

MS960Y MST SOLUTIONS

SINGAPORE POLYTECHNIC 2019 / 2020 Semester 1 MST

Module Name: Foundation Mathematics

Module Code: MS960Y

Course: Polytechnic Foundation Programme

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| No.  | SOLUTION                                                                                                                                                                                                                                                     |
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| 1(a) | $\left(\frac{4xy^2h^6}{7}\right)\left(\frac{x^3}{12h}\right)$ $=\left(\frac{1}{21}\right)(xy^2h^6)\left(\frac{x^3}{h}\right)$ $=\frac{x^{3+1}y^2h^{6-1}}{21}$ $=\frac{x^4y^2h^5}{21}$                                                                        |
| 1(b) | $(4d^4)^{-2}(2kd^{-1})^3$ $=\left(\frac{1}{4d^4}\right)^2\left(\frac{2k}{d}\right)^3$ $=\left(\frac{1}{16d^8}\right)\left(\frac{8k^3}{d^3}\right)$ $=\frac{k^3}{2d^{11}}$                                                                                    |
| 1(c) | $\left(\sqrt{25(w+x)^{3a}}\right)^6\left(\frac{5^{-4}}{(w+x)^2}\right)$ $=\left(5(w+x)^{\frac{3a}{2}}\right)^6\left(\frac{5^{-4}}{(w+x)^2}\right)$ $=\left(5^6(w+x)^{9a}\right)\left(\frac{5^{-4}}{(w+x)^2}\right)$ $=5^{6-4}(w+x)^{9a-2}$ $=25(w+x)^{9a-2}$ |
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| No.  | SOLUTION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
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| 2(a) | $\begin{aligned} & (5x+3y)^2 - (5x-3y)^2 \\ &= ((5x+3y) + (5x-3y))((5x+3y) - (5x-3y)) \\ &= (10x)(6y) \\ &= 60xy \end{aligned}$                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| 2(b) | $\begin{array}{r} x^2 + x + 4 \\ x-2 \overline{) x^3 - x^2 + 2x + 3} \\ \underline{-(x^3 - 2x^2)} \phantom{+ 3} \\ x^2 + 2x \phantom{+ 3} \\ \underline{-(x^2 - 2x)} \phantom{+ 3} \\ 4x + 3 \\ \underline{-(4x - 8)} \\ 11 \end{array}$ <p><math>\therefore Q(x) = x^2 + x + 4 \quad \text{and} \quad R = 11</math></p>                                                                                                                                                                                                                                                                    |
| 2(c) | $\begin{aligned} & 12p^4 - 4p^3 - 8p^2 \\ &= 4p^2(3p^2 - p - 2) \\ &= 4p^2(3p+2)(p-1) \end{aligned}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 2(d) | <p>Let <math>f(x) = 3x^3 + ax^2 + bx + 12</math>,<br/> as <math>(x-3)</math> is a factor of <math>3x^3 + ax^2 + bx + 12</math>,<br/> <math>f(3) = 0</math><br/> <math>3(3)^3 + a(3)^2 + b(3) + 12 = 0</math><br/> <math>81 + 9a + 3b + 12 = 0</math><br/> <math>9a + 3b = -93</math><br/> <math>3a + b = -31</math><br/> Since <math>3x^3 + ax^2 + bx + 12</math> has a remainder of -12 when divided by <math>(x+1)</math>,<br/> <math>f(-1) = -12</math><br/> <math>3(-1)^3 + a(-1)^2 + b(-1) + 12 = -12</math><br/> <math>-3 + a - b + 12 = -12</math><br/> <math>a - b = -21</math></p> |

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|           | $3a + b = -31 \quad (1)$ $a - b = -21 \quad (2)$ $(1) + (2): 4a = -52$ $a = -13$ $\text{Subst } a = -13 \text{ in (1): } 3(-13) + b = -31$ $b = -31 + 39 = 8$                                                                     |
| 3(a)(i)   | $\frac{x^2 + 8x + 12}{3x} \div \frac{x + 6}{6x^3}$ $= \frac{x^2 + 8x + 12}{3x} \times \frac{6x^3}{x + 6}$ $= \frac{(x + 6)(x + 2)}{3x} \times \frac{6x^3}{x + 6}$ $= 2x^2(x + 2)$                                                 |
| 3(a)(ii)  | $\frac{1}{3 - x} - \frac{x}{x^2 - 9}$ $= \frac{1}{3 - x} - \frac{x}{(x + 3)(x - 3)}$ $= \frac{1}{3 - x} + \frac{x}{(x + 3)(3 - x)}$ $= \frac{x + 3}{(x + 3)(3 - x)} + \frac{x}{(x + 3)(3 - x)}$ $= \frac{2x + 3}{(x + 3)(3 - x)}$ |
| 3(a)(iii) | $2 + \frac{2}{\frac{x - 1}{3}}$ $3 + \frac{3}{x - 1}$ $= \frac{2(x - 1)}{x - 1} + \frac{2}{x - 1}$ $= \frac{3(x - 1)}{x - 1} + \frac{3}{x - 1}$ $= \frac{2x - 2 + 2}{\frac{3x - 3 + 3}{x - 1}}$                                   |

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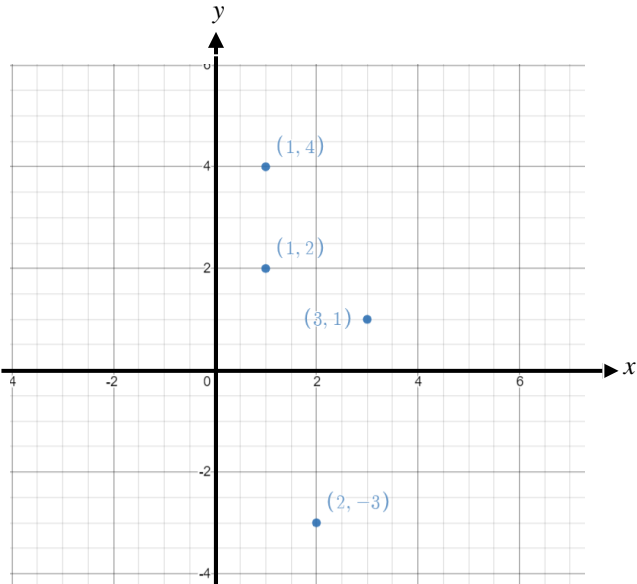
|   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
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|   | $\frac{2x}{x-1} = \frac{x-1}{3x} \cdot \frac{2x}{x-1}$ $= \frac{2x}{3x}$ $= \frac{2}{3}$                                                                                                                                                                                                                                                                                                                                                                                             |
| 4 | $\frac{5x^2 + 3 - x}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$ <p>Multiply every term by <math>x(x^2 + 1)</math>,</p> $5x^2 + 3 - x = A(x^2 + 1) + (Bx + C)x$ $5x^2 + 3 - x = Ax^2 + A + Bx^2 + Cx$ <p>Subst <math>x = 0</math>:</p> $3 = A$ <p>Comparing coefficients of <math>x^2</math>, <math>5 = A + B</math></p> $5 = 3 + B$ $B = 2$ <p>Comparing coefficients of <math>x</math>,</p> $-1 = C$ $\frac{5x^2 + 3 - x}{x^3 + x} = \frac{3}{x} + \frac{2x - 1}{x^2 + 1}$ |

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| 5(a)      |  <p>y is <b>not</b> a function of x<br/>because for the x-value of 1, there are two possible y-values (2 and 4).</p>                                                                       |
| 5(b)(i)   | $g(-3) = (-3)^2 + 2$ $= 11$                                                                                                                                                                                                                                                   |
| 5(b)(ii)  | $g(x-2) = (x-2)^2 + 2$ $= x^2 - 4x + 4 + 2$ $= x^2 - 4x + 6$                                                                                                                                                                                                                  |
| 5(b)(iii) | <p>Shifted 2 units to the right: <math>g(x-2) = (x-2)^2 + 2 = x^2 - 4x + 6</math></p> <p>Stretched vertically by a factor of 3: <math>3g(x-2) = 3[x^2 - 4x + 6]</math></p> <p>Shifted 5 units upwards: <math>3g(x-2) + 5 = 3(x^2 - 4x + 6) + 5</math></p> $= 3x^2 - 12x + 23$ |
| 6(a)      | $f(-1) = 0$ $f(-0.5) = -0.125$                                                                                                                                                                                                                                                |
| 6(b)      | <p><math>f'(-1)</math> is gradient of tangent line at <math>x = -1</math>.</p> $f'(-1) = 0$                                                                                                                                                                                   |

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| 6(c) | <p>The domain is <math>\{x \mid -\infty &lt; x &lt; \infty\}</math>.</p> <p>The range is <math>\{f(x) \mid -\infty &lt; f(x) &lt; \infty\}</math>.</p>                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                   |
| 6(d) | <p>Based on the given graph, the roots are <math>x = -1</math> and <math>x = 0</math>.</p> <p>Since the polynomial is of degree 3, one of the roots has to be a repeated root.</p> $f(x) = (x)(x+1)^2 \quad \text{or} \quad f(x) = (x)^2(x+1)$ <p align="right">[reject because <math>f(-0.5) = -0.125</math>]</p> $\begin{aligned} \therefore f(x) &= (x)(x+1)^2 \\ &= (x)(x^2 + 2x + 1) \\ &= x^3 + 2x^2 + x \end{aligned}$ |                                                                                                                                                                                                                                                                                                   |
| 7(a) | <p>Monthly earnings = Basic salary + commission</p> $\text{Monthly earnings} = 800 + \frac{m}{100}(50000)$ $3300 = 800 + \frac{m}{100}(50000)$ $2500 = 500m$ $m = 5$                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                   |
| 7(b) | <p><b>1. Understand the problem</b></p> <ul style="list-style-type: none"> <li>State the given conditions and quantities.</li> </ul>                                                                                                                                                                                                                                                                                          | Johnson's May earnings = Willy's May earnings                                                                                                                                                                                                                                                     |
|      | <p><b>2. Devise a plan</b></p> <ul style="list-style-type: none"> <li>Identify which are the relevant concepts that can be applied.</li> </ul>                                                                                                                                                                                                                                                                                | <p>Monthly earnings = Basic salary + commission</p> <p>Let the sales amount be <math>x</math>,</p> <p><u>Scenario 1 (sales amount &gt; 30000)</u></p> $800 + \frac{5}{100}x = 1300 + \frac{4.5}{100}(x - 30000)$ <p><u>Scenario 2 (sales amount &lt; 30000)</u></p> $800 + \frac{5}{100}x = 1300$ |
|      | <p><b>3. Implement the plan</b></p> <ul style="list-style-type: none"> <li>Carry out the</li> </ul>                                                                                                                                                                                                                                                                                                                           | <p><u>Scenario 1 (sales amount &gt; 30000)</u></p> $800 + 0.05x = 1300 + 0.045x - 1350$                                                                                                                                                                                                           |

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|    | <p>plan, showing each step clearly.</p>                                                                                                            | <p> <math>850 = -0.005x</math><br/> <math>x = -17000</math> (reject)<br/> <u>Scenario 2 (sales amount &gt; 30000)</u><br/> <math>800 + 0.05x = 1300</math><br/> <math>0.05x = 500</math><br/> <math>x = 10000</math> </p> |
| 7c | <p><b>4. Look back</b></p> <ul style="list-style-type: none"> <li>Substitute your answer back and check if it satisfies the conditions.</li> </ul> | <p>             Willy's June salary = 1300<br/>             Johnson's June salary = <math>800 + \frac{5}{100}(10000) = 1300</math> </p>                                                                                   |

