SINGAPORE POLYTECHNIC
2022/2023 SEMESTER ONE MID-SEMESTER TEST

Foundation Year

FOUNDATION MATHEMATICS Time allowed: 1hour 40min

Instructions:

1. The Singapore Polytechnic Examination rules are to be complied with.
2. This paper consists of **4** printed pages.
3. Unless otherwise stated, all decimal answers given should be correct to 3 significant figures.
4. Answer **ALL** the questions in the answer booklet provided.

1. Simplify the given expressions and express all answers in positive exponent form:

(a) $\left(\frac{x^5y^4}{64z^0}\right)\left(\frac{8y^3}{x^3}\right)$ [4]

(b) $\left(3p^2q^{-2}\right)^2\left(27p^2q^{-5}\right)^{-1}$ [5]

(c) $\left(\sqrt{\frac{36x^4}{y^{12}}}\right)\times\frac{y^6}{2x}$ [6]

2. (a) Simplify $(2a+b)^2-(2a-b)^2$. [4]

(b) Find the quotient and remainder when x^3+2x^2-x+5 is divided by $x+1$. [6]

(c) Factorise $(x+5)^3-9x-45$ completely. [4]

(d) The expression $2x^3+ax^2-7x+b$ has a factor of $x-2$ and leaves a remainder of -10 when divided by $x+3$. Determine the values of a and b. [11]

3. Perform the indicated operations and simplify your answers.

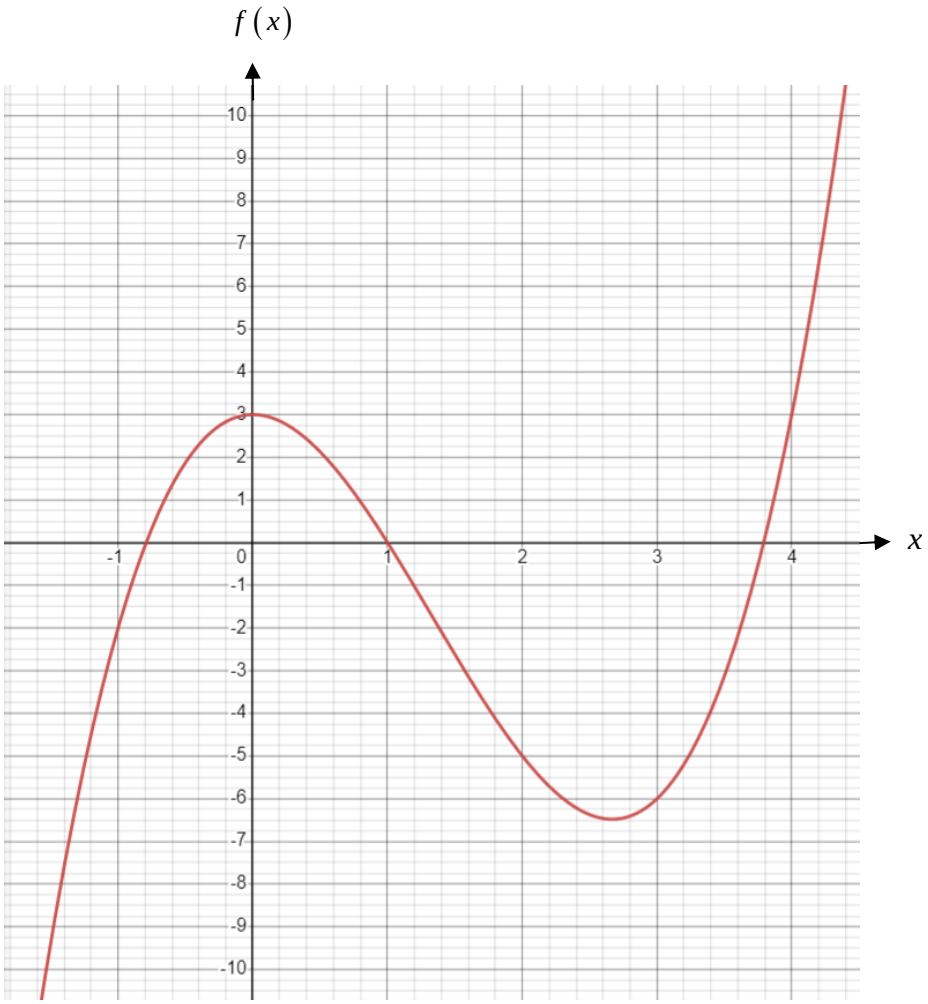
(a) $\frac{12a^9b^8}{a^2+3a-4}\times\frac{a-1}{3a^2b^5}$ [6]

(b) $\frac{x+3}{x^2-25}-\frac{1}{x+5}$ [4]

(c) $\dfrac{\frac{1}{u}-\frac{1}{3u}}{\frac{1}{u}-\frac{1}{6u}}$ [5]

4. Express $\dfrac{2x^2+7x-1}{(x+1)(x^2+5)}$ in partial fractions. [10]

5. A graph of the function $f(x)$ is shown below.



(a) State the values of $f(-1)$ and $f(2)$. [2]

(b) What are the values of x when $f(x)=0$. [3]

(c) Determine the value of $f'(0)$ and interpret its meaning. [2]

(d) State the interval of x when $f(x)>0$ **and** $f'(x)<0$. [1]

(e) Will $f'(1)$ be greater than, equal to or less than $f'(4)$. Explain your answer. [2]

(f) State the domain and range of $f(x)$. [2]

6. (a) Given function $g(x)=(x-4)^2+3$.

(i) Evaluate $g(6)$. [2]

(ii) Determine $g(k+1)$. [3]

(b) Describe how the graph of $g(x)=(x-4)^2+3$ can be obtained from the graph of $f(x)=x^2$. Hence, sketch the graph $g(x)$, label clearly the axes and coordinates of the vertex . [5]

7. You run a canoe-rental business on a reservoir. Currently, you charge \$12 per canoe and rent out 36 canoes each day. An industry journal says that, for every fifty-cent increase in rental price, the average business can expect to lose two rentals a day.

Let *x* be the number of fifty-cent price hikes. The daily revenue $R(x)$ will be a function of *x*.

(a) Show that the daily revenue is given by $R(x)=-x^2-6x+432$. [4]

(b) What is $R(0)$? Interpret this value. [2]

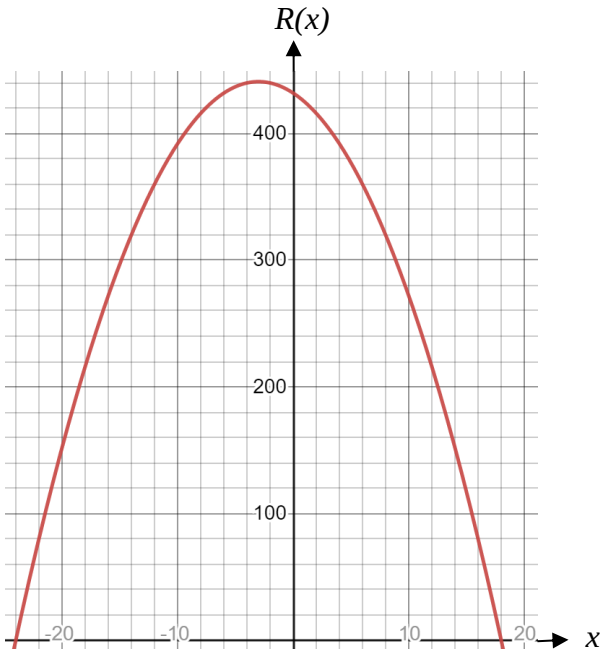
(c) What does it mean when x is negative? [2]

(d) Fill in the table for $x=-3$ and $x=0$.

x	-4	-3	-2	-1	0	1	2
R (*x*)	440		440	437		425	416

[2]

(e) The graph of $R(x)$ is given below. How much should you charge per canoe to get maximum daily revenue?



[2]

(f) What does $x = -24$ mean? [1]

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