sol: $Var(Y_t) = Var(\phi Y_{t-1} + e_t)$

$= \phi^2 Var(Y_{t-1}) + Var(e_t)$

$= \phi^2 Var(Y_{t-1}) + \sigma^2$

Verificando p/ $t-2$:

$\Rightarrow Var(\phi^2 Y_{t-2} + e_t)$

$= \phi^4 Var(Y_{t-2}) + 2\sigma^2$

p/ $t-3$

$\Rightarrow Var(\phi^4 Y_{t-3} + e_t)$

$= \phi^8 Var(Y_{t-3}) + 4\sigma^2$

Podemos observar que conforme deslocamos t, a constante ϕ cresce em potência, tq

$$\lim_{s \to \infty} Var(Y_{t-s}) = \phi^{2s} Var(Y_{t-s}) + s\sigma^2$$

$$= \infty$$

Portanto, o processo é não estacionário $\square$

4.18) AR(1) $Y_t = \phi Y_{t-1} + e_t$; $-1 < \phi < 1$. 4

c constante $\neq 0$ e $W_t = Y_t + c\phi^t$

a) Mostrar que $E(W_t) = c\phi^t$

Sol: $E(W_t) = E(Y_t + c\phi^t)$

$$= \underbrace{E(Y_t)}_{\text{Estacionária} = 0} + \underbrace{E(c\phi^t)}_{\hookrightarrow \text{Constante}}$$

$$= c\phi^t$$

(b) W_t satisfaz estacionariedade em $W_t = \phi W_{t-1} + e_t$

Sol.: Tomando $\phi(Y_{t-1} + c\phi^{t-1}) + e_t$

$$= \phi Y_{t-1} + c\underbrace{\phi\phi^{t-1}}_{\phi^t} + e_t ,$$

Sabemos que $Y_{t-1} + e_t \propto Y_t - e_t$

Então

$$\phi\left(\frac{Y_t - e_t}{\phi}\right) + c\phi^t + e_t$$

$$= Y_t + c\phi^t \equiv W_t$$

c) Não, pois W_t depende de t ∎

4.24) e_t Processo de ruído branco c/ média zero e variância constante.

Seja $Y_0 = c_1 e_0$ e $Y_1 = c_2 Y_0 + e_1$. Daí
$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t \quad p/\ t > 1$$
AR(2); Iniciado em tempo $t=0$.

ⓐ Mostrar que a média do processo é 0.

Sol.:
$$E(Y_0) = E(c_1 e_0)$$
$$= c_1 \underbrace{E(e_0)}_{=0} = 0;$$

$$E(Y_1) = E(c_2 Y_0 + e_1)$$
$$= c_2 E(Y_0) + \underbrace{E(e_1)}_{0} = 0$$

$$E(Y_t) = E(\phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t)$$
$$= \phi_1 \underbrace{E(Y_{t-1})}_{=0} + \phi_2 \underbrace{E(Y_{t-2})}_{=0} + \underbrace{E(e_t)}_{=0}$$
$$= 0$$

$\square$

(b) P/ valores particulares de ϕ_1 e ϕ_2;
Escolher c_1 e c_2 tq $Var(Y_0) = Var(Y_1)$
e lag 1 autocorrelação entre Y_1 e Y_0 parece com
um processo estacionário $AR(2)$ c/ parâmetros ϕ_1, ϕ_2

Sol.:
$$Var(Y_0) = c_1^2 \, Var(e_0) = c_1^2 \sigma^2$$
$$Var(Y_1) = c_2^2 \, Var(Y_0) + Var(e_0)$$
$$= c_2^2 c_1^2 \sigma^2 = \sigma^2(1 + c_1^2 c_2^2)$$

Daí, montamos o sistema

$c_1^2 \sigma^2 = \sigma^2 (1 + c_1^2 c_2^2)$. P/ $0 \neq c_1, c_2, \sigma^2$;

Se $\sigma^2 = 1$; $c_1^2 = 1 + c_1^2 c_2^2 \equiv c_1^2(1 - c_2^2) = 1$;

$\Leftrightarrow c_1^2 = \dfrac{1}{1 - c_2^2} \equiv c_1 = \dfrac{1}{\sqrt{1 - c_2^2}}$

$$Cov(Y_0, Y_1) = Cov(c_1 e_0, c_2 c_1 e_0 + e_1)$$
$$= Cov(c_1 e_0, c_2 c_1 e_0) + Cov(c_1 e_0, e_1)$$
$$= c_1^2 c_2 \sigma^2 + \underbrace{c_1 Cov(e_0, e_1)}_{0} = c_1^2 c_2 \sigma^2$$

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$\mathcal{E}_1$

$$\rho_1 = \frac{c_1^2 c_2 \sigma^2}{\sqrt{(c_1^2)^2}} = \frac{\cancel{c_1^2} c_2 \cancel{\sigma^2}}{\cancel{c_1^2}} = \boxed{c_2}$$

Portanto,

$$c_2 = \frac{\phi_1}{1 - \phi_2}$$

$$c_1 = \frac{1}{\sqrt{1 - c_2^2}}$$

(c) Partindo do processo Y_t gerado, 8

podemos mudar a média e a variância do processo com a transformada $\frac{Y_t \sqrt{\gamma_0}}{c_2} + \mu$ inserindo os nossos valores desejados