



中国科学技术大学  
University of Science and Technology of China



GAMES 102在线课程

# 几何建模与处理基础

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中国科学技术大学



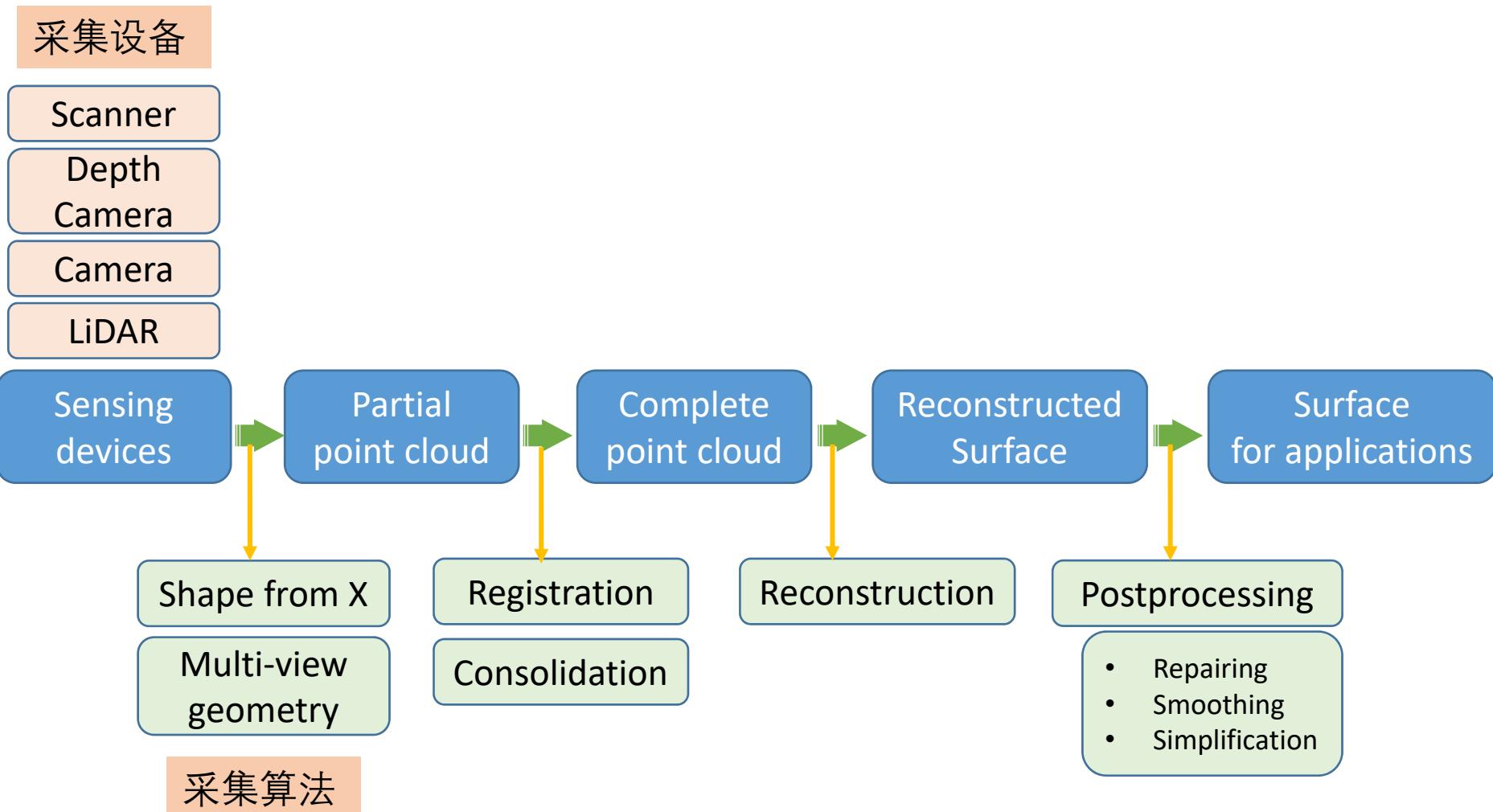
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GAMES 102在线课程：几何建模与处理基础

# 几何建模

# 回顾：曲面重建



# 建模(modeling): 设计与重建

- **曲面设计(Design)**

- 不存在的物体：通过人工交互凭空设计出新的物体
  - CAGD (NURBS)、mesh modeling
- 存在的物体：通过人工交互编辑修改构建出新的物体
  - Editing, deformation

- **曲面重建(Reconstruction)**

- 存在的物体：对其采集并进行数字化构建
- 也称为：逆向工程、扫描重建
  - Reverse engineering, scanning

# Design Modeling

- Design from zero
  - Create a shape by a set of 3D design operators such as extrusion or revolution etc.
- Design from a given shape
  - Select a base shape
  - Select editing elements and editing operators
  - Deform the shape to obtain a new shape

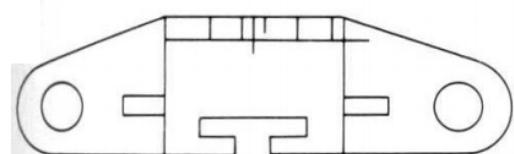
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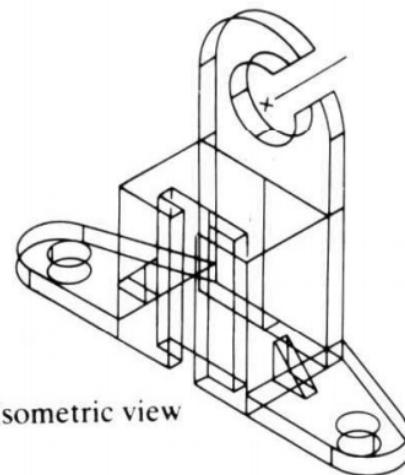
# Geometric Modeling Techniques

- Wireframe Modeling
- Surface Modeling
  - Analytical Surface
  - Free-form, Curved, Sculptured Surface
- Solid Modeling
  - Parametric Modeling
  - Feature Based Modeling
  - Constructive Solid Geometry (CSG)
  - Boundary Representation (B-Rep)

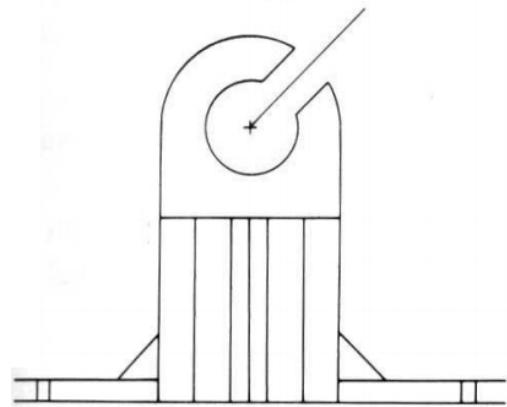
# Wireframe Modeling (工程制图)



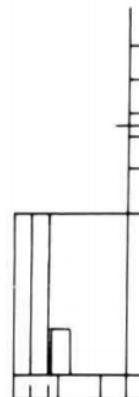
Top view



Isometric view

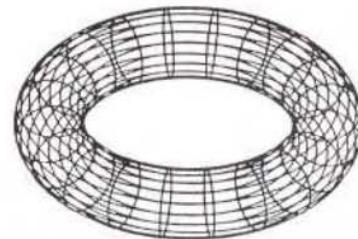
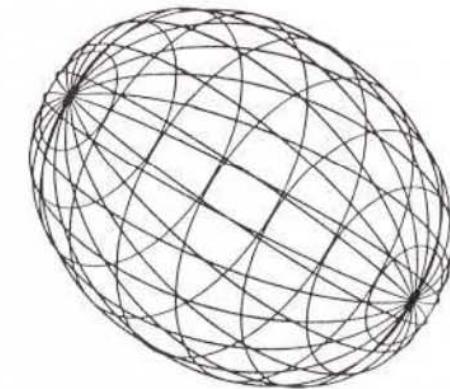


Front view

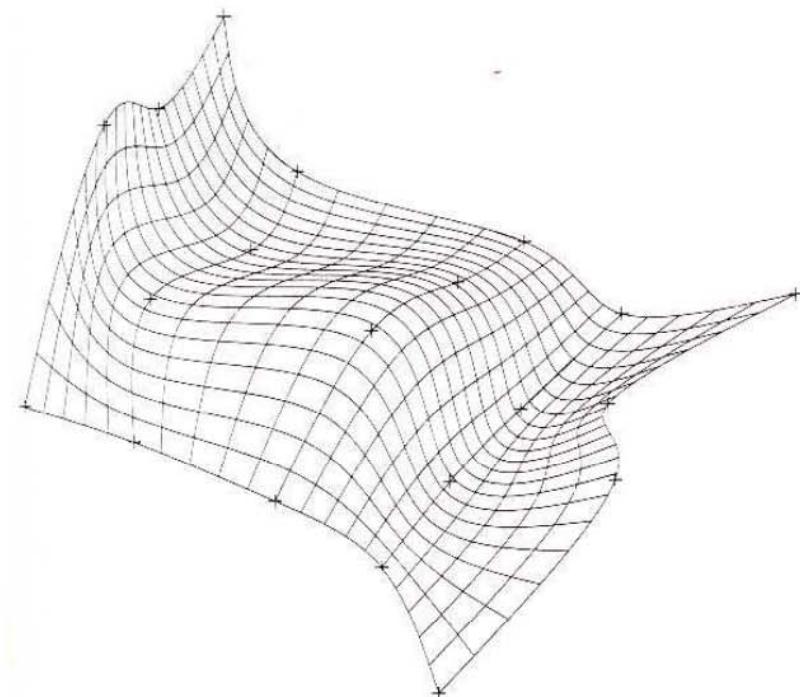


Right view

# Surface Modeling (NURBS)



Analytical Surfaces



Free-form, Curved, or  
Sculptured Surface



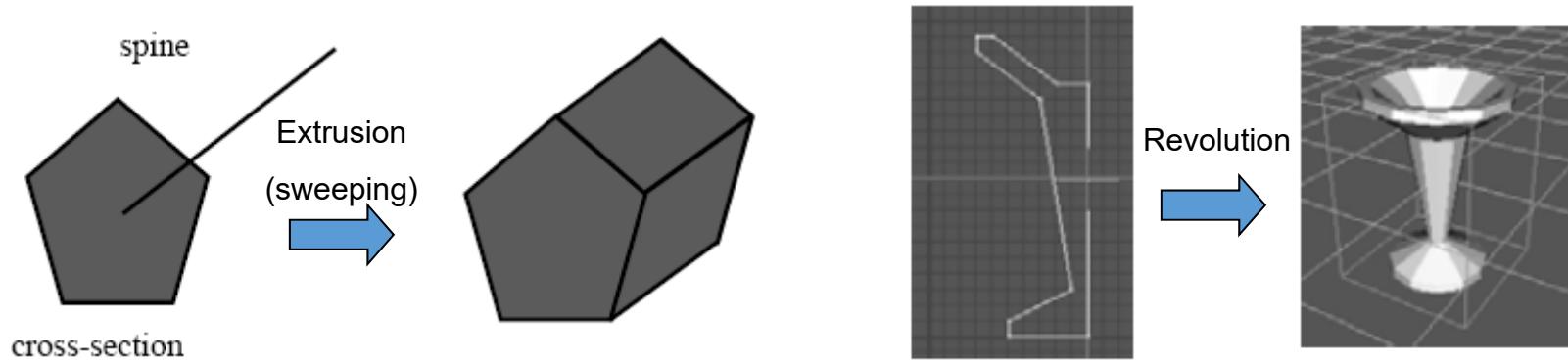
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# Solid Modeling

# Solid Modeling

- Modeling by a set of geometric operators
  - Geometric operators: extrusion, revolution, sweeping, lofting, etc.
  - Parametric: various semantic parameters
  - Feature-based: various components such as holes, grooves, fillets, and chamfers.



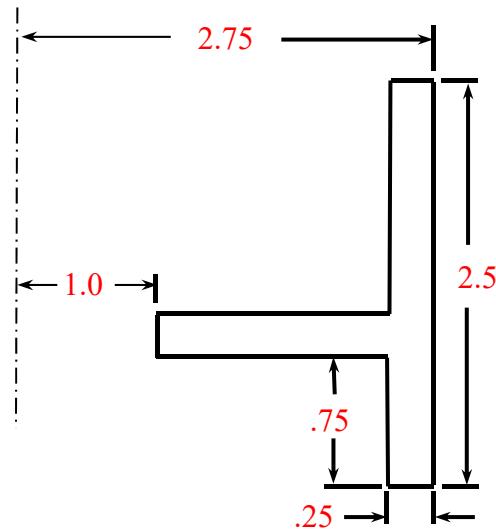
- Commercial software
  - SolidWorks, Creo (PTC), Inventor & Fusion 360 by Autodesk, Unigraphics, Catia, .....

# Concepts of Parametric Modeling

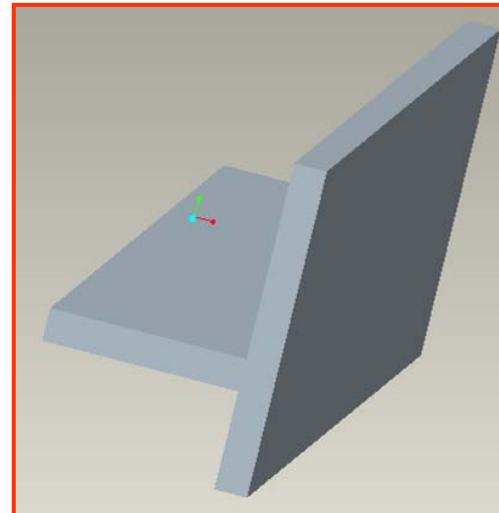
- Sketching
  - Take the word sketch literally. A sketch should be just that, a sketch.
  - When sketching it is not necessary to create geometry with accuracy. Lines, arcs, and additional geometry need not be created with exact dimensions in mind.
- Dimensioning
  - When dimensions are added, the sketch will change size and shape. This is the essence of Parametric Modeling.
- Features
  - Create a 2D sketch and dimension it.
  - Create a feature from the sketch by extruding, revolving, sweeping, and lofting.

# Extruding and revolving

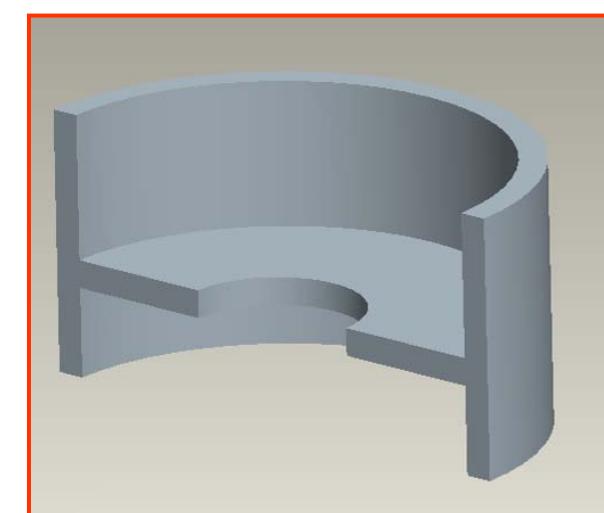
- Extrusion: 平移, 沿直线路段挤出 (扫掠sweeping)
- Revolution: 旋转曲面, 沿圆弧挤出



Sketching: profile



Extruding

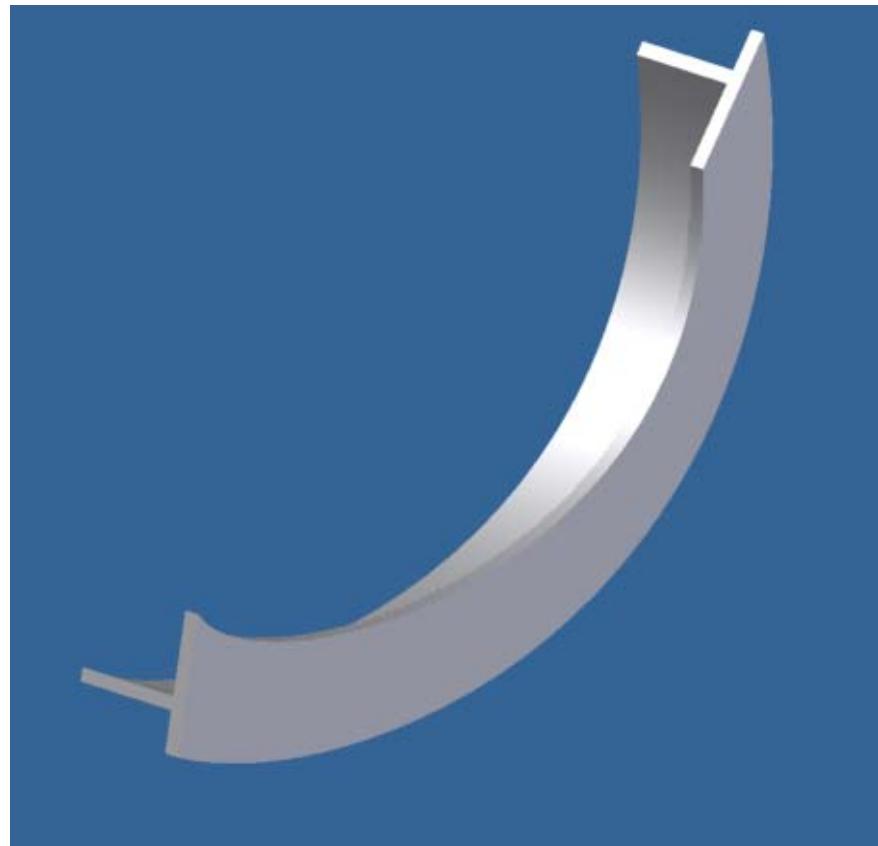
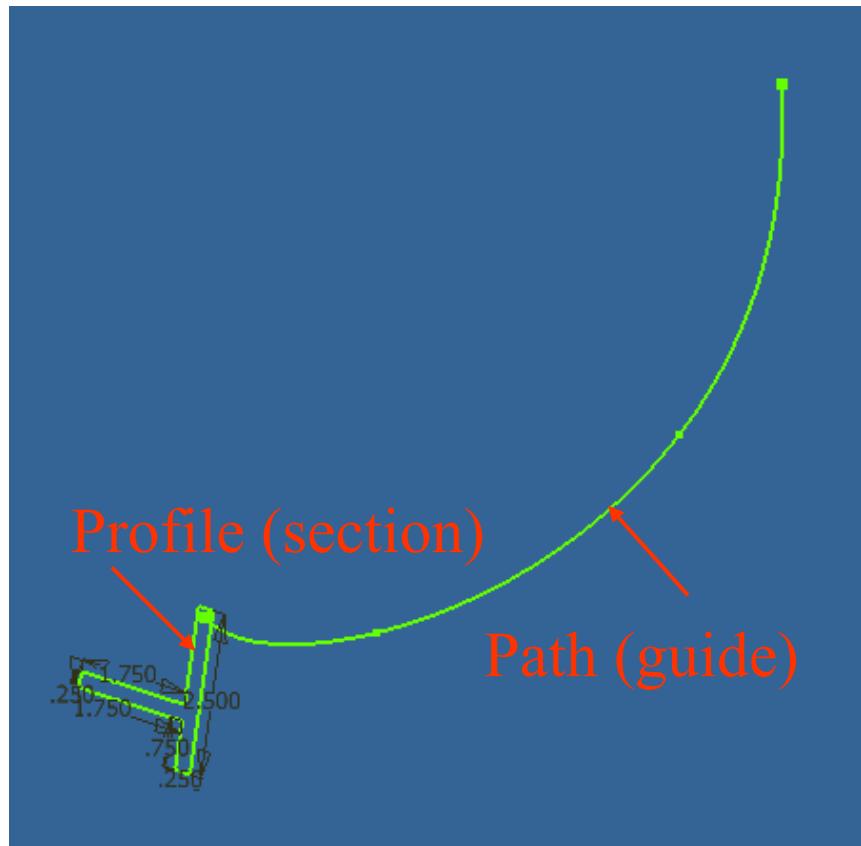


Revolving

Courtesy of Ken Youssefi

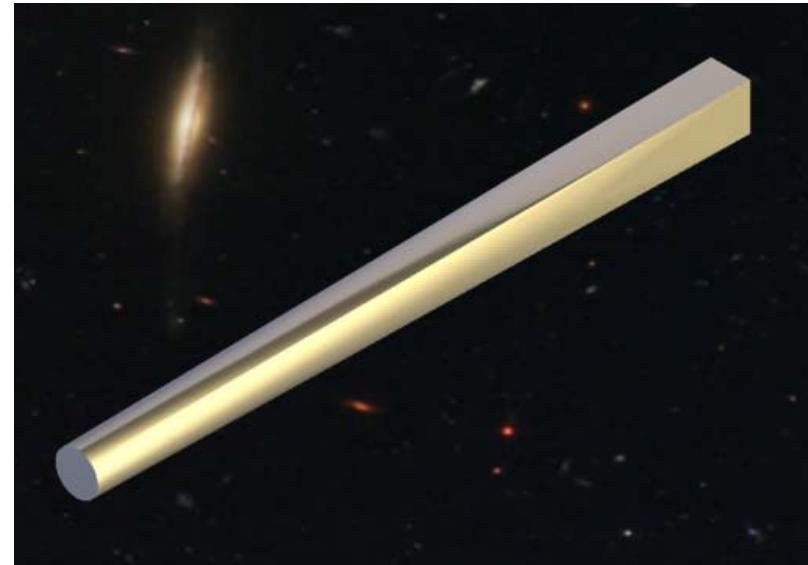
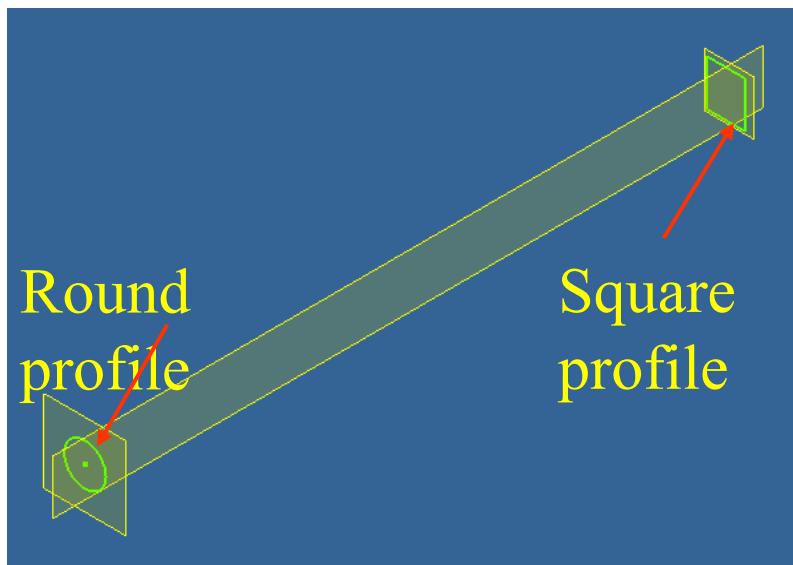
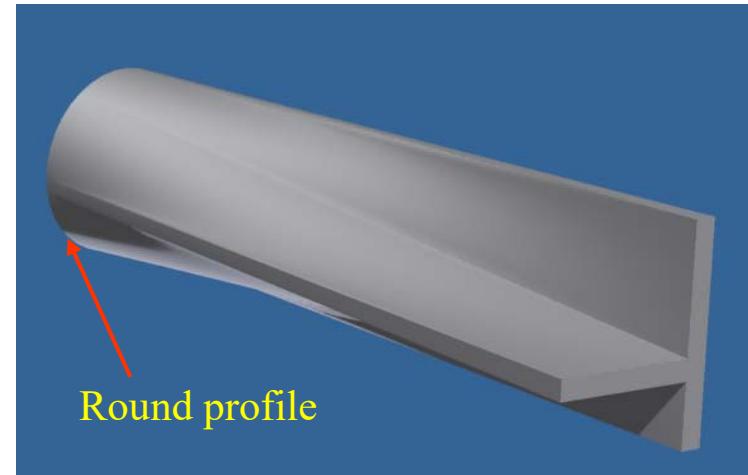
# Sweeping

- A sweep feature requires a profile and a path. The profile will follow the path to create the solid.

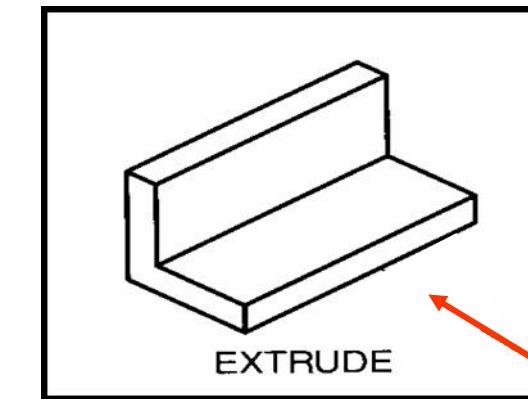


# Lofting

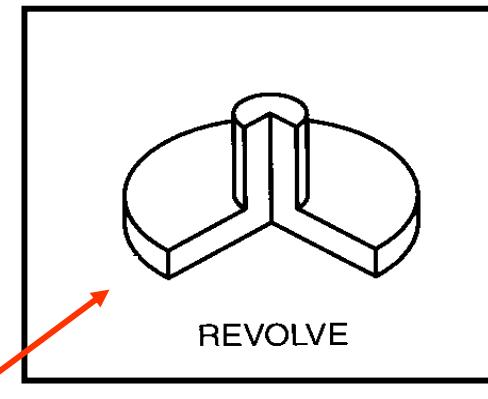
- Sweeping with different profiles
  - Sections (profiles) do not have to be sketched on parallel planes
  - All sections must be either closed or open



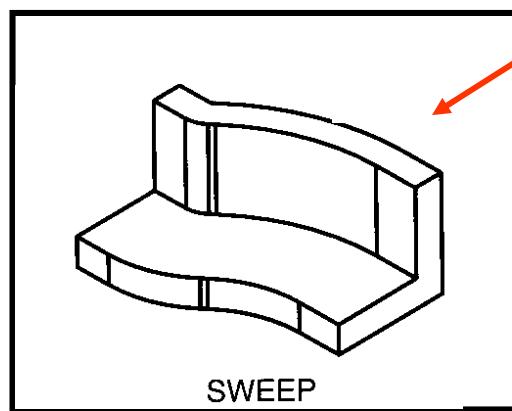
# Creating Features from Sketches



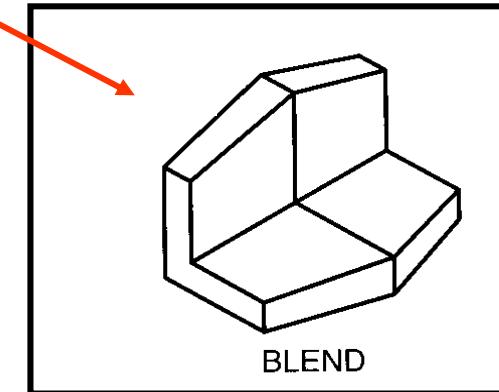
EXTRUDE



REVOLVE



SWEEP

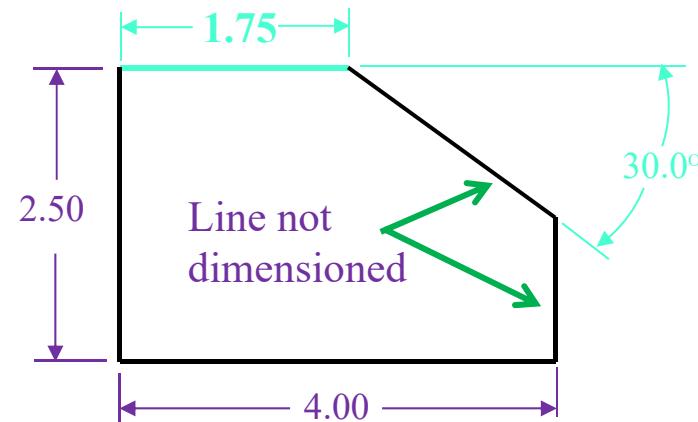
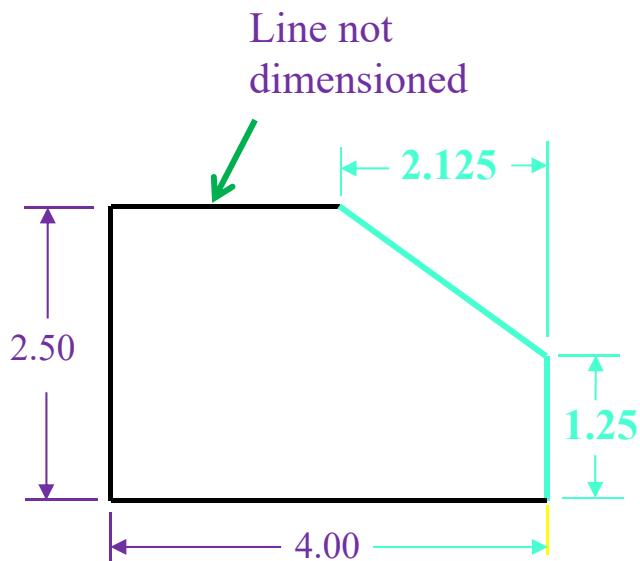


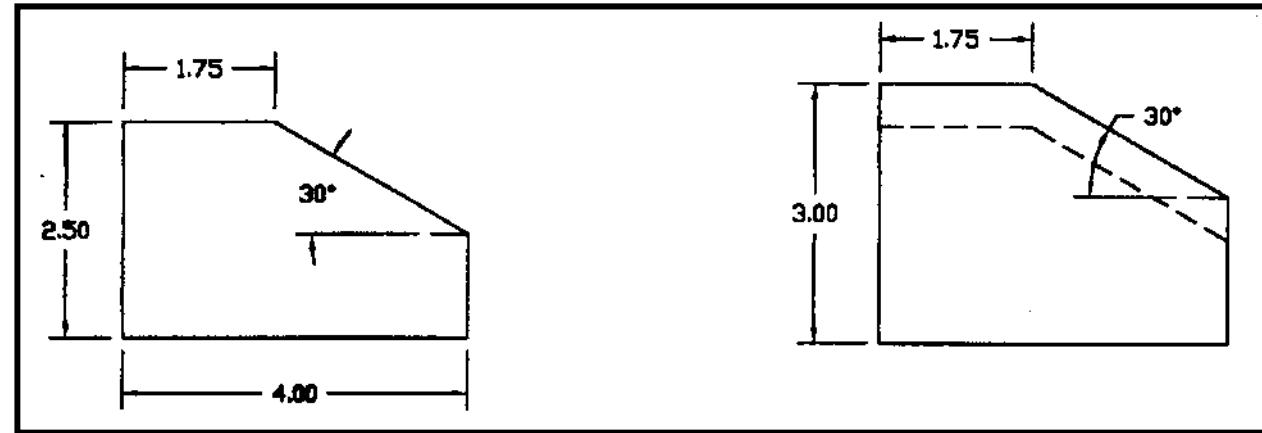
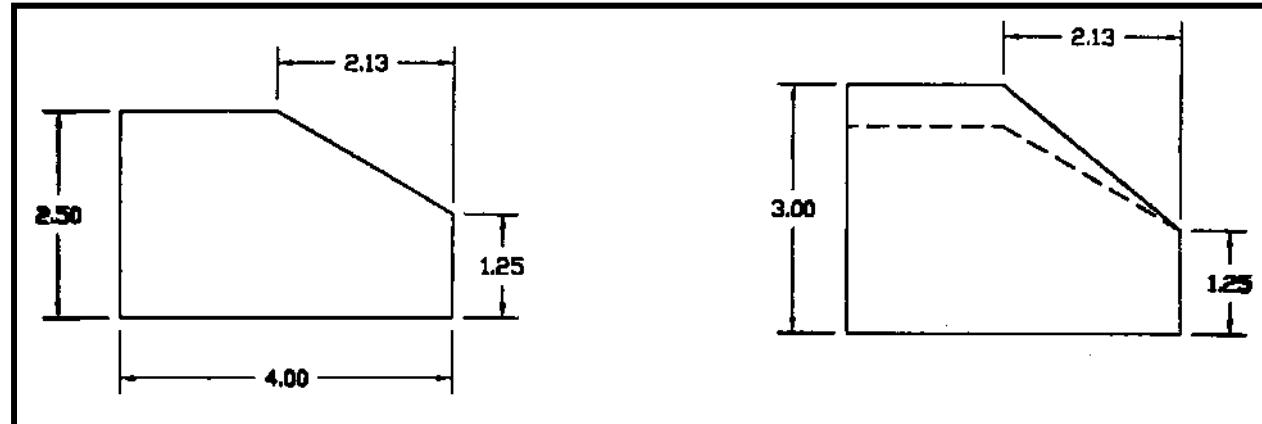
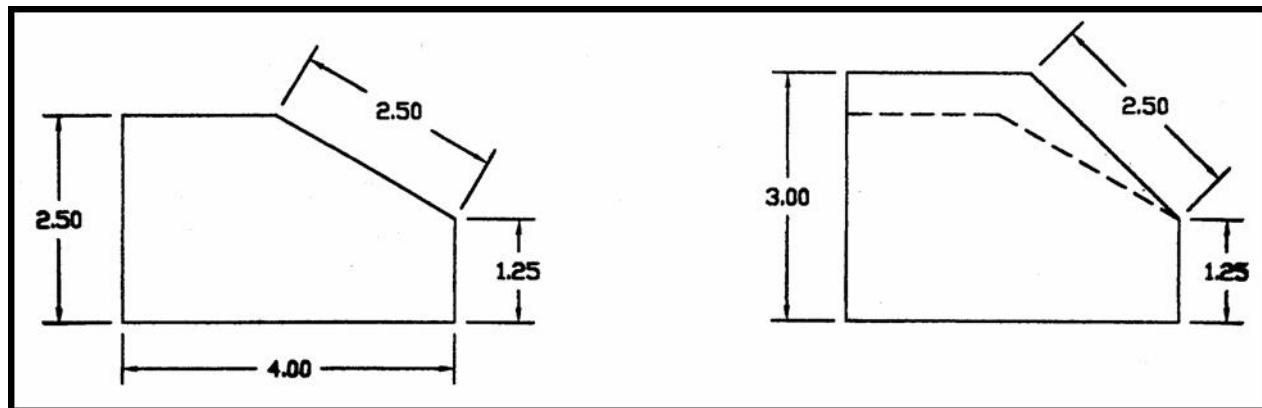
BLEND

*Loft* in  
SolidWorks

# Design Intent (constraints)

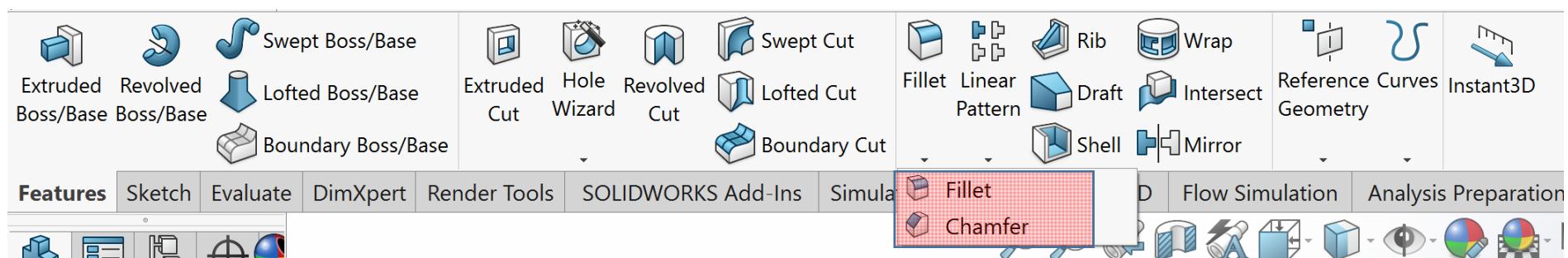
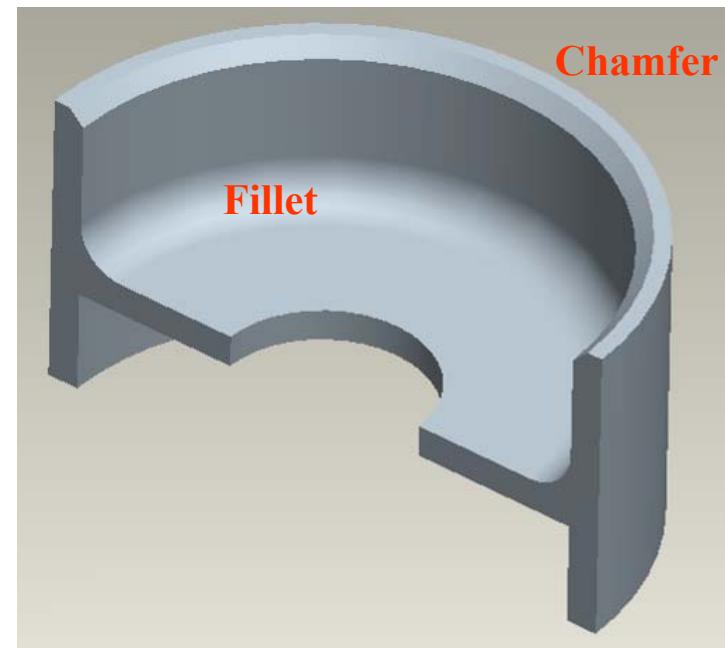
- How your model will react when dimension values are changed?





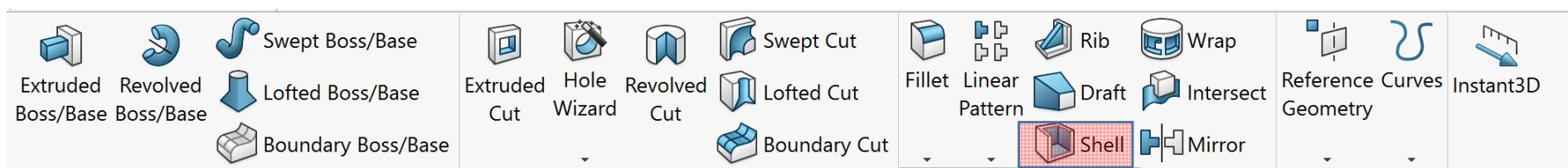
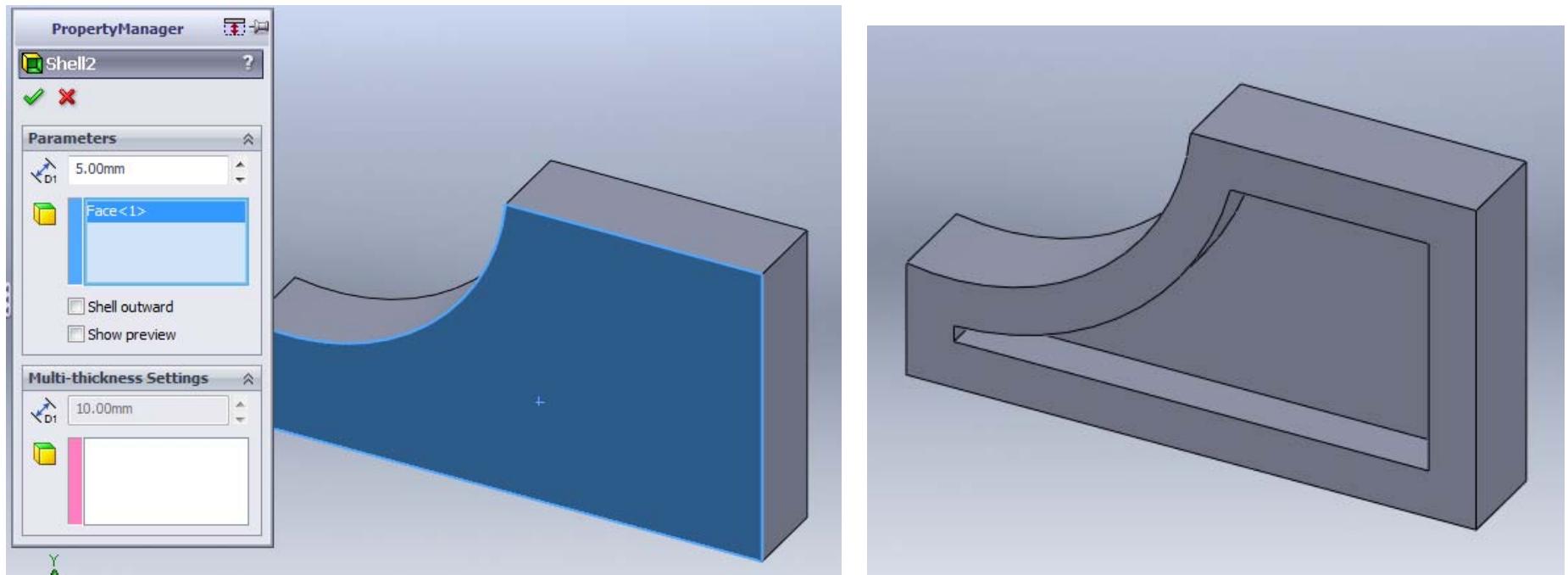
# Applied Feature: Chamfer and Fillet

- Applied feature does not require a sketch.
- It is applied directly to the model.

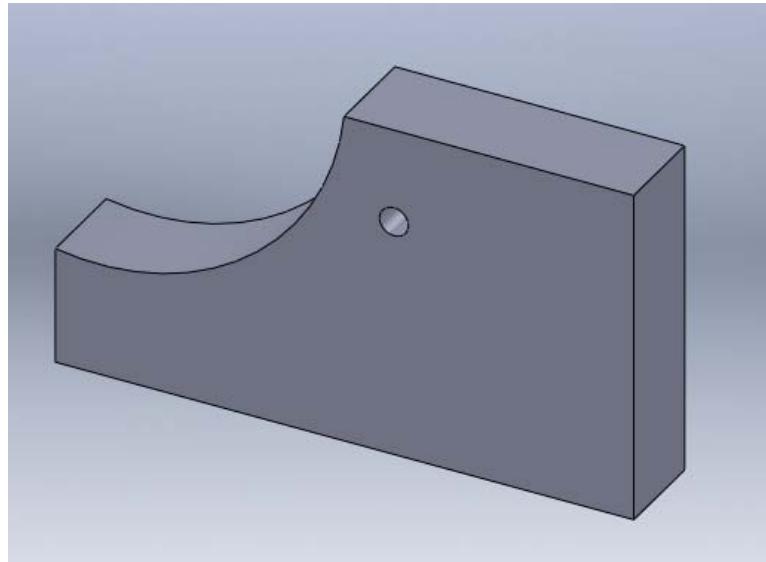


# Applied Feature: Shell

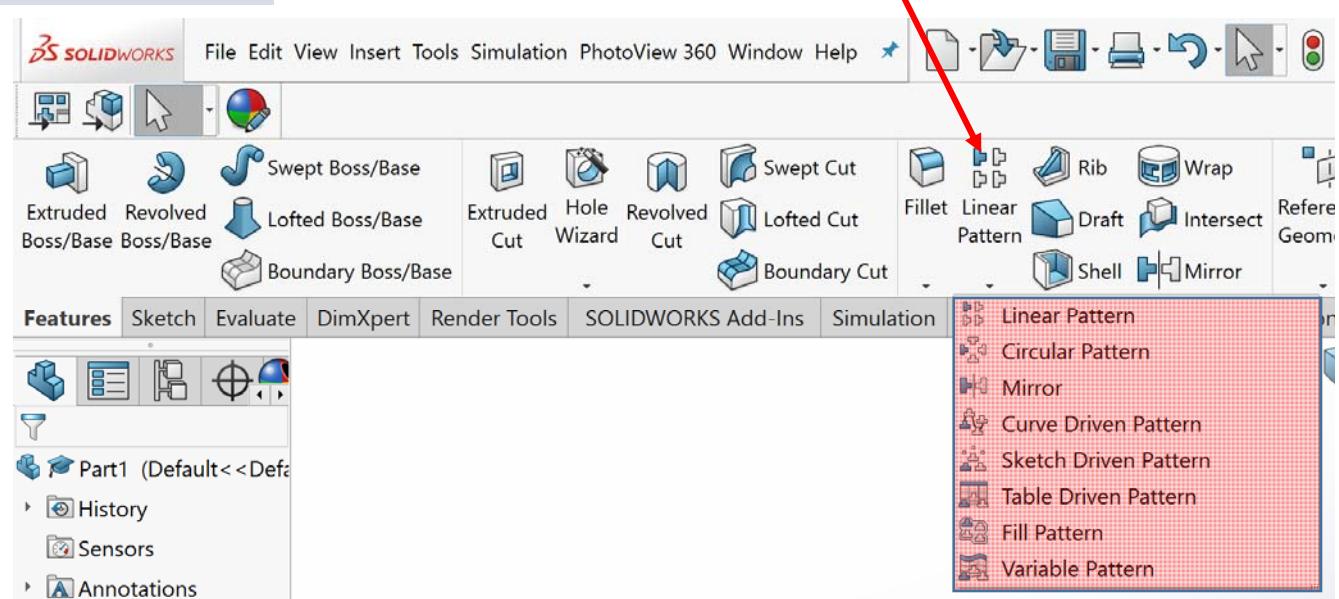
- Shell – hollowing out a solid



# Applied Feature: Patterns



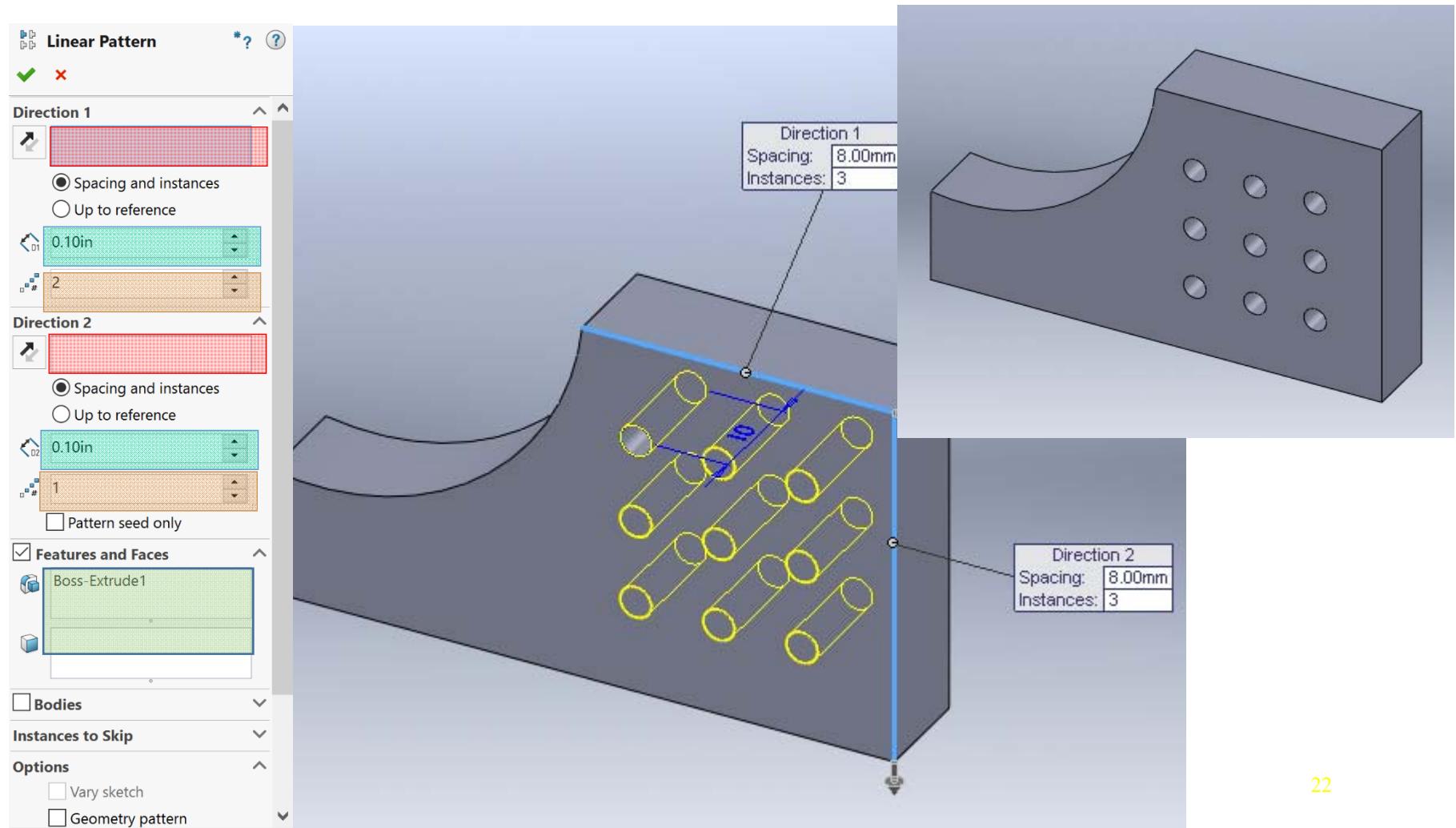
Linear (rectangular) pattern



# *Applied Features - Patterns*

## Linear (rectangular) pattern

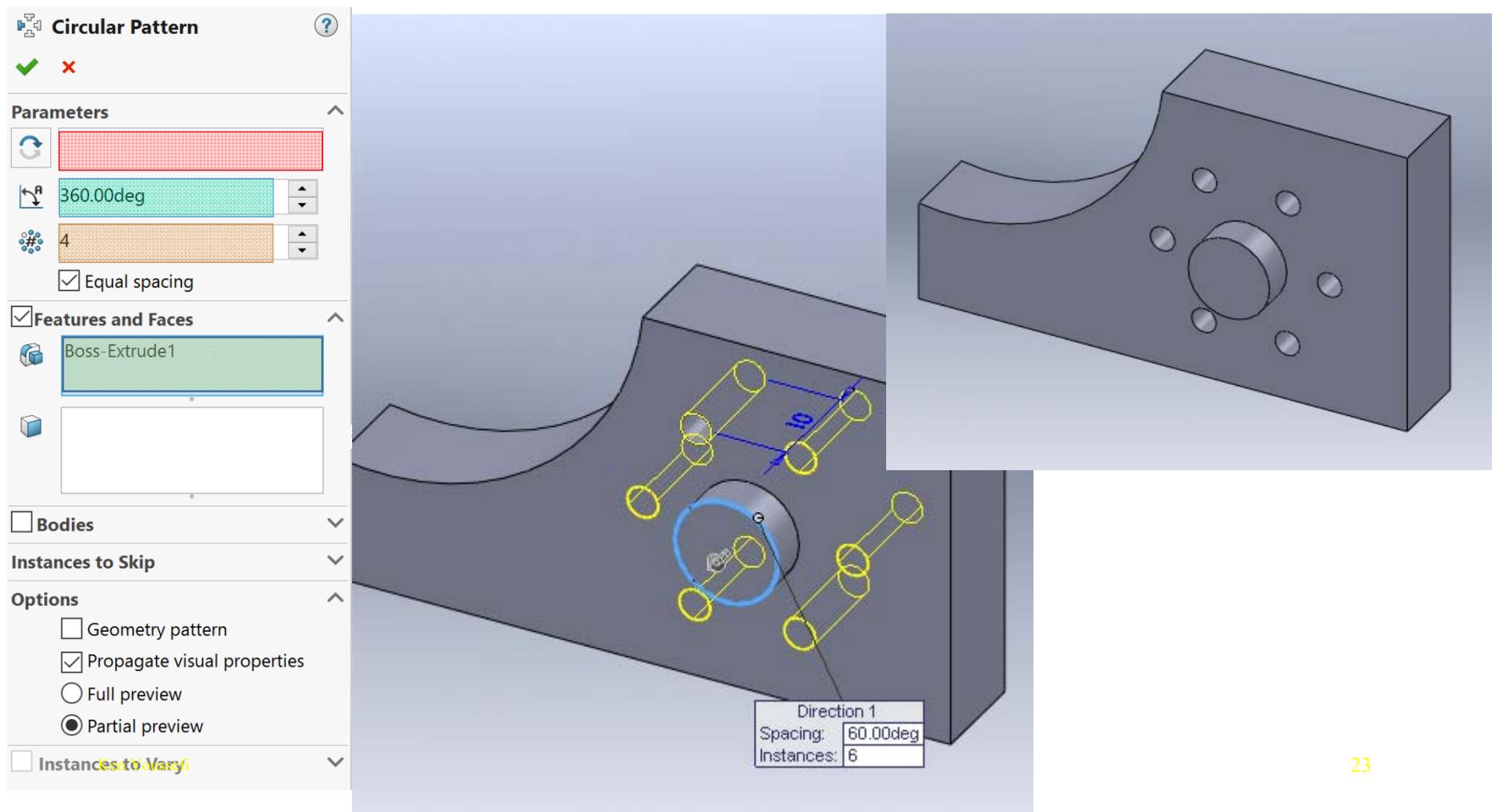
1. Select direction 1 and 2
2. Select spacing in dir. 1 and 2
3. Select # of features in dir. 1 and 2
4. Select feature to pattern



# *Applied Features - Patterns*

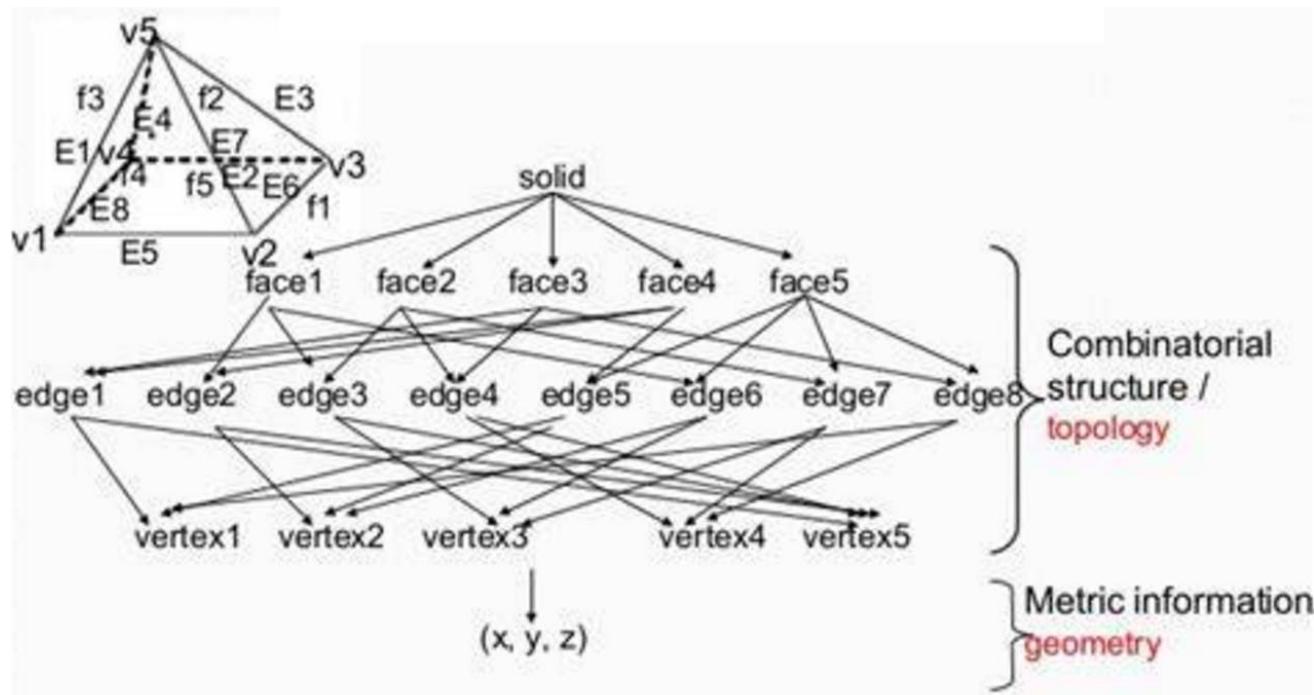
## Circular (polar) pattern

1. Select axis of rotation
2. Select spacing between features
3. Select # of features
4. Select feature to pattern



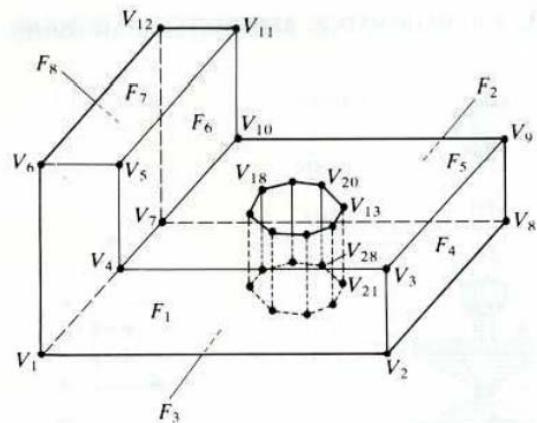
# Boundary Representation (B-Rep) Modeling

- BREP defines an object by their spatial boundaries.
  - Topology: faces, edges, and vertices.
  - Geometry: surfaces, curves, and points.
- Data structure: Object list, surface list, polygon list, boundary list, line list, point list...

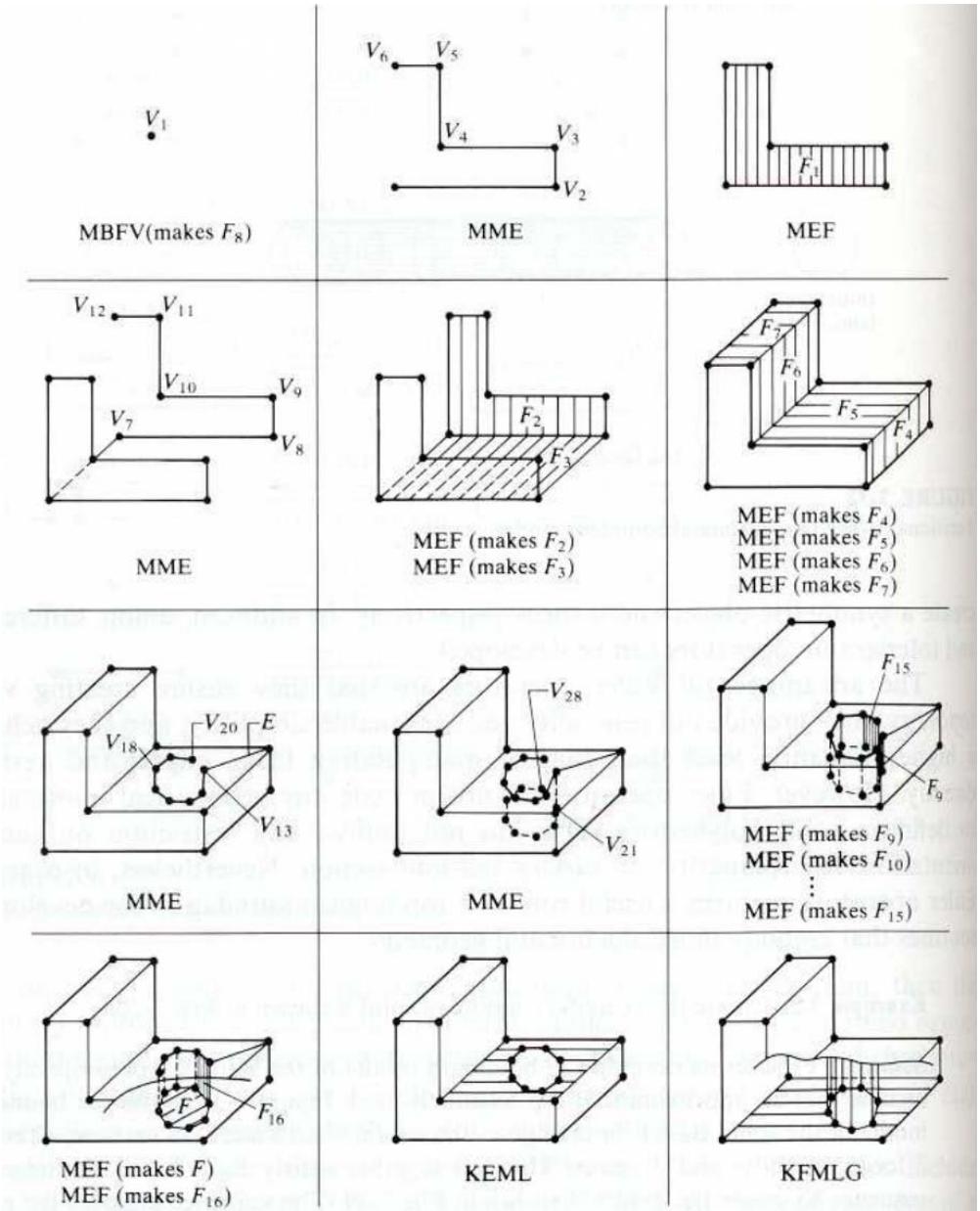


# B-Rep

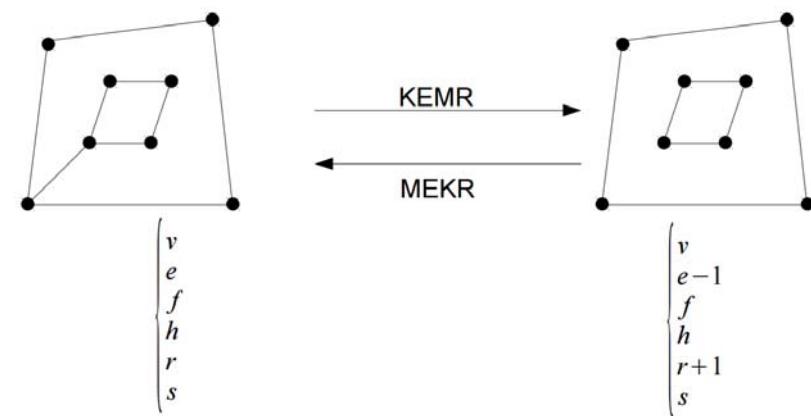
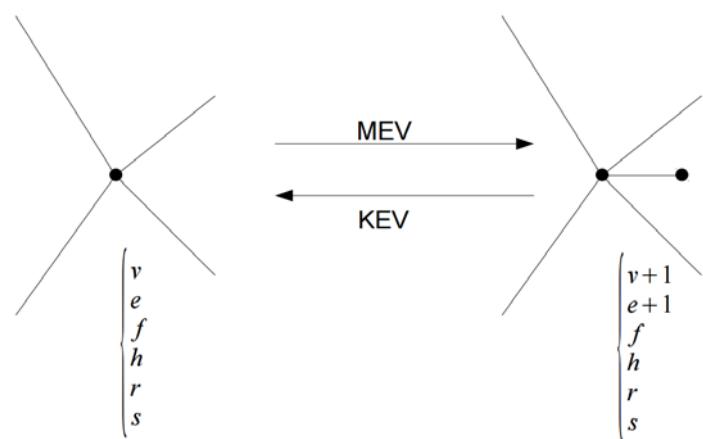
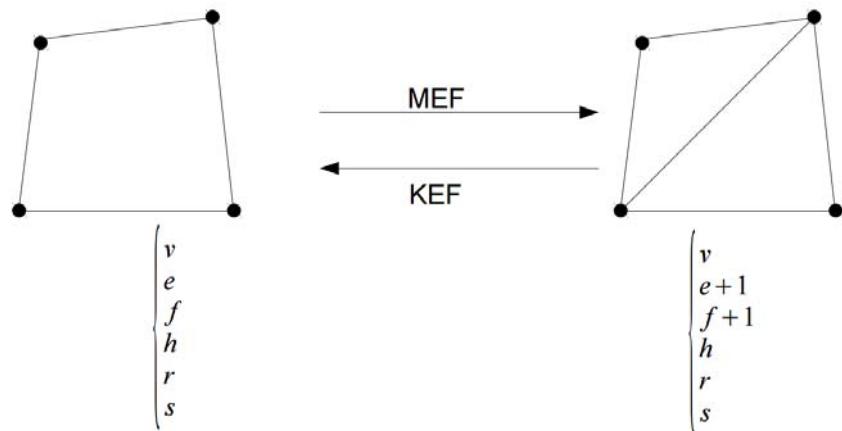
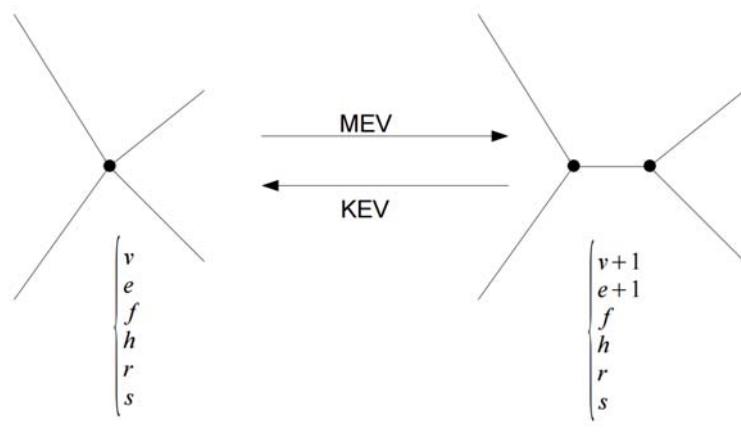
- Euler operators
  - MBFV: Make body, face, vertex
  - MME: Make multi edges
  - MEF: Make edge, face
  - KEML: Kill edge, make loop
  - KFMLG: Kill face, make loop, genus
- Validity Check:  $V - E + F = 2$



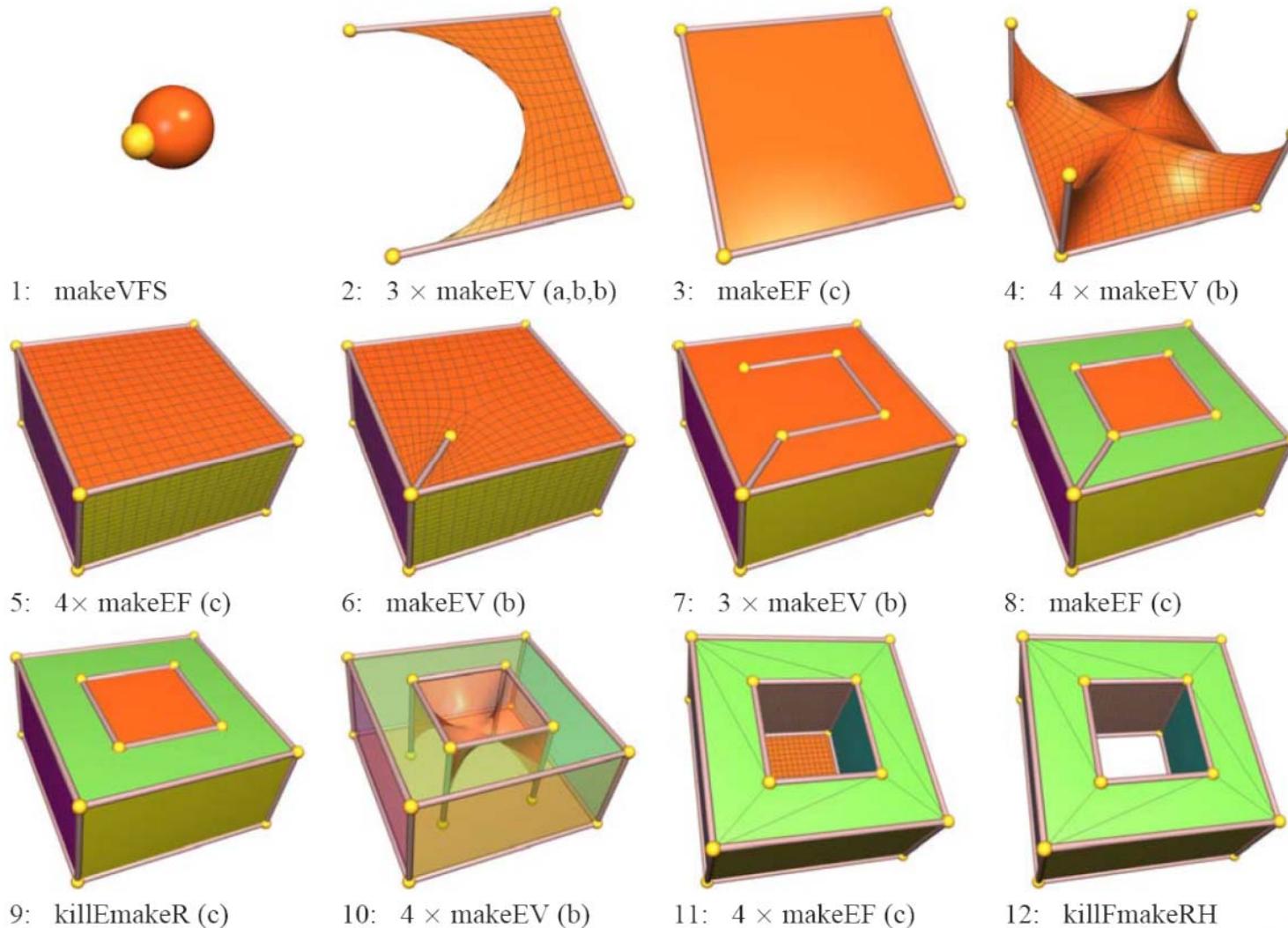
**Body, Face, Polygon  
(Edge Loop), Edge, Vertex**



# Examples of Euler Operators

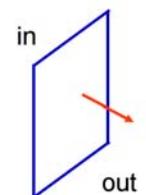


# Modeling Example of using Euler operators

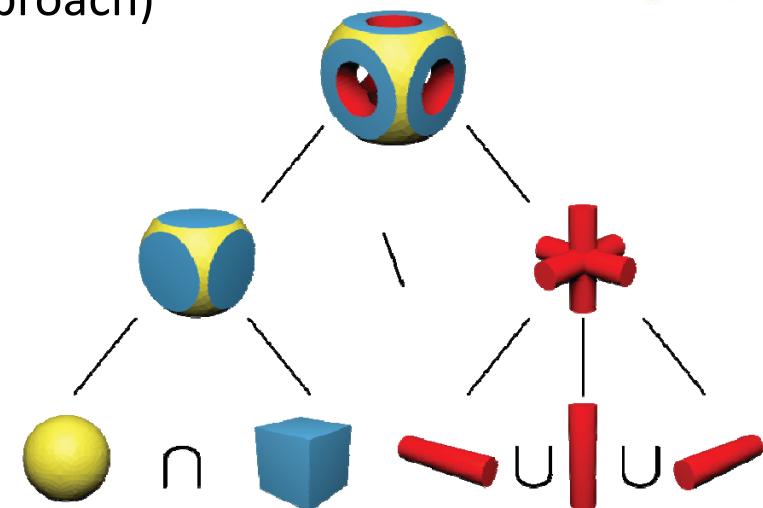
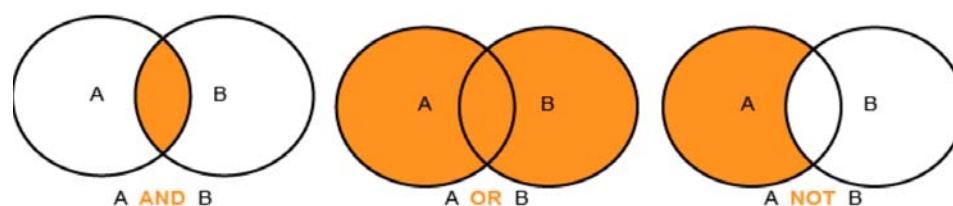


# Constructive Solid Geometry (CSG) Modeling

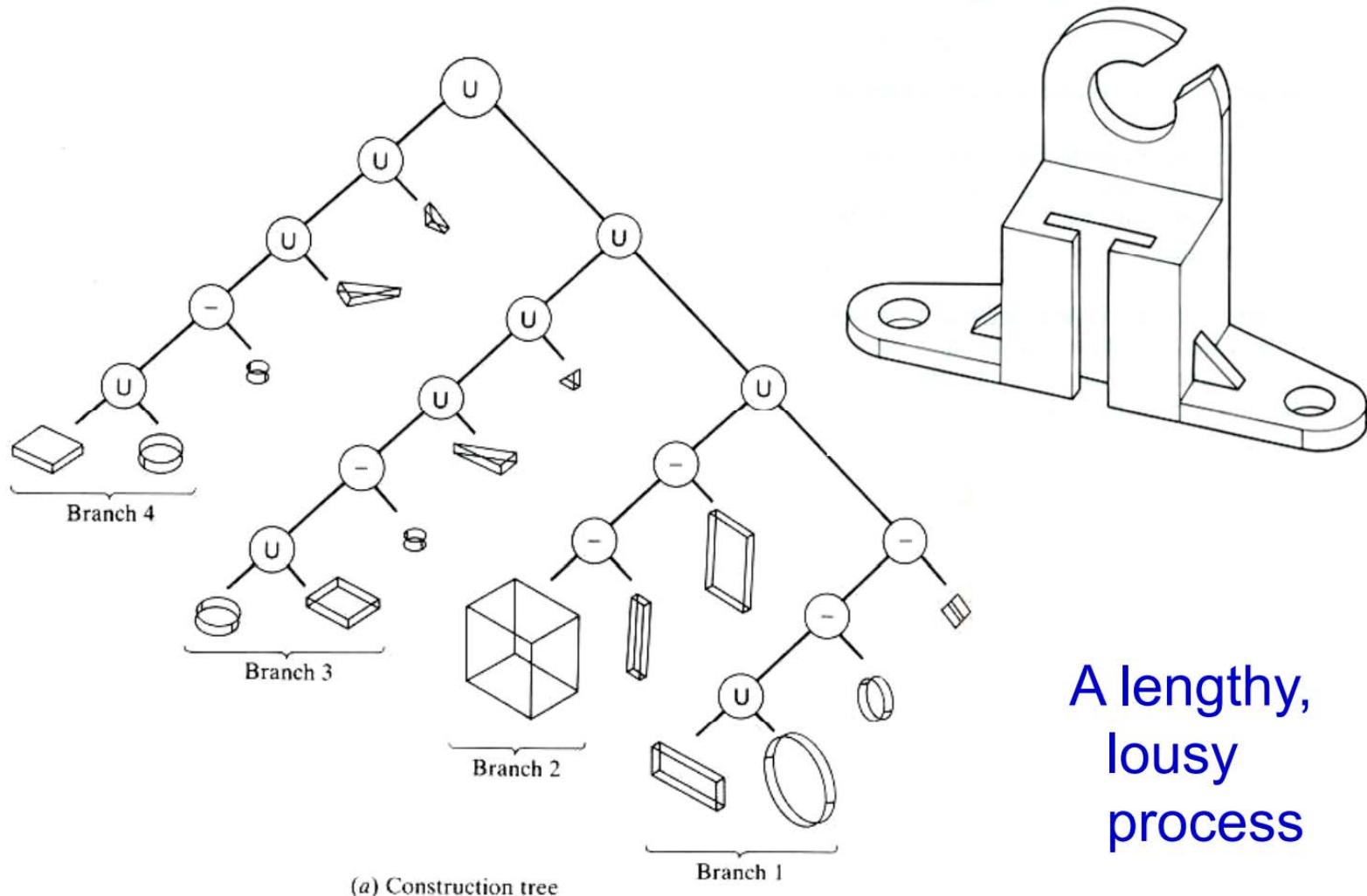
- Predefined primitive objects: cuboids, cylinders, prisms, pyramids, spheres, and cones.
- Boolean operations: union, intersection, difference (subtraction)
- Two types
  - Primitive based CSG: It is a popular CSG scheme which is based on bounded solid primitives, R-sets.
  - Half-space based CSG: This CSG scheme uses unbounded **half-spaces**.
- CSG tree structure (building process/approach)



Boolean AND, OR, and NOT



# A CSG model

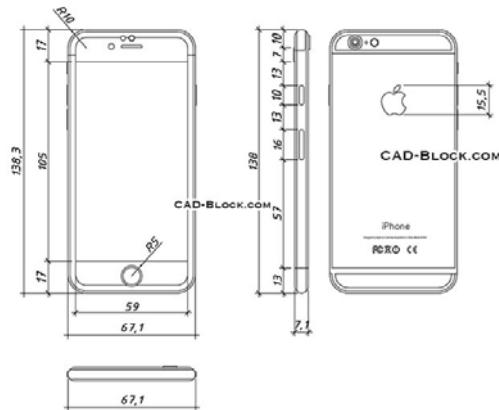


# B-Rep VS. CSG

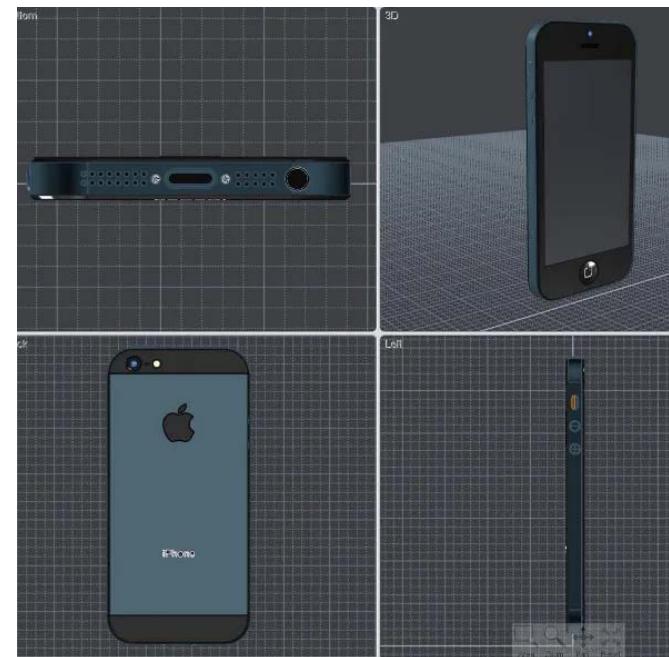
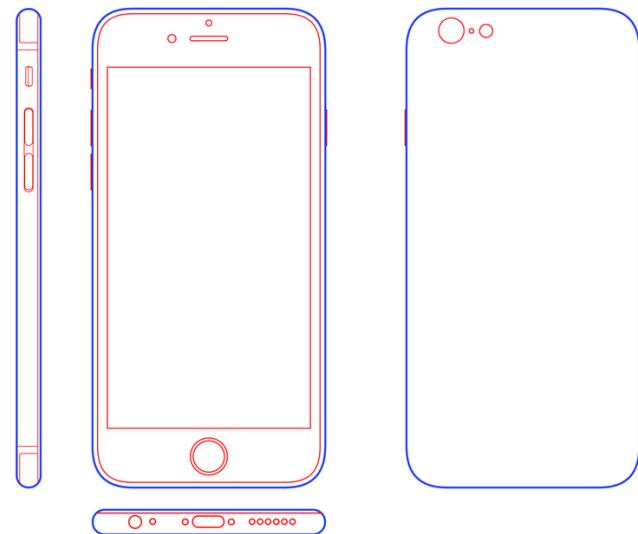
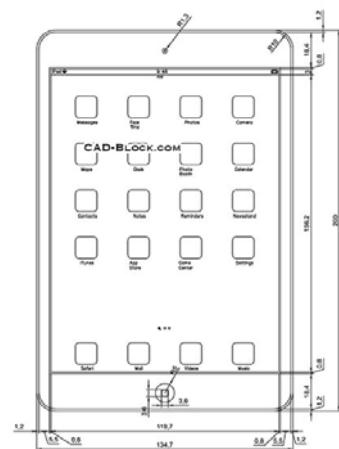
| <b>Boundary Representation (BREP)</b>  | <b>Constructive Solid Geometry (CSG)</b>   |
|--|--|
| BREP describes only the oriented surface of a solid as a data structure composed of vertices, edges, and faces.  | A solid is represented as a set of Boolean expression of primitive solid objects, of a simpler structure.  |
| A BREP object is easily rendered on a graphic display system.  | A CSG object is always valid in the sense that its surface is closed and orientable and encloses a volume, provided the primitives are authentic in this sense.                                |
| For B-rep, we review the possible surface types, the winged-edge representation schema, and the Euler operators. | For CSG, The basic operations include classifying points, curves, and surfaces concerning a solid; detecting redundancies in the representation; and approximating CSG objects systematically. |

# CAD Design Example

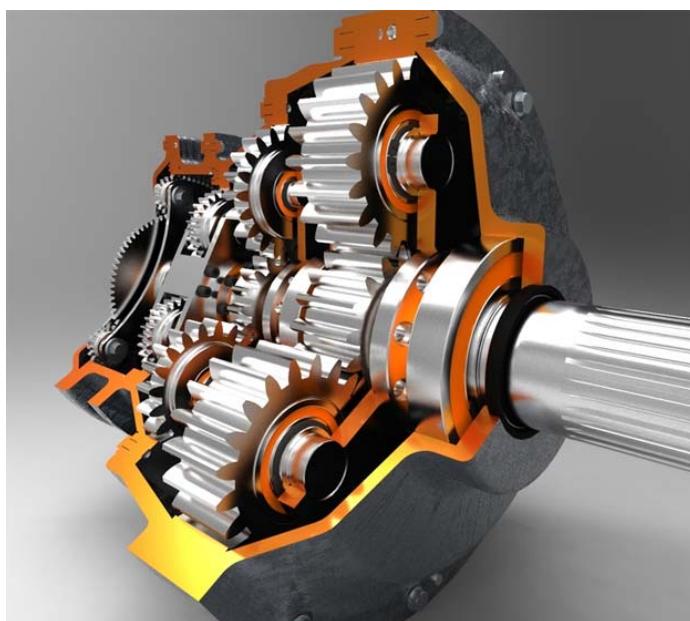
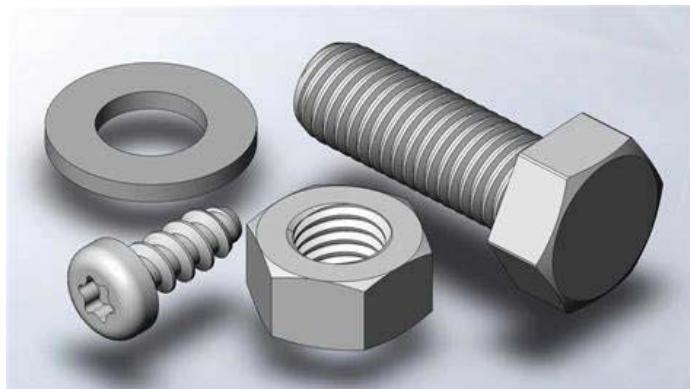
CAD-BLOCK.COM iPhone 6S



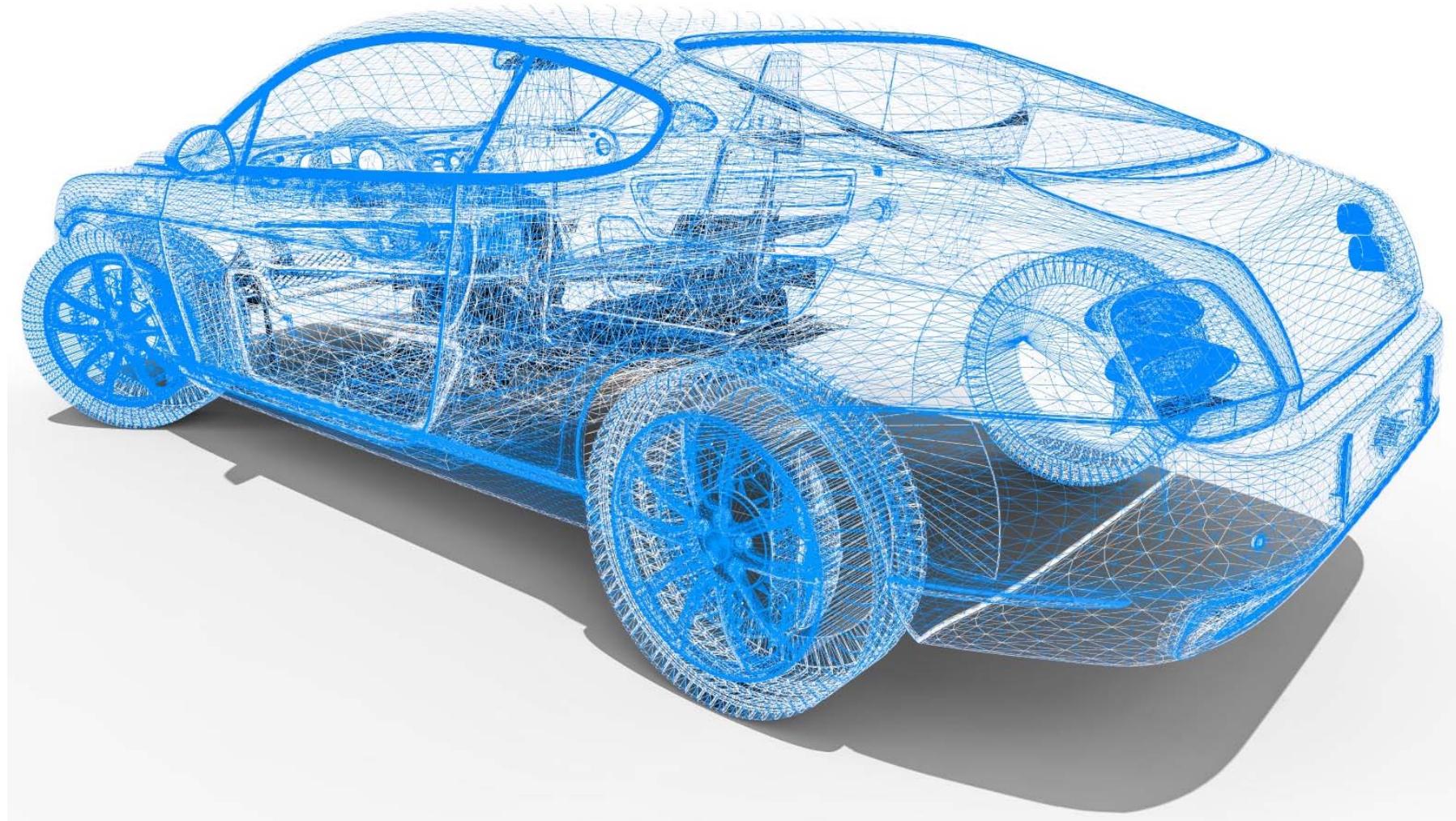
iPad Mini 5.3" x 7.87"



# Examples of CAD models



# Examples of CAD models

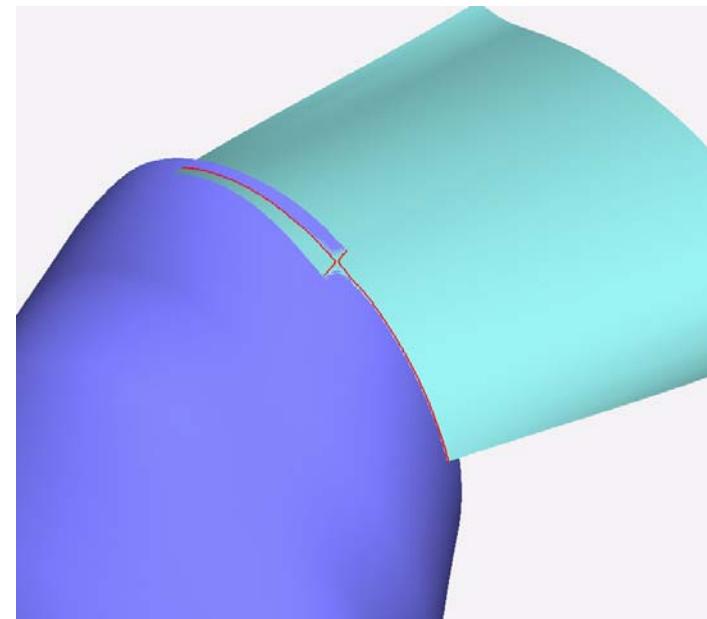
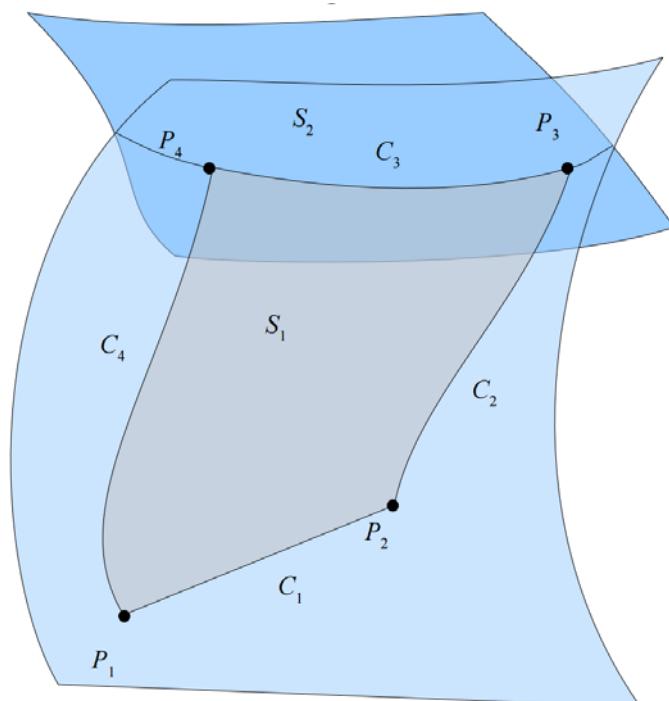


# Examples of CAD models



# Challenges in CAD systems

- Intersections & Topology robustness/precision
- Parametric design: non-linear optimization



# 从“中国制造”到“中国智造”

制造业是国民经济的主体，是立国之本、兴国之器、强国之基



国务院2015：《中国制造2025》

- 设计顶层规划及战略目标
- 制定“三步走”路线图

- 第一步到2025年，迈入世界制造强国行列
- 第二步到2035年，达到世界制造强国中等水平
- 第三步到2049年，进入世界制造强国前列

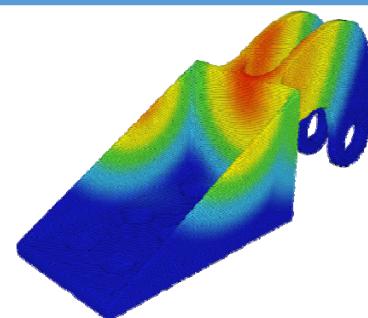
# 挑战：国产自主工业设计软件

工业制造的核心要素是工业设计软件

计算机辅助设计(CAD)



计算机辅助分析(CAE)



计算机辅助制造(CAM)



CAD 软件  
➤ AutoCAD  
➤ SolidWorks

几何设计

CAE 软件  
➤ Ansys  
➤ Abaqus

性能分析

CAM 软件  
➤ MasterCAM  
➤ Pro/NC

加工制造

- 立志：国产自主工业设计软件
- 破解：“卡脖子”难题！



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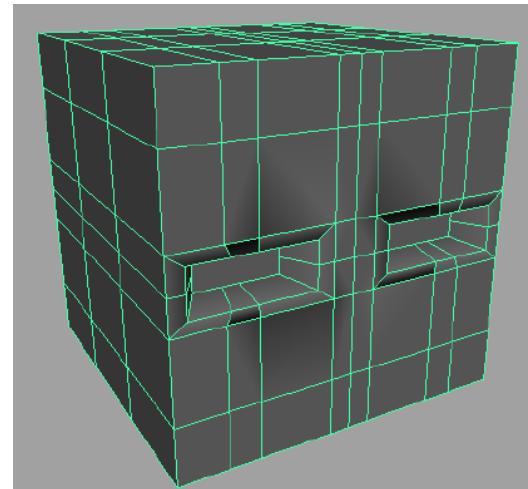
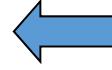
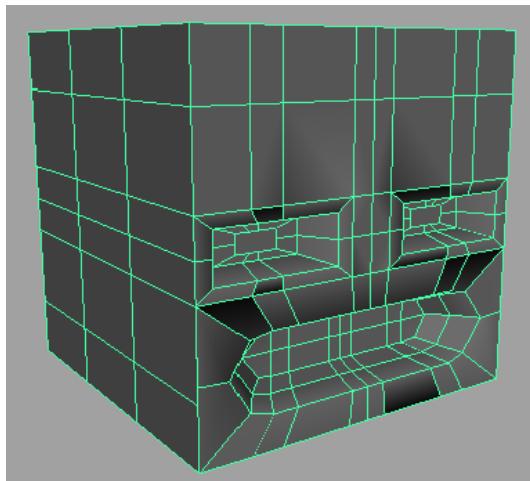
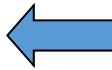
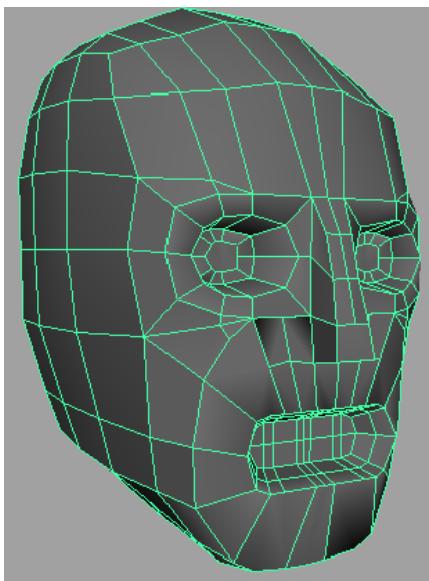
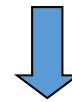
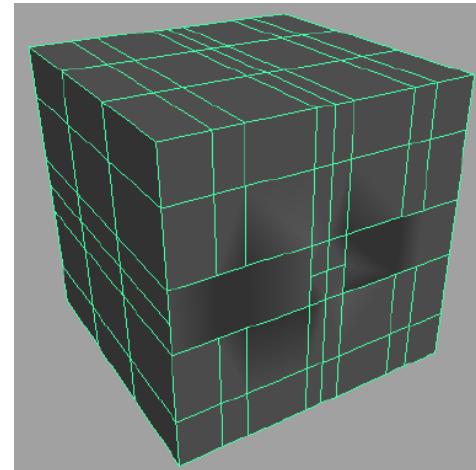
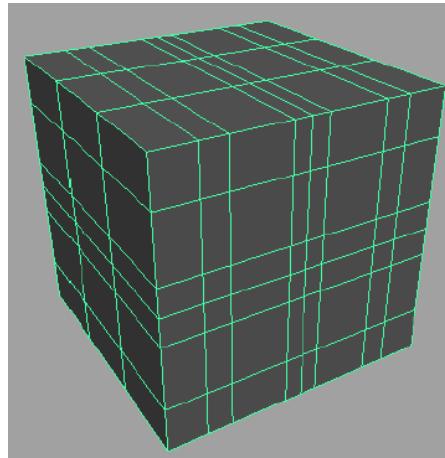
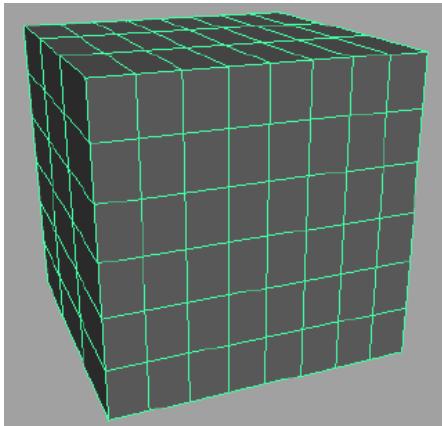


# Surface Editing

# Design Modeling

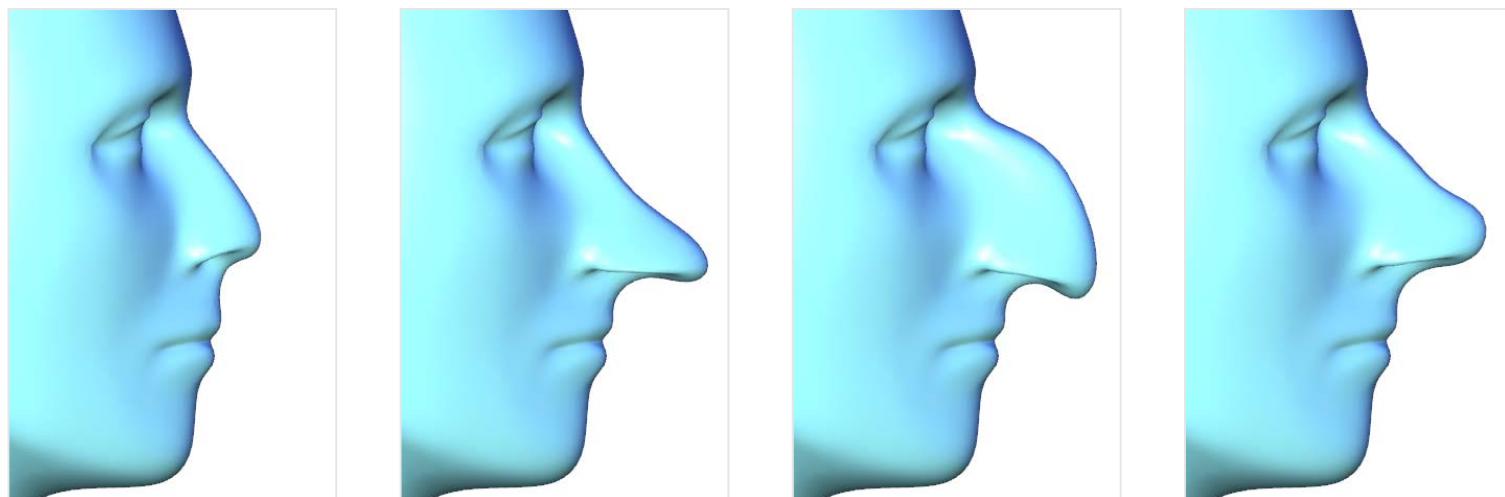
- Design from zero
  - Create a shape by a set of 3D design operators such as extrusion or revolution etc.
- Design from a given shape (Mesh editing)
  - Select a base shape
  - Select editing elements and editing operators
  - Deform the shape to obtain a new shape

# Mesh Surface Editing



# Interactive shape modeling

- Modeling is an interactive, iterative process
  - Tools need to be intuitive (interface and outcome)
  - Allow quick experimentation
  - Preserve shape properties

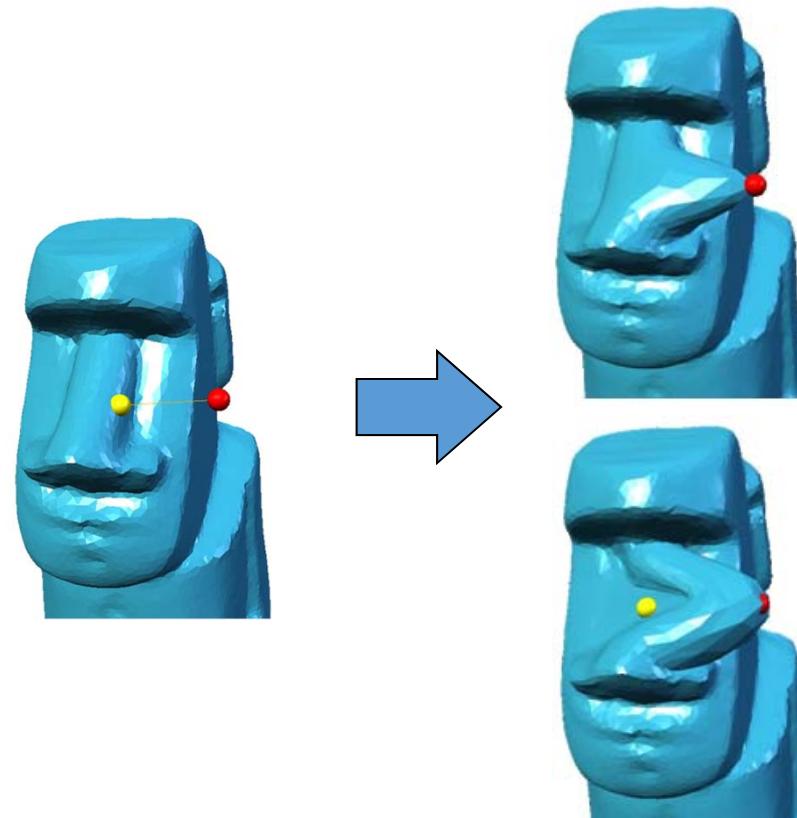


# User Interaction (UI)

- User specifications: Intuitive, easy-to-use
  - Dragging handles -- vertices, features, ROI, ...
- Deformed shape: adapt to user intents

## Challenges:

- Operations
- Preservation of properties
- Semantics



# Methodology

- Set a proxy  $P$  for the input shape  $S$ 
  - $P$  is simpler/easier/more semantic than  $S$
  - Find a map between  $P$  and  $S$ :  $S=g(P)$ 
    - $g$  is also called an **embedded space**
- User interacts and deforms the proxy  $P$  to  $P'$ 
  - Find a deformation map:  $P'=h(P)$
- Compute a new shape  $S'$  from  $P'$ :  $S'=g(P')$

Key Problem:

- find a good map  $g$

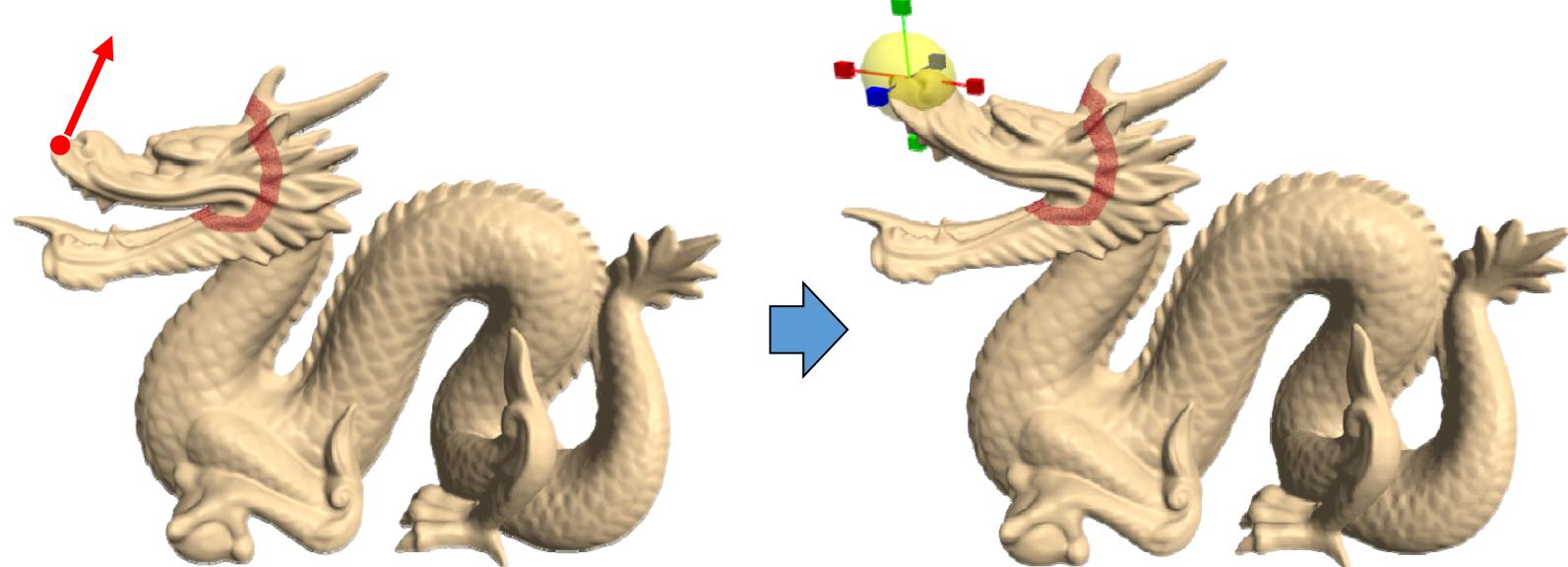
# Proxy Shapes (handles)

- Points
  - Points, vertices, ...
- Lines/Curves
  - Sketches, skeletons, silhouettes, wires, ...
- Meshes
  - Bezier nets, lattices, cages, ...
- Other shapes
  - Deformation transfer: learning deformation from other shapes

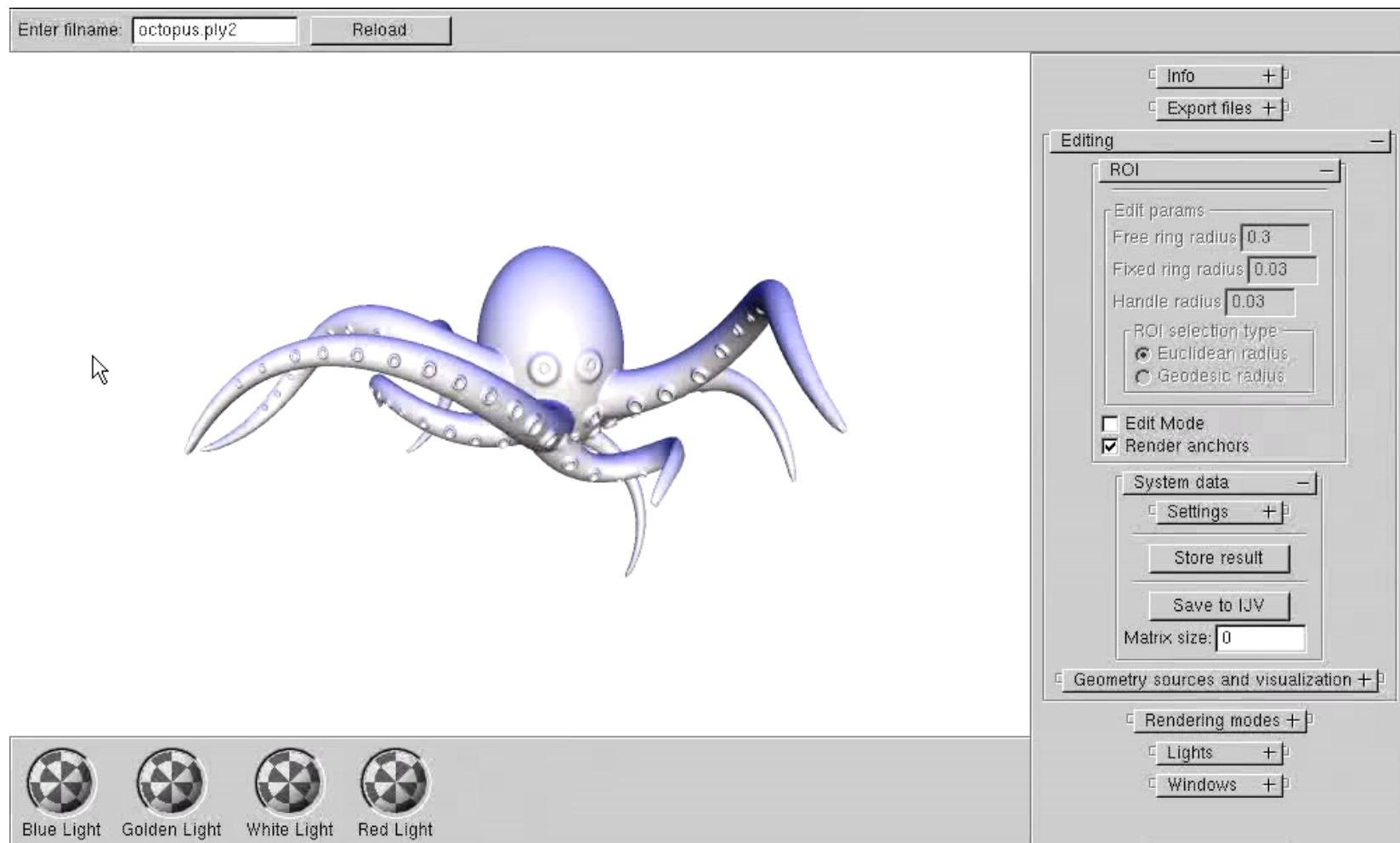
# 1. Point Proxy

# Point based editing

- Fix some vertices
- Drag one or more vertices

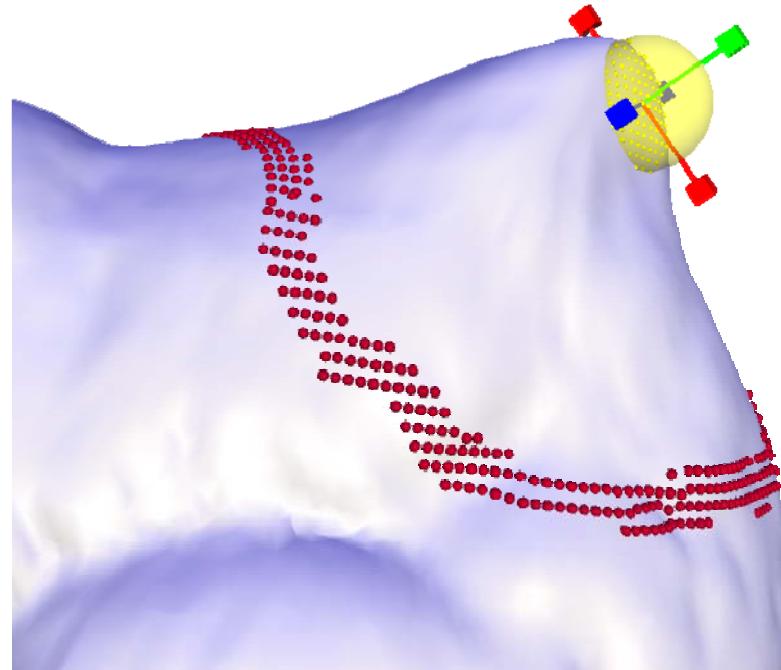


# User Interfaces



# User Interfaces

- ROI is bounded by a belt (static anchors)
- Manipulation through handle(s)

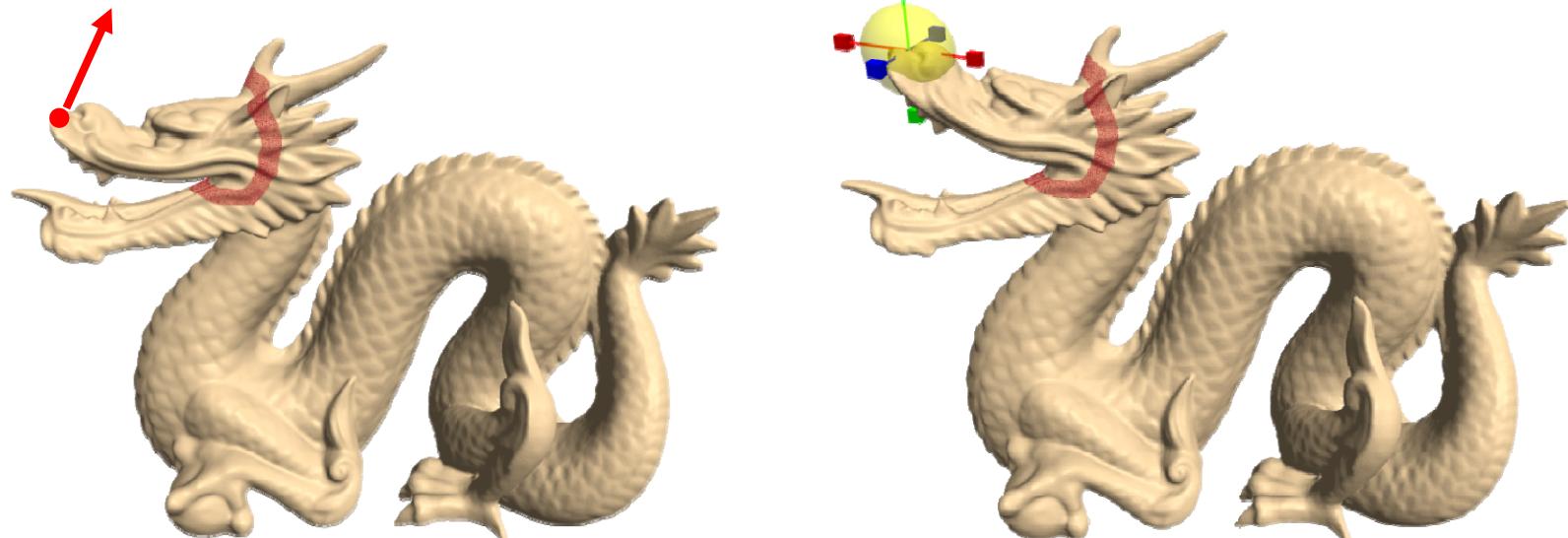


本质数学问题：数据插值问题

# Interpolation Problem

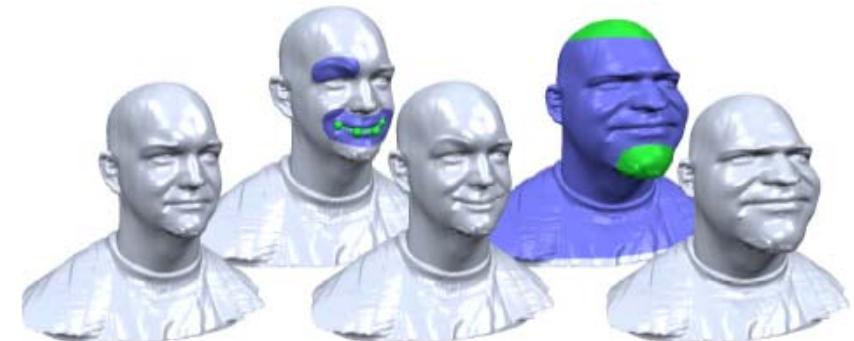
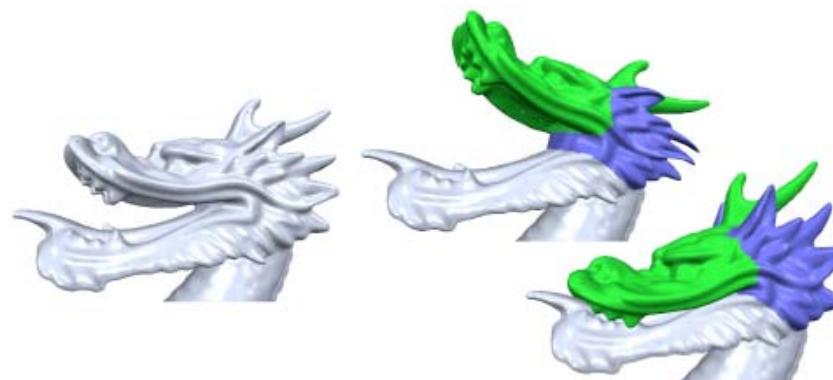
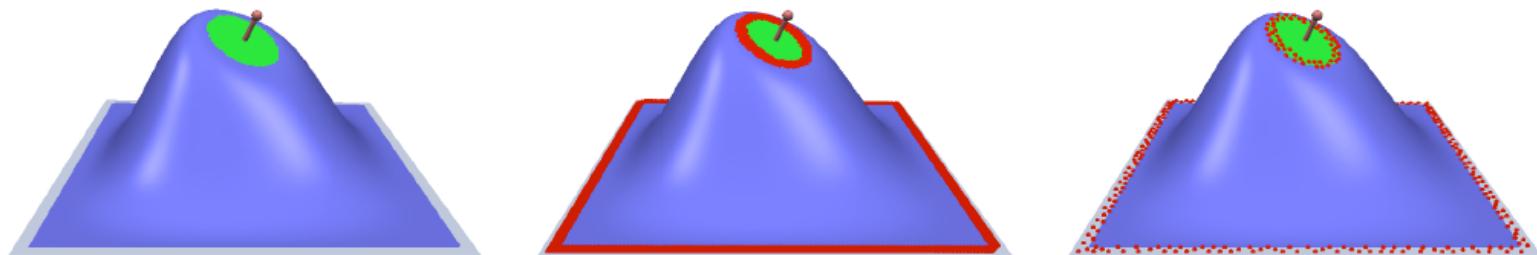
$$\min E(f)$$

$$\text{s. t. } f(x_i) = y_i, i = 1, 2, \dots, n$$



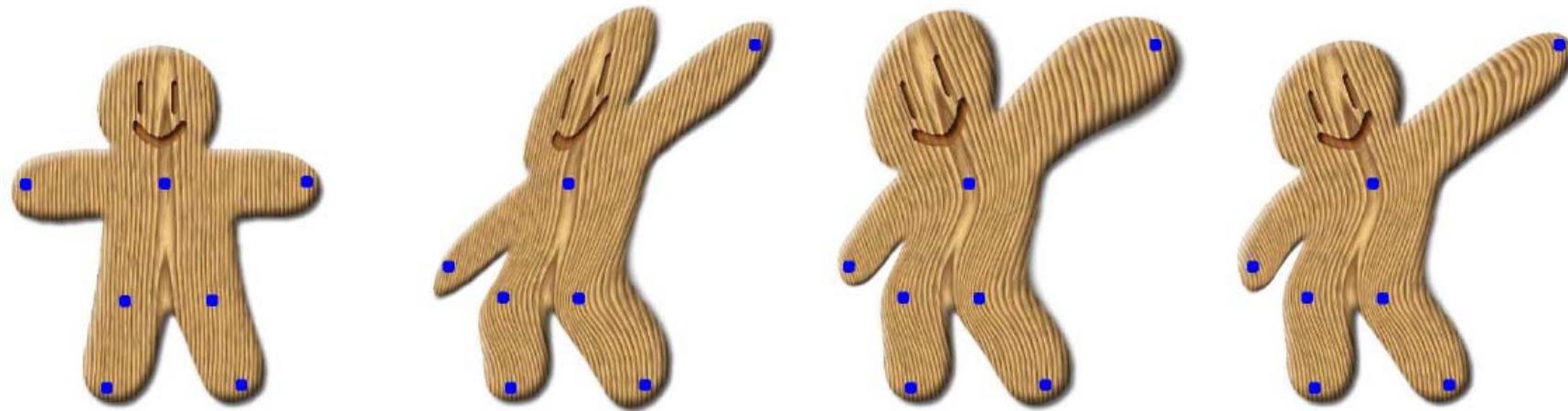
# 1.1 RBF-based Editing

[2005]



# 1.2 Moving Least Squares Method

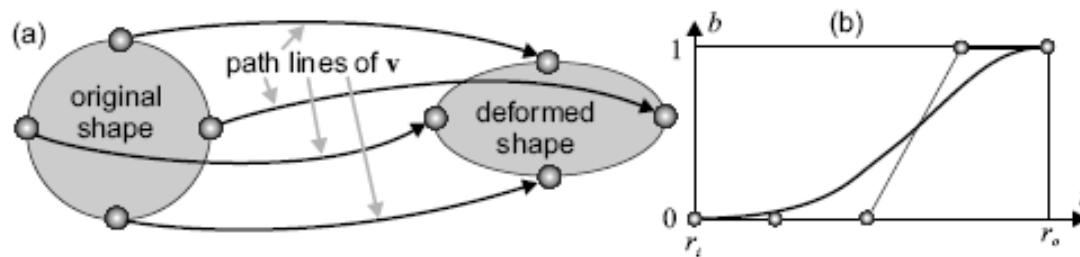
[Siggraph 2006]



# 1.3 Vector Field Based Deformations

[Siggraph 2006]

- Basic model: Moving vertex along the deformation orbit – defined by the path lines of a vector field  $v$ .



- Given a time-dependent vector field  $\mathbf{V}(\mathbf{X}, t)$ , a *Path Line*  $\mathbf{X}(t)$  in space is an integral.

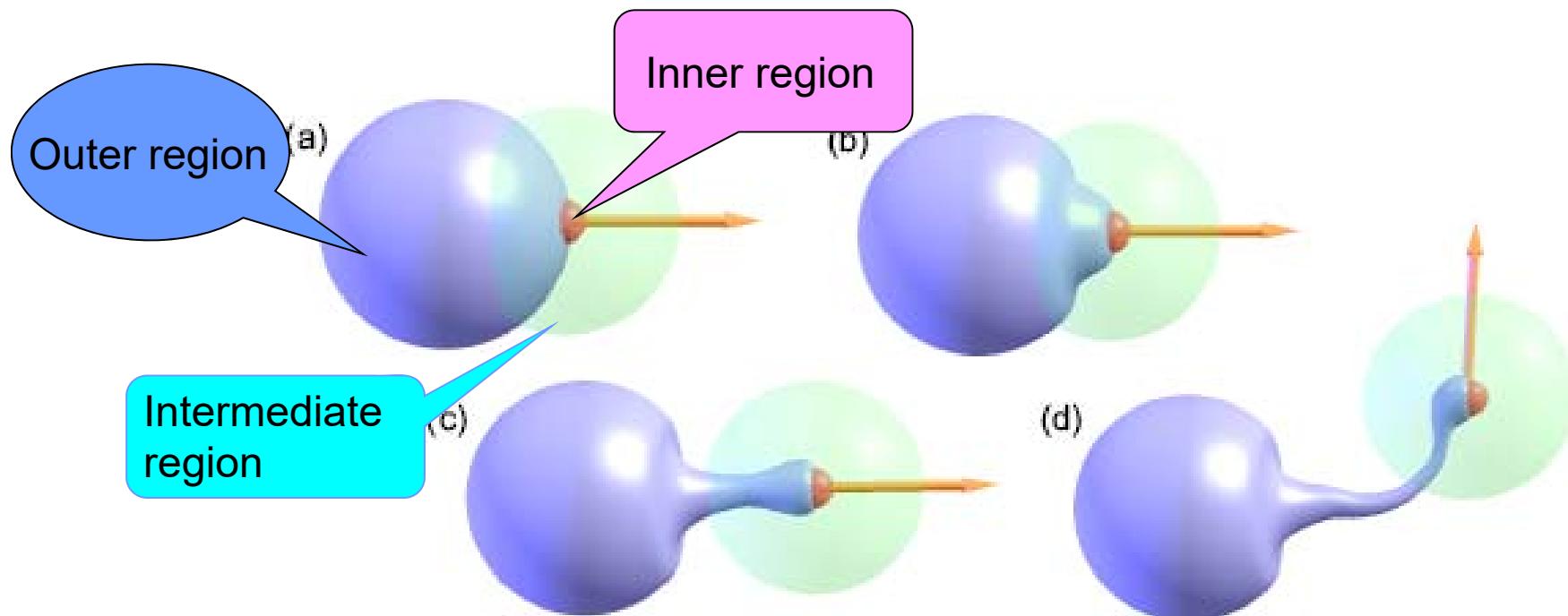


# Vector Field Selection

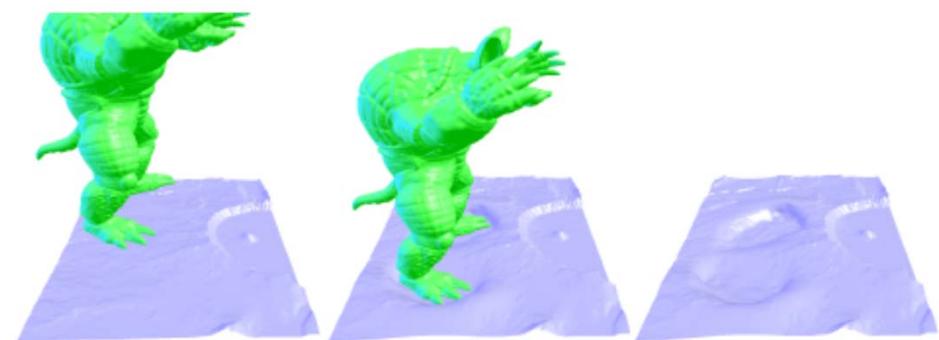
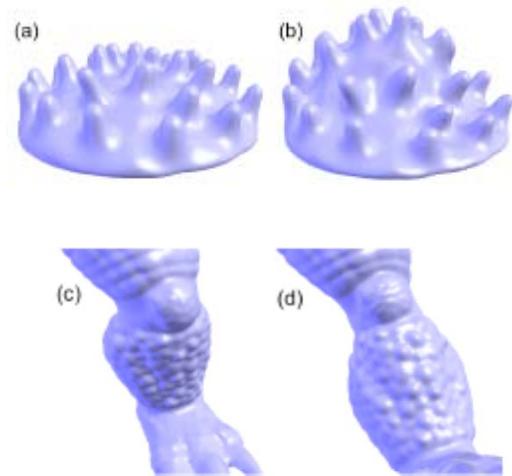
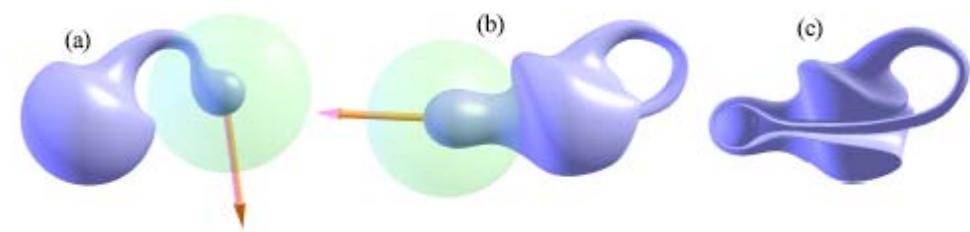
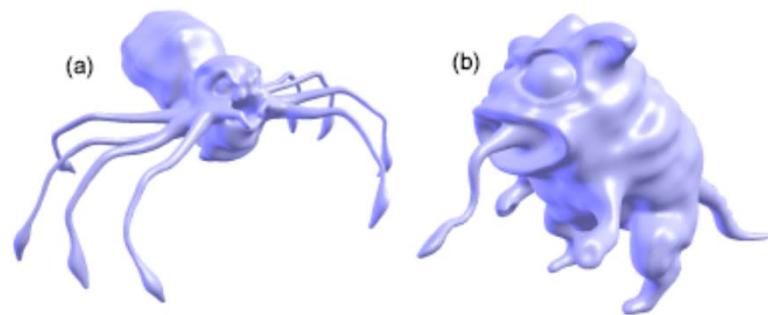
- Deformation Request:
  - No self-intersection
  - Volume-preserving
  - Details-preserving
  - Smoothness of shape in deformation
- Divergence-free Vector Field:  $\mathbf{v}=(V_1, V_2, V_3)$

$$\operatorname{div} \mathbf{V} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} = 0$$

# Piecewise Field for Deformation



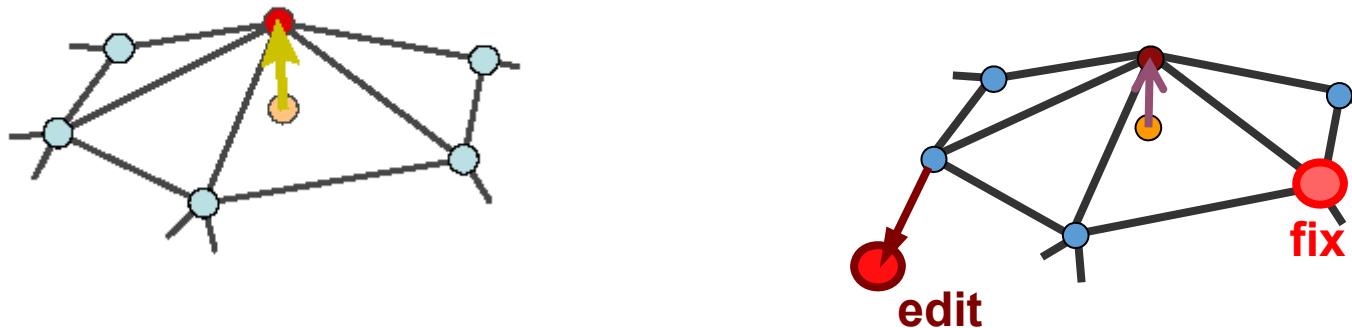
# Examples



# 1.4 Laplacian Editing

[Sorkine et al. SGP 2004]

- Preserve local detail – Laplacian coordinates
- Representation with **sparse** matrices
- Efficient **linear** surface reconstruction



$$v_i = \sum_{j \in N(i)} w_j v_j + \delta_i$$

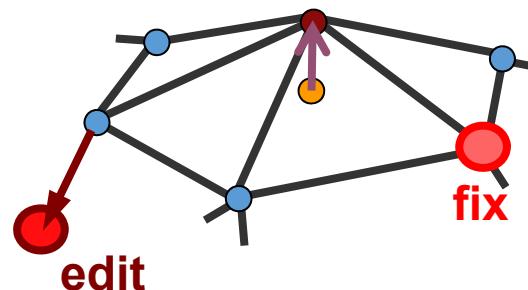
# Variational Viewpoint

- Laplacian Approximation

$$\tilde{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \left( \|L\mathbf{x} - \delta^{(x)}\|^2 + \sum_{j \in C} \omega_j^2 \|x_j - c_j\|^2 \right).$$

- Gradient Approximation

$$\min_{\phi} \int \int_{\Omega} \|\nabla \phi - \mathbf{w}\|^2 dA,$$



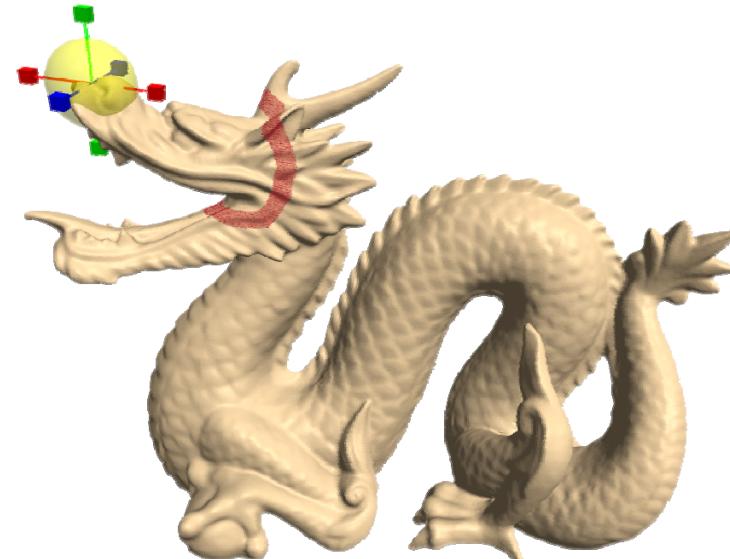
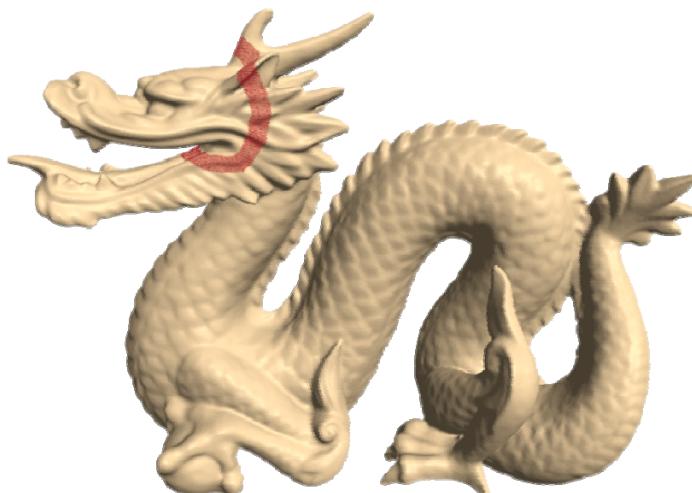
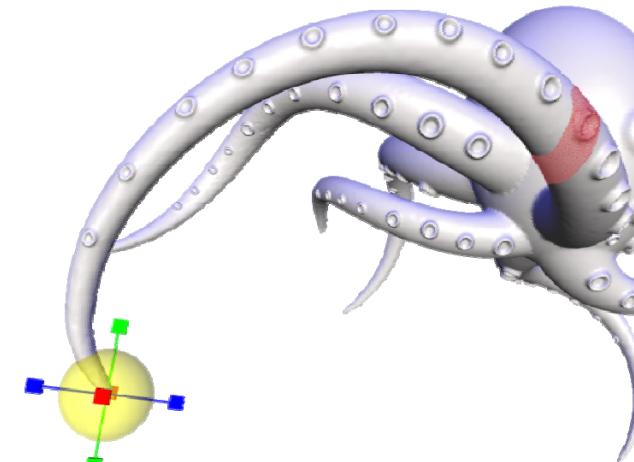
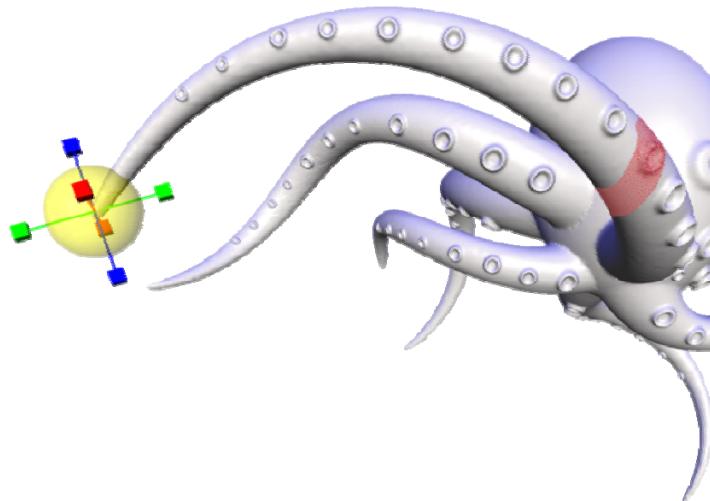
# Editing framework

- The spatial constraints will serve as modeling constraints
- Reconstruct the surface every time the modeling constraints are changed

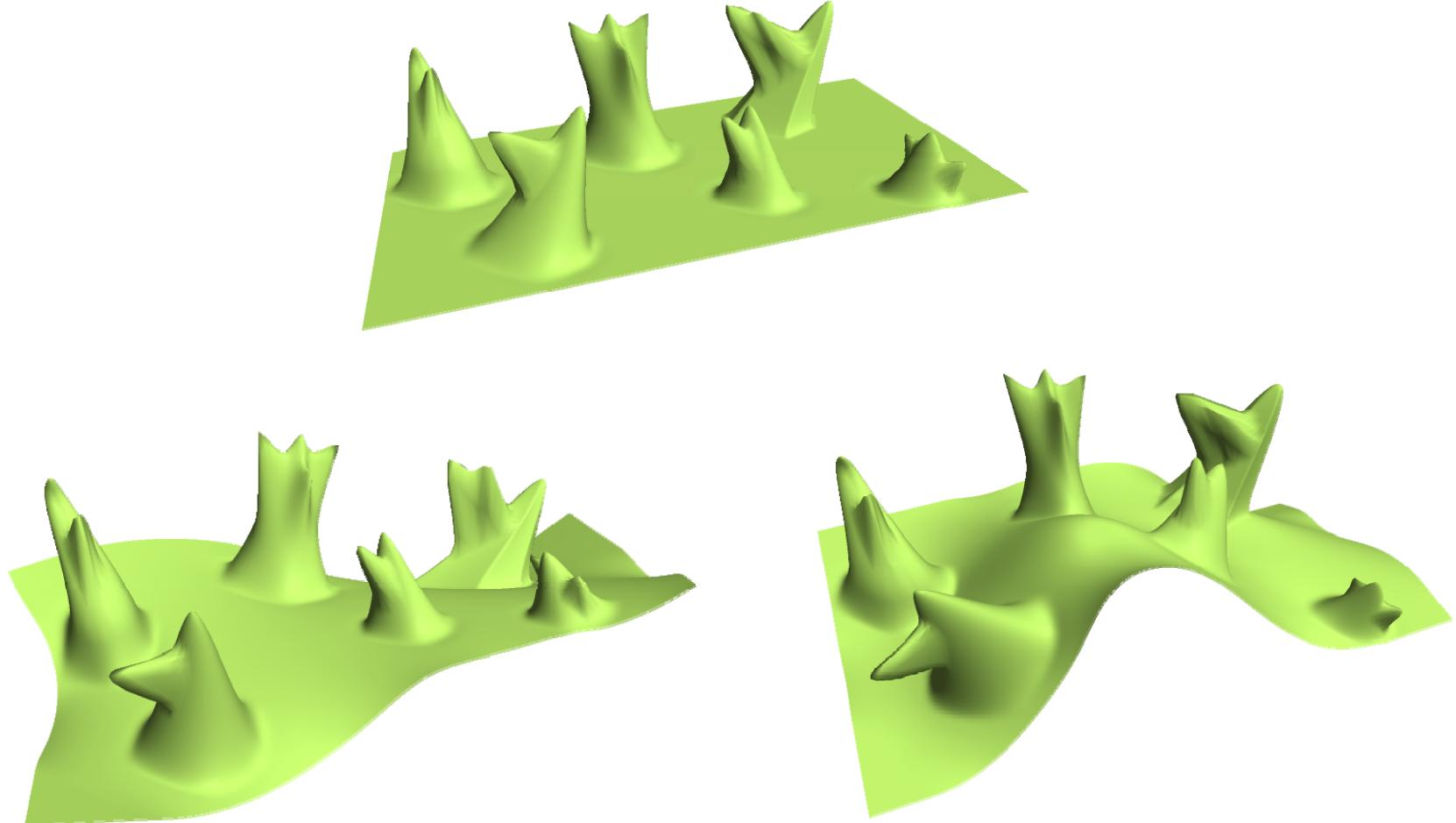
Detail constraints:  $L\mathbf{x} = \boldsymbol{\delta}$

Modeling constraints:  $x_j = c_j, \quad j \in \{j_1, j_2, \dots, j_k\}$

# Results

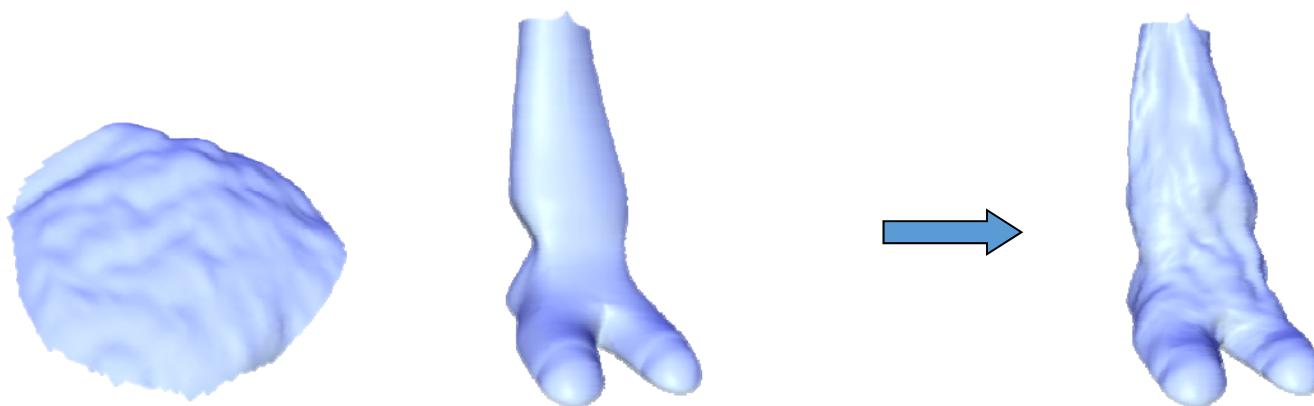


# Results

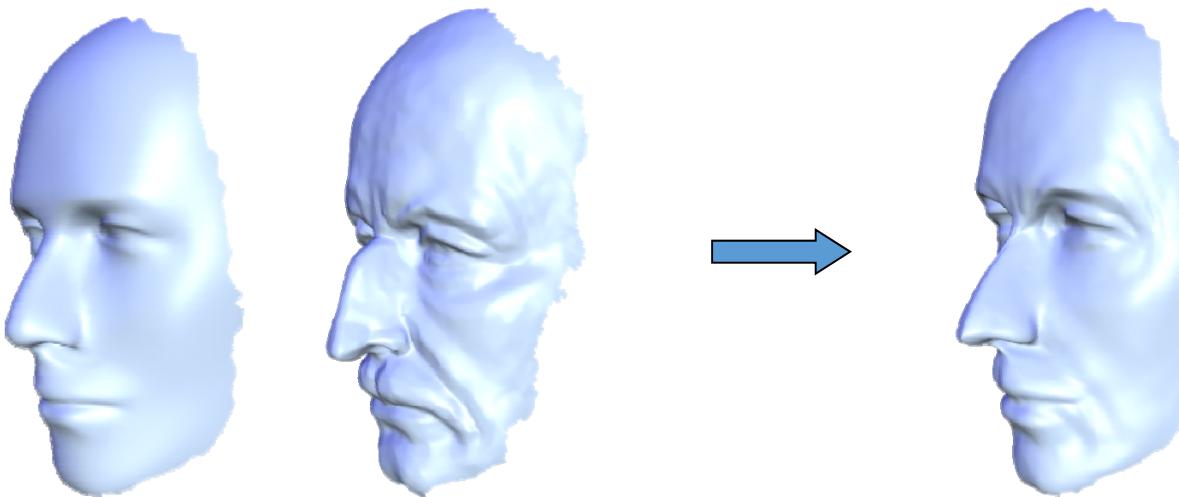


# Detail transfer and mixing

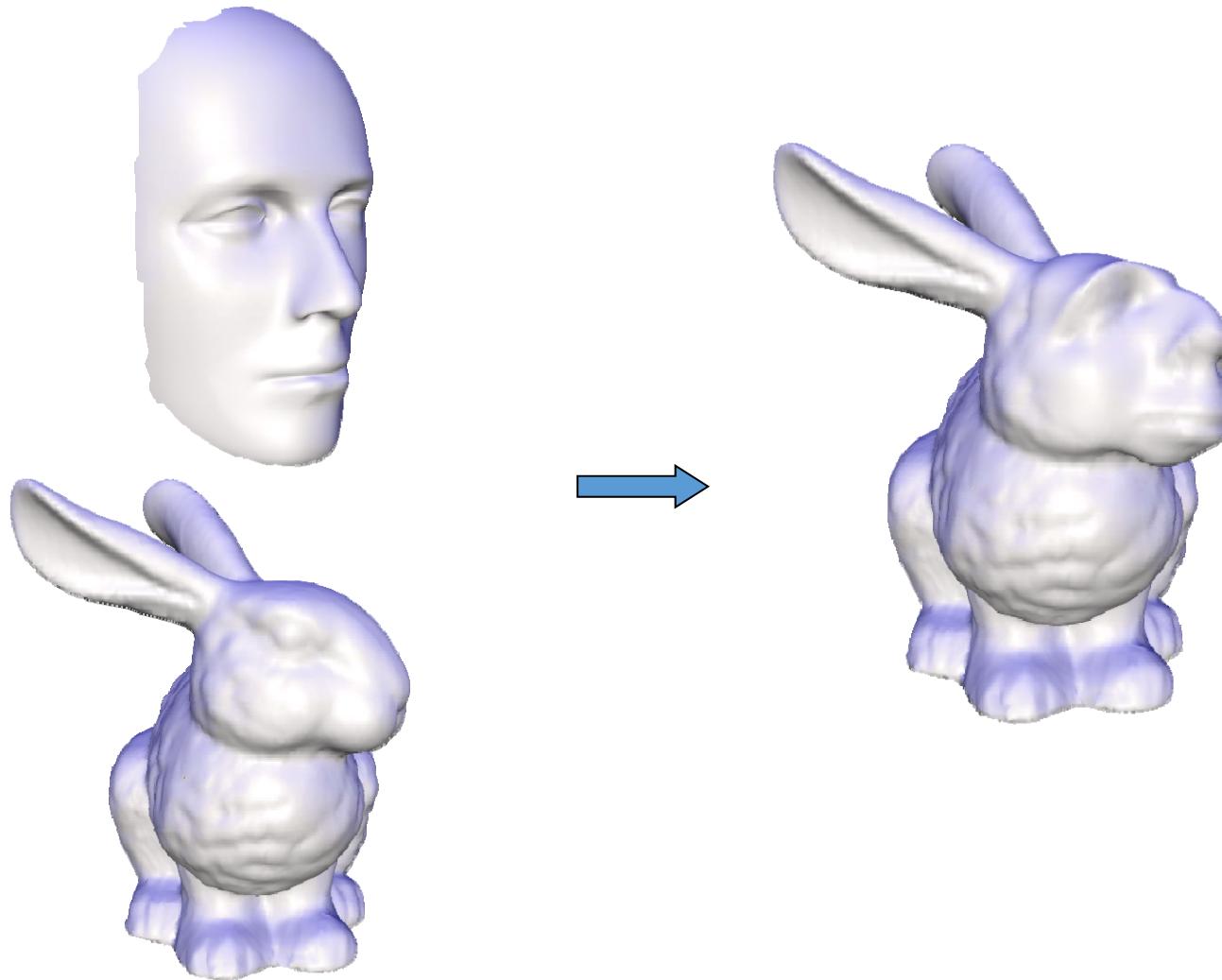
- “Peel“ the coating of one surface and transfer to another



# Examples

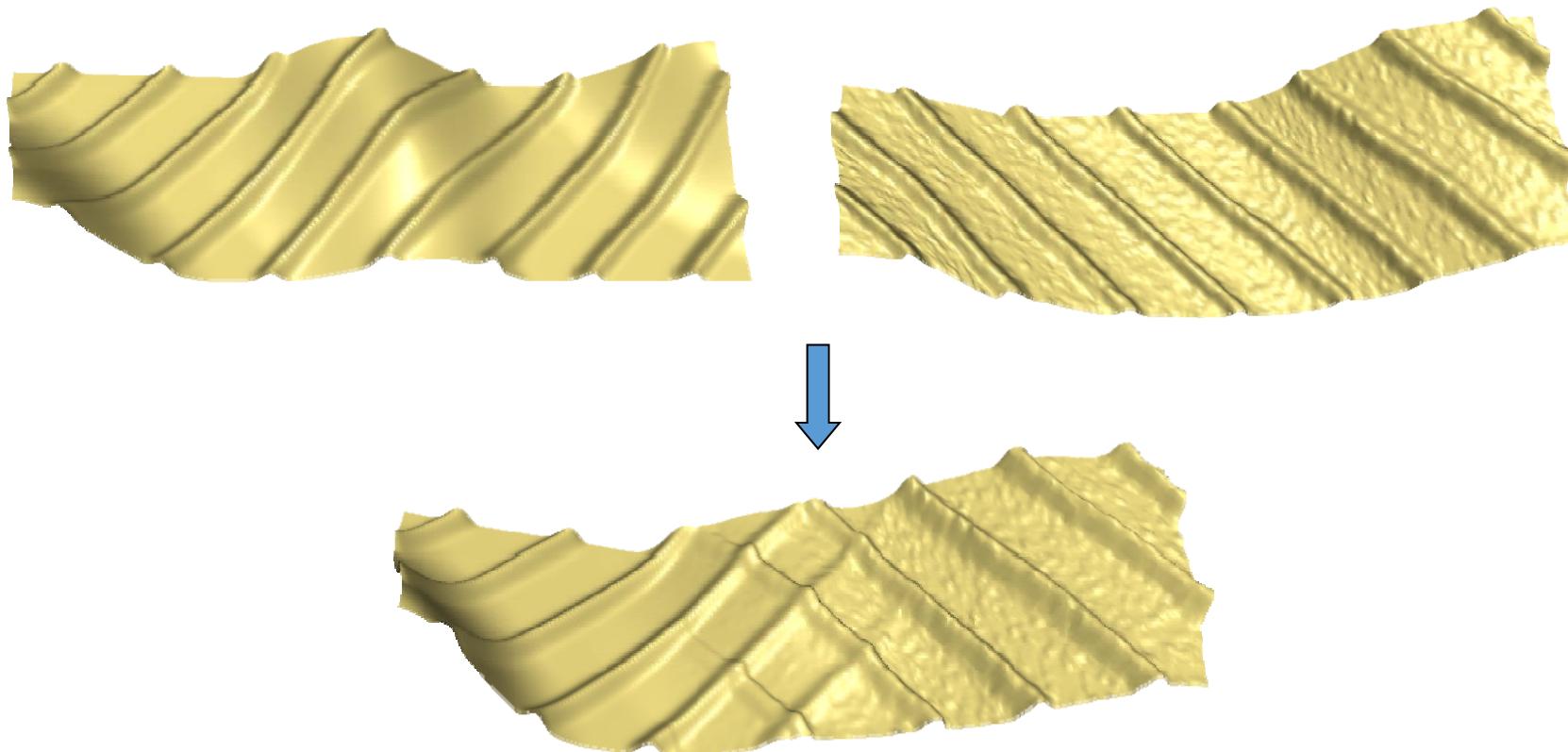


# Examples



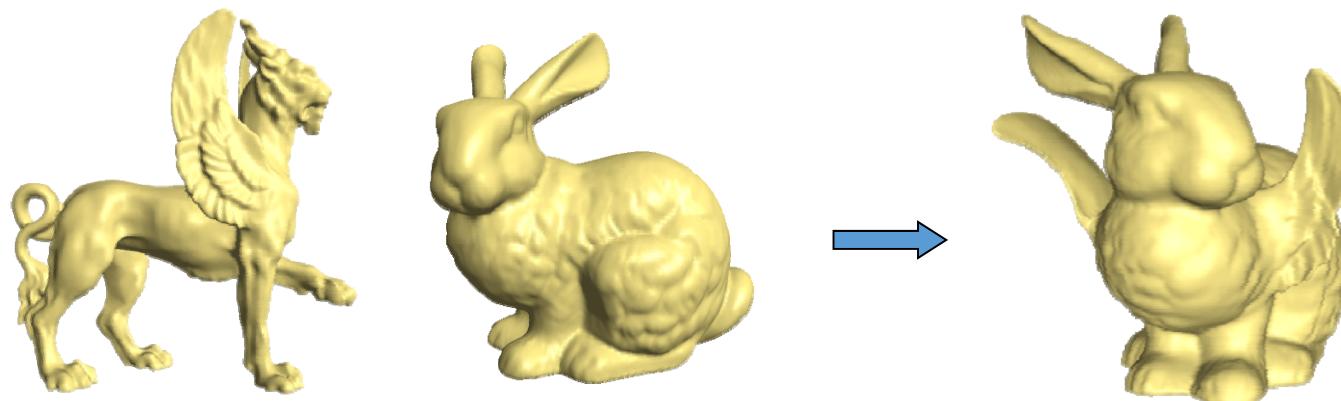
# Mixing Laplacians

- Taking weighted average of  $\delta_i$  and  $\delta'_i$



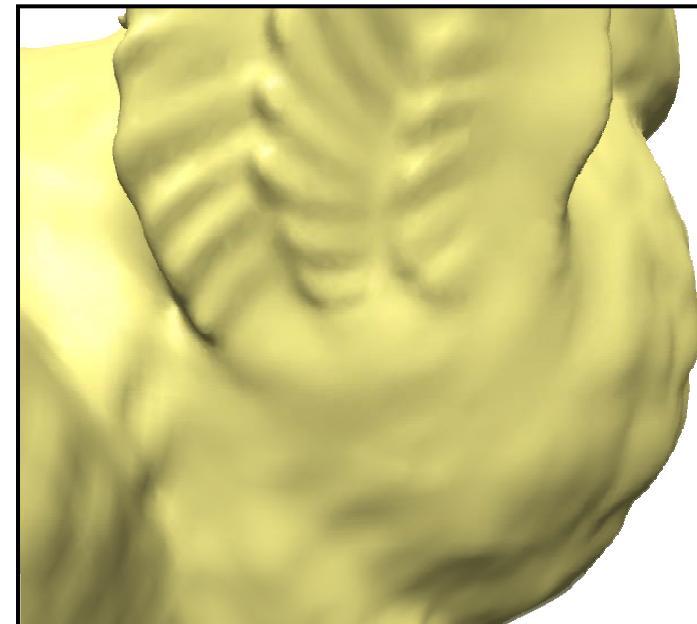
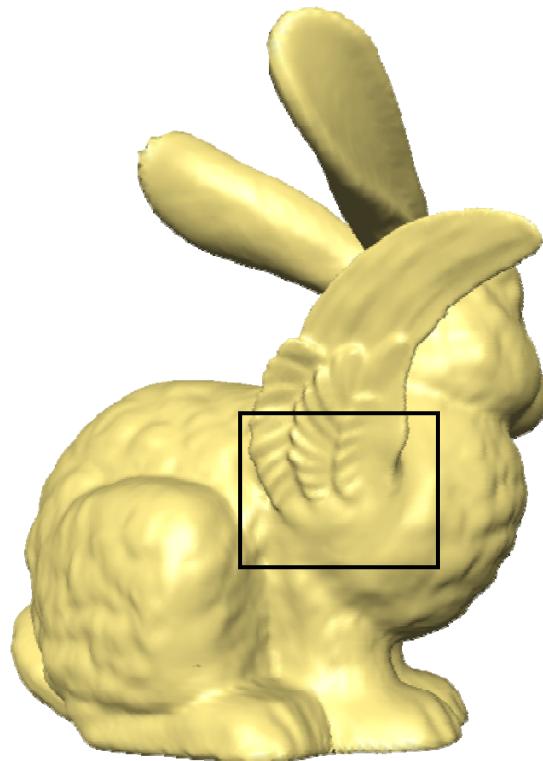
# Mesh transplanting

- The user defines
  - Part to transplant
  - Where to transplant
  - Spatial orientation and scale
- Topological stitching
- Geometrical stitching via Laplacian mixing



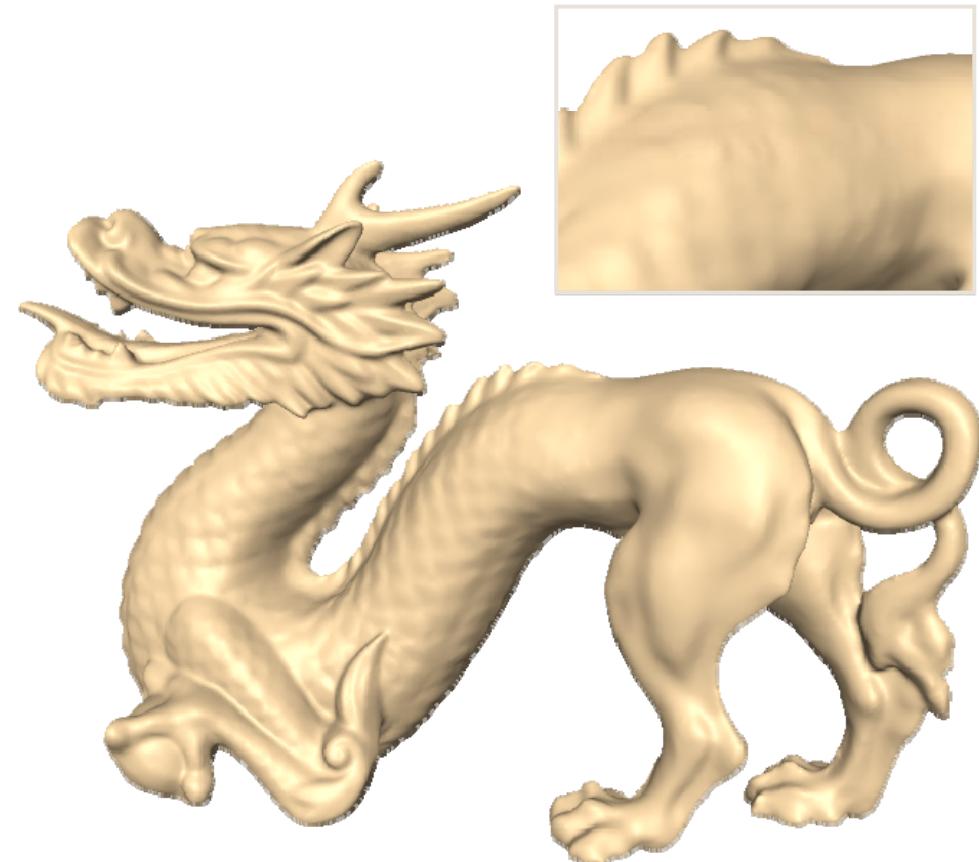
# Mesh transplanting

- Details gradually change in the transition area



# Mesh transplanting

- Details gradually change in the transition area



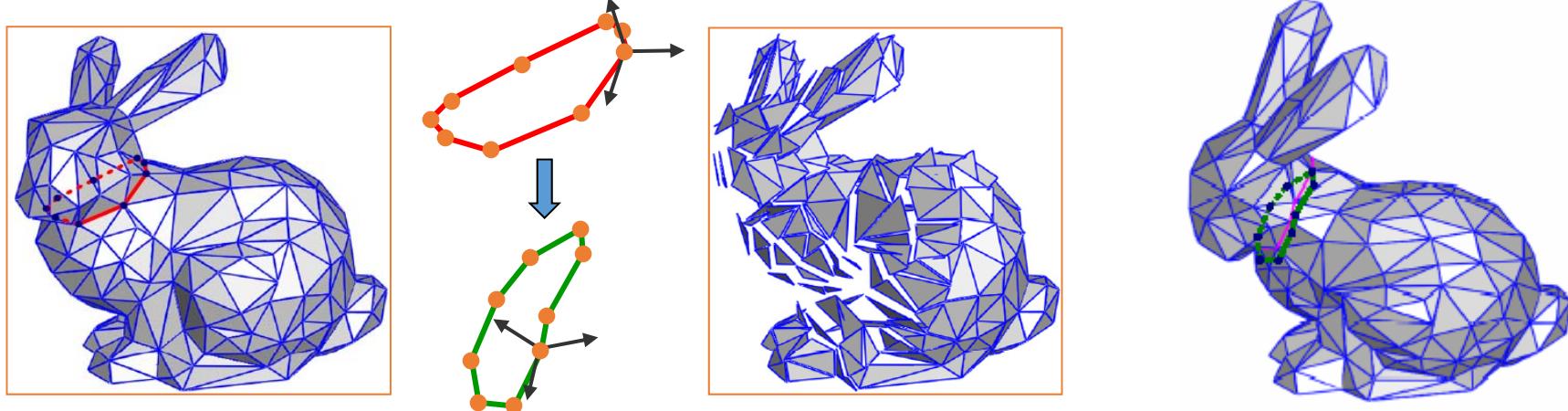
# Invariance – solutions

- Explicit transformation of the differential coordinates prior to surface reconstruction
  - Lipman, Sorkine, Cohen-Or, Levin, Rössl and Seidel [SMI 04], “Differential Coordinates for Interactive Mesh Editing”,
    - Estimation of rotations from naive reconstruction
  - Yu, Zhou, Xu, Shi, Bao, Guo and Shum [SIGGRAPH 04], “Mesh Editing With Poisson-Based Gradient Field Manipulation”,
    - Propagation of handle transformation to the rest of the ROI using geodesic distances
  - Zayer, Rössl, Karni and Seidel [EG 05], “Harmonic Guidance for Surface Deformation”,
    - Propagation of handle transformation to the rest of the ROI using harmonic functions

# 1.5 Poisson Mesh Editing

Yu et al. Mesh Editing With Poisson-Based Gradient Field Manipulation. Siggraph 2004.

- The representation: the gradients of the functions X, Y, Z on each triangle of the mesh
- Deformation: propagate the transformation of the handle onto the ROI using geodesic distances



# Poisson editing – images

- Inspiration: [Poisson Image Editing](#) [Pérez et al. 03]



- Reconstruct a function from its gradients via the Poisson equation:

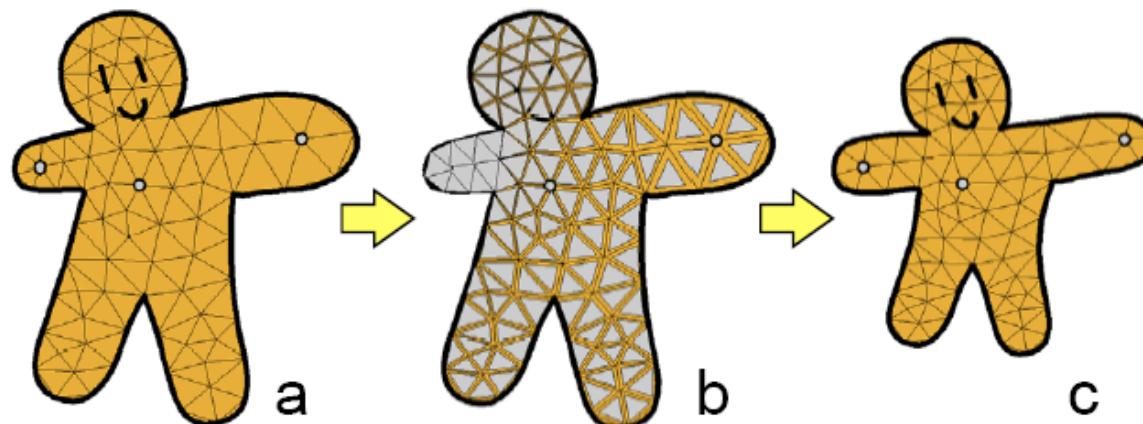
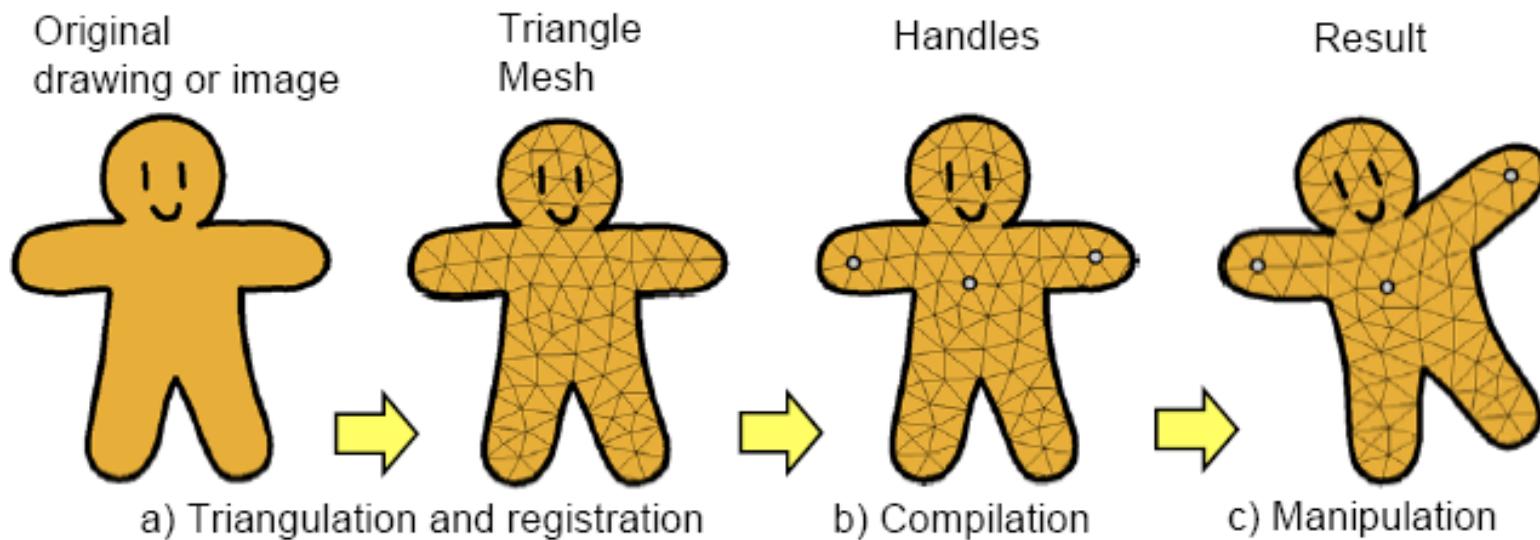
$$\arg \min_f \int_{\Omega} \|\nabla f - \mathbf{w}\|^2, \quad \text{s.t. } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$



$$\Delta f = \operatorname{div} \mathbf{w} \quad \text{with} \quad f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

# 1.6 As-rigid-as-possible Deformation

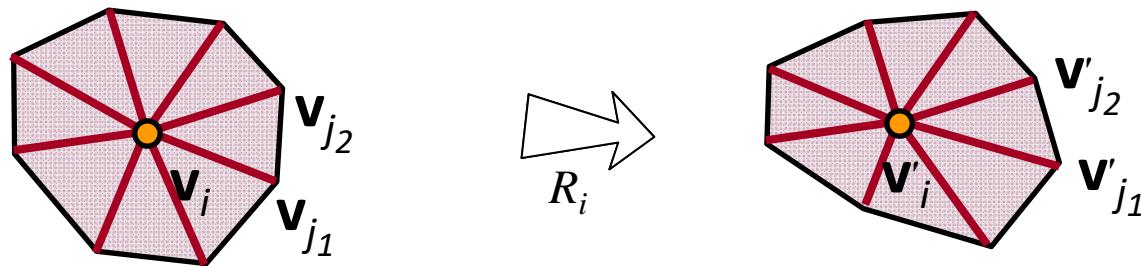
[Siggraph 2003]



# 1.7 ARAP Modeling

[Sorkine and Alexa, As-Rigid-As-Possible Surface Modeling. SGP 2007]

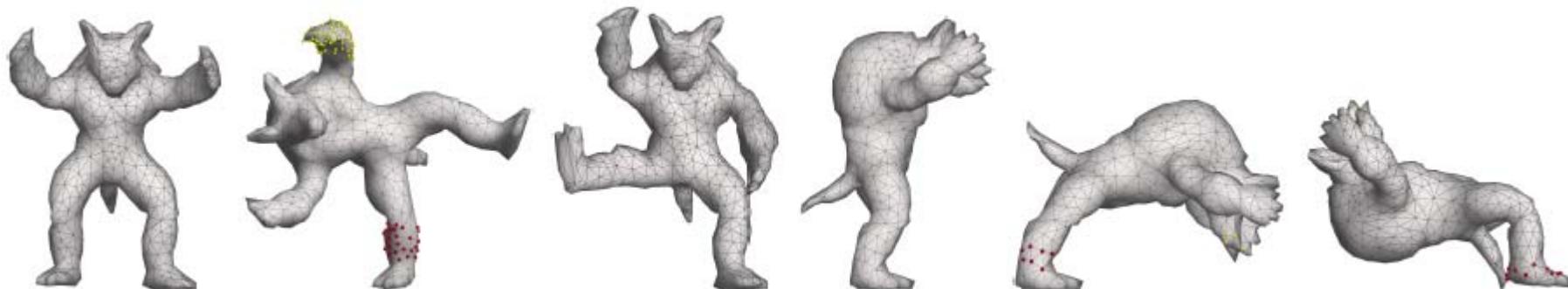
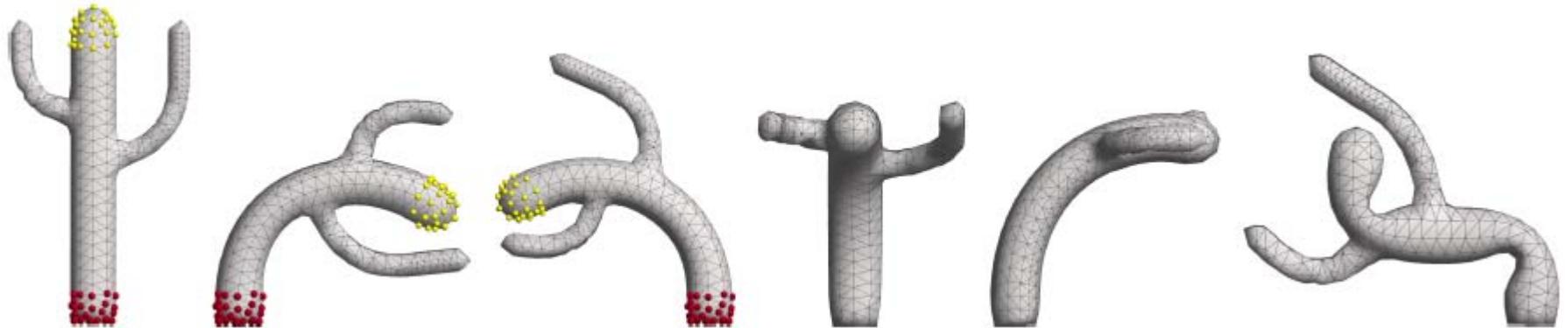
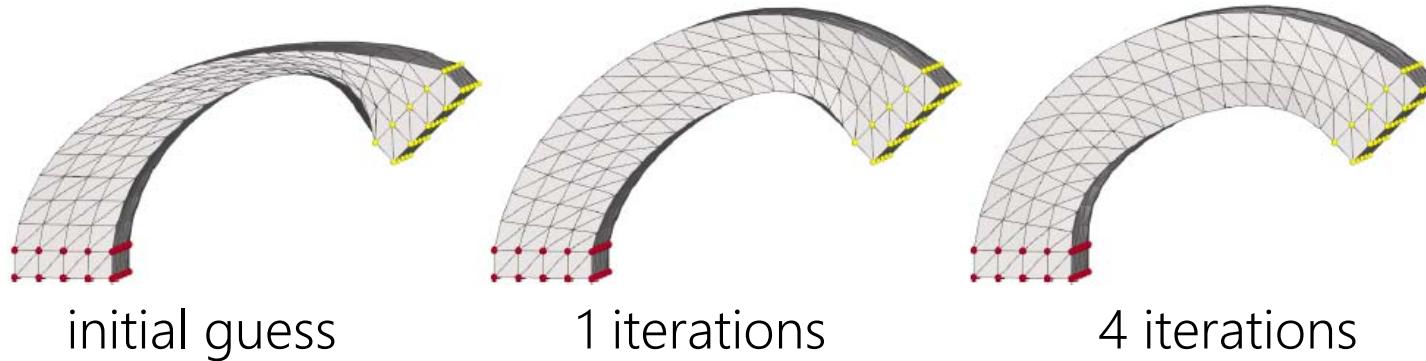
- Ask all star edges to transform rigidly by some rotation  $R$ , then the shape of the cell is preserved



$$\min_{\mathbf{v}'} \sum_{i=1}^n \sum_{j \in N(i)} \|(\mathbf{v}'_i - \mathbf{v}'_j) - R_i(\mathbf{v}_i - \mathbf{v}_j)\|^2$$

$$s.t. \mathbf{v}'_j = \mathbf{c}_j, j \in C$$

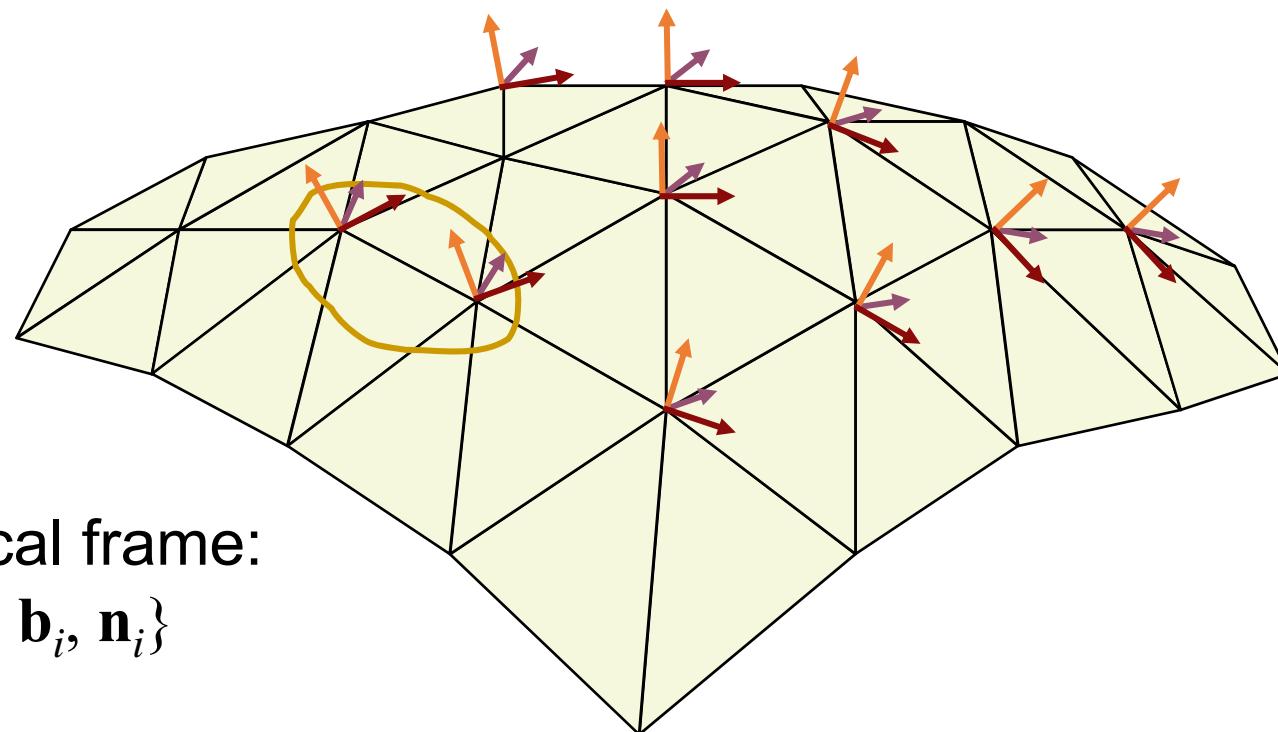
# ARAP Modeling



# 1.8 Linear Rotation-invariant Coordinates

[Lipman et al. Siggraph 05]

- Keep a local frame at each vertex
- Prescribe changes to some selected frames



Local frame:

$$\{\mathbf{a}_i, \mathbf{b}_i, \mathbf{n}_i\}$$

# Frame-based deformations

- Encode the differences between adjacent frames
- Solve for the new frames in least-squares sense

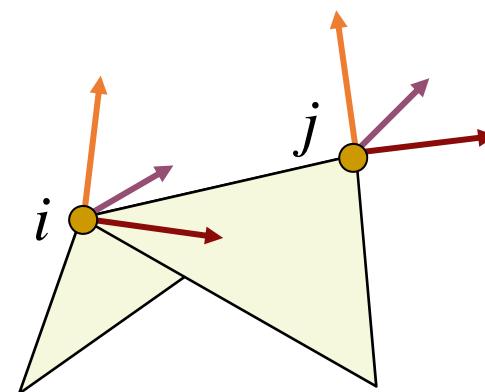
$$\mathbf{a}_i - \mathbf{a}_j = \alpha_1 \mathbf{a}_i + \alpha_2 \mathbf{b}_i + \alpha_3 \mathbf{n}_i$$

$$\mathbf{b}_i - \mathbf{b}_j = \beta_1 \mathbf{a}_i + \beta_2 \mathbf{b}_i + \beta_3 \mathbf{n}_i$$

$$\mathbf{n}_i - \mathbf{n}_j = \gamma_1 \mathbf{a}_i + \gamma_2 \mathbf{b}_i + \gamma_3 \mathbf{n}_i$$

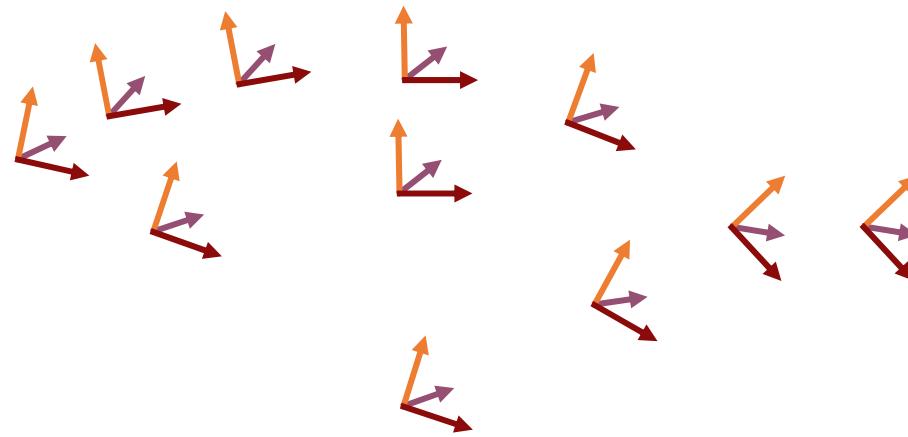
... ...

*constraints*



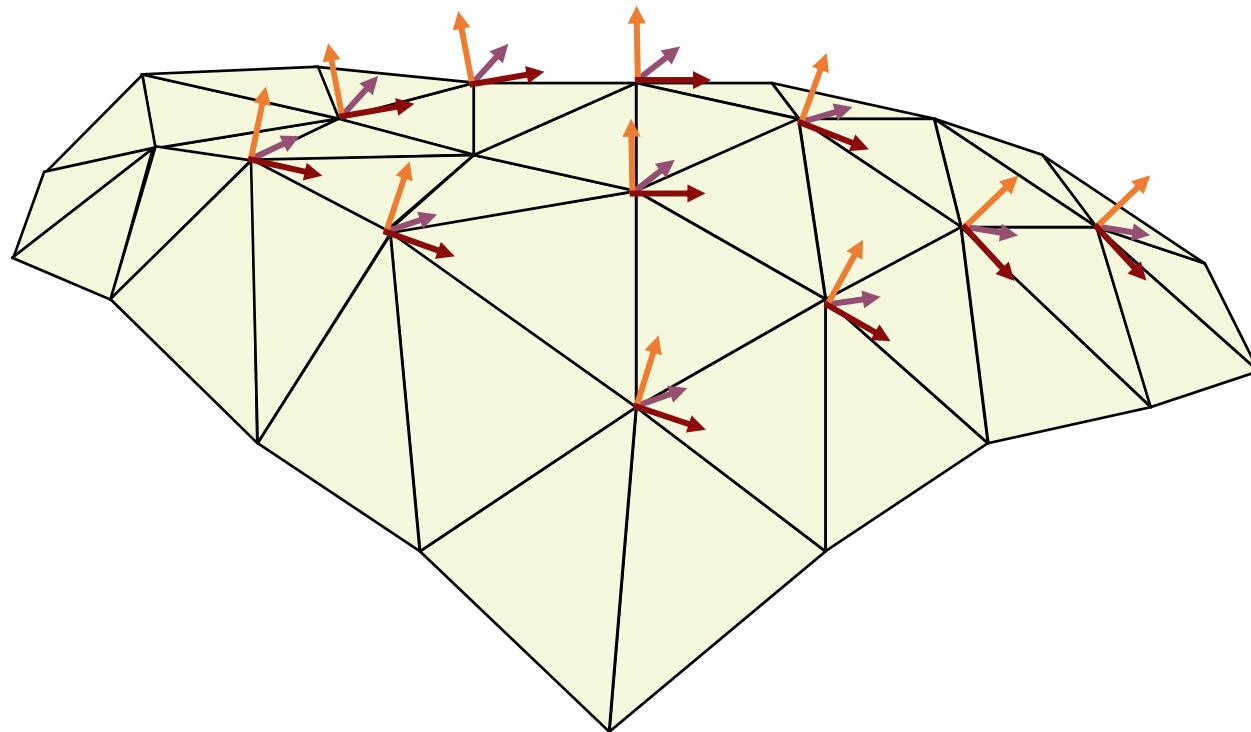
# Frame-based deformations

- Reconstruction:
  - After having the frames, solve for positions

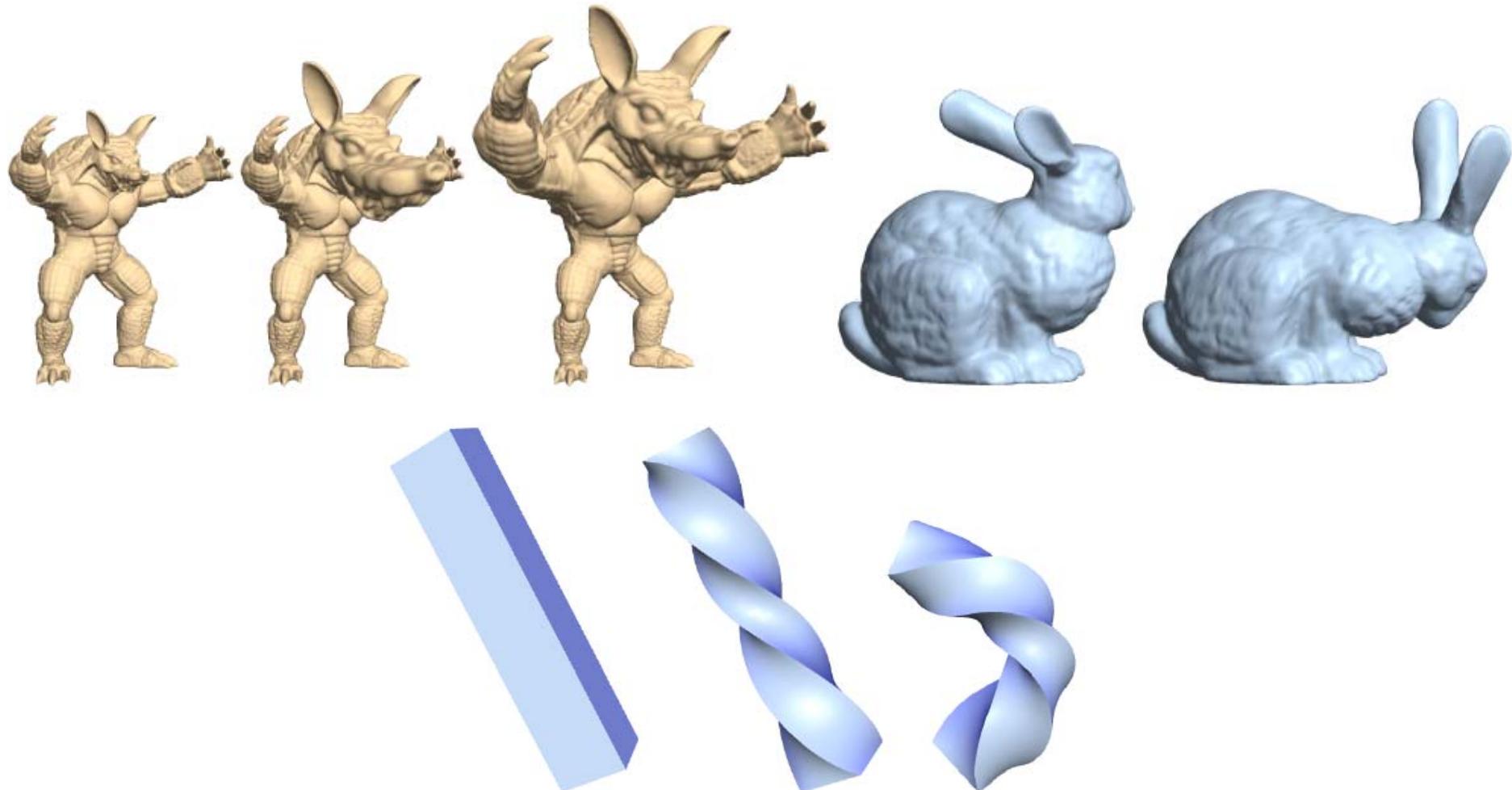


# Frame-based deformations

- Reconstruction:
  - After having the frames, solve for positions



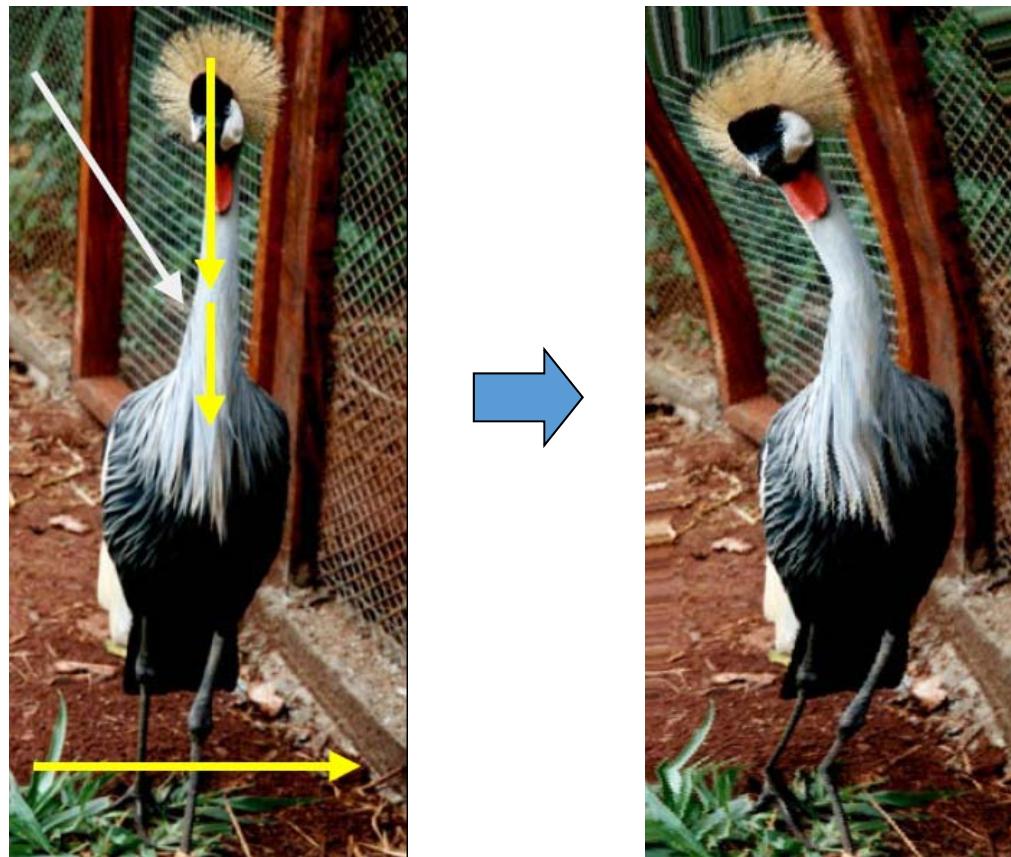
# Results



## 2. Curve Proxy

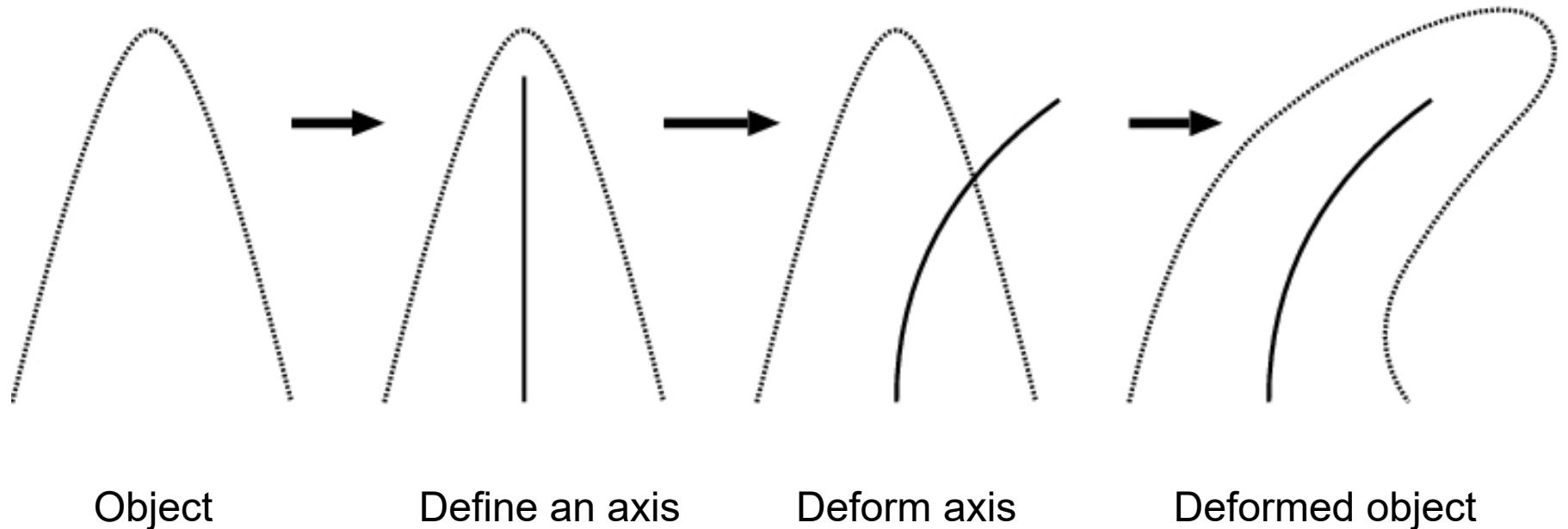
# User Specifications

- Line or curve features

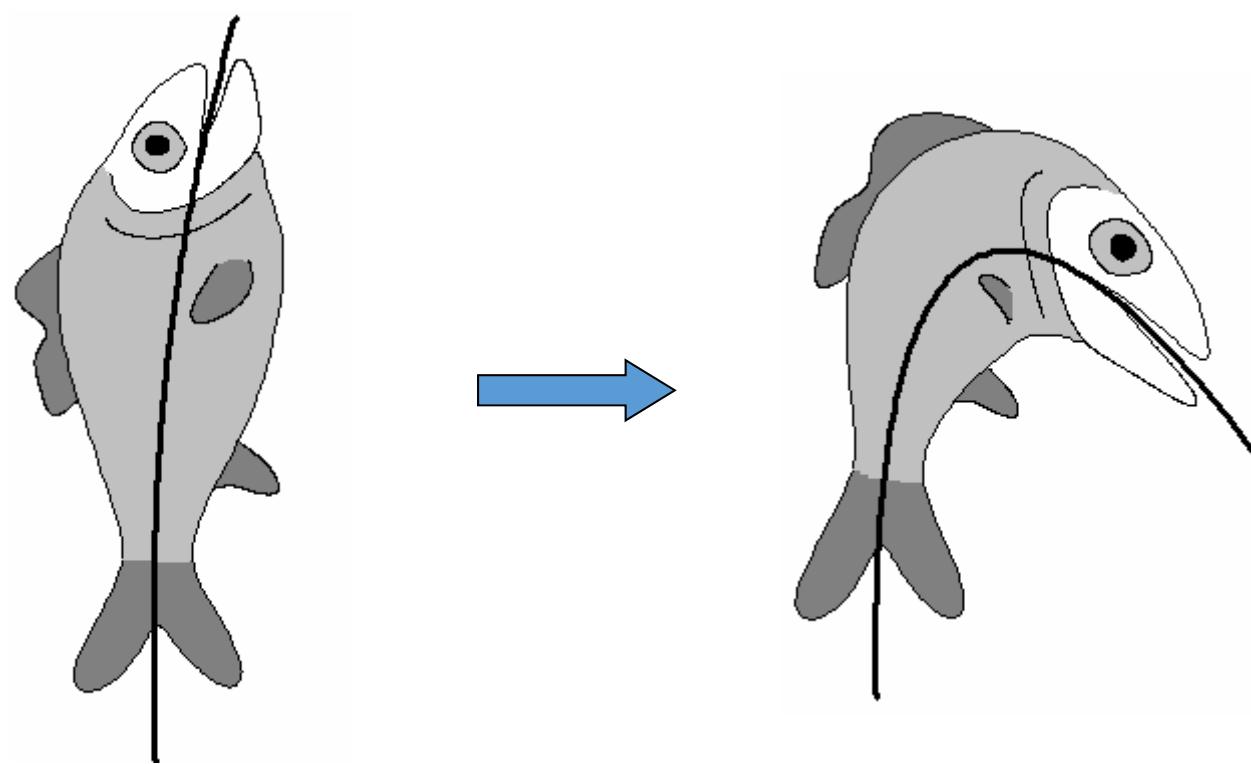


## 2.1 Skeleton based

- Axial Deformation [1994]

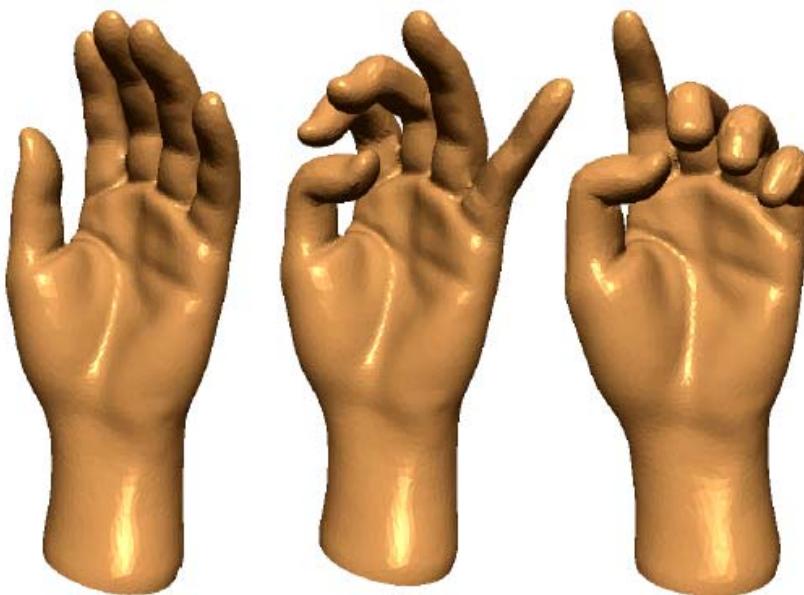
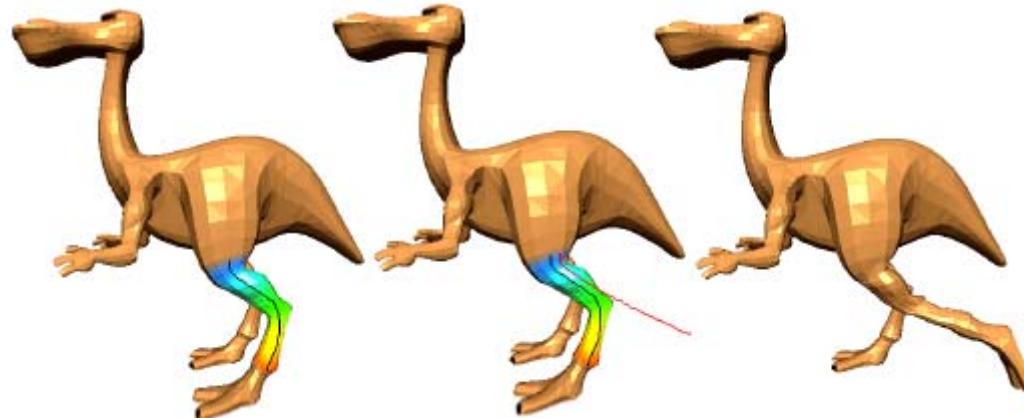


# Axial Deformation

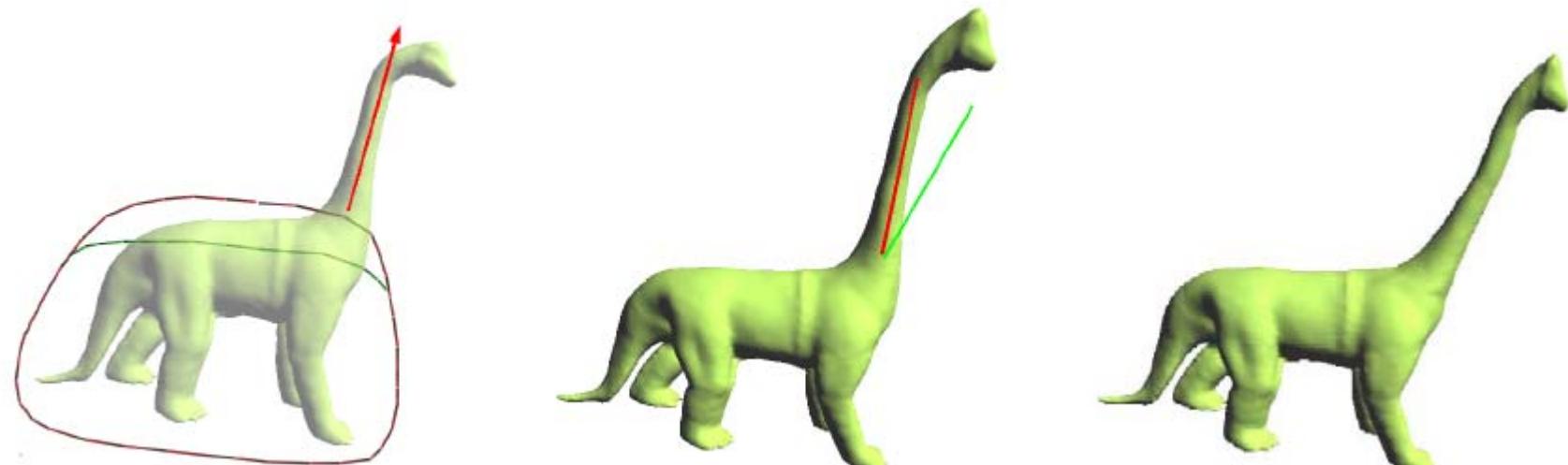
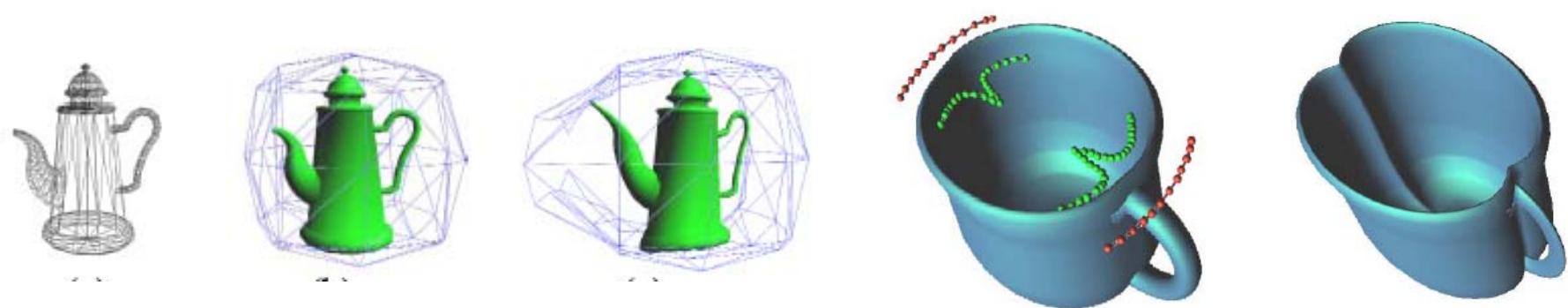


## 2.2 Sketching Deformations

[2005]



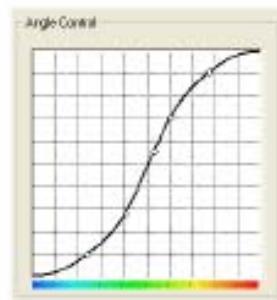
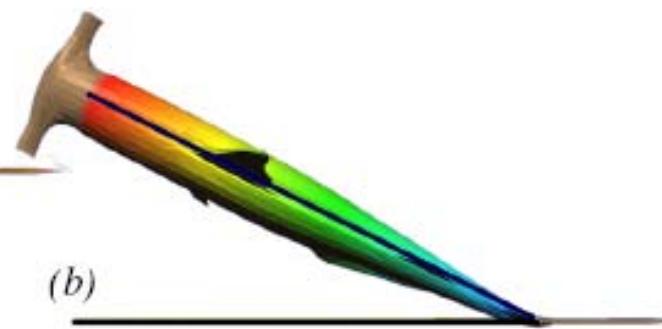
# Sketching Deformation



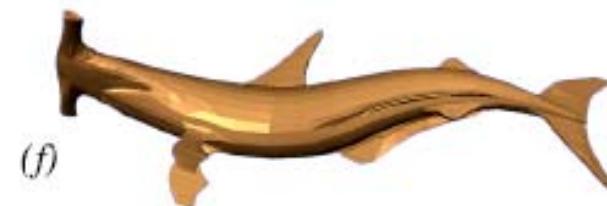
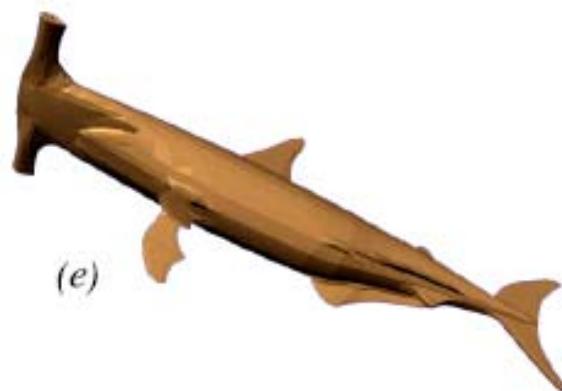
# Skeleton-based Deformation



[2002]

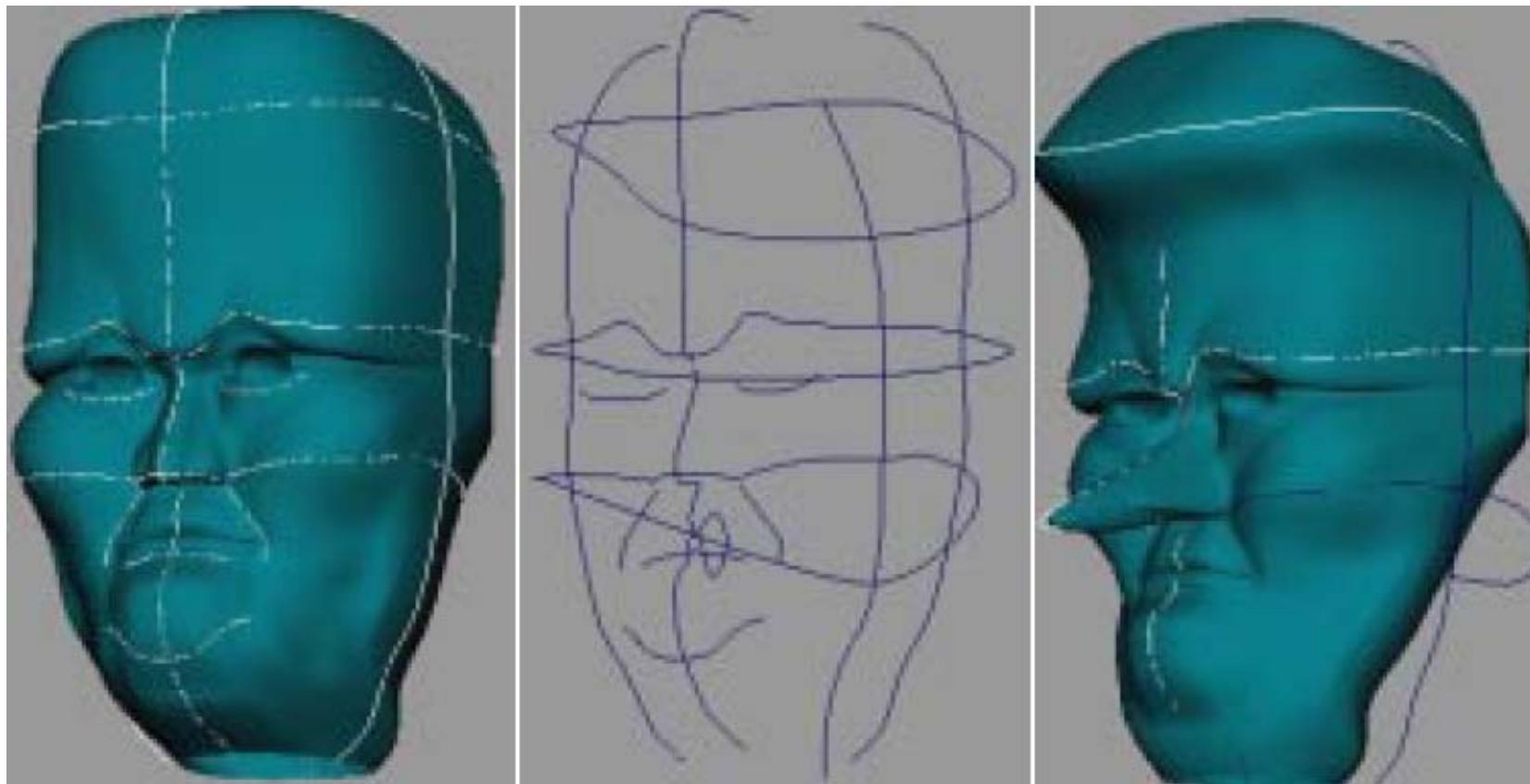


(c)



## 2.3 Wires based

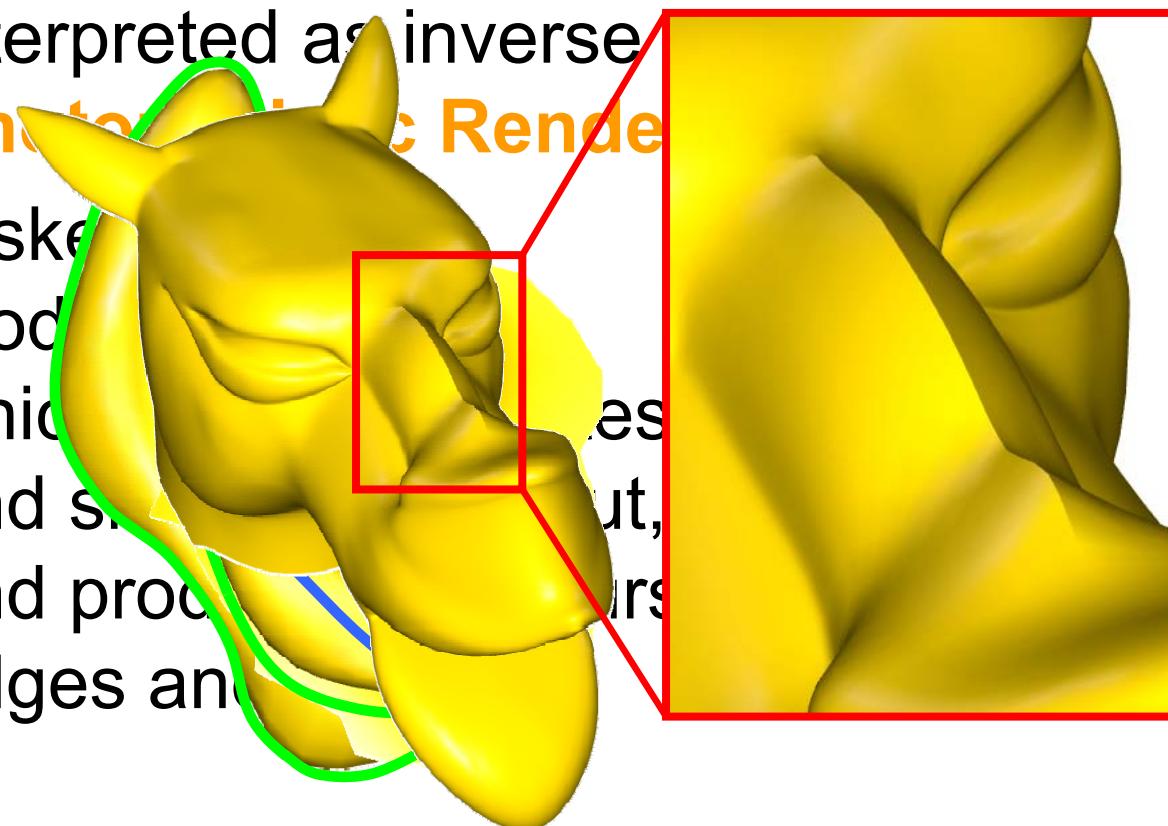
[Siggraph 1999]



## 2.4 Silhouette sketching based

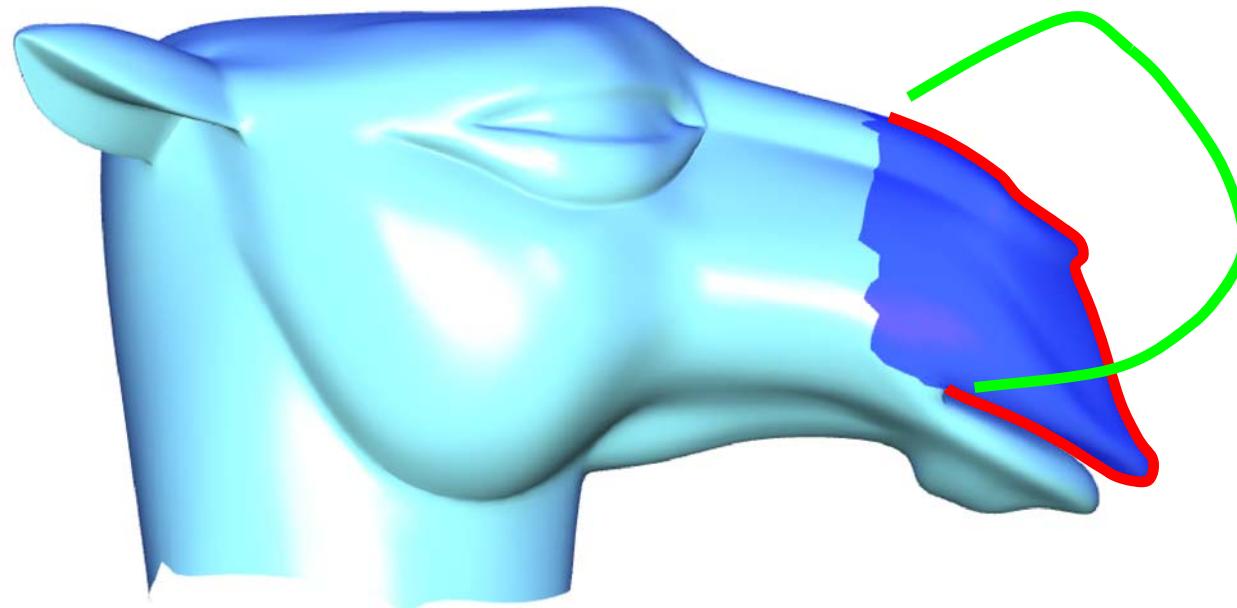
[Nealen et al. Siggraph 05]

- Silhouette sketching  
Silhouette shape can be interpreted as inverse  
**Photometric Render**
- A sketch model which and smooths and processes ridges and



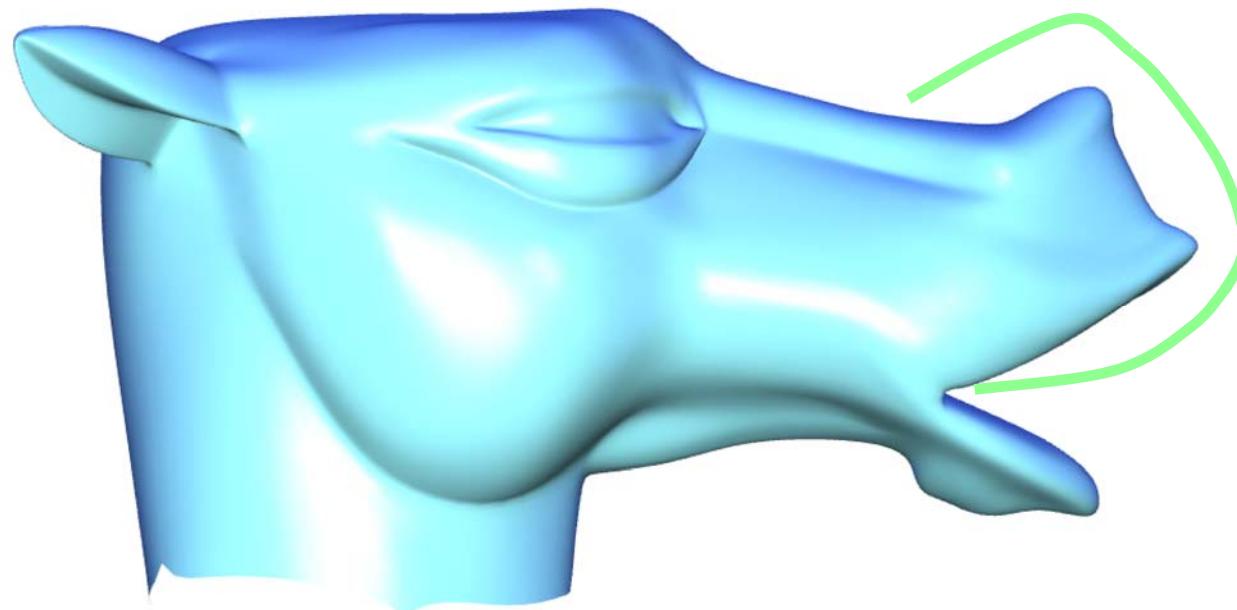
# Silhouette Sketching

- Approximate sketching
  - Balance weighting between detail and positional constraints



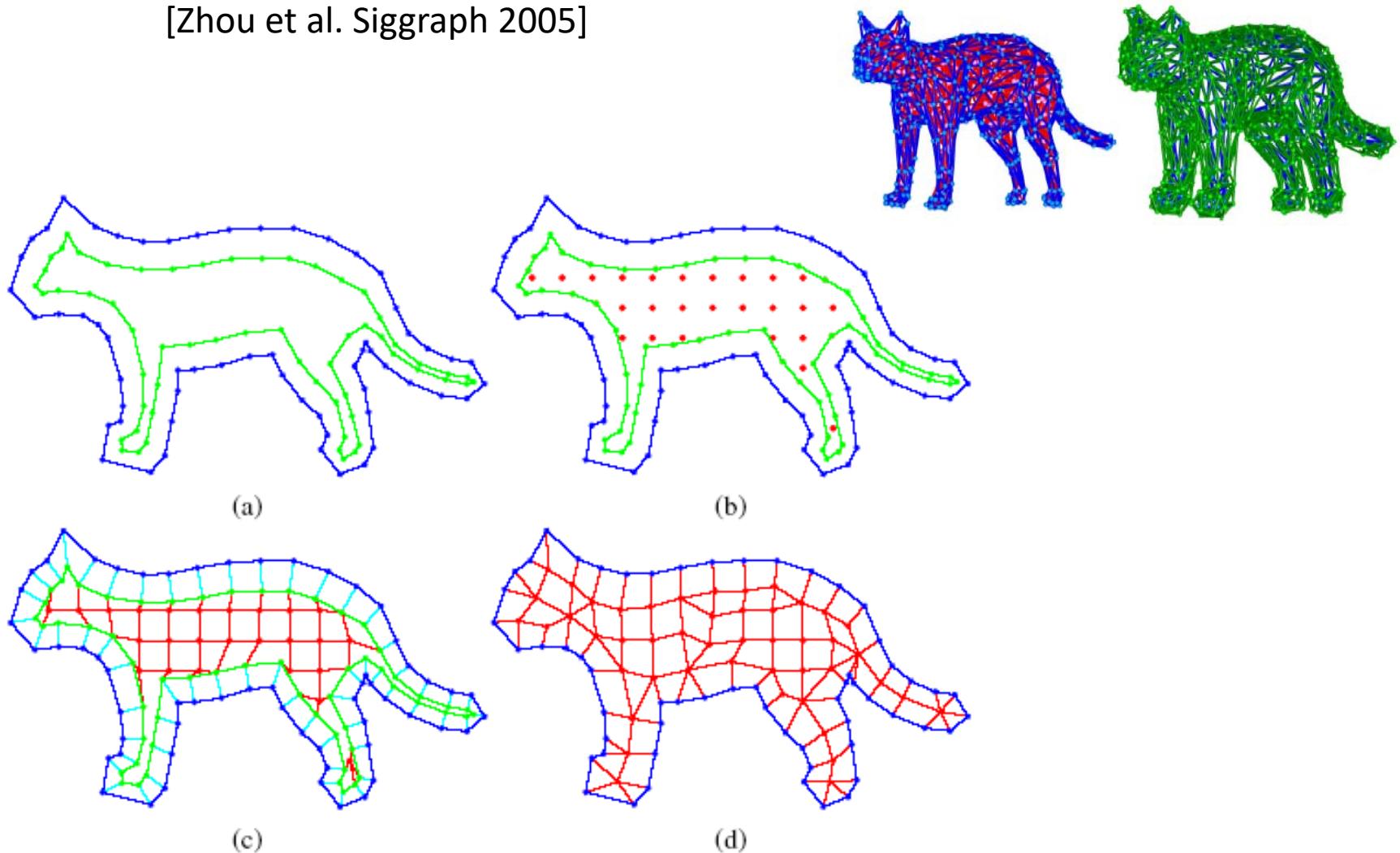
# Silhouette Sketching

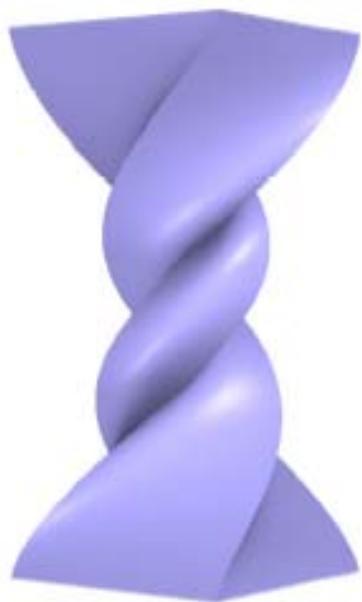
- Approximate sketching
  - Balance weighting between detail and positional constraints



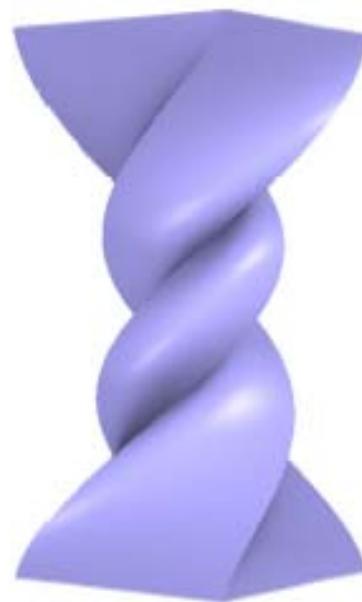
# 2.5 Volumetric Graph Laplacian

[Zhou et al. Siggraph 2005]

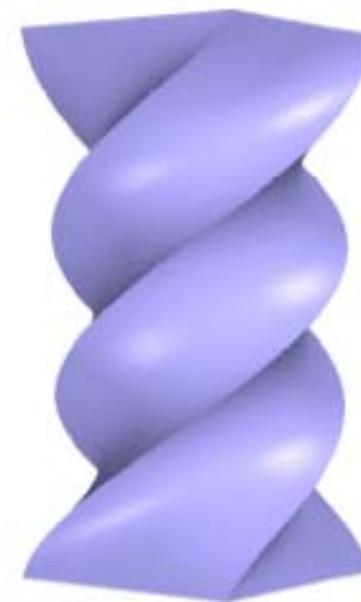




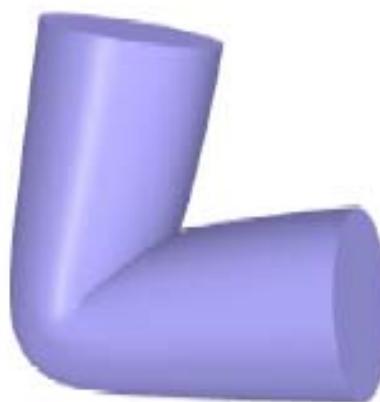
(a) Laplacian surface



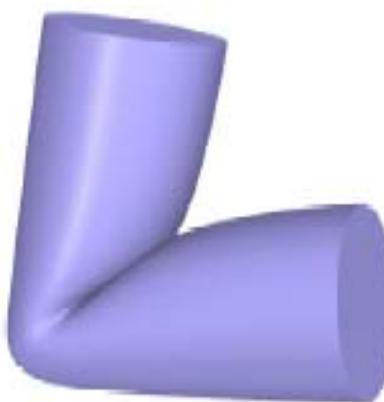
(b) Poisson mesh



(c) VGL



(a) Laplacian surface

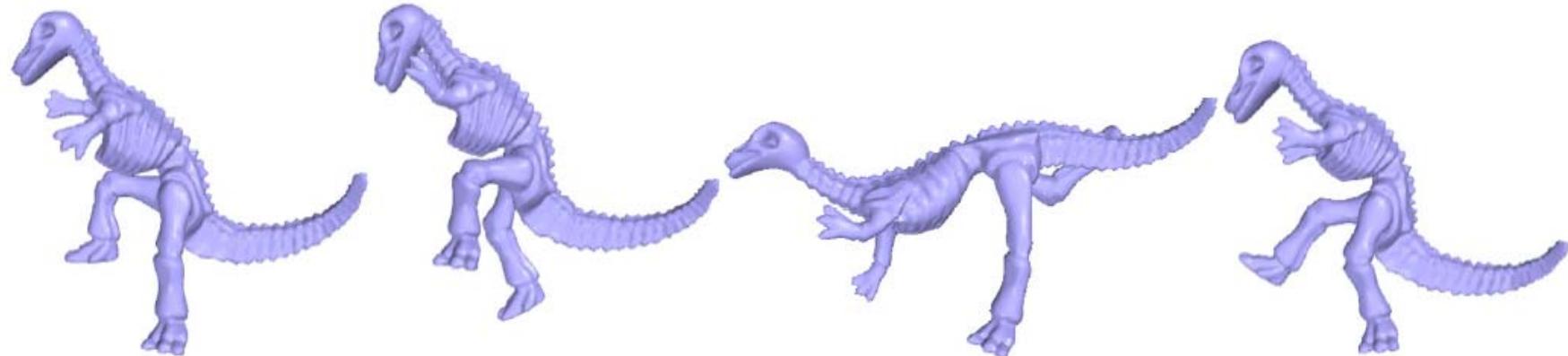
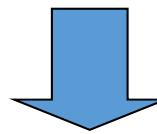
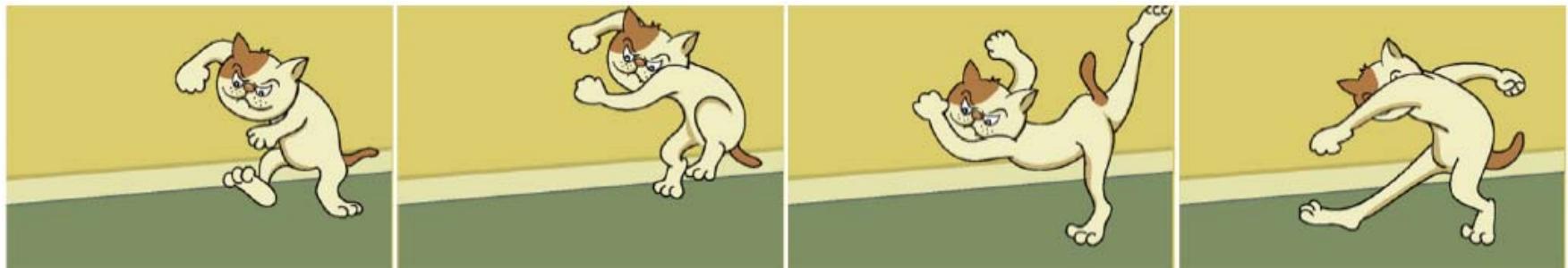


(b) Poisson mesh



(c) VGL

# Animation Modeling from 2D Cartoons



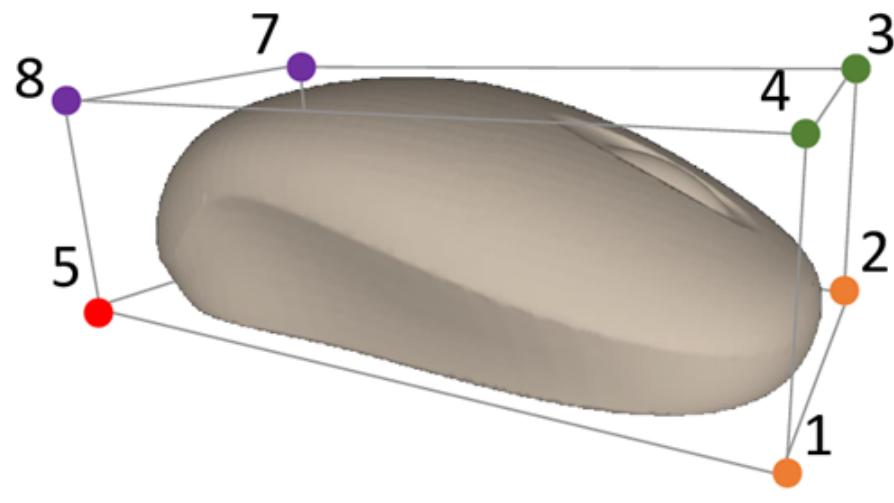
# Short Summary: curve proxy

- Curve features
- Skeletons
- Silhouette
- Free-form sketches
- Wires
- 2D image contours
- ...

# 3. Mesh Proxy

# User Specifications

- Define a mesh proxy for the shape: generally a simpler, coarse mesh which encloses the shape
  - The shape is embedded into the mesh proxy
- Deform the mesh proxy using point/line base methods
- Advantages:
  - You can deform arbitrary objects
  - Independent of object representation

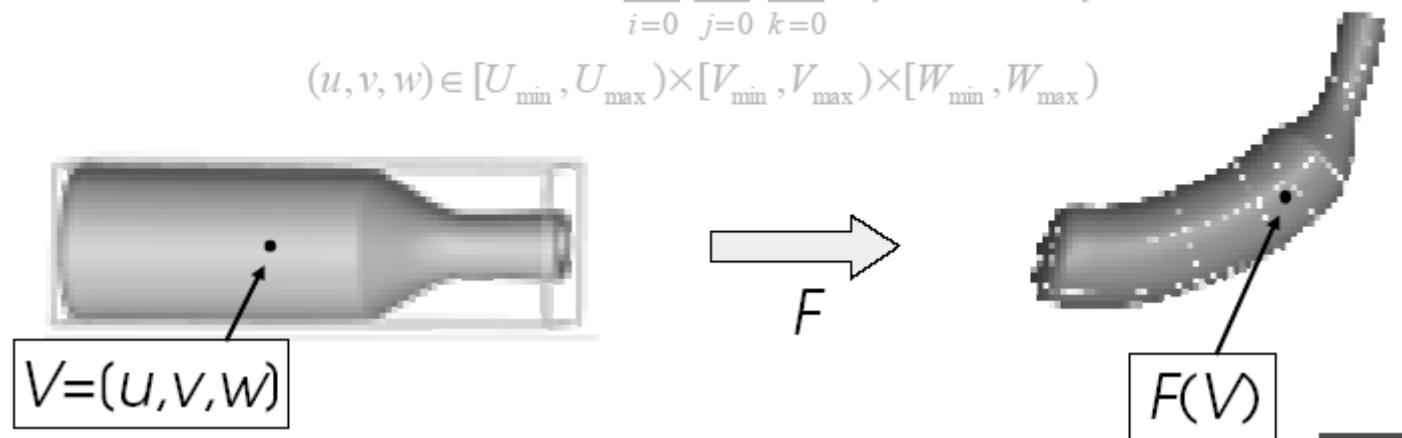


# 3.1 Free-form Deformation (FFD)

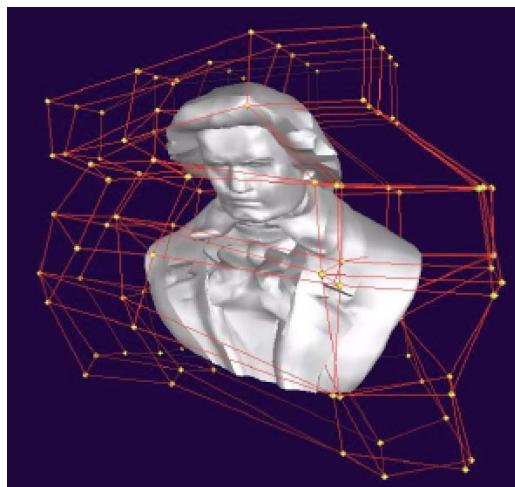
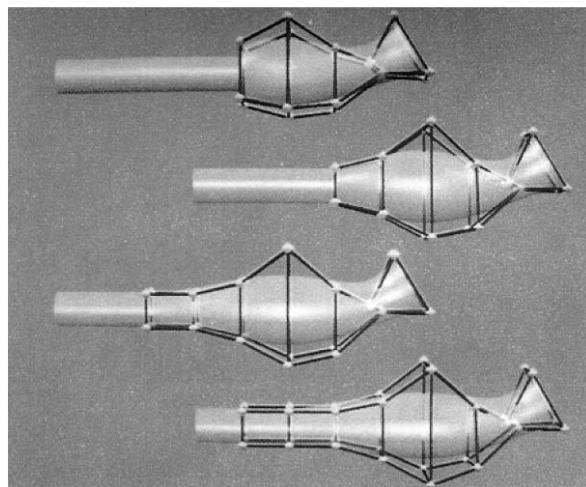
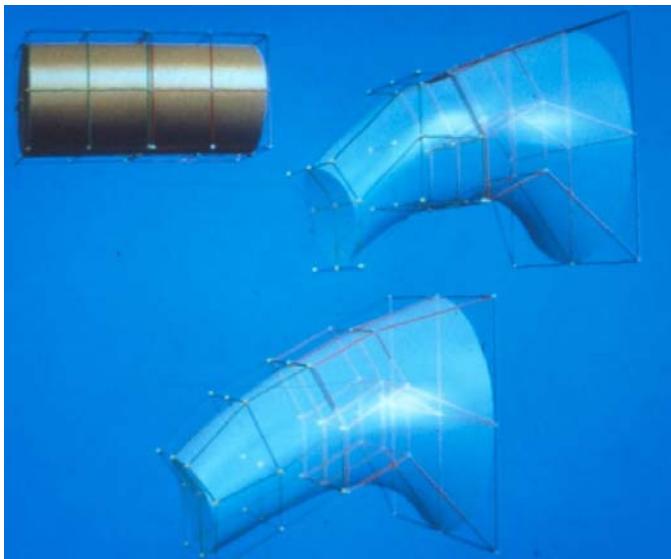
[Sederberg et al. 86]

- Proxy: a **lattice** mesh
  - more easily parametrized than the object
  - Deformation defined by the tensor Bezier solid

$$X_{new} = F(u, v, w) = \sum_{i=0}^l \sum_{j=0}^m \sum_{k=0}^n P_{ijk} B_i(u) B_j(v) B_k(w)$$
$$(u, v, w) \in [U_{min}, U_{max}] \times [V_{min}, V_{max}] \times [W_{min}, W_{max}]$$

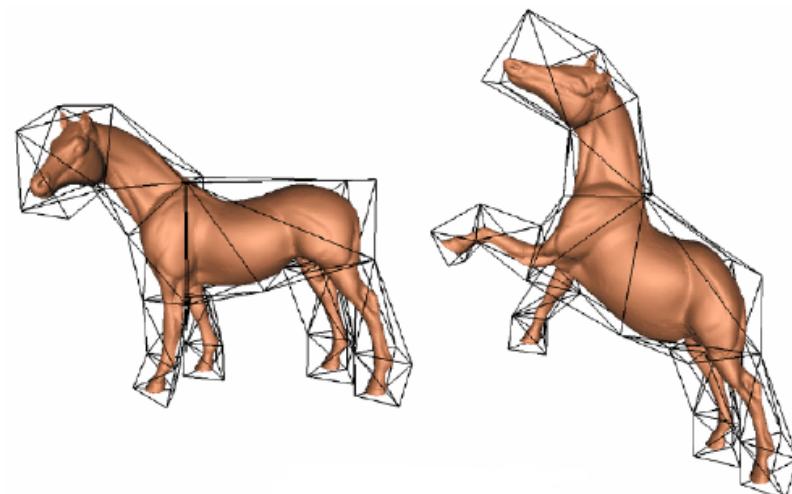


# FFD Examples

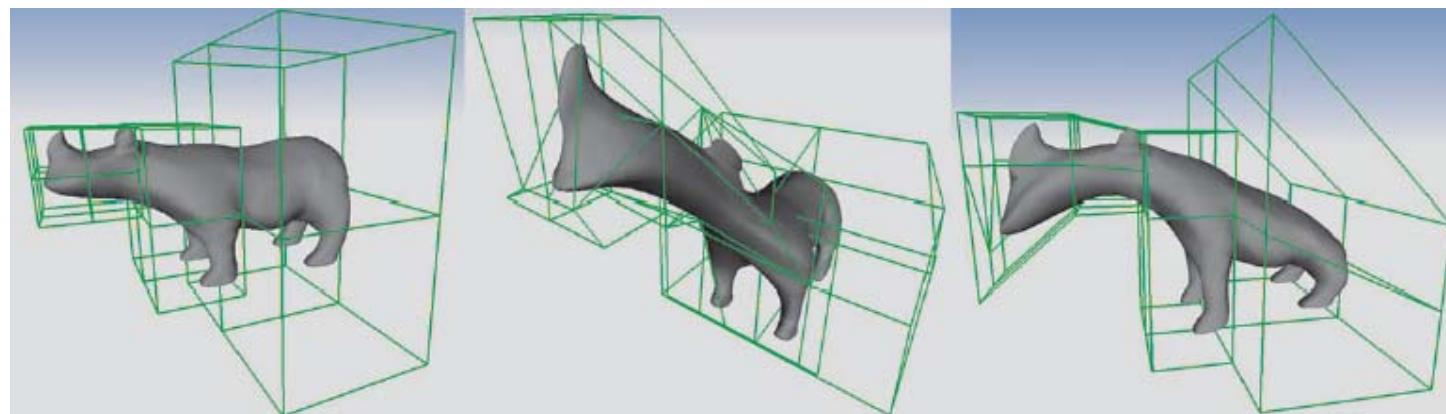


## 3.2 Cage-based deformation

- Proxy: cage
  - Barycentric coordinates
  - Many works...

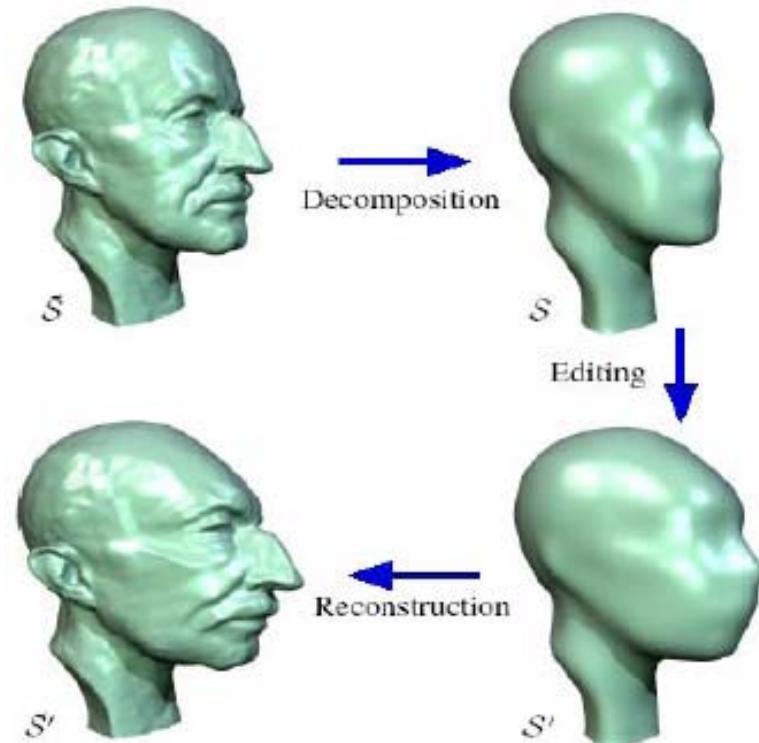


[Ju et al. 2005]

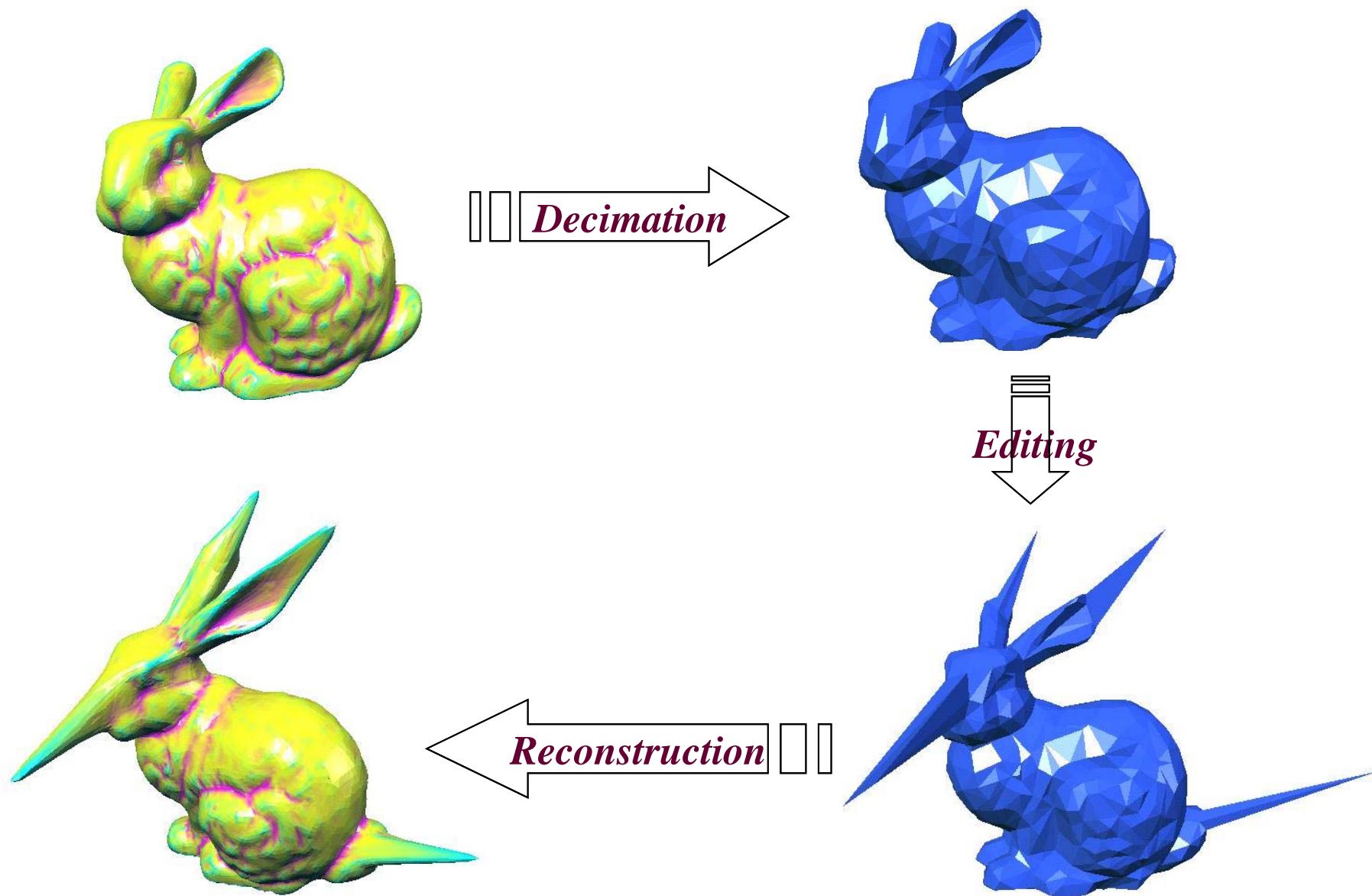


### 3.3 Multiresolution Editing

- Proxy: simplified shape
- Pros
  - Preserving details, scalable
- Cons
  - Instable reconstruction for large deformation
  - Resampling problem
- Invariant variables
  - Detail information



[Kobbelt et al. 2003]



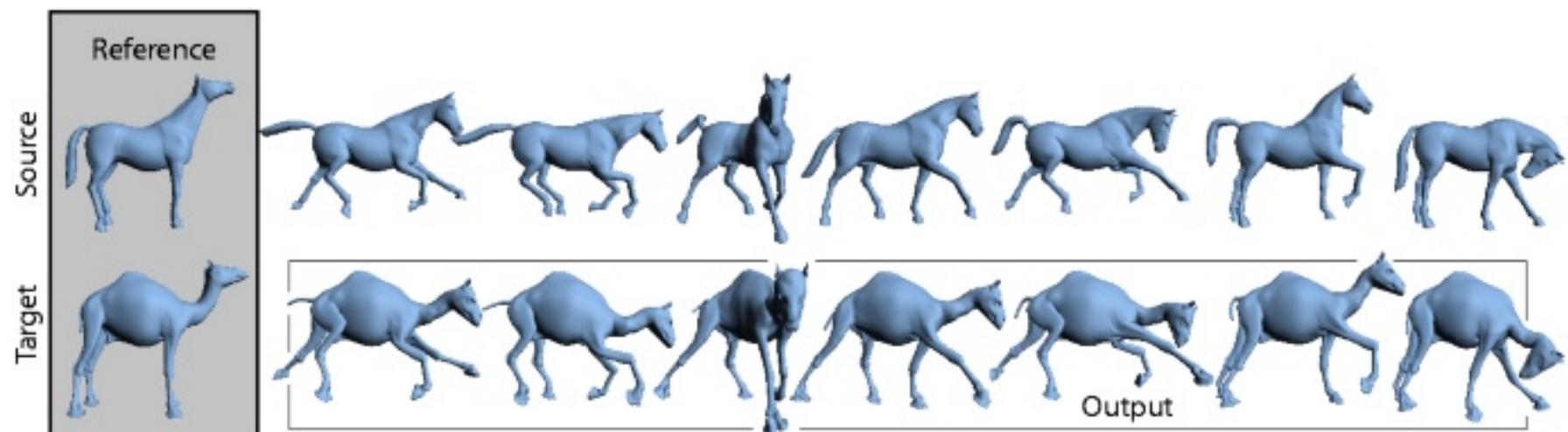
# Short Summary: mesh proxy

- Lattice
- Cage
- Simplified shape
- ...

# 4. Deformation Transfer

# Deformation Transfer

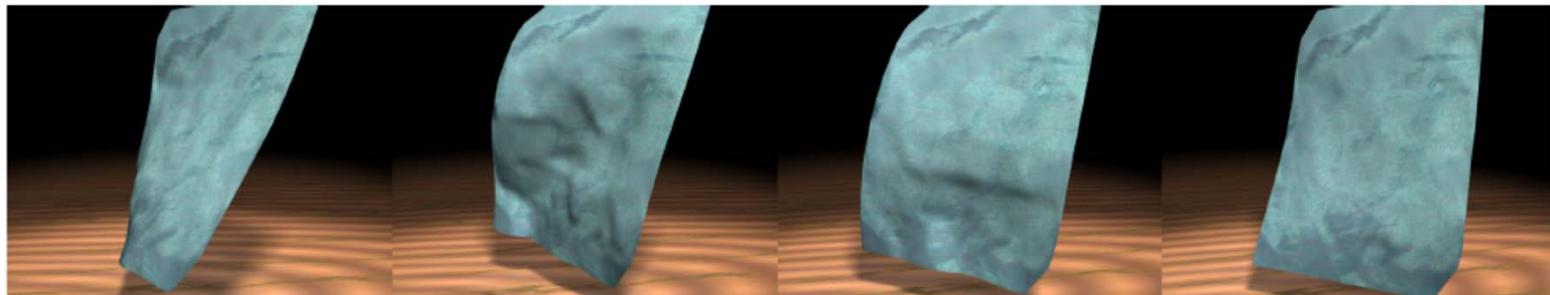
Siggraph 2004



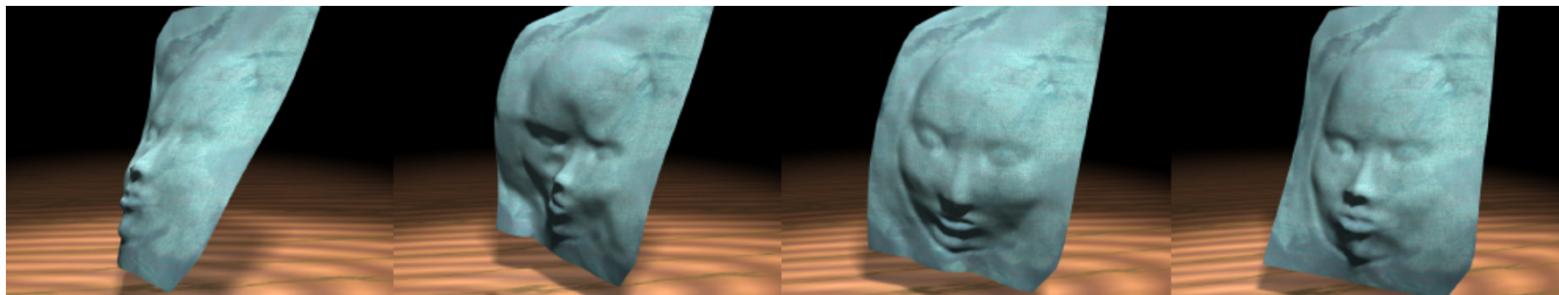
# Editing Arbitrary Deforming Surface Animations

Siggraph 2006

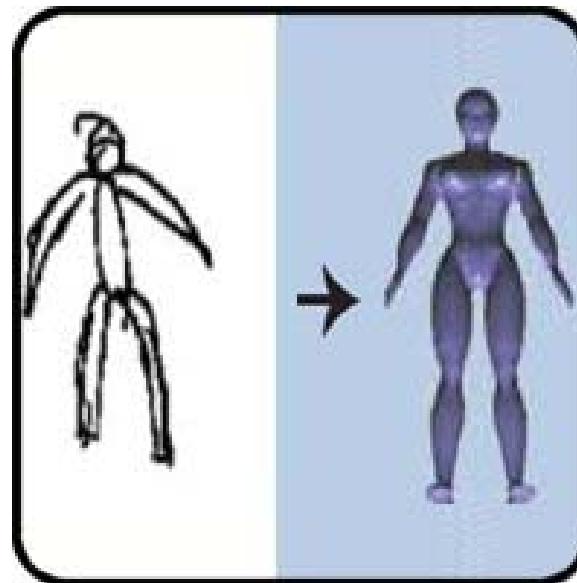
Deforming Surface



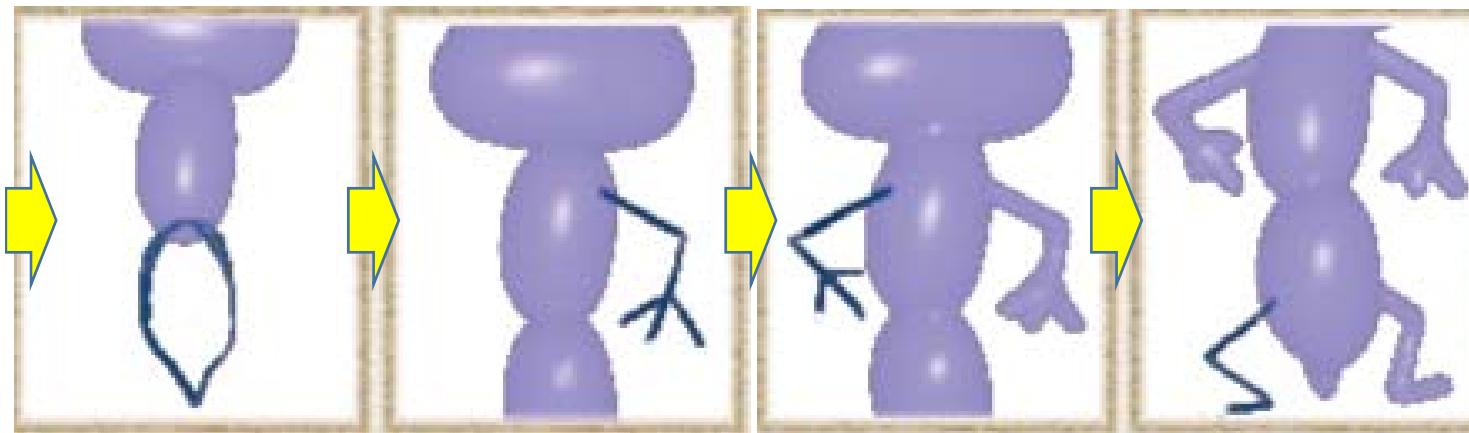
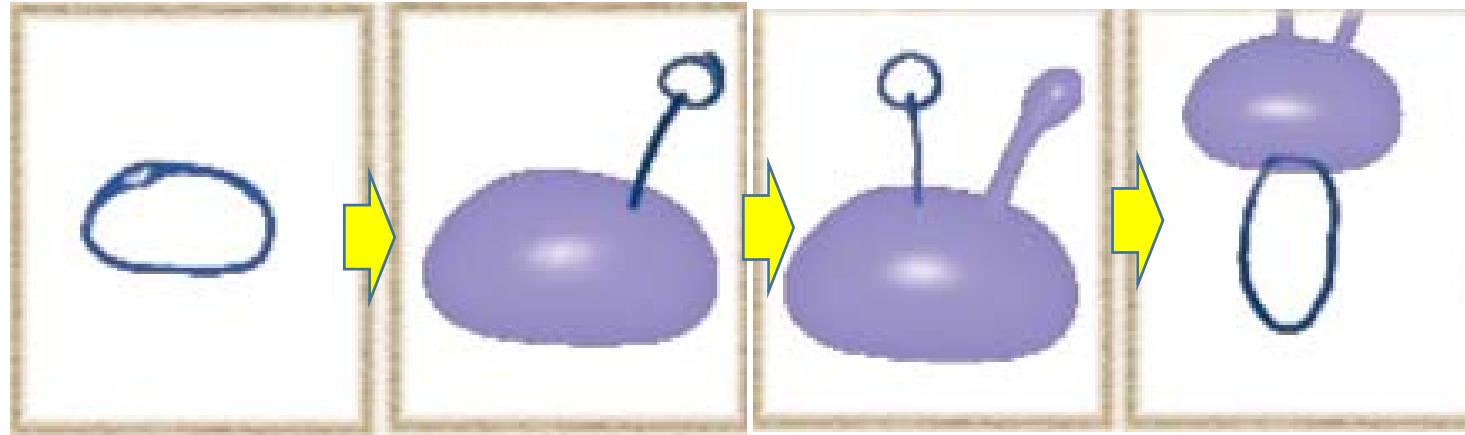
Editing Surface



## 5. Sketch-based Modeling (基于草图的建模)

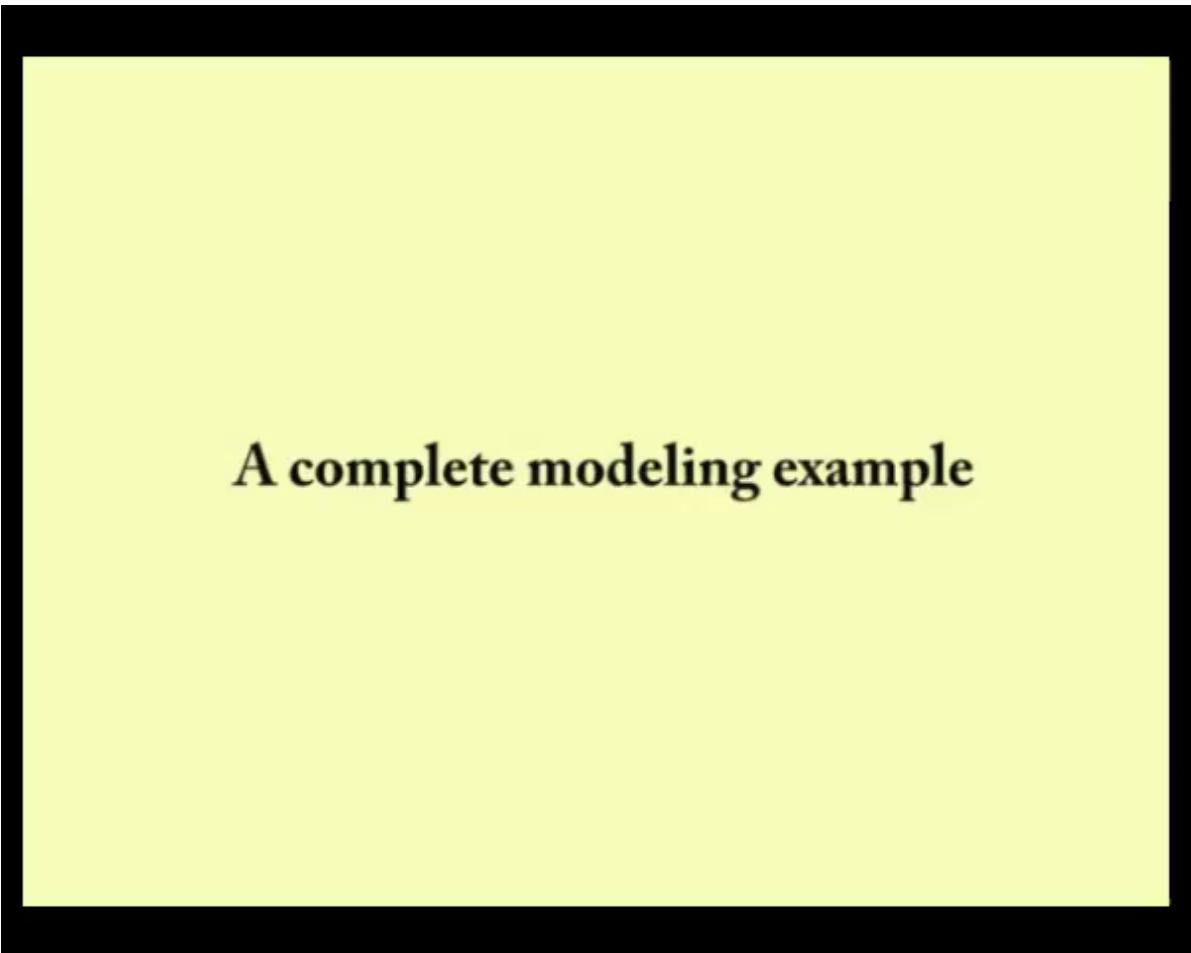


# An Example



# B-Mesh

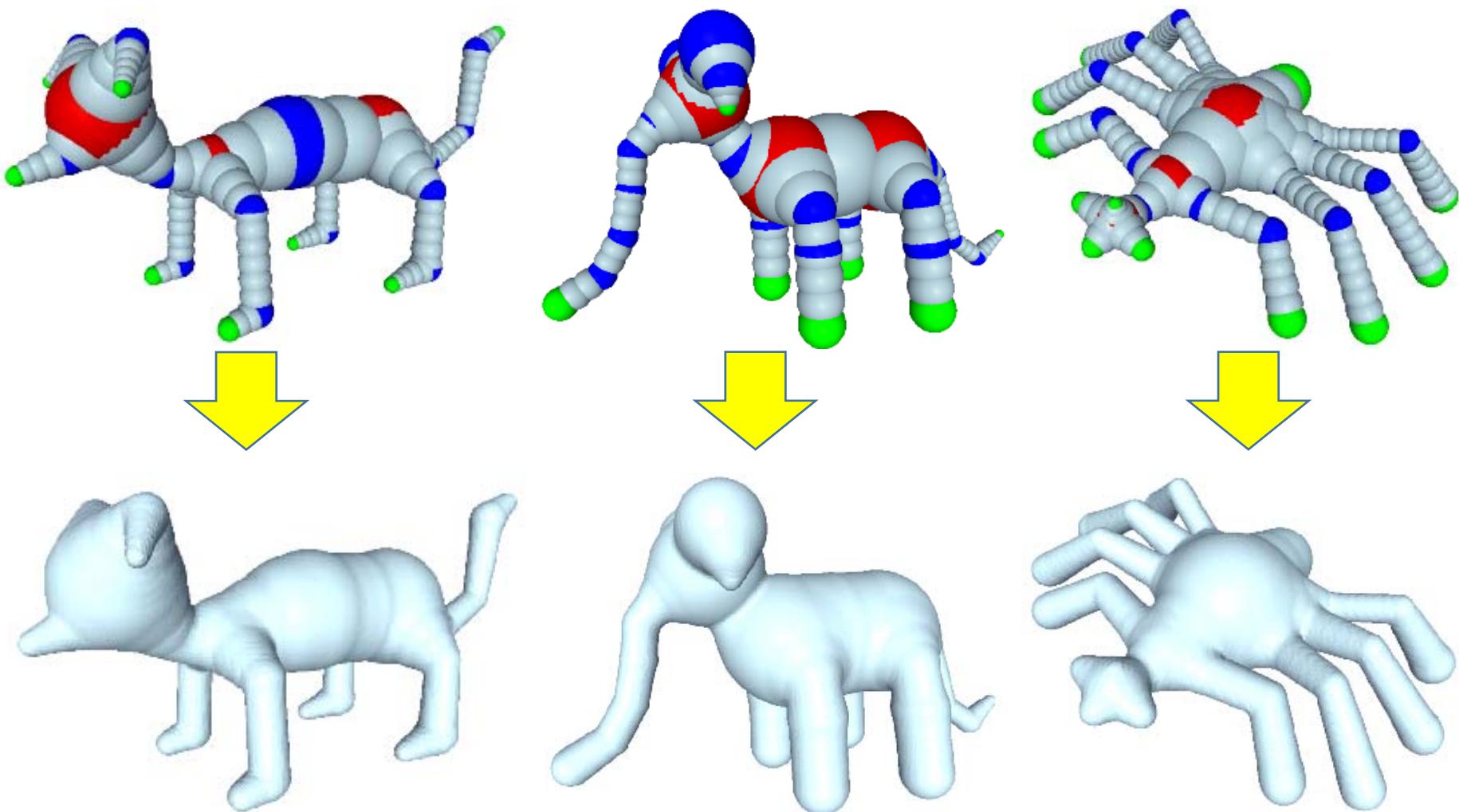
[Ji et al. 2010]



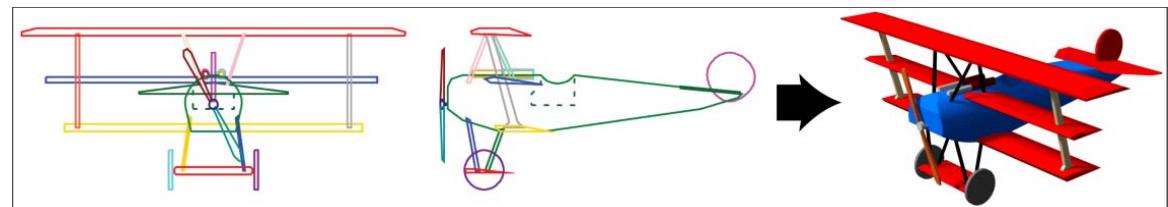
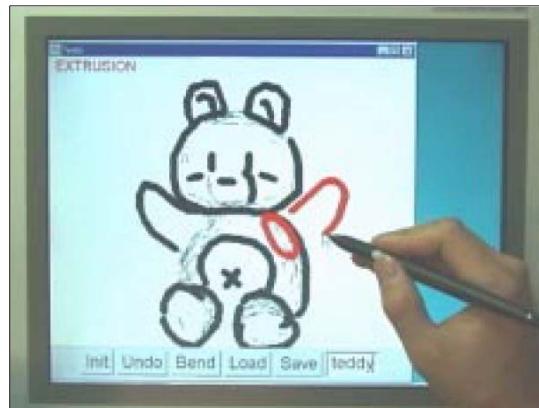
A complete modeling example

# B-Mesh

[Ji et al. 2010]

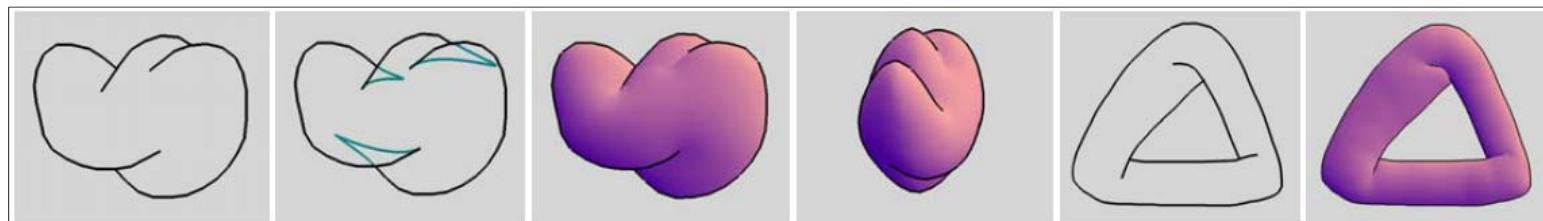


# Sketch-based Modeling Systems



[Rivers et al. 2010]

[Igarashi et al. 1999] ●



[Karpenko et al. 2006]

# More...



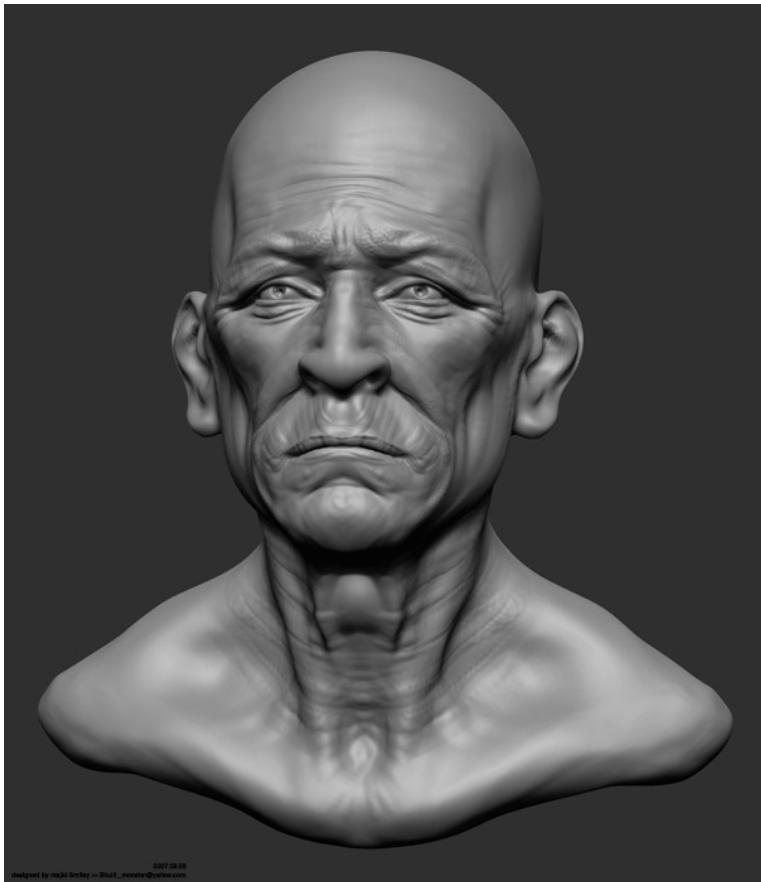
**Sketch2Scene** [Xu et al. 2013]



**Sketch-to-Design** [Xie et al. 2013]

# 6. Sculpturing Modeling

# Highly Detailed Meshes



# Motivation: Sculpturing

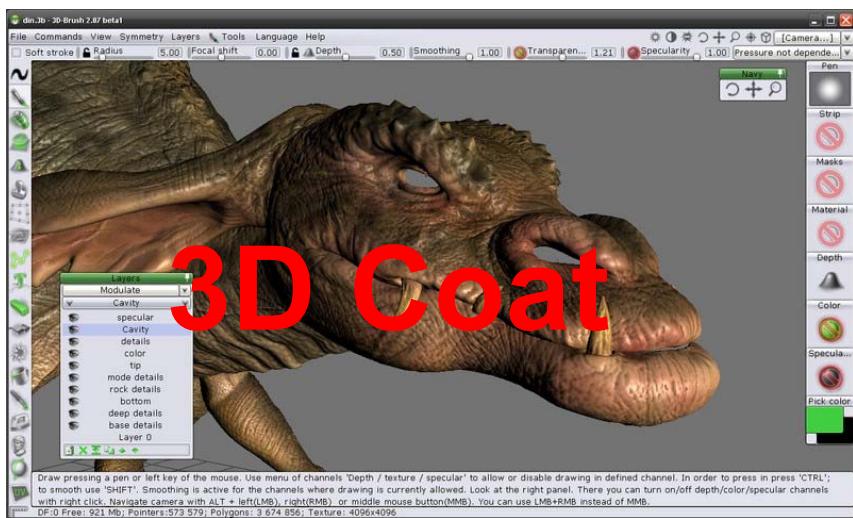


# Sculpturing Modeling

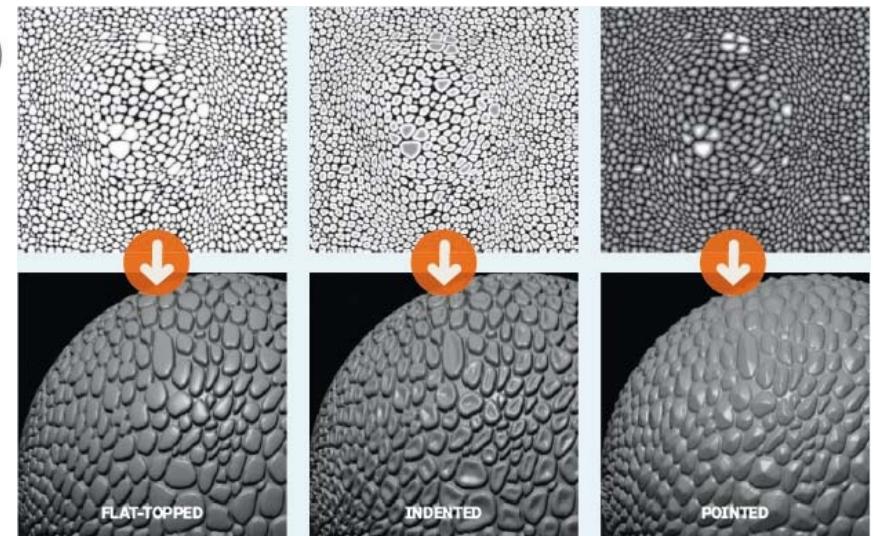
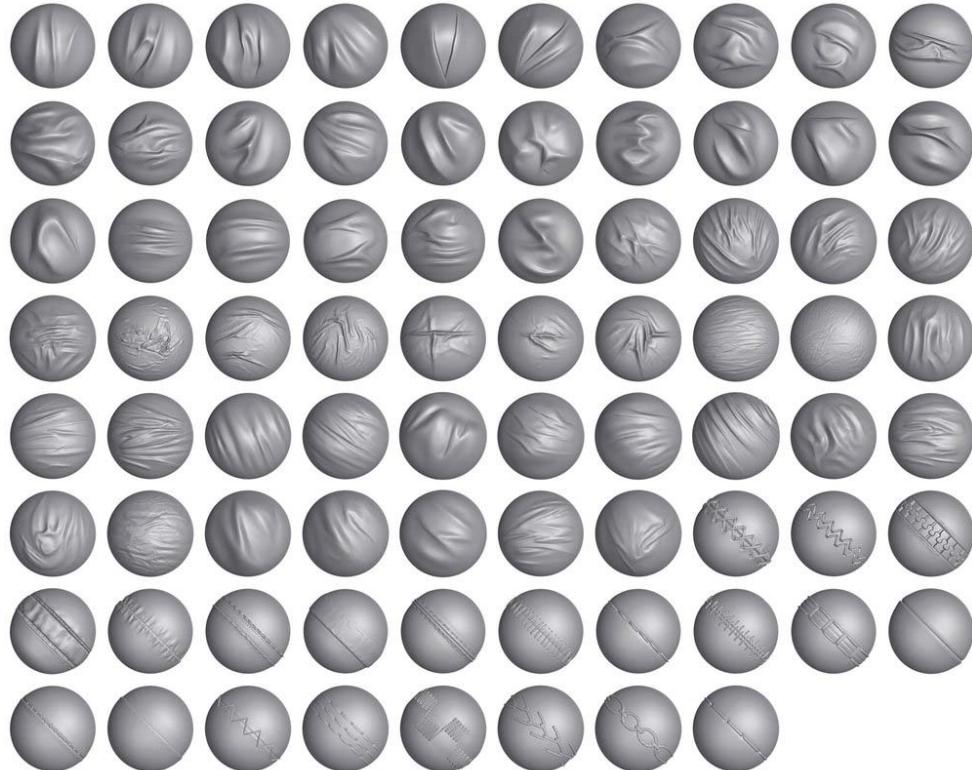
- A series of sculpting/brush operators



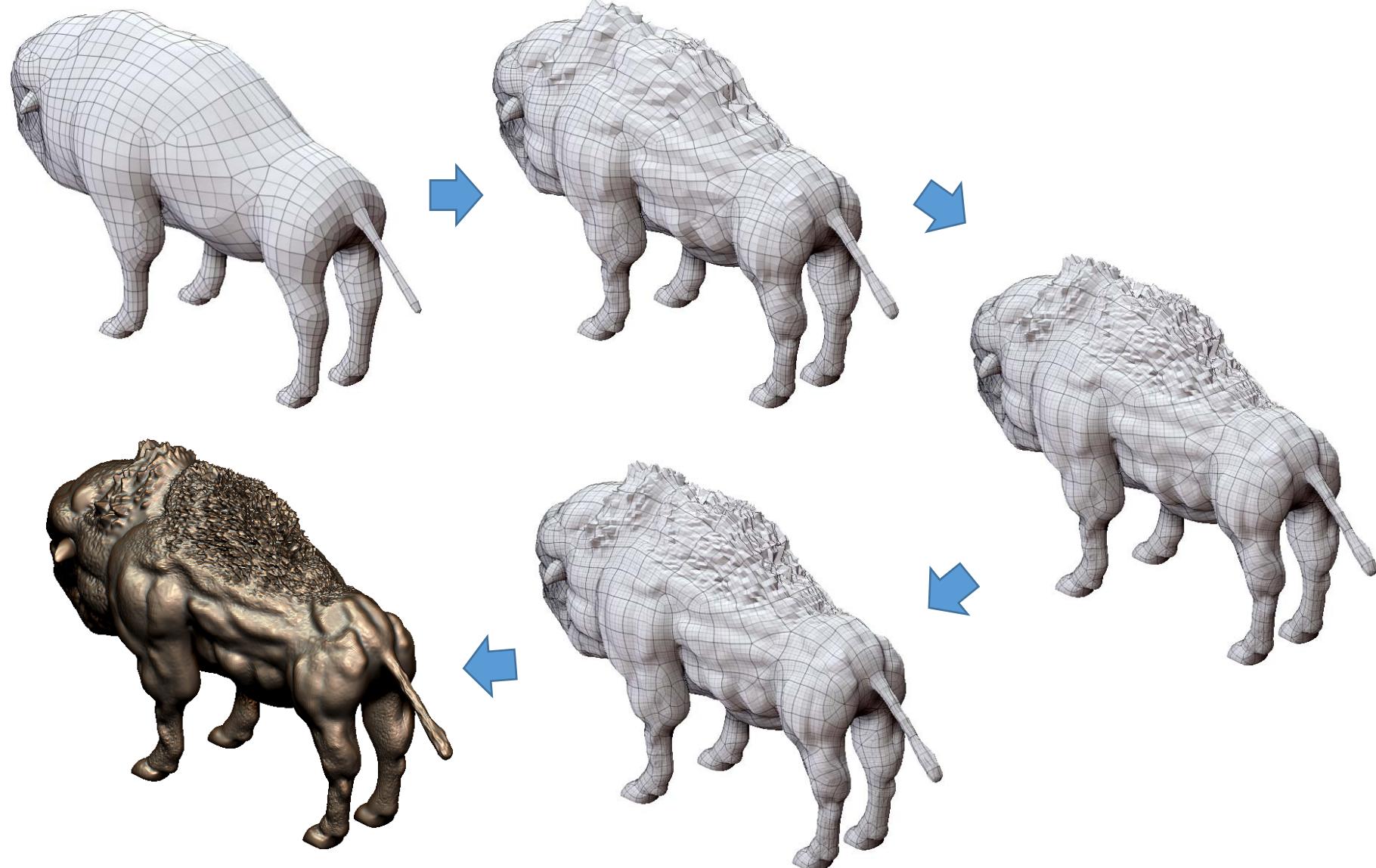
# Sculpting Modeling Tools



# 3D Brushes



# Sculpting Modeling



# Summary: Shape Editing

- Points
  - Points, vertices, ...
- Lines/Curves
  - Sketches, skeletons, silhouettes, wires, ...
- Meshes
  - Bezier nets, lattices, cages, ...
- Other shapes
  - Deformation transfer: learning deformation from other shapes
- Physically-based shape deformation (\*)
  - Material-based, FEM, simulation



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# Data-Driven Modeling

# 三维几何模型数据库



# 1. Modeling by Examples

[Funkhouser et al. Siggraph 2004]

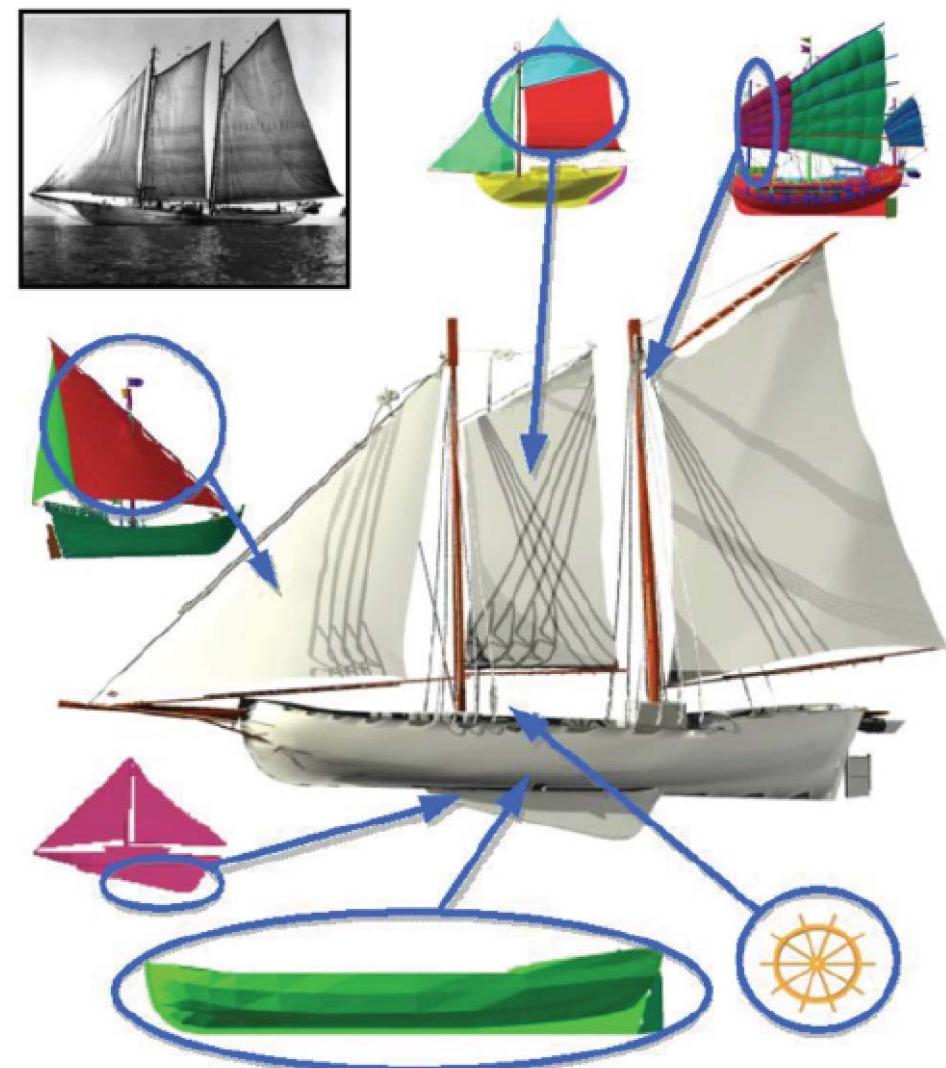
1. Segment into parts
2. Search 3D database for parts
3. Compose parts into a model



# Modeling by Examples

[Funkhouser et al. Siggraph 2004]

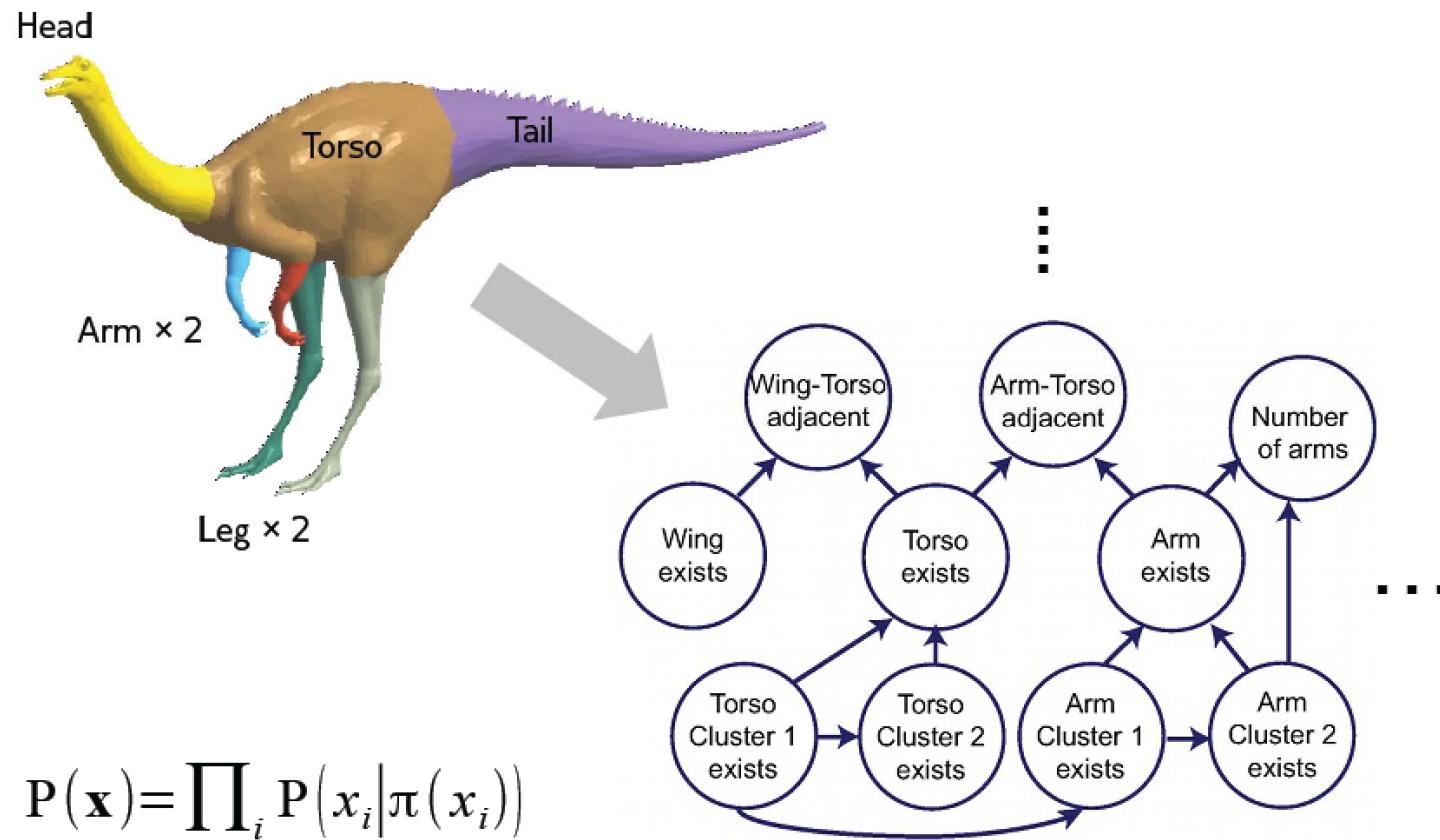
1. Segment into parts
2. Search 3D database for parts
3. Compose parts into a model



# 2. Probabilistic Reasoning

[Chaudhuri et al. Siggraph 2011]

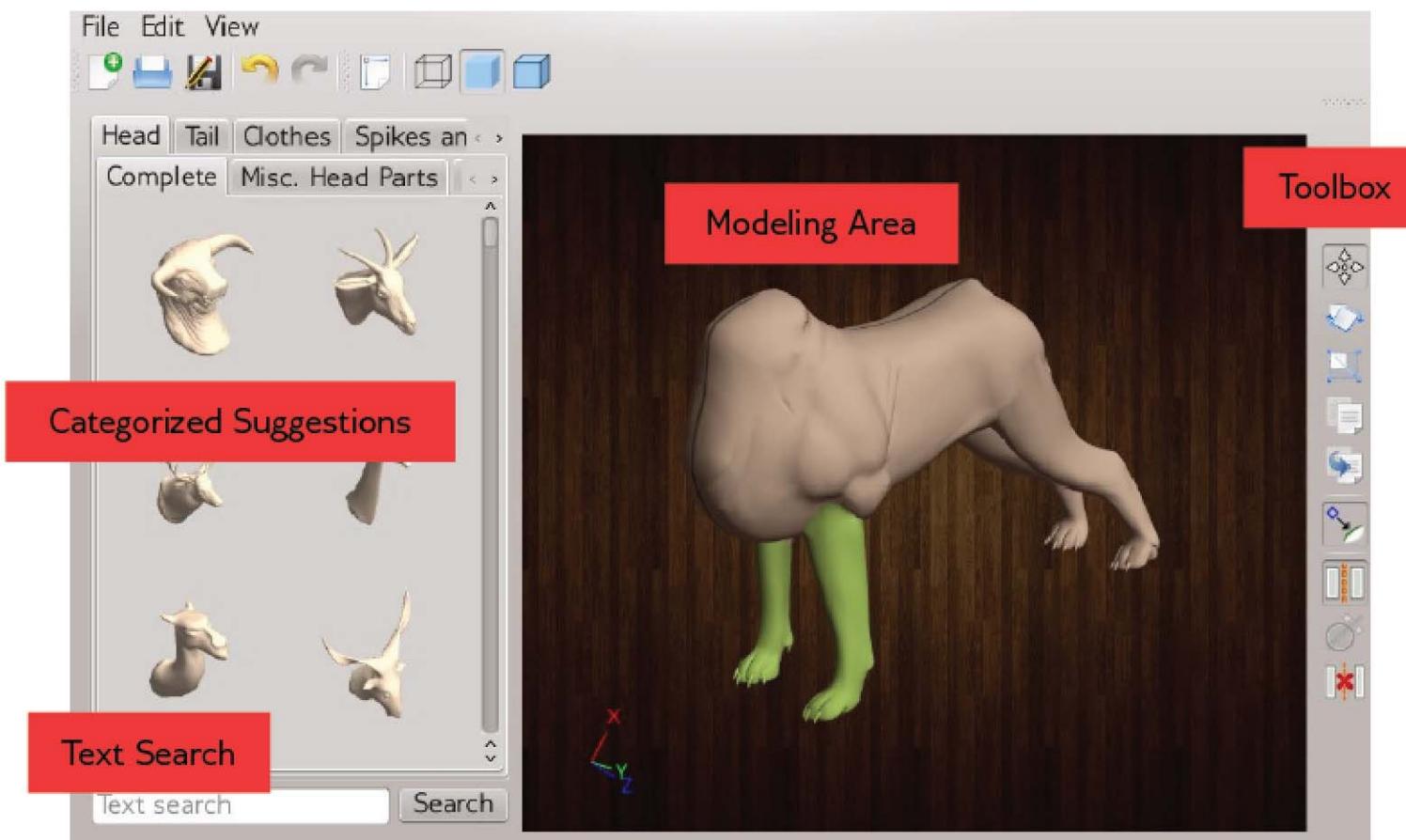
- Design space



# 2. Probabilistic Reasoning

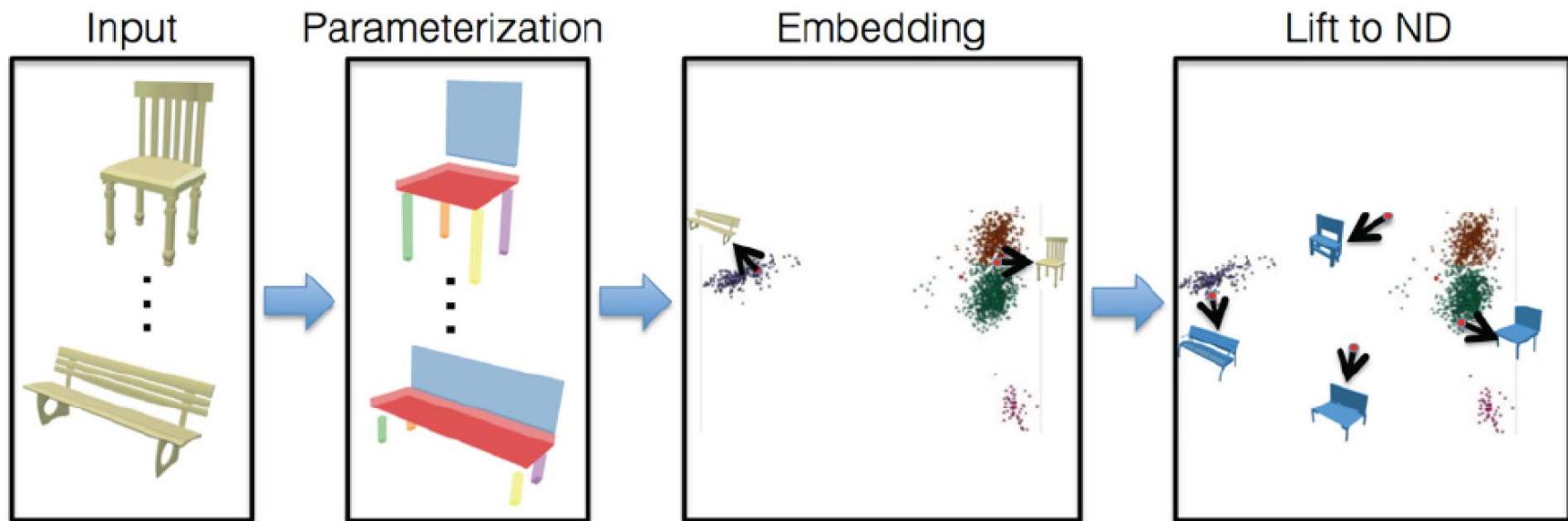
[Chaudhuri et al. Siggraph 2011]

- Exploring the design space



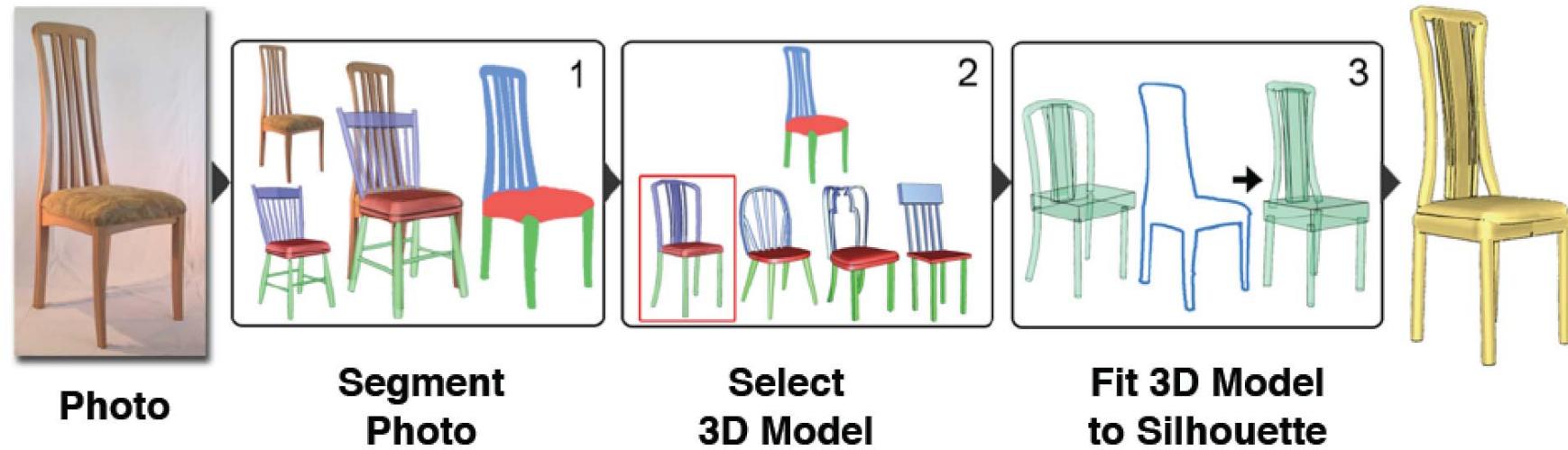
# 3. ShapeSynth: Exploration as Design Tool

[Averkiou et al. Siggraph 2014]



# 4. Photo-inspired Modeling

[Xu et al. Siggraph 2012]

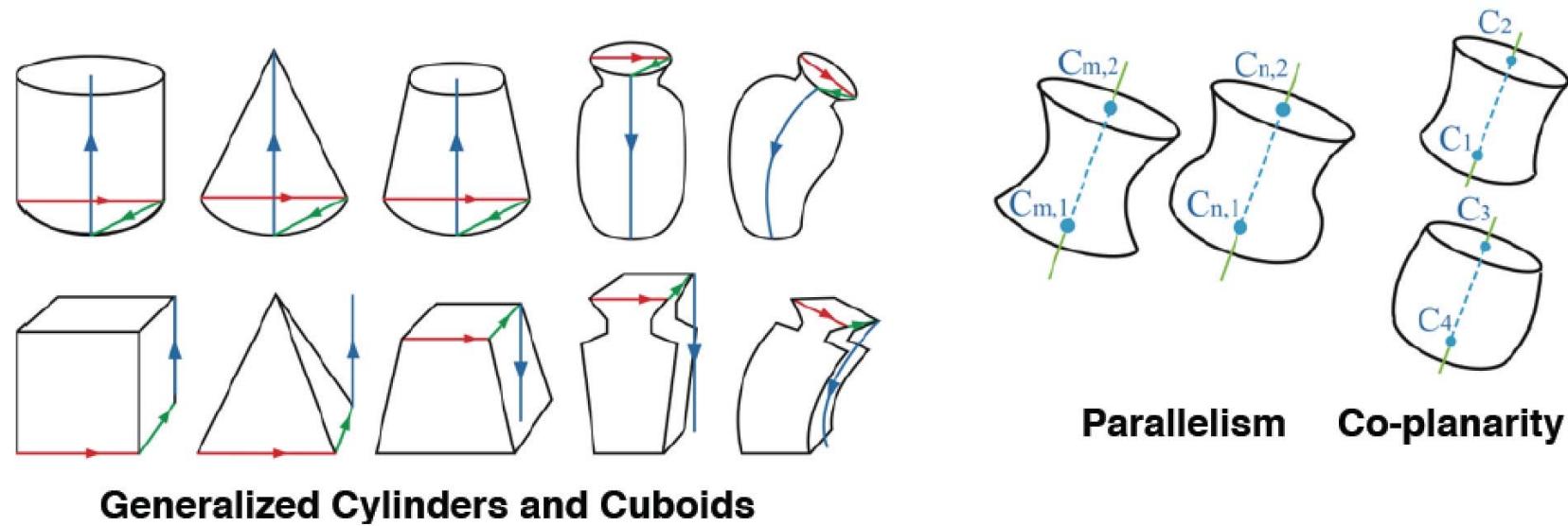
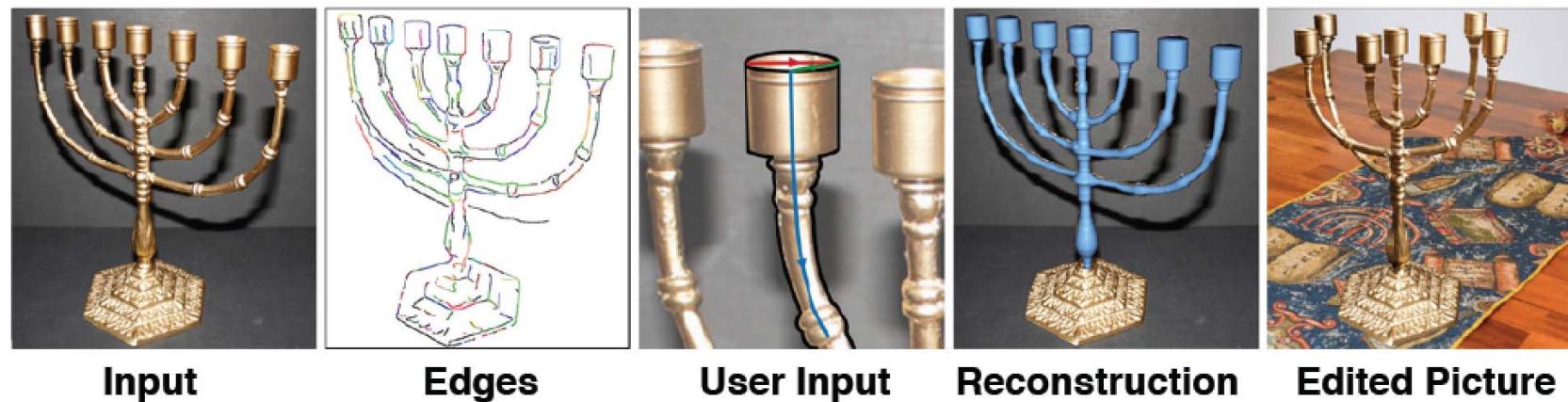


# Examples



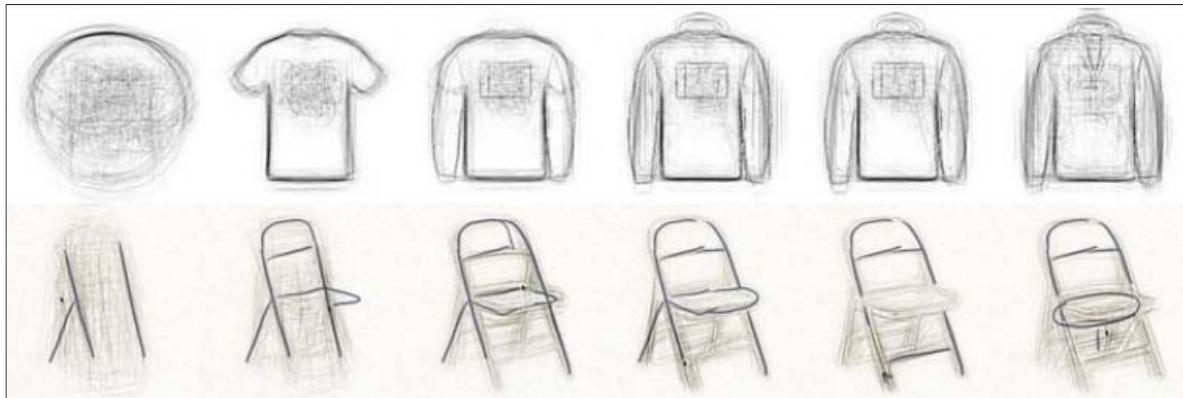
# 5. Primitive-based Modeling

Chen et al. 2013

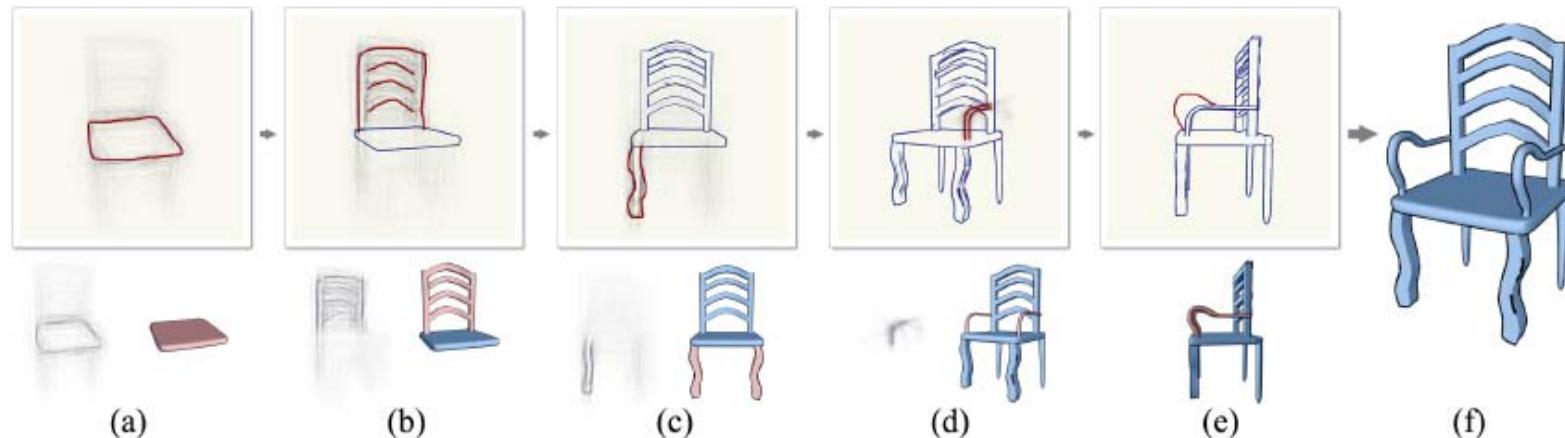


# 6. Shadow Guidance

Fan et al. 2013

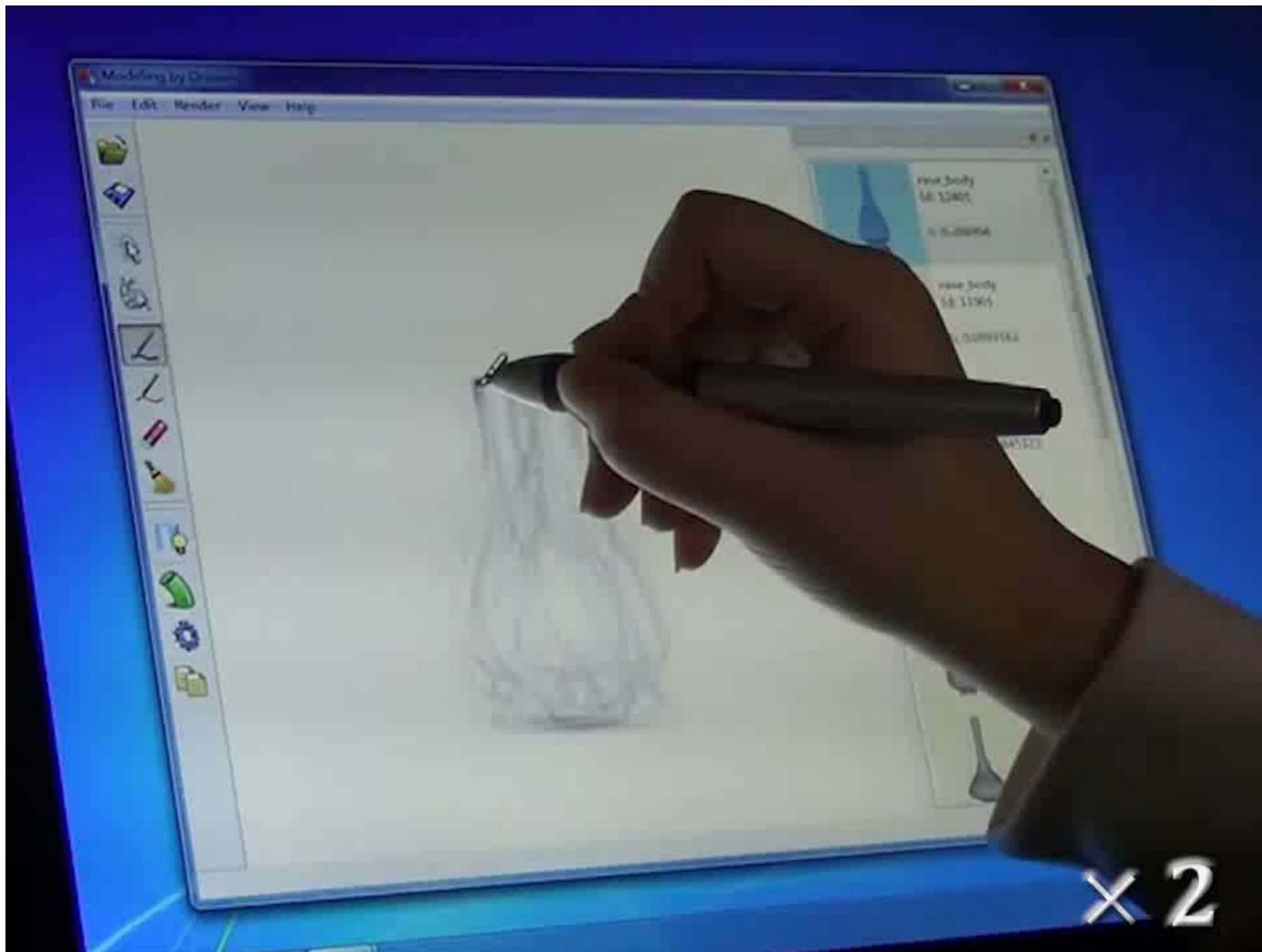


**ShadowDraw** [Lee et al. 2011]



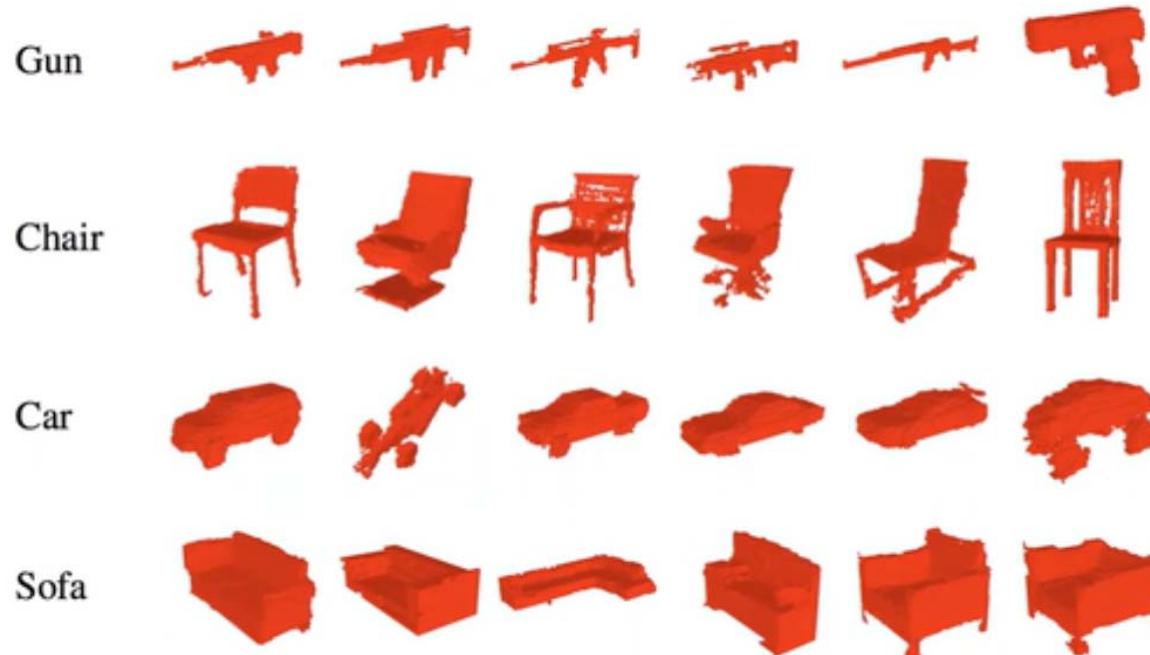
# 6. Shadow Guidance

Fan et al. 2013



# 7. Learning Generative Models of 3D Shapes (3D-GAN)

- Emergence of various 3D-GANs for shape generation



3D volumetric shapes generated from random latent vectors

Chinagraph 2020 Key note talk by Hao Zhang:  
<https://www.bilibili.com/video/BV1Pa4y1x7au?p=3>



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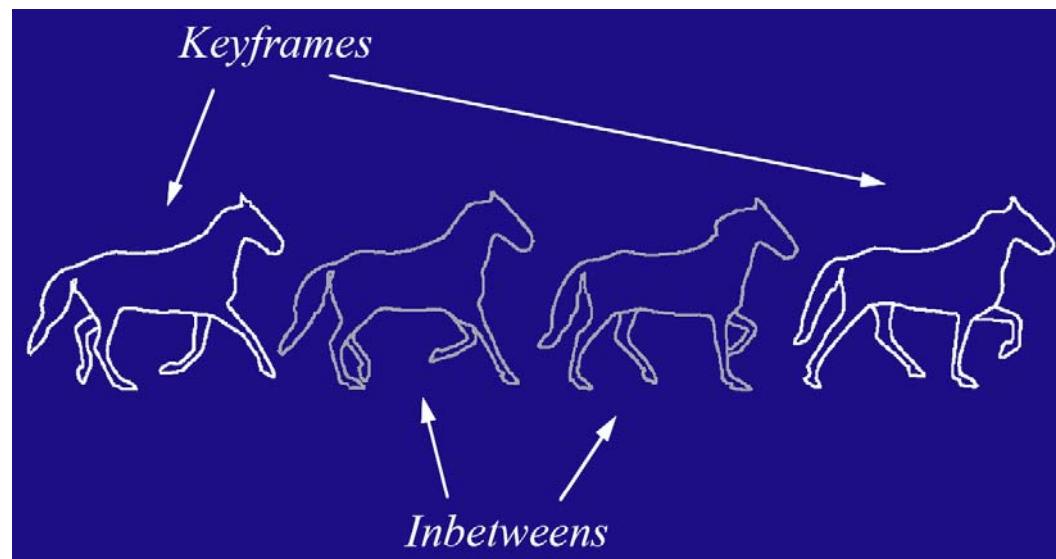
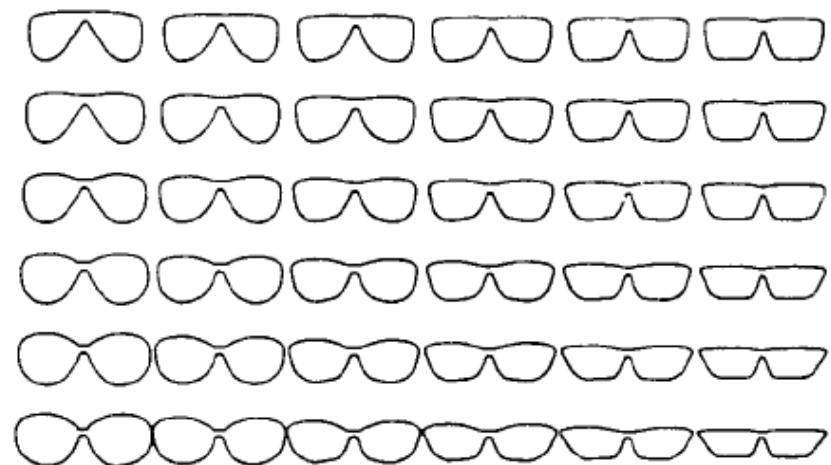
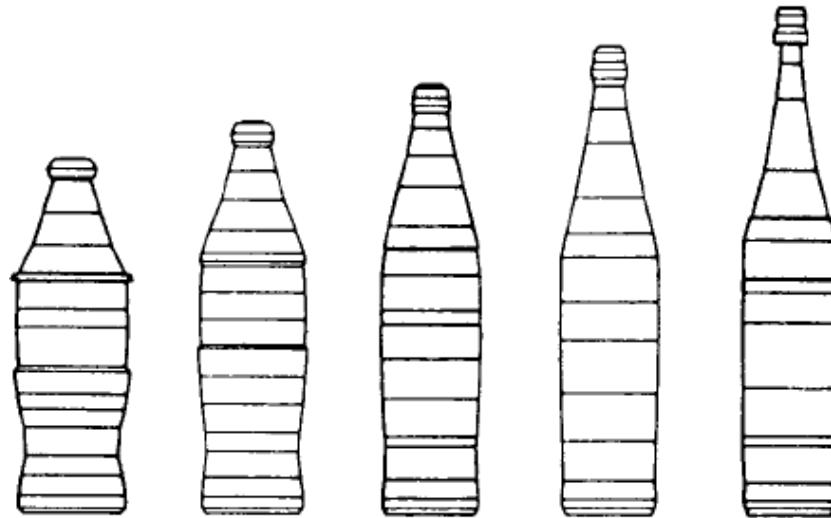
# Shape Morphing

# Shape Morphing

- Given two objects produce sequence of intermediate objects that gradually evolve from one object to the other
  - Interpolate object shapes
  - Interpolate object attributes
    - Color, texture, normal, etc.

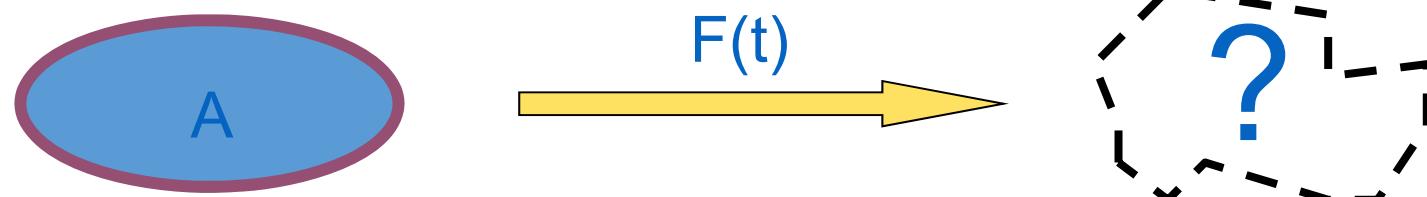


# Applications

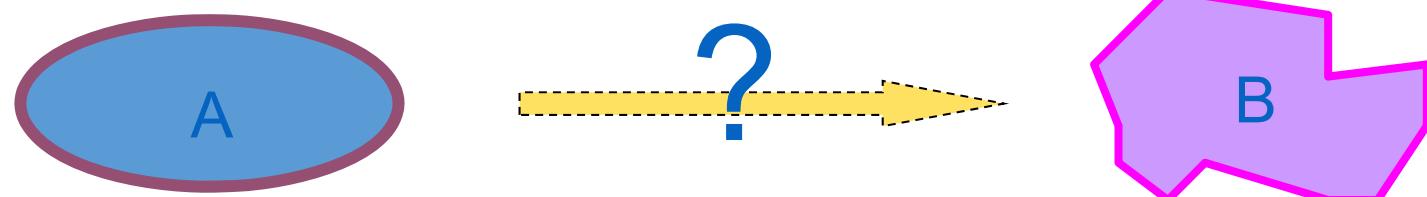


# Shape Transformations

- Warping: Unary Op
  - Given Object A and  $F(t)$ , find Object B



- Morphing: Binary Op
  - Given Object A and Object B, find  $F(t)$



# Terminologies

- Morphing
- Metamorphosis
- Shape blending
- Shape averaging
- Shape interpolation
- Shape transition

# Rules for Good Morphing

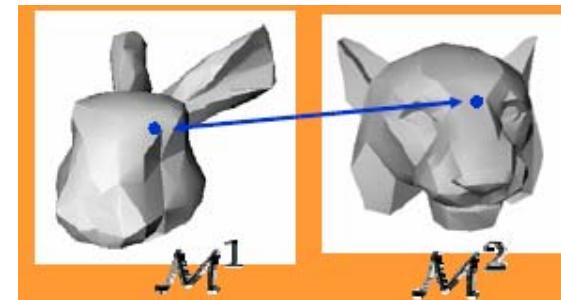
- Natural
  - Keep as much as possible of the two shapes during the transformation
    - Volume, curvature, area, etc...
    - Subjective aesthetic criteria
- User control
  - intuitive
  - not too heavy
  - can be adapted to user's knowledge

# Morphing

- Input: two meshes source & target
  - Frames at  $t_0$  and  $t_n$
- Output: sequence of intermediate meshes
  - Frames  $t_1$  to  $t_{n-1}$
- Intermediate mesh:
  - For each point on source/target model specify location at time  $t_i$  consistent with source & target

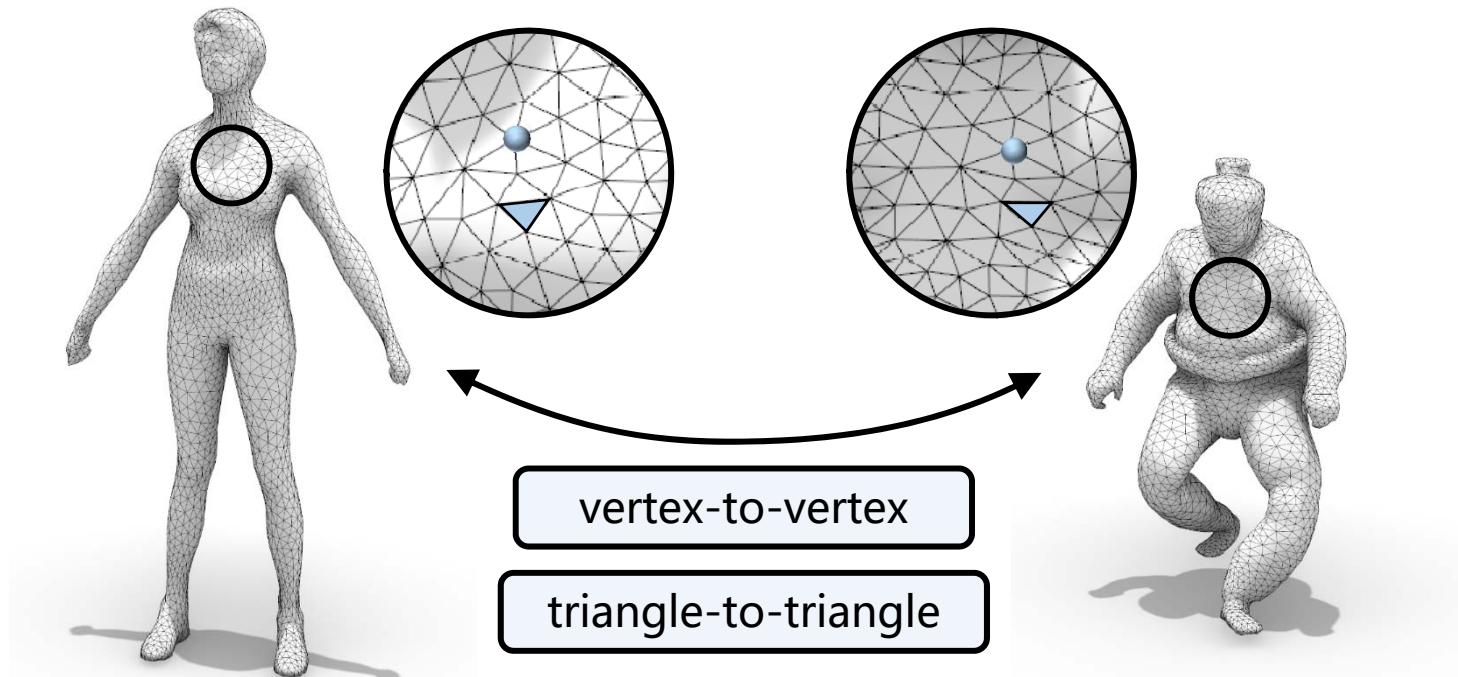
# Morphing: Two Sub-Problems

- Correspondence problem
  - Compatible meshes
  - For each point on source/target meshes find corresponding point on second mesh  
= Parameterization
- Path problem
  - Inbetween shapes
  - Specify trajectory in time for each point
    - For mesh – specify vertex trajectory



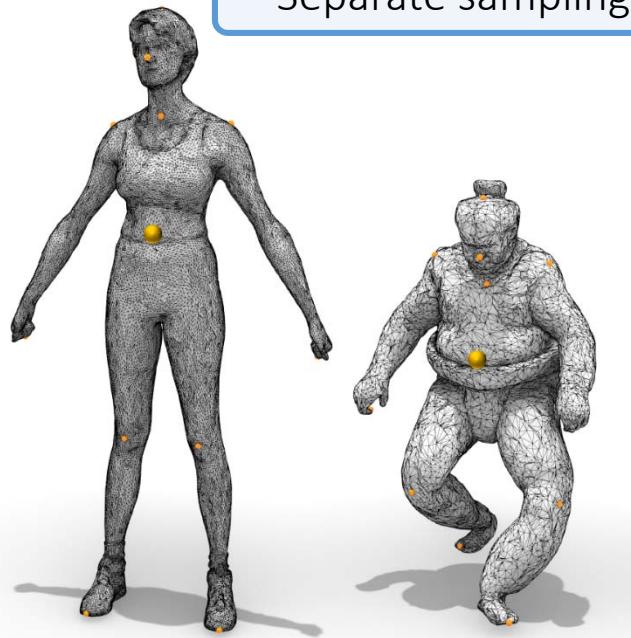
# 1. Vertex Correspondence

- Compatible meshes (consistent meshes, inter-map)
  - Each vertex on source mesh mapped to vertex on target (and vice versa)
  - Have common connectivity



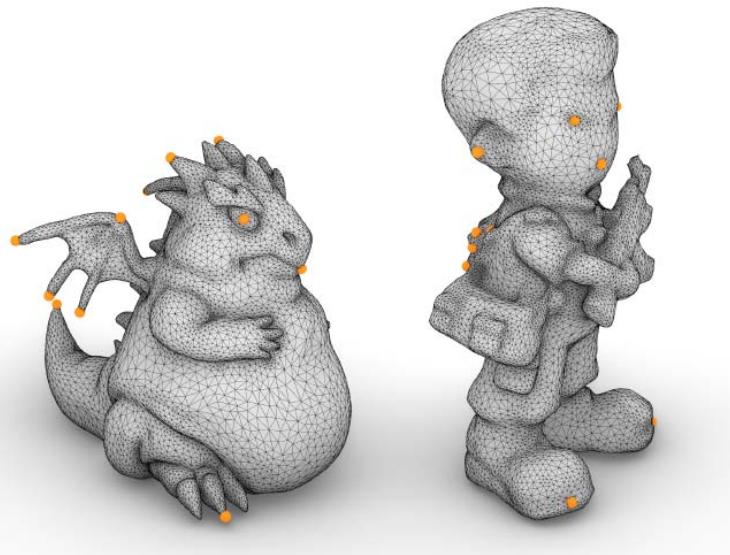
# Challenges

Separate sampling



Homogeneous models

Different geometries



Heterogeneous models

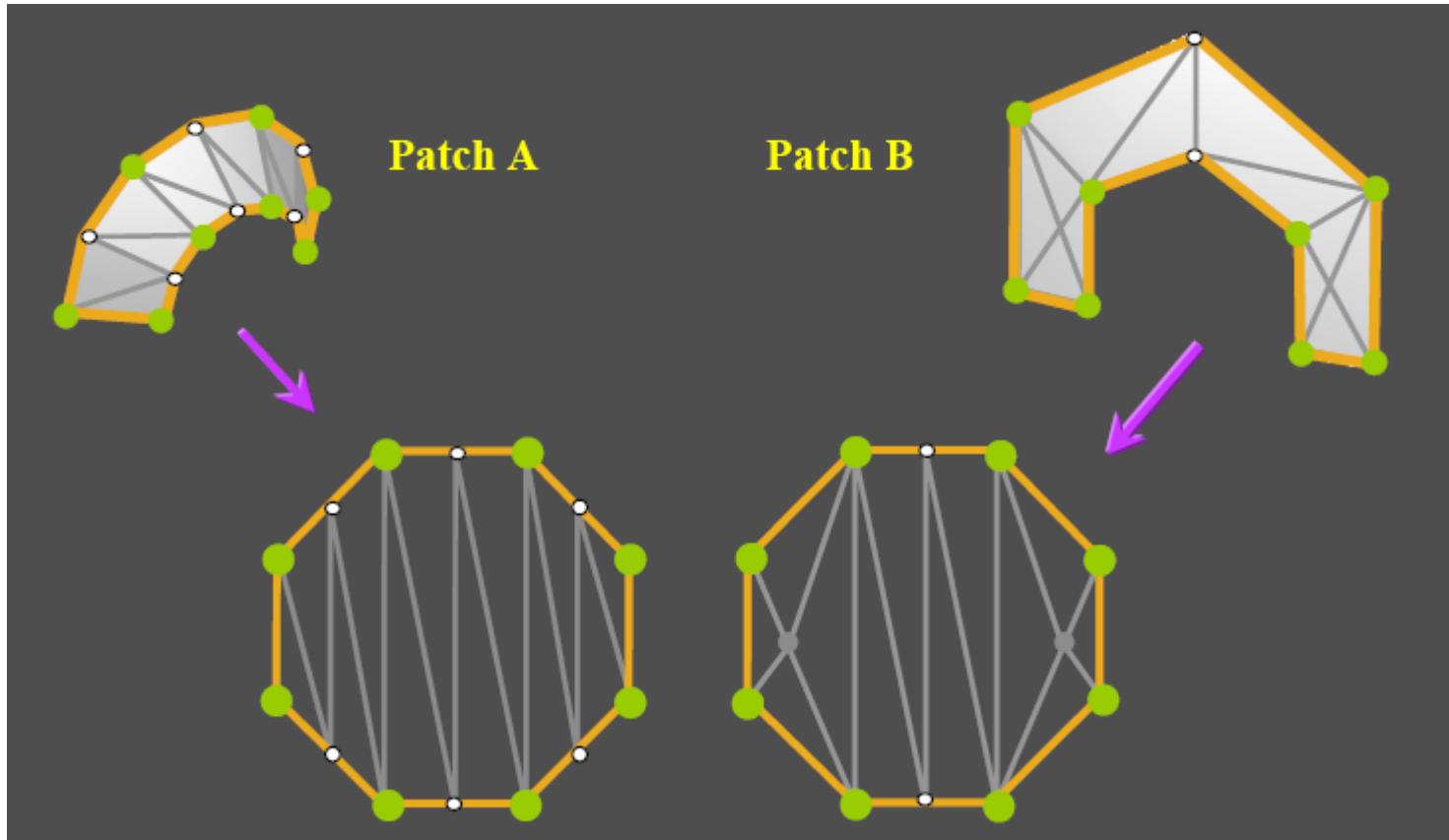
# Correspondence: Parameterization

- To compute map between source mesh S and target mesh T parameterize both on common domain D:

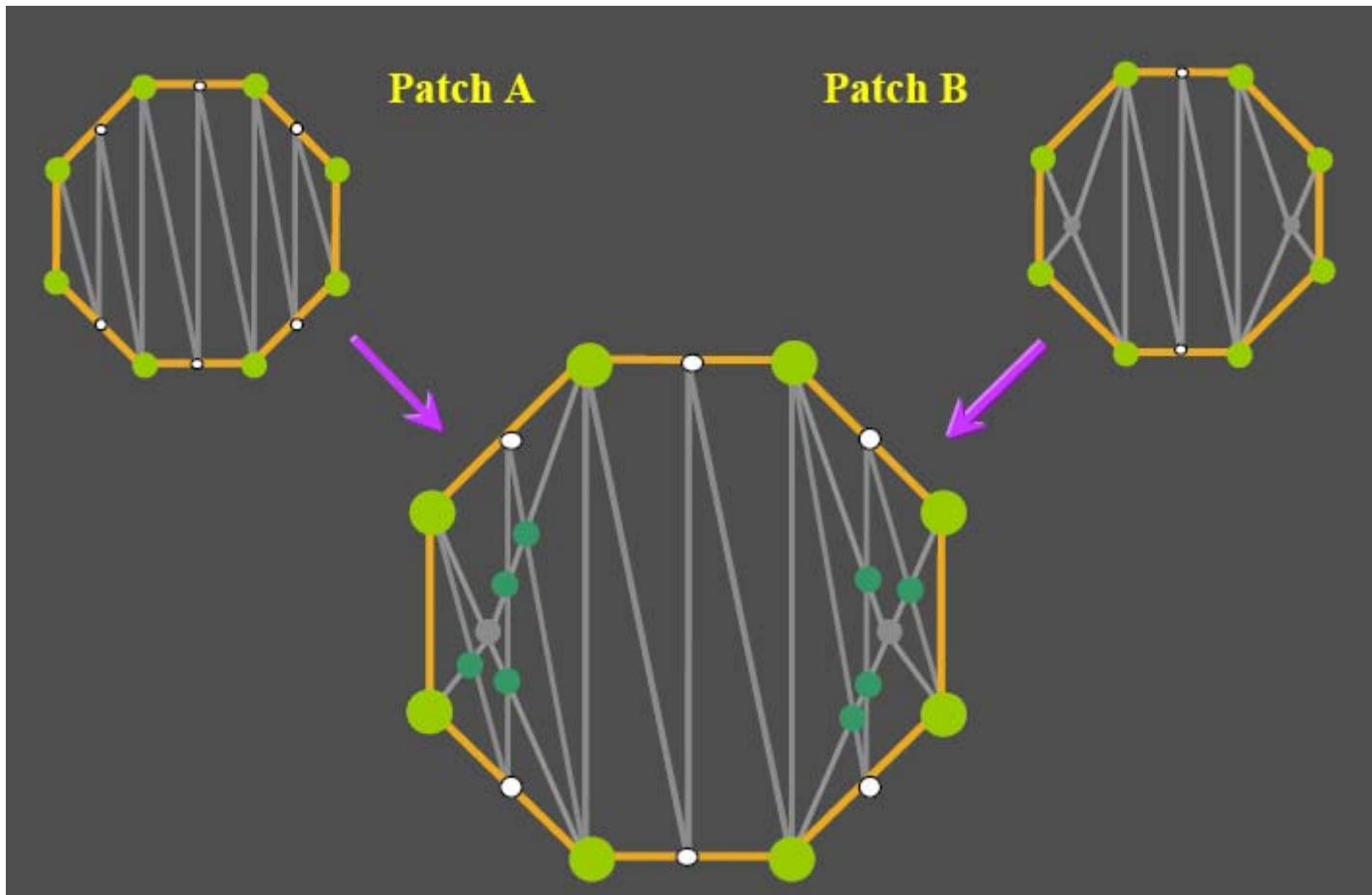
$$\begin{aligned} F_s &: S \rightarrow D \\ F_t &: S \rightarrow D \\ F_{st} &= F_t^{-1}F_s \end{aligned}$$

- Common domain options
  - 2D patch(es) – works for genus 0 + boundary
    - Use convex boundary (why?)
  - Sphere
  - Base mesh

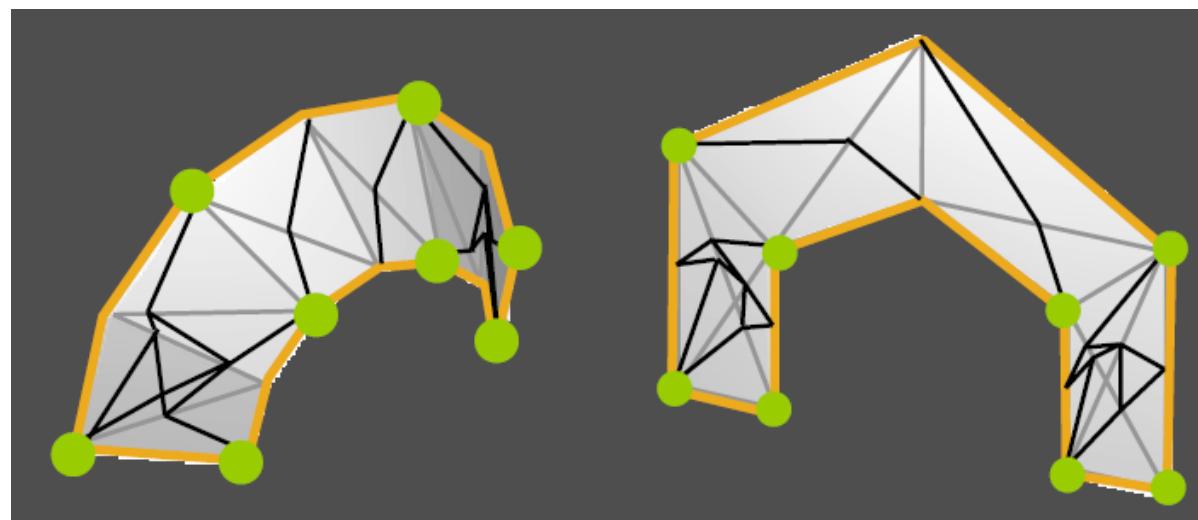
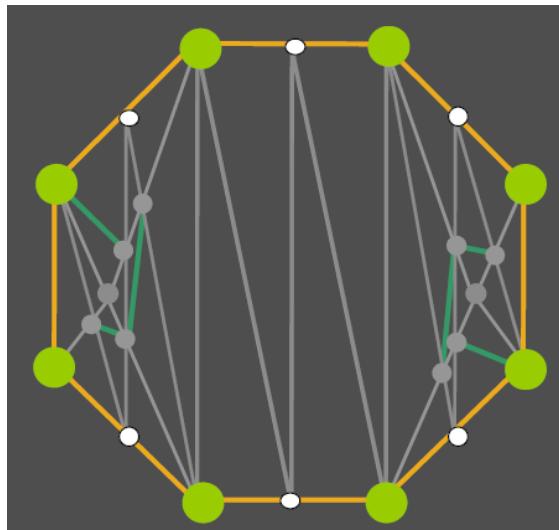
# 1.1 Parameterization: Planar domain



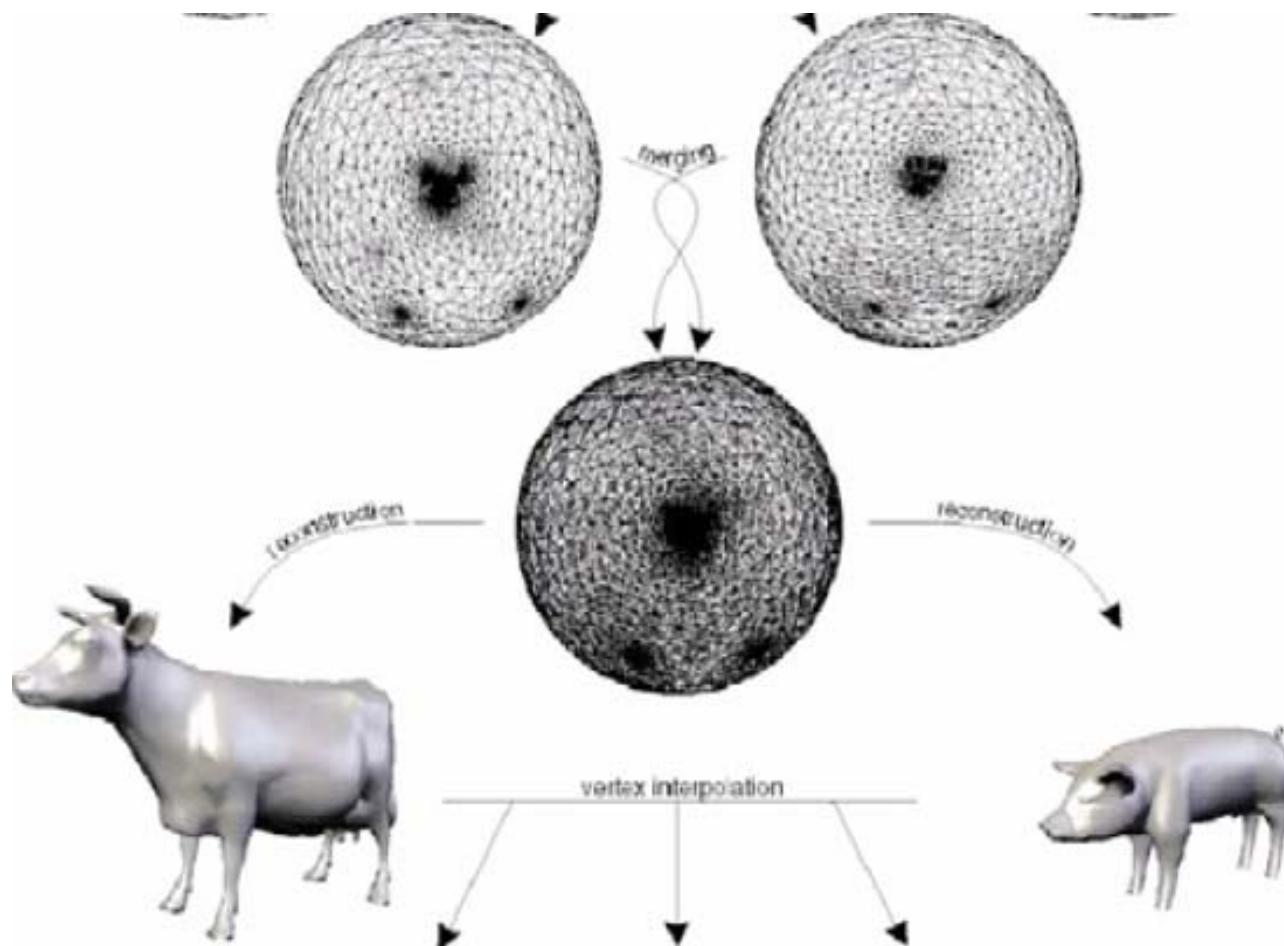
# 1.1 Parameterization: Planar domain



## 1.1 Parameterization: Planar domain



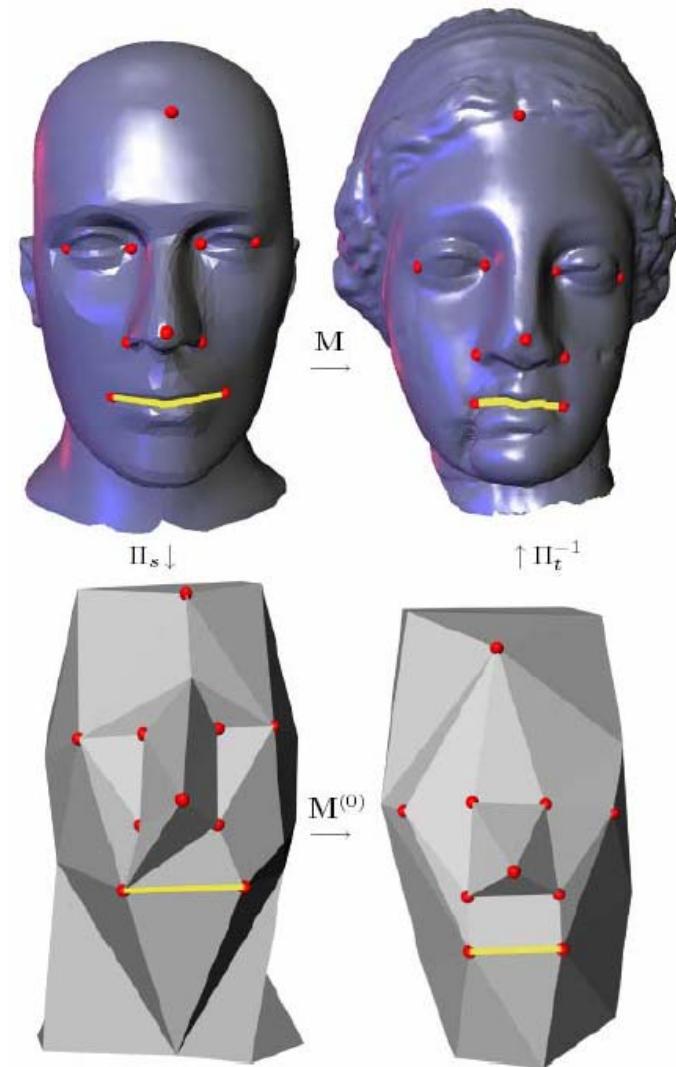
## 1.2 Parameterization: Sphere domain



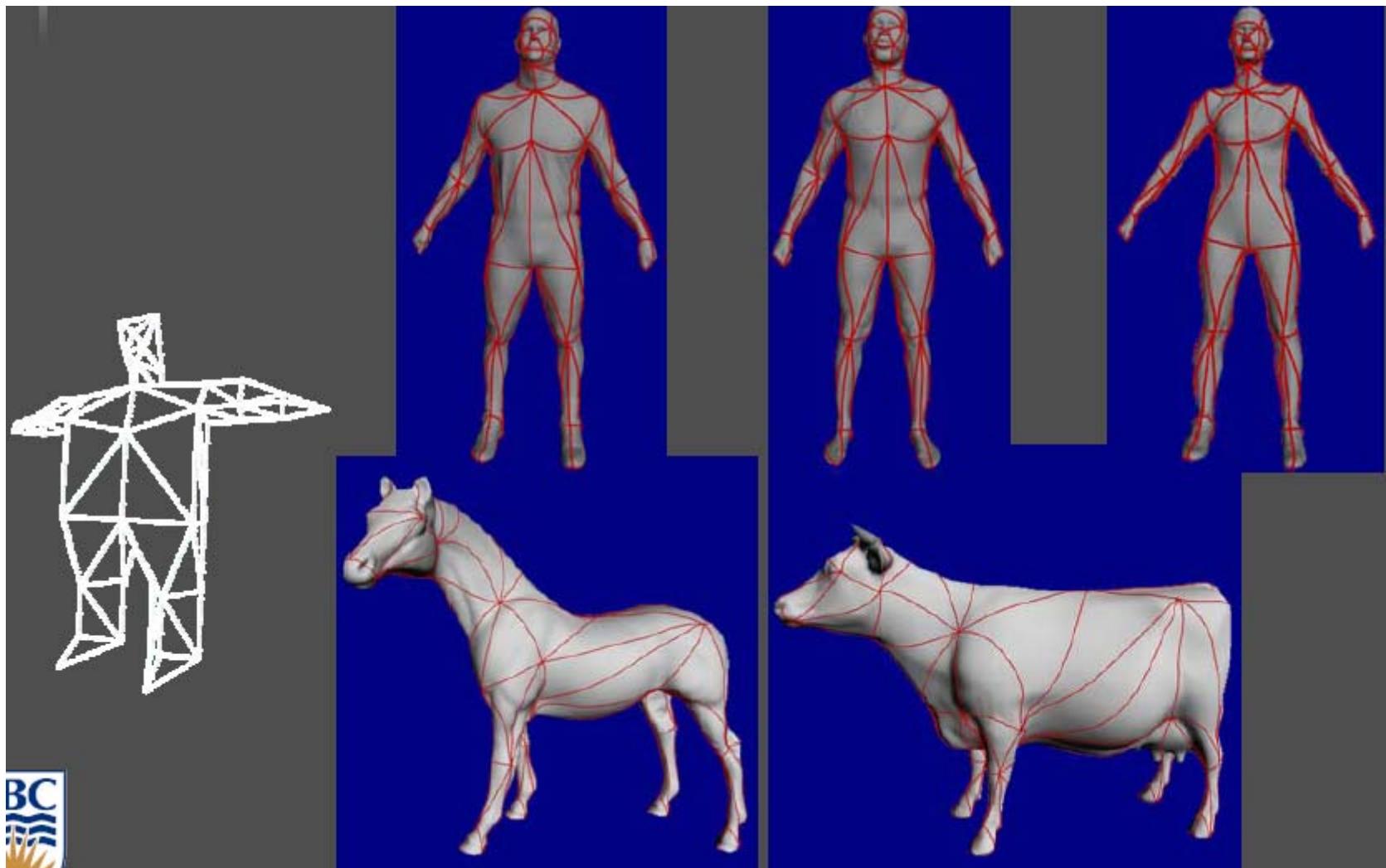
# 1.3 Parameterization: Base mesh

Lee et al. 1999

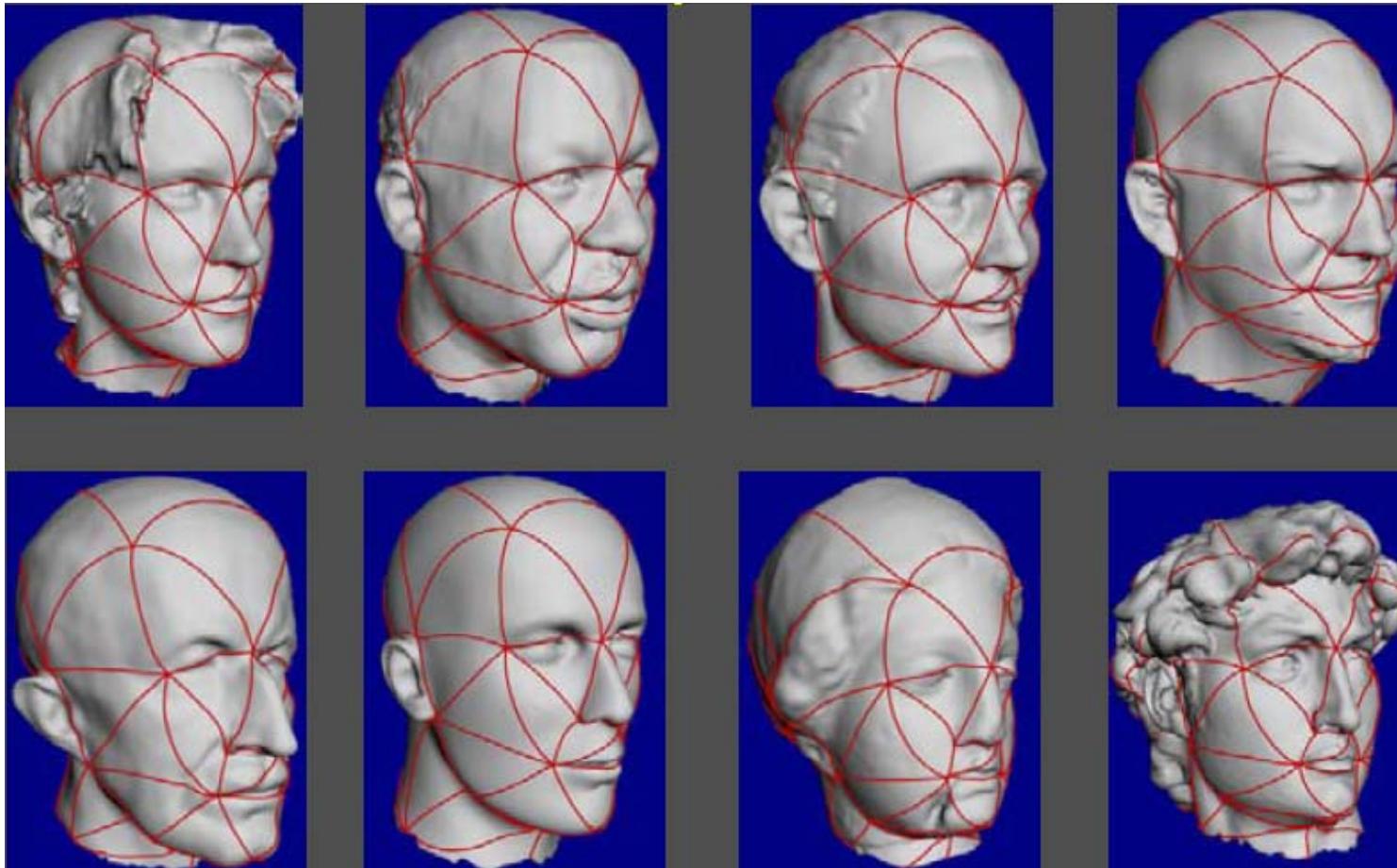
- MAPS



## 1.3 Parameterization: Base mesh

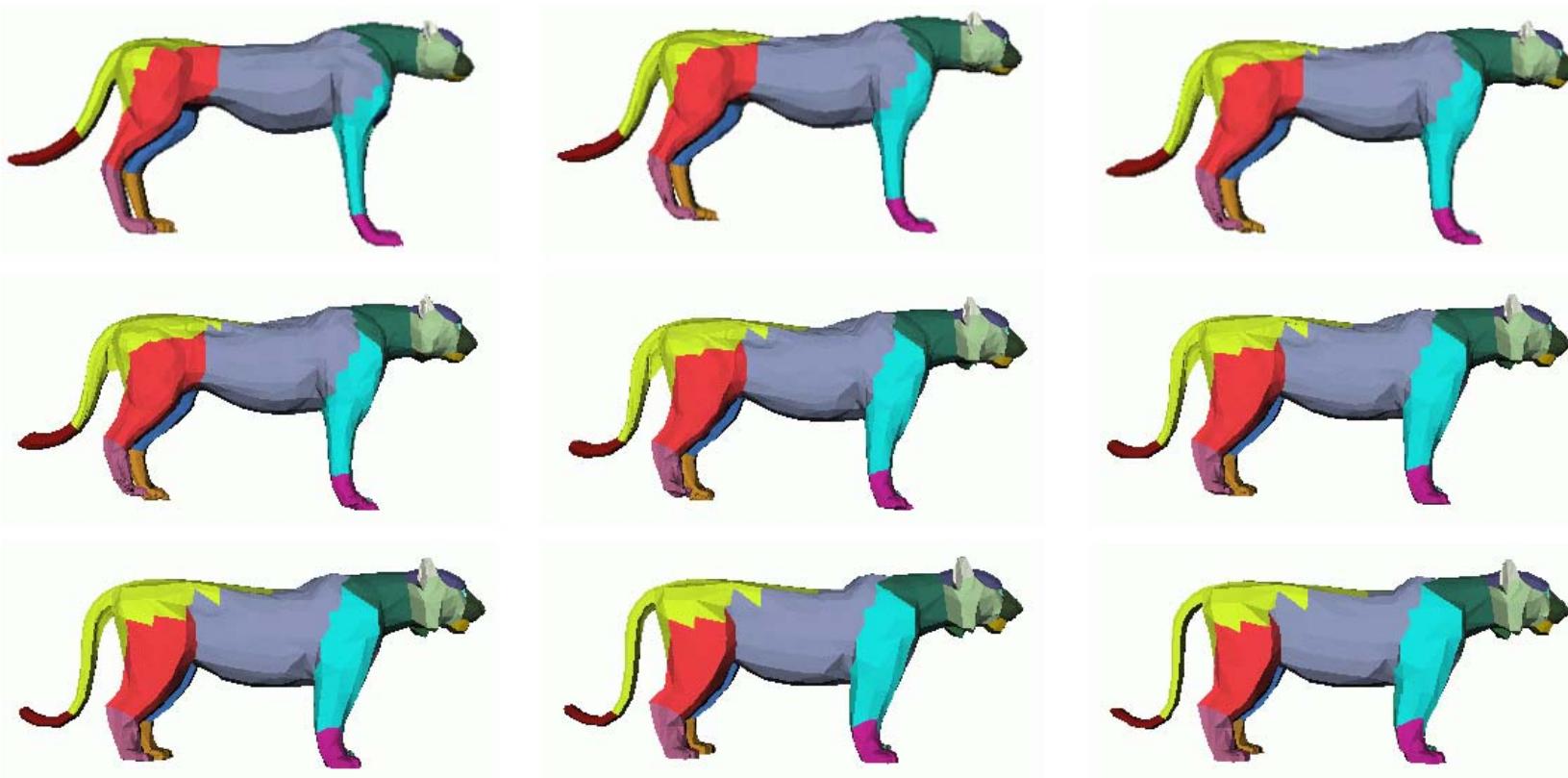


## 1.3 Parameterization: Base mesh



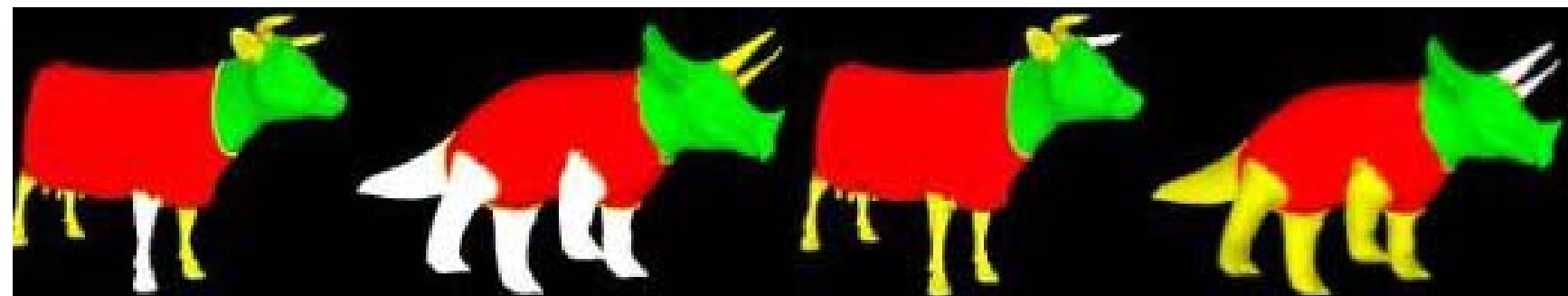
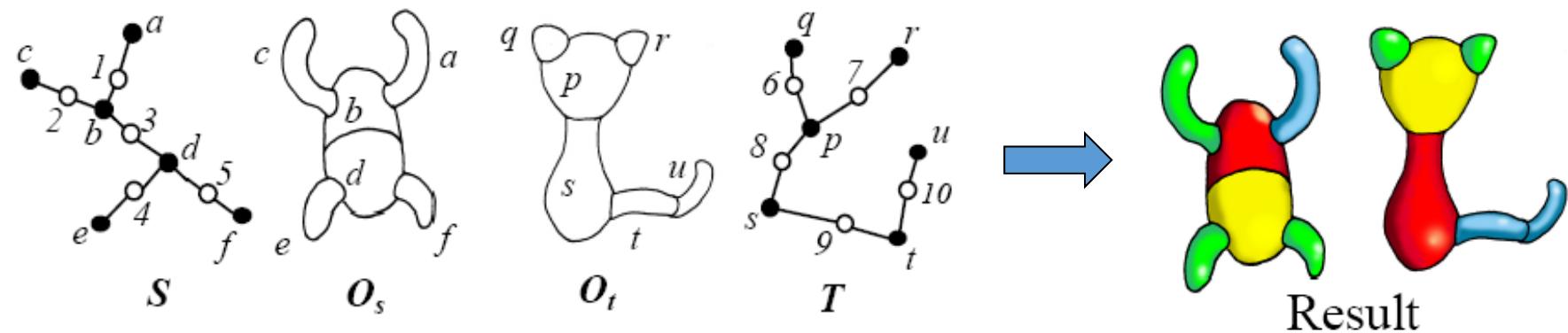
## 1.4 Decomposition Based

[Shlafman et al. 2002]



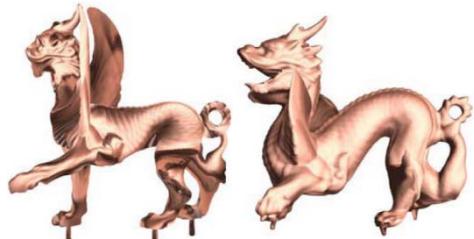
# 1.5 Component Based

[Zhao et al. 2003]



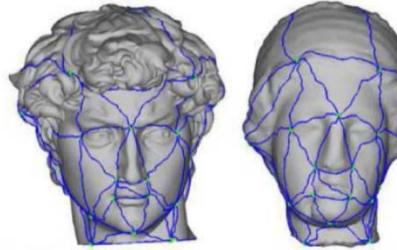
# 1.6 Many Recent Works

Overlaying-based

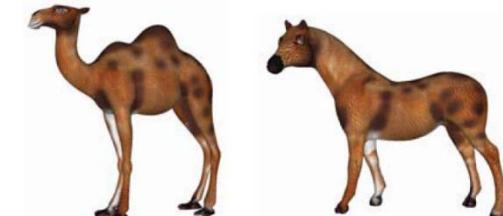


[Schreiner et al. 2004]

Subdivision-based

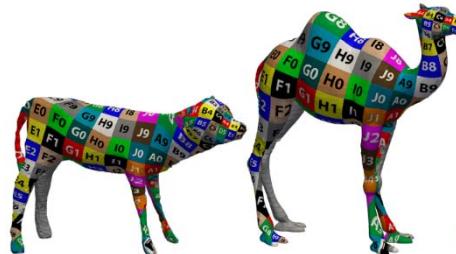


[Kwok et al. 2012]



[Kraevoy et al. 2004]

Common domain-based



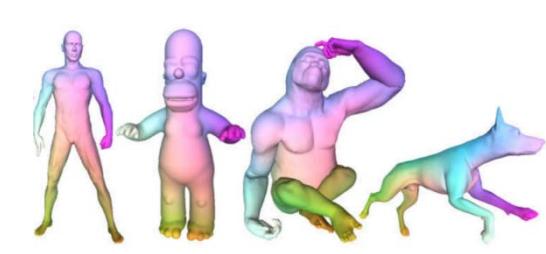
[Aigerman et al. 2014]



[Aigerman et al. 2015]



[Aigerman et al. 2016]

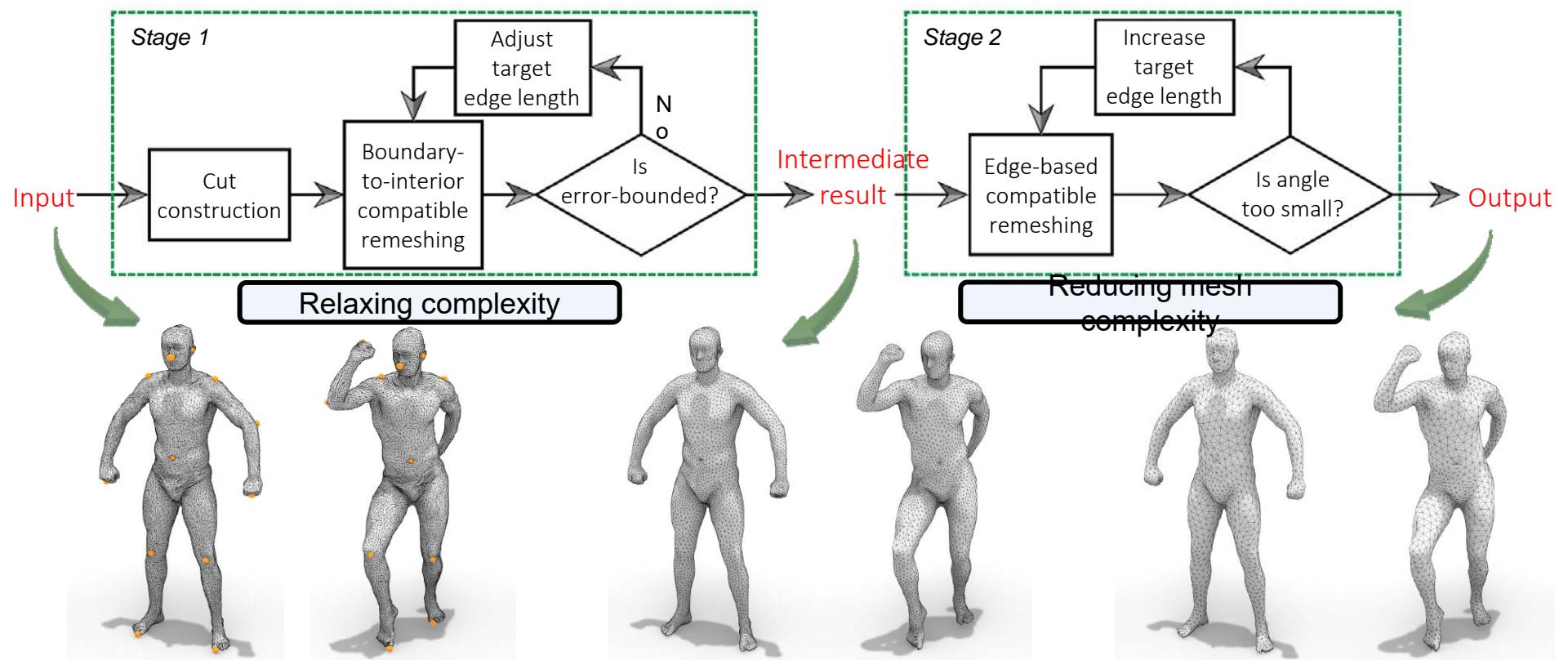


[Mandad et al. 2016]

# 1.7 Error-Bounded Compatible Remeshing

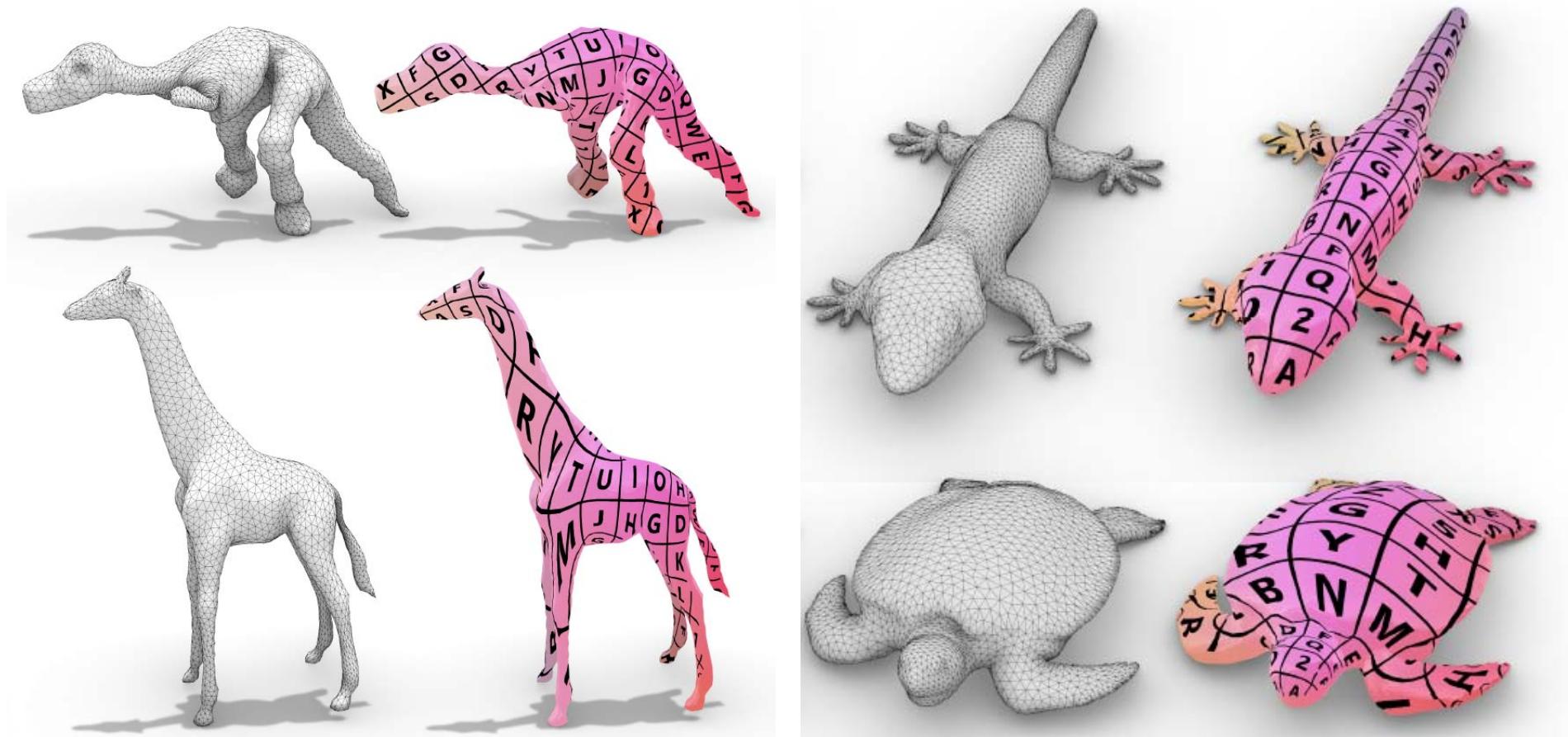
Yang et al. Error-Bounded Compatible Remeshing. Siggraph 2020.

- Optimization based method



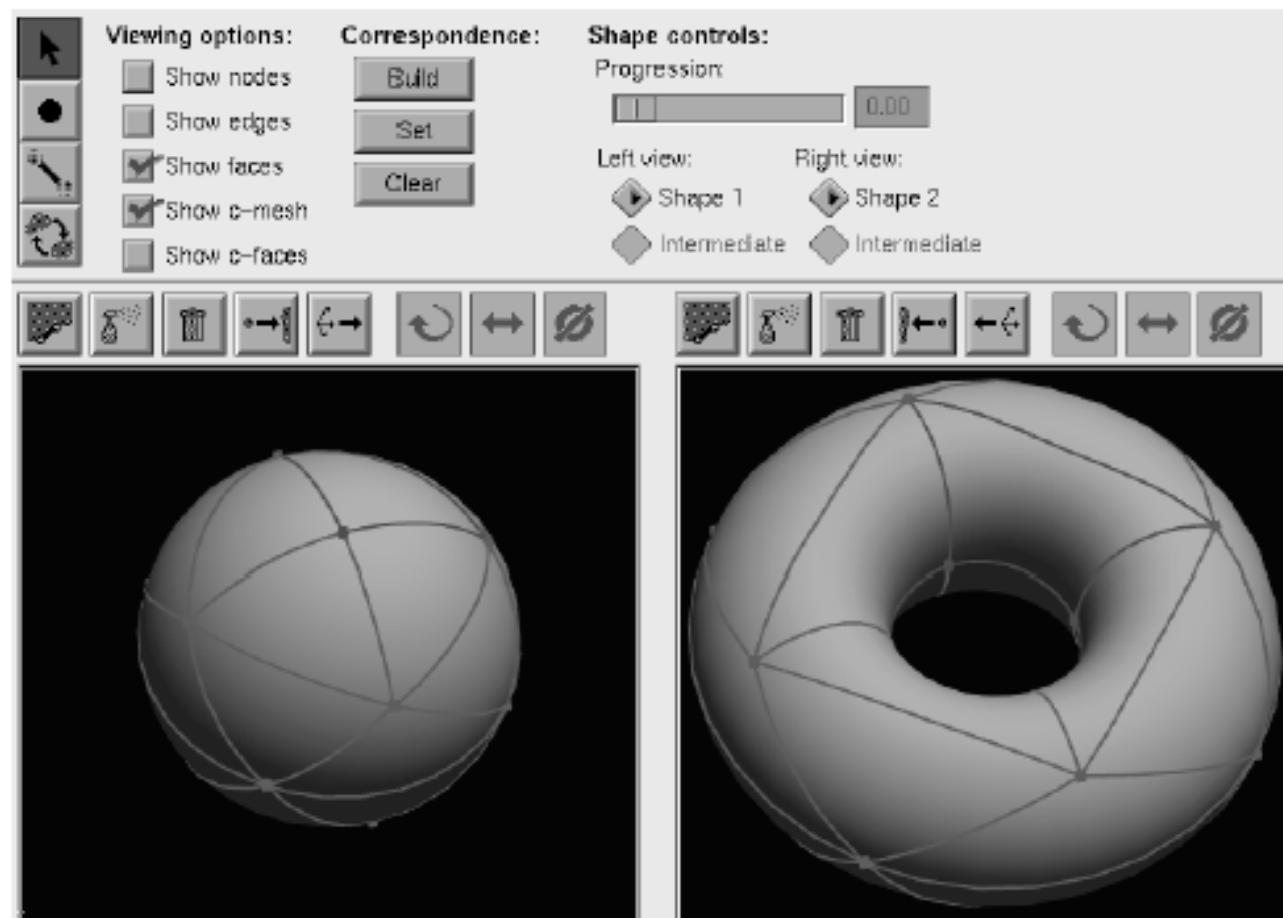
# 1.7 Error-Bounded Compatible Remeshing

Yang et al. Error-Bounded Compatible Remeshing. Siggraph 2020.



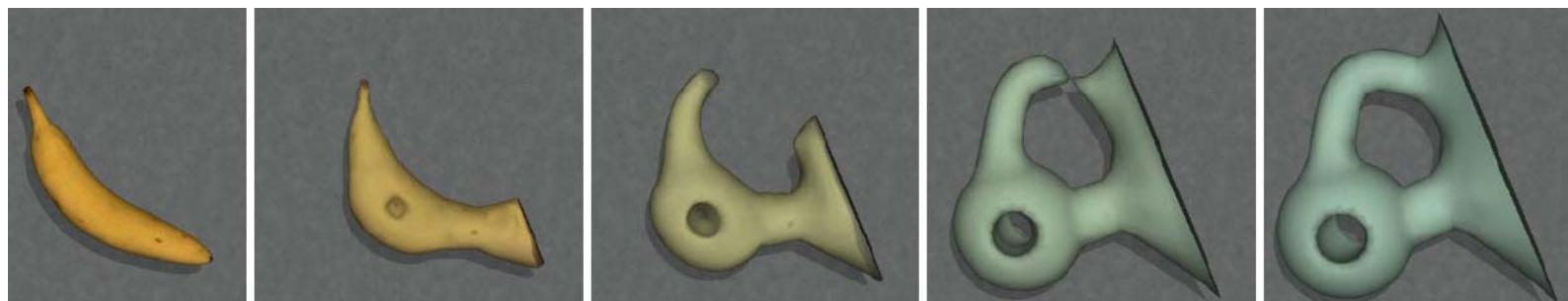
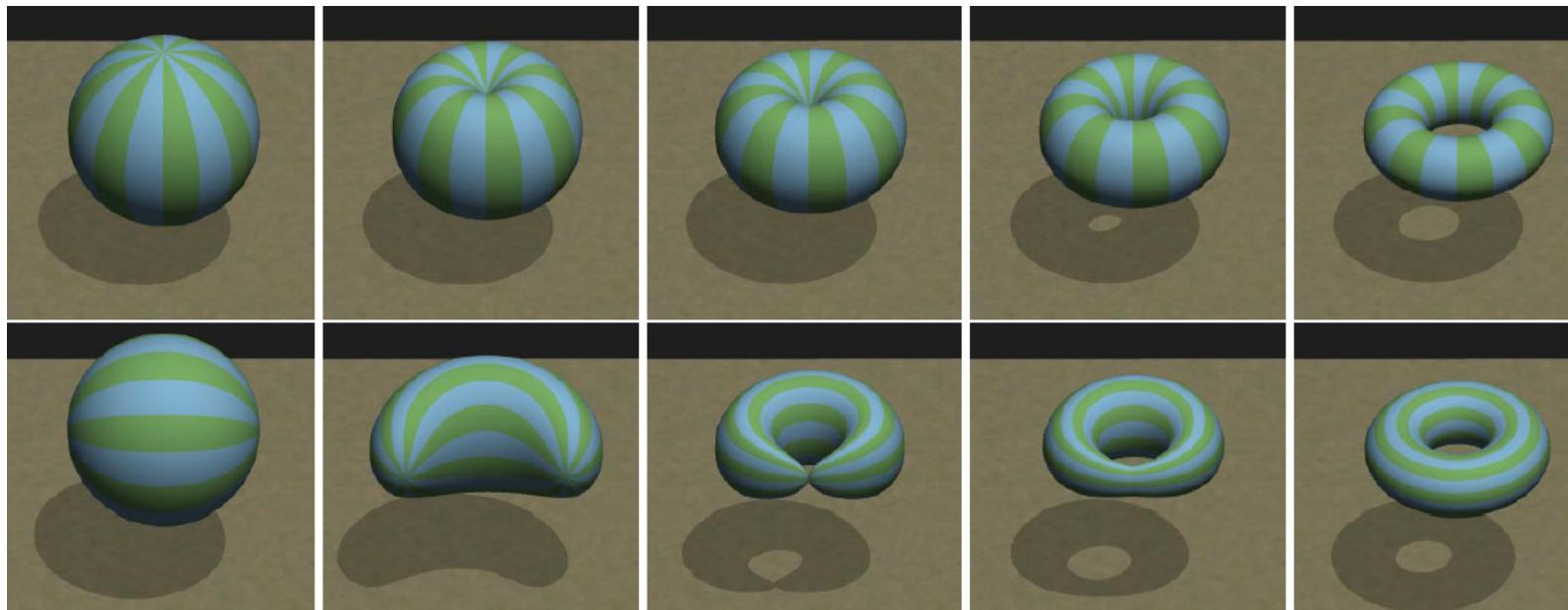
# 1.8 Different Topologies

[DeCarlo et al. 1996]



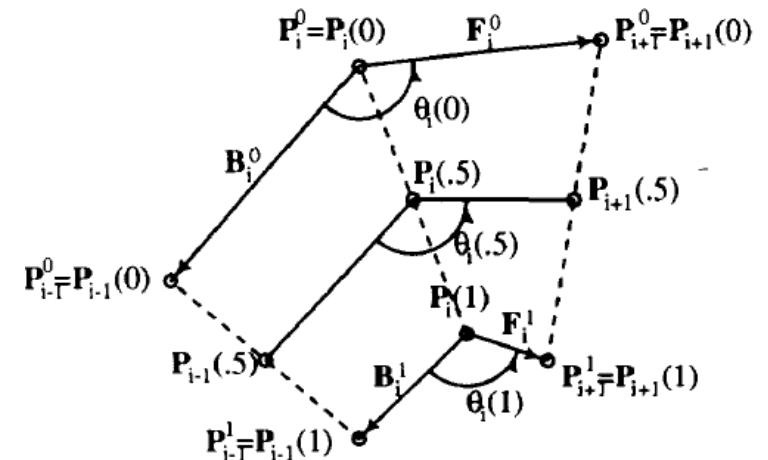
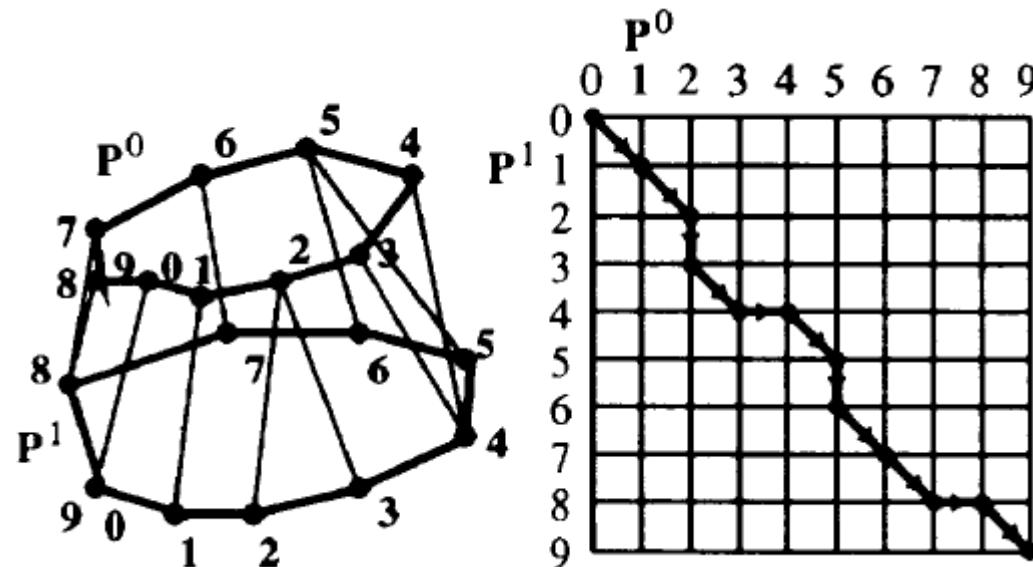
# 1.8 Different Topologies

[DeCarlo et al. 1996]



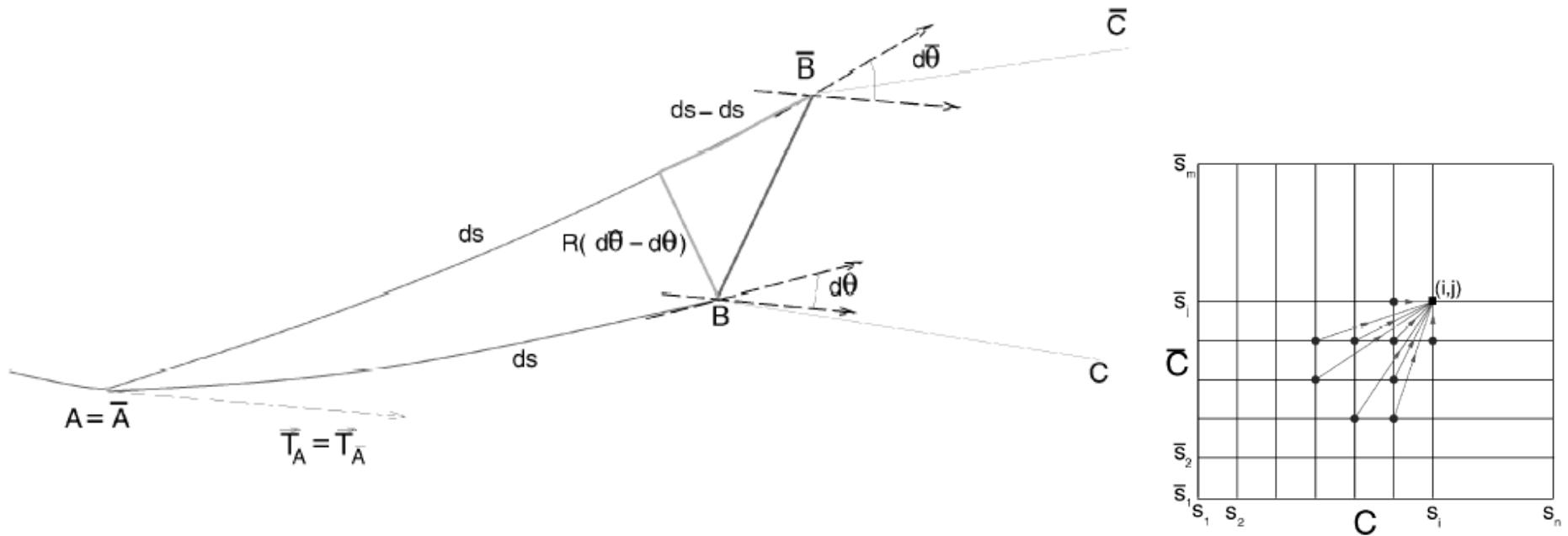
# More: Correspondences between planar shapes – Matching

- Physically Based Method [Sederberg et al. 1992]



# More: Correspondences between planar shapes – Matching

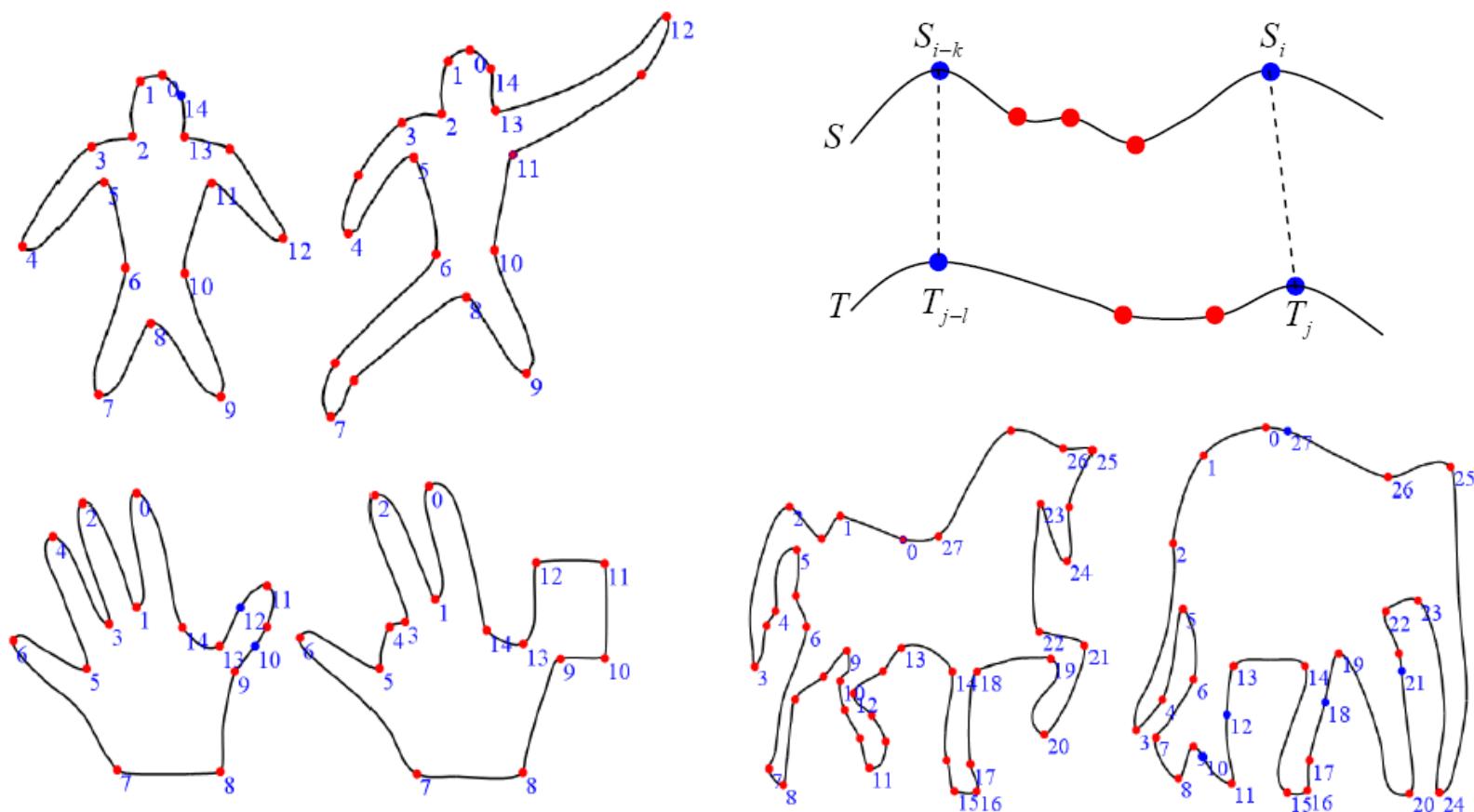
- Curve Aligning [Sebastian et al. 2003]



$$\mu[g] = \int_C \left| \frac{\partial}{\partial s} (\bar{C}(\bar{s}) - C(s)) \right|^2 ds + R \int_C (\kappa(s) - \bar{\kappa}(\bar{s}))^2 ds,$$

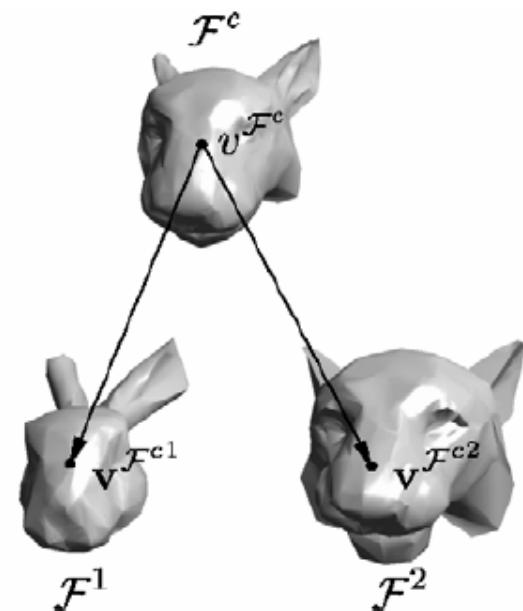
# More: Correspondences between planar shapes – Matching

- Perceptually Based Method [Liu et al. 2004]



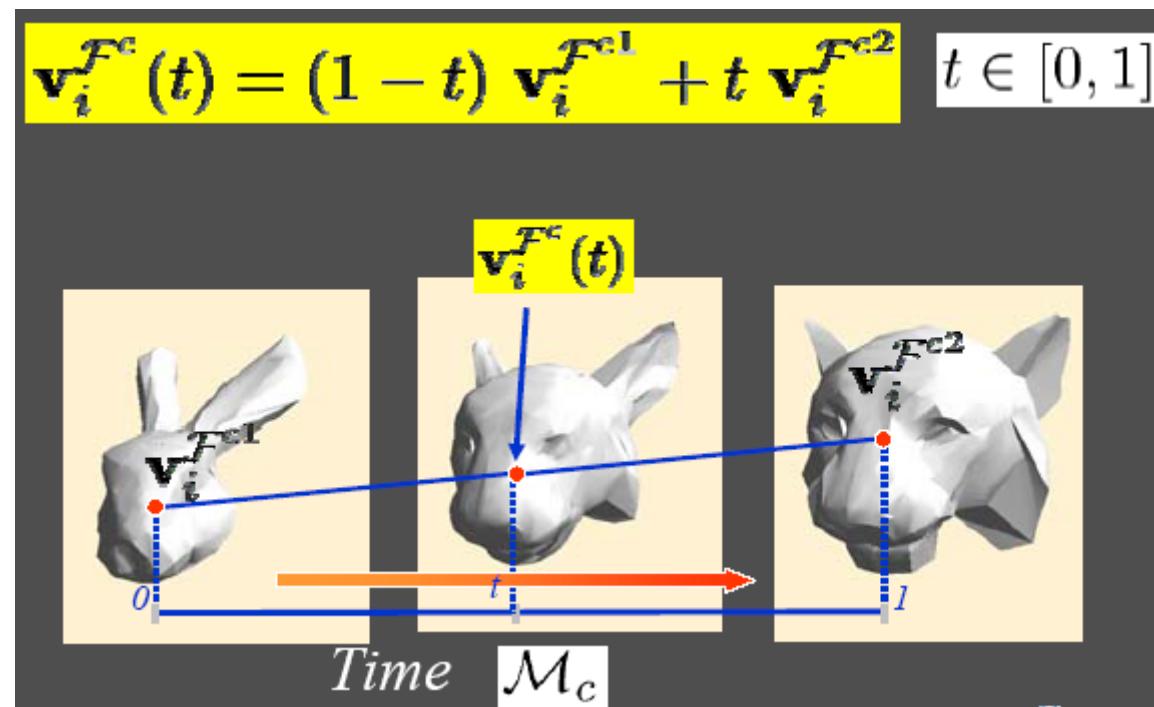
## 2. Vertex Path (Trajectory)

- Input:
  - All vertices on source & target have one-to-one correspondence with each other
  - Each vertex has two 3D coords  $v^{\mathcal{F}c1}$  (source) and  $v^{\mathcal{F}c2}$  (target)
- Output: generate the intermediate shapes from two shapes



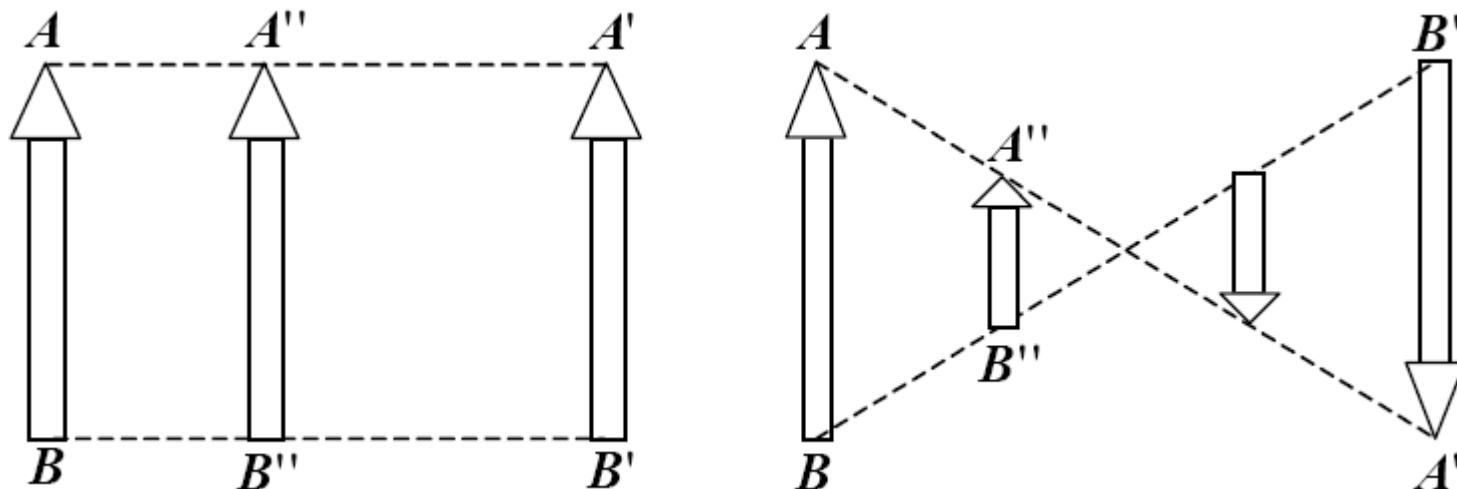
# Simplest Method: Linear Interpolation

- Linear interpolation between corresponding points
- Work well for many cases
  - many drawbacks

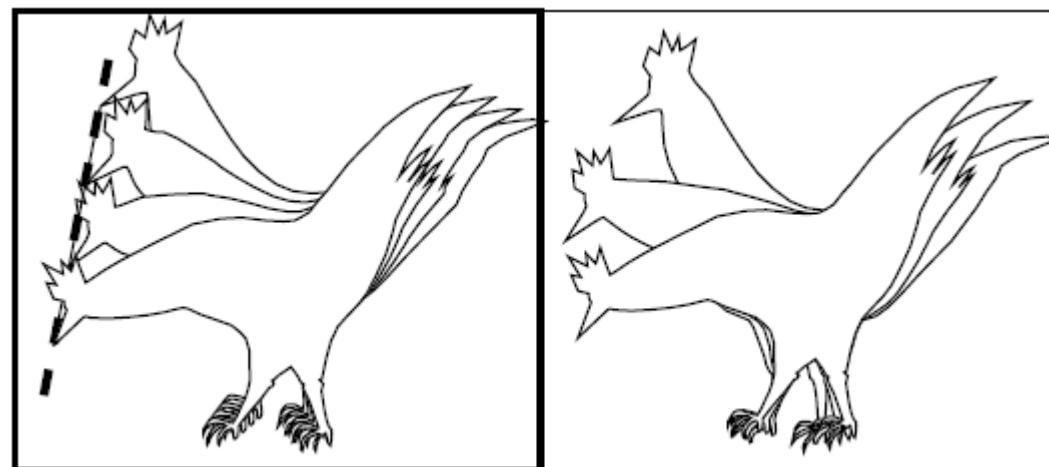
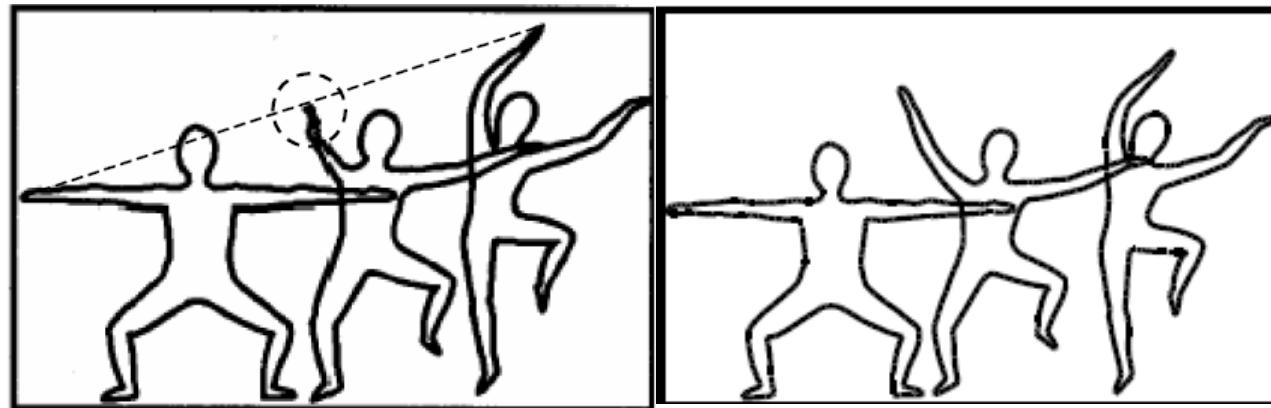


# Linear Interpolation

- Simple and easy
- Drawbacks
  - Shrinkage



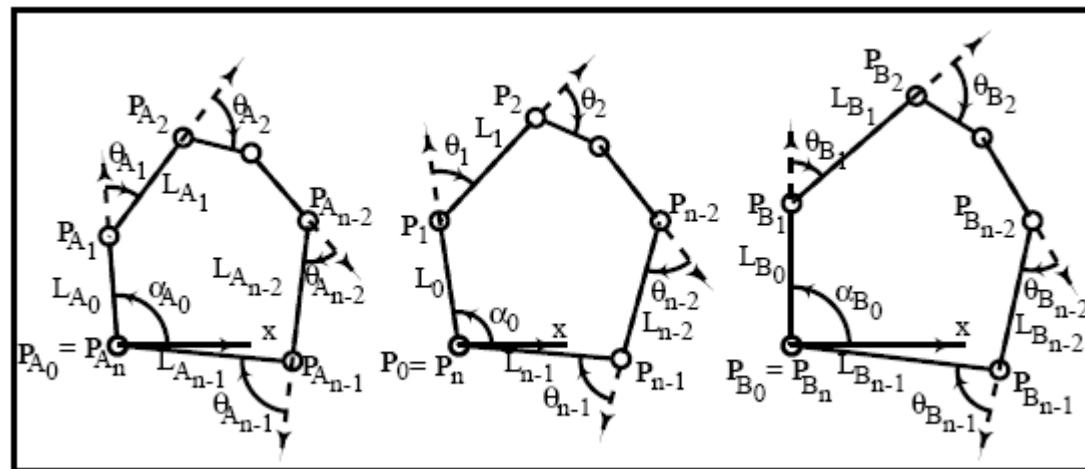
# Unnatural Results in Linear Interpolation



# Intrinsic Approach

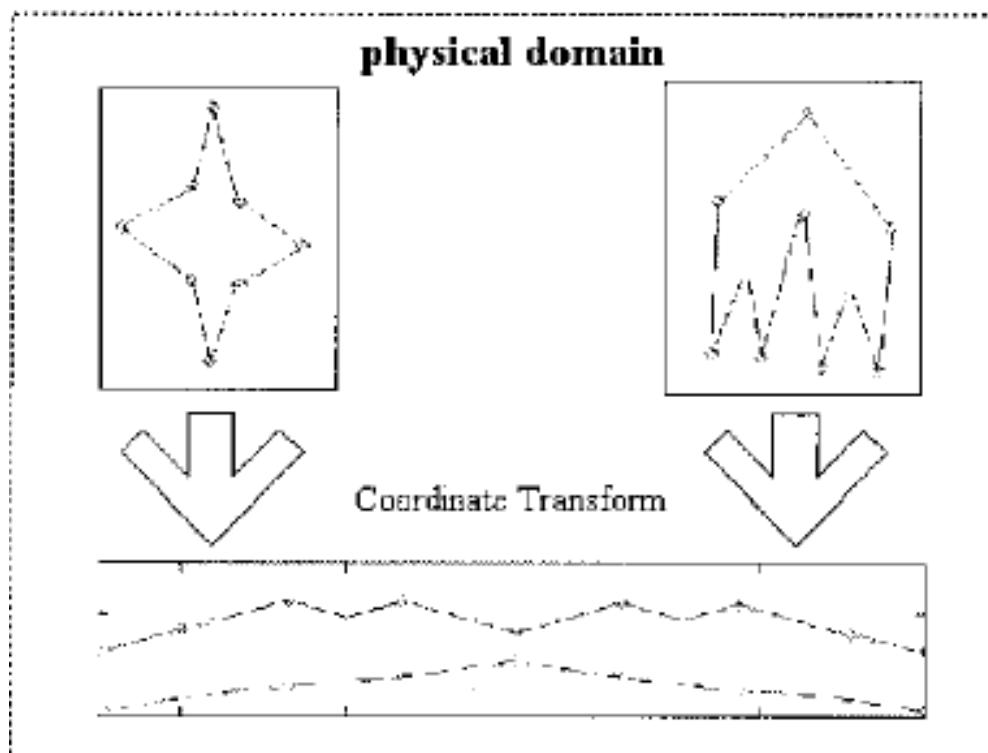
Sederberg et al. 1993

- Intrinsic variables
  - edge lengths
  - turning angles



# Fourier Approach

Chen et al. 2001

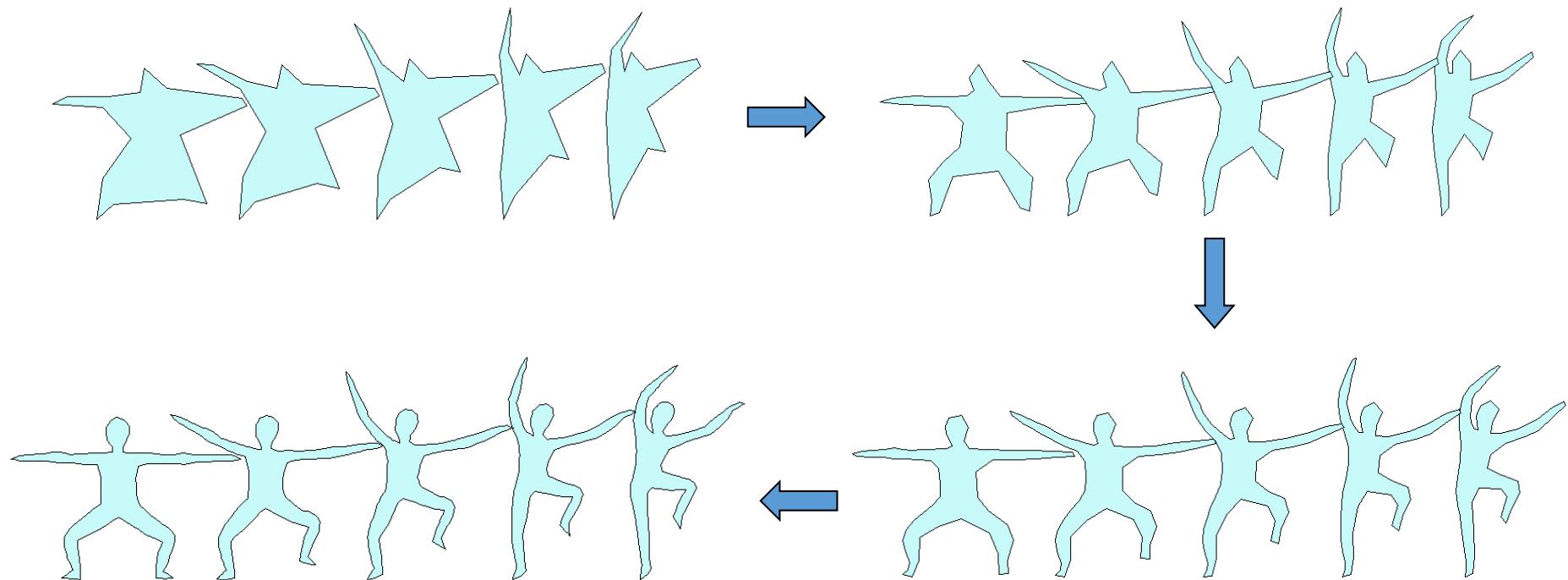


$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \begin{bmatrix} \cos(2\pi k t) \\ \sin(2\pi k t) \end{bmatrix}$$

# Wavelet Approach

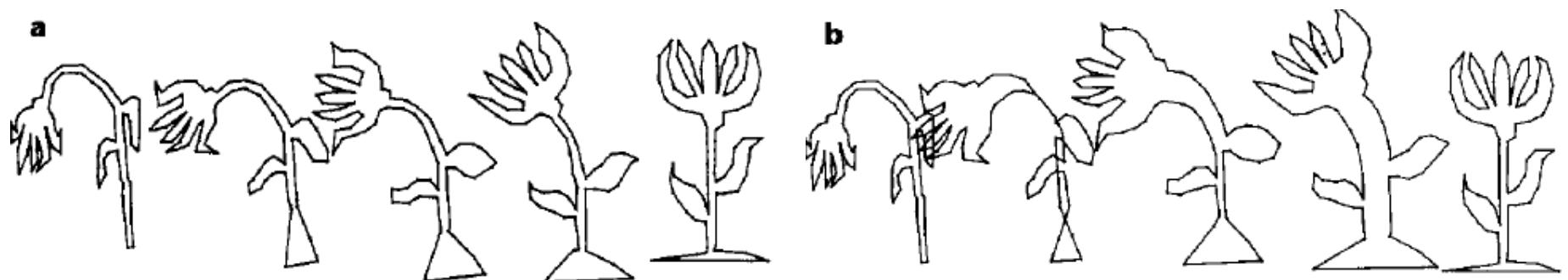
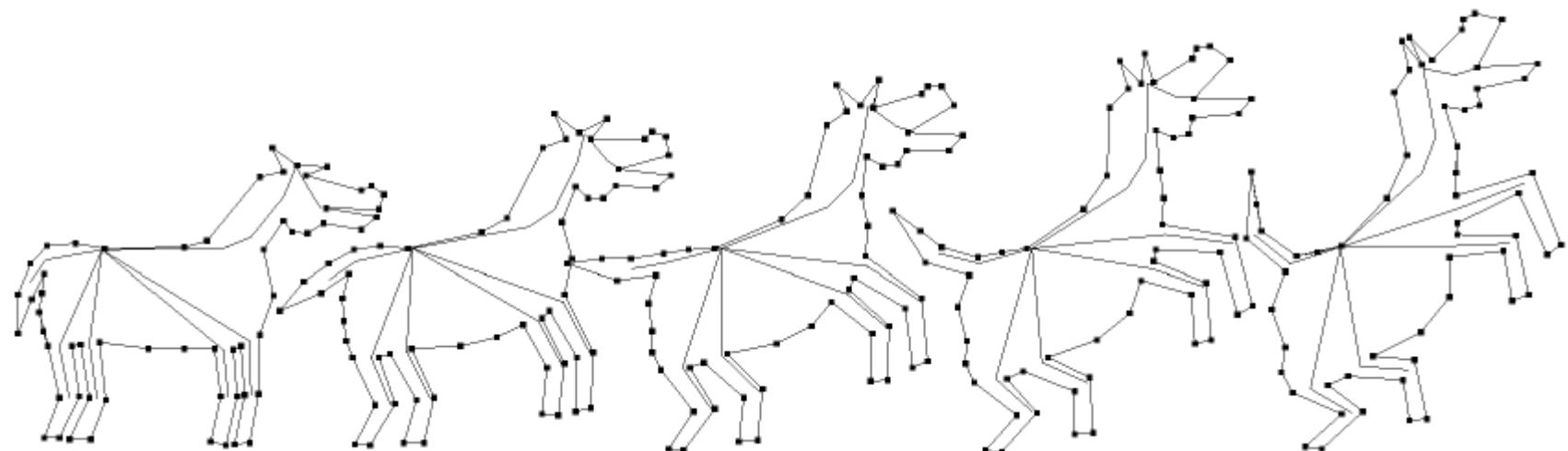
Zhang et al. 2000

- Wavelet decomposition



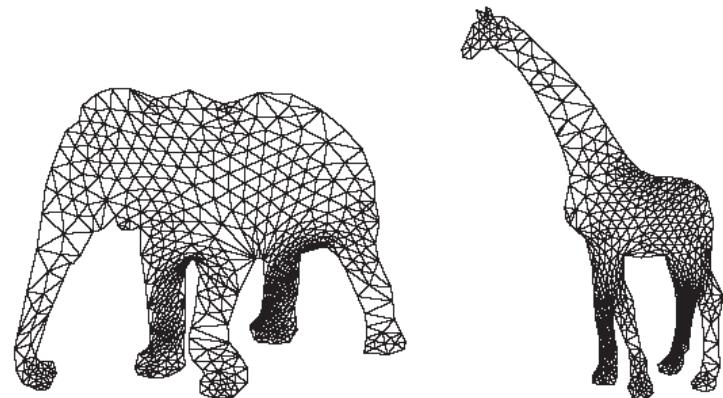
# Star Skeleton Representation

[Shapira et al. 1995]



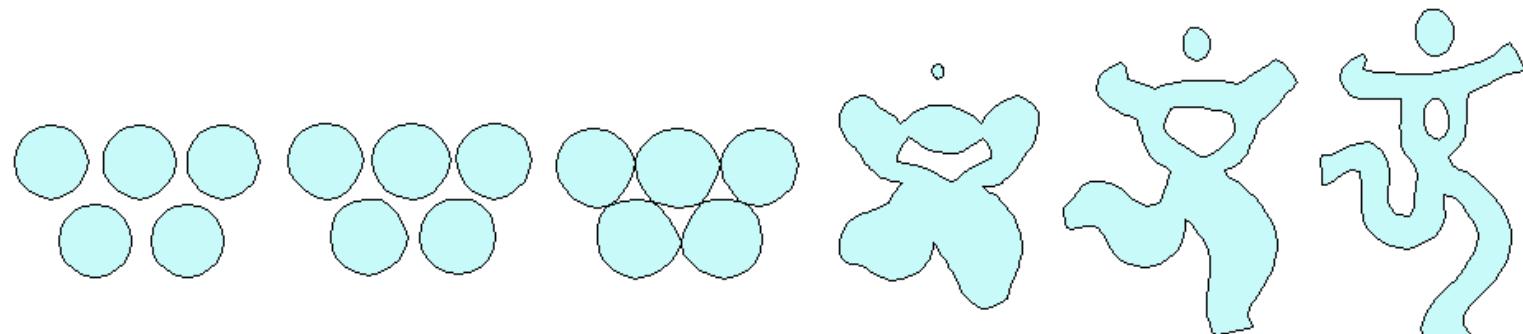
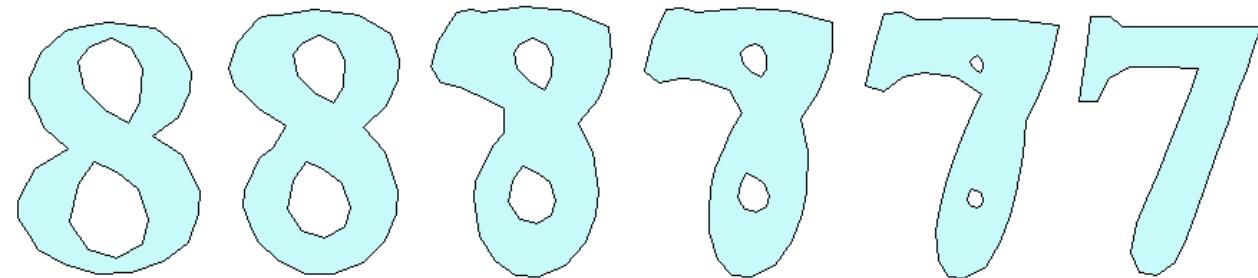
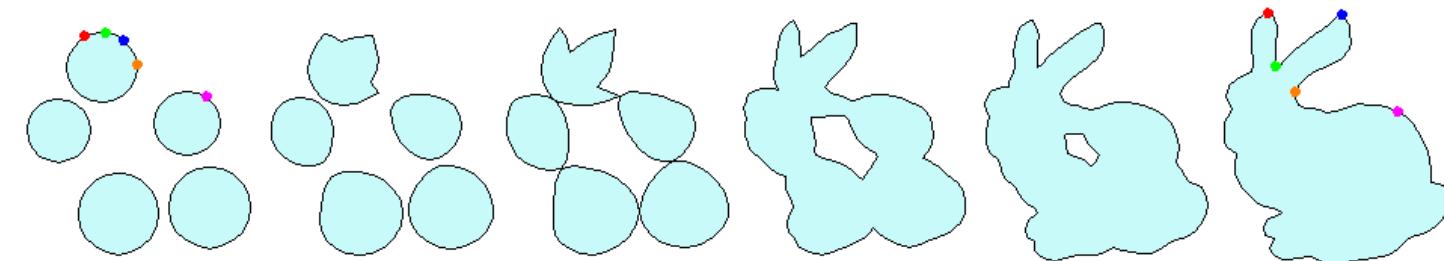
# Interior Based Approach

- Based on compatible triangulation
  - [Gotsman and Surazhsky, 1999-2001]
  - As-rigid-as-possible [Alexa et al. 2000]



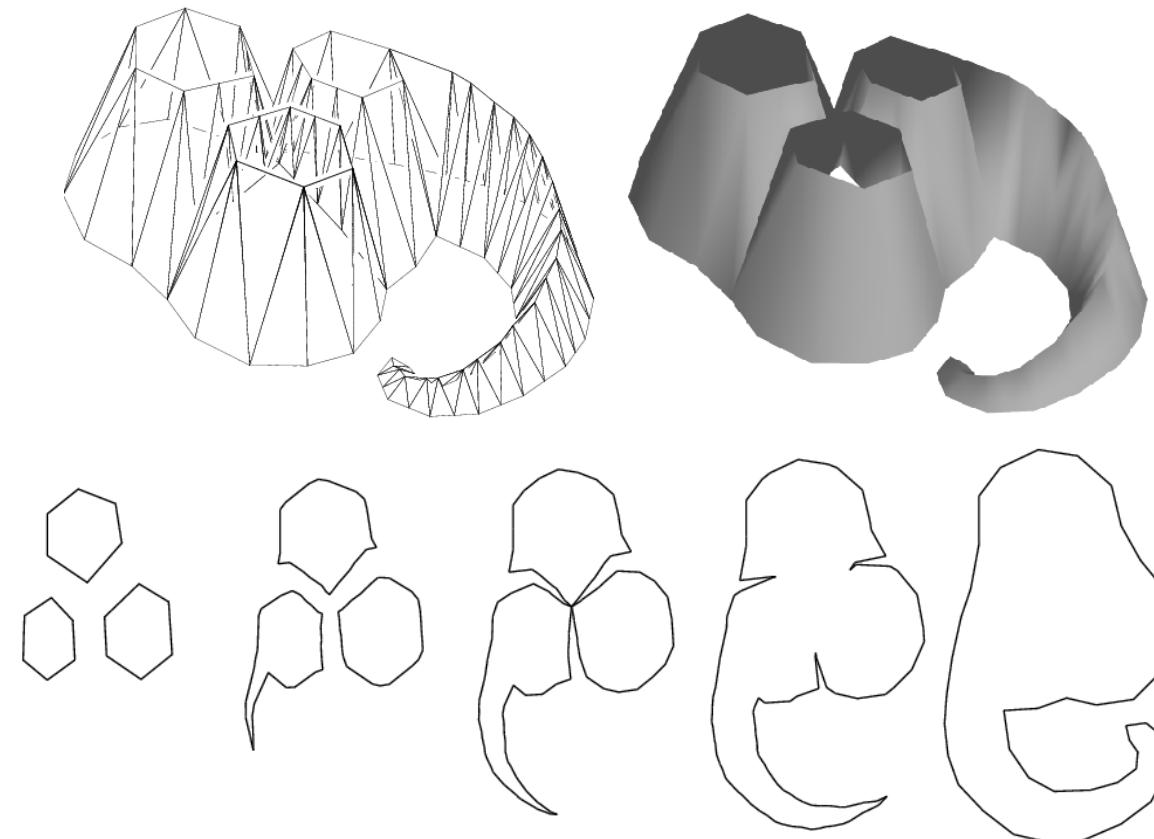
# Morphing between Different Topologies

Liu et al. 2005



# Implicit Approaches

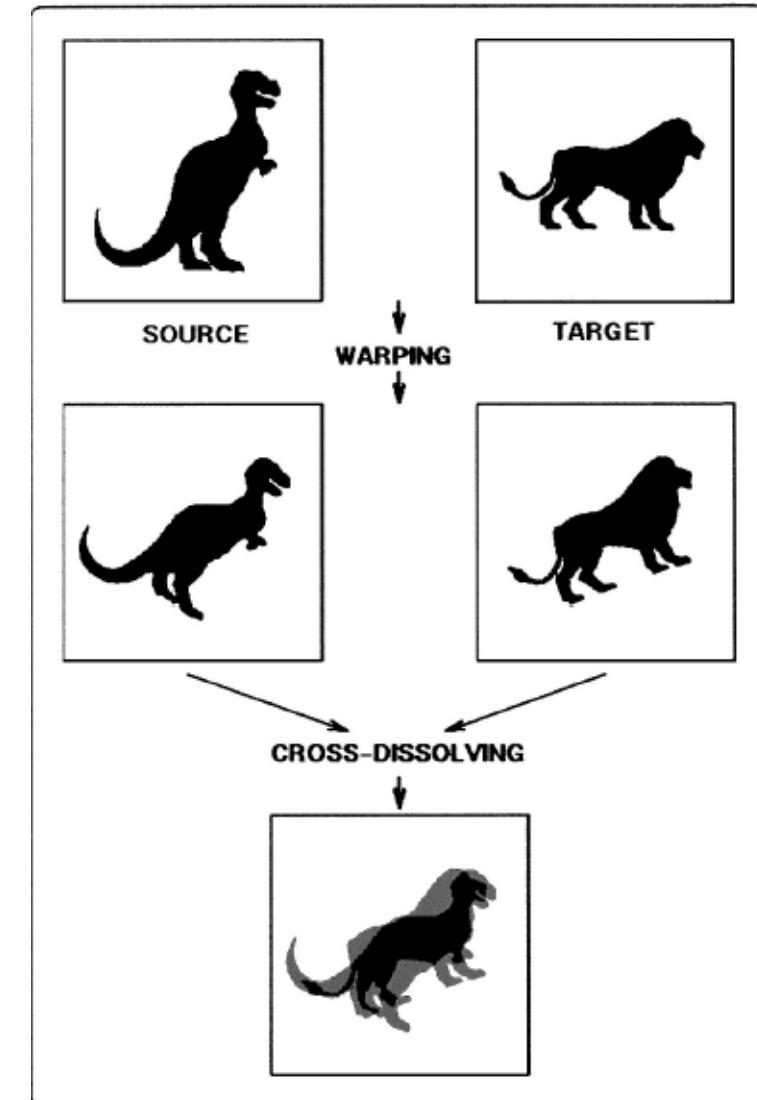
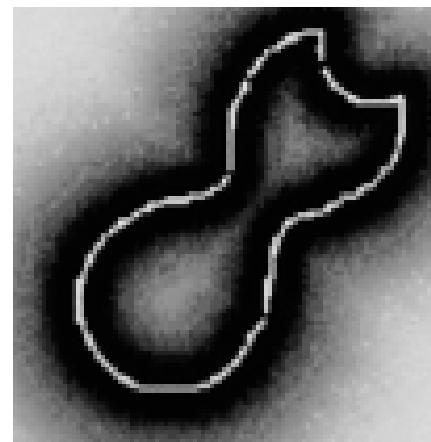
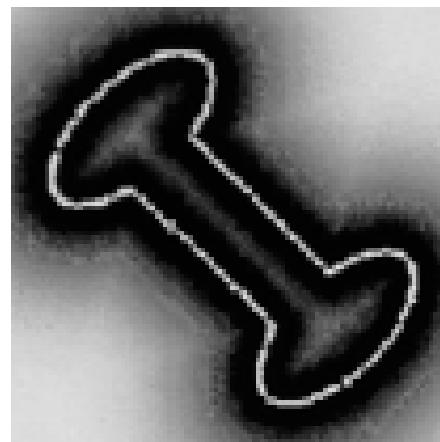
- Construct a 4D function which interpolates two shapes (with iso-value 0 and 1 respectively)



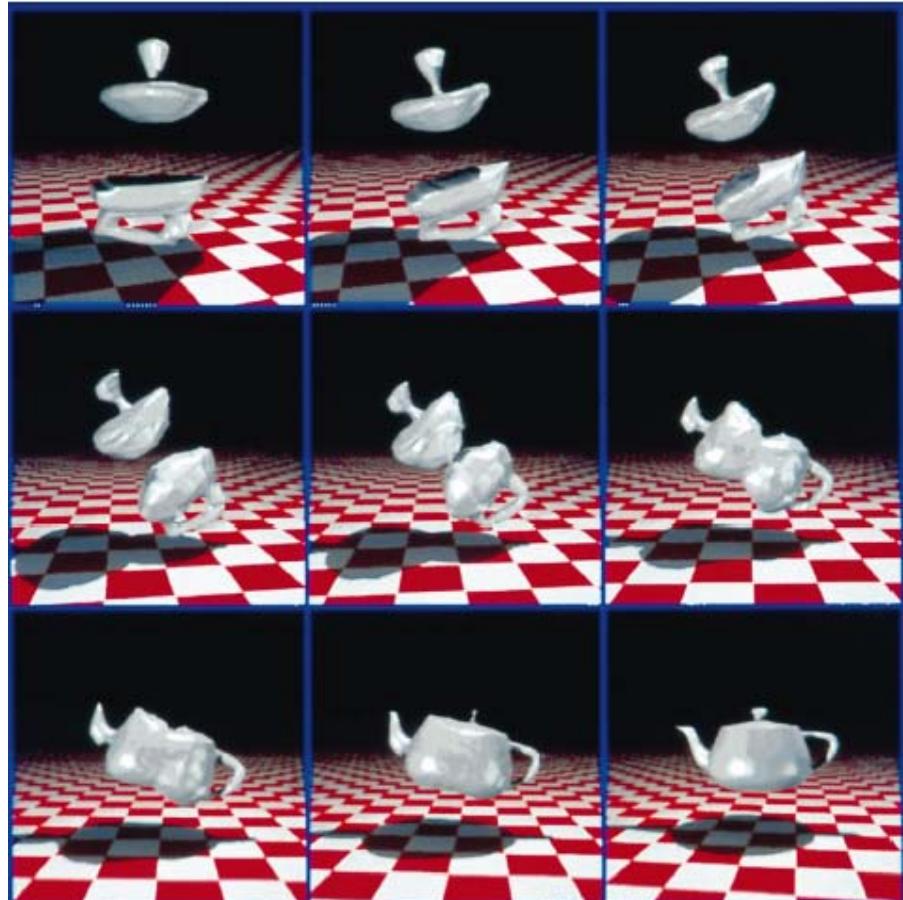
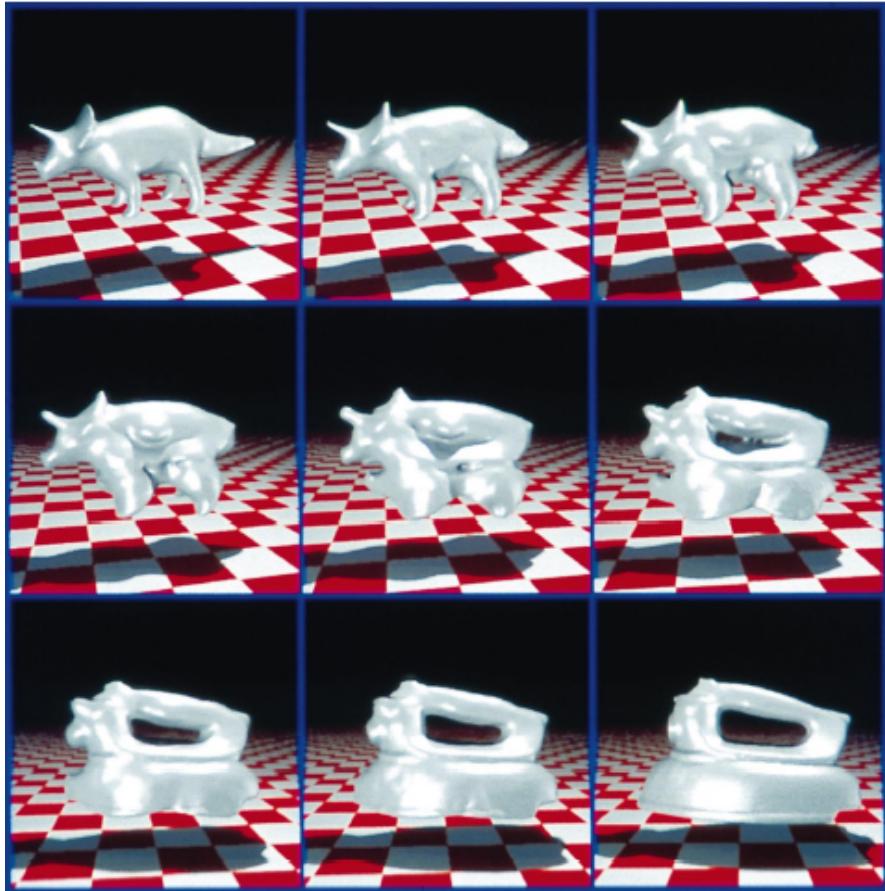
# Distance Field

[Cohen-Or et al. 1998]

- Distance field of a shape

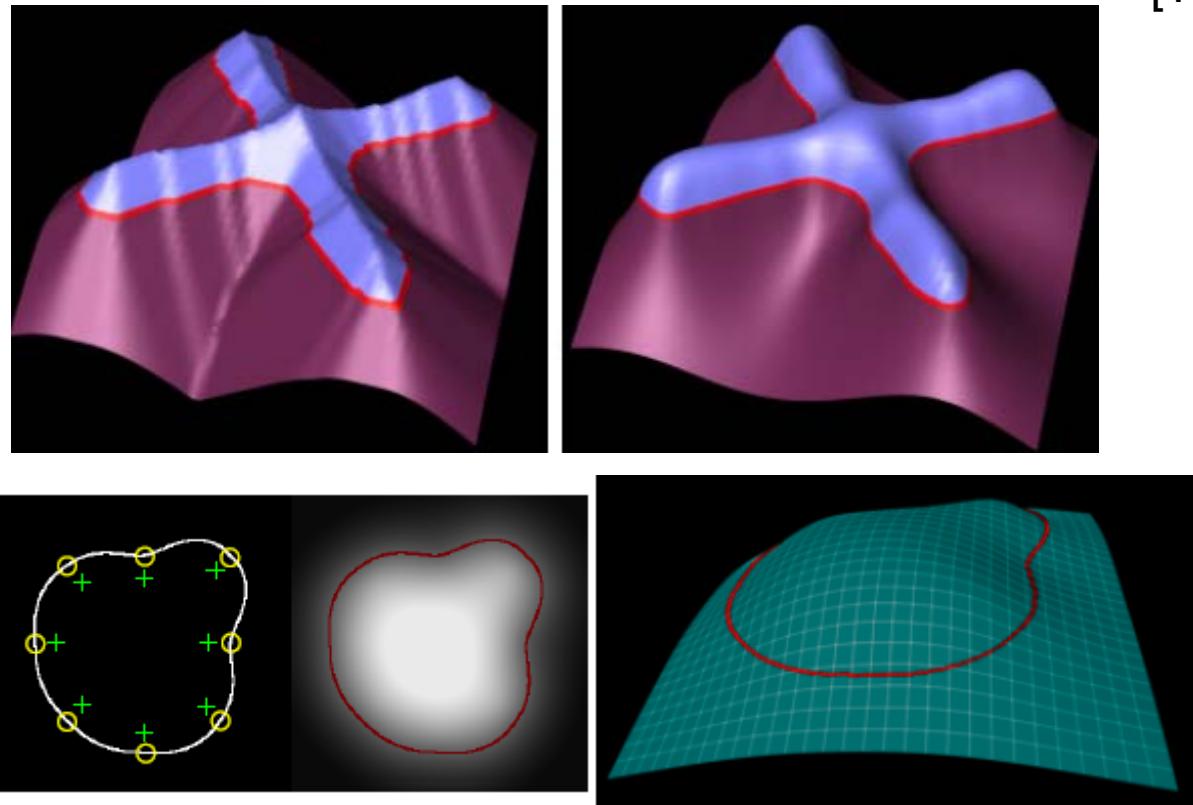


# Distance Field



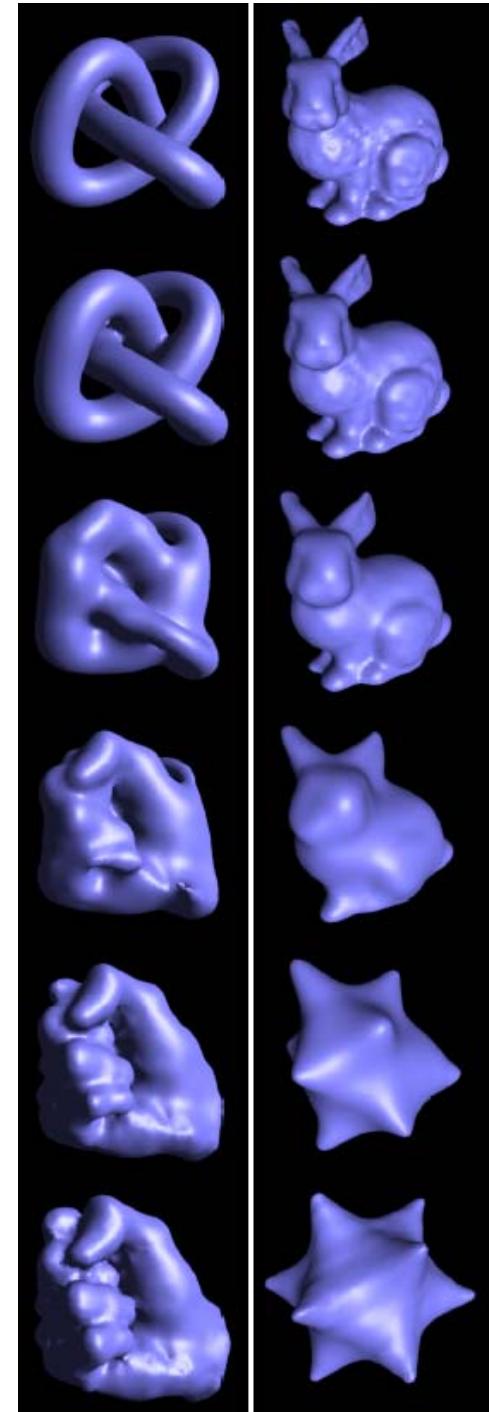
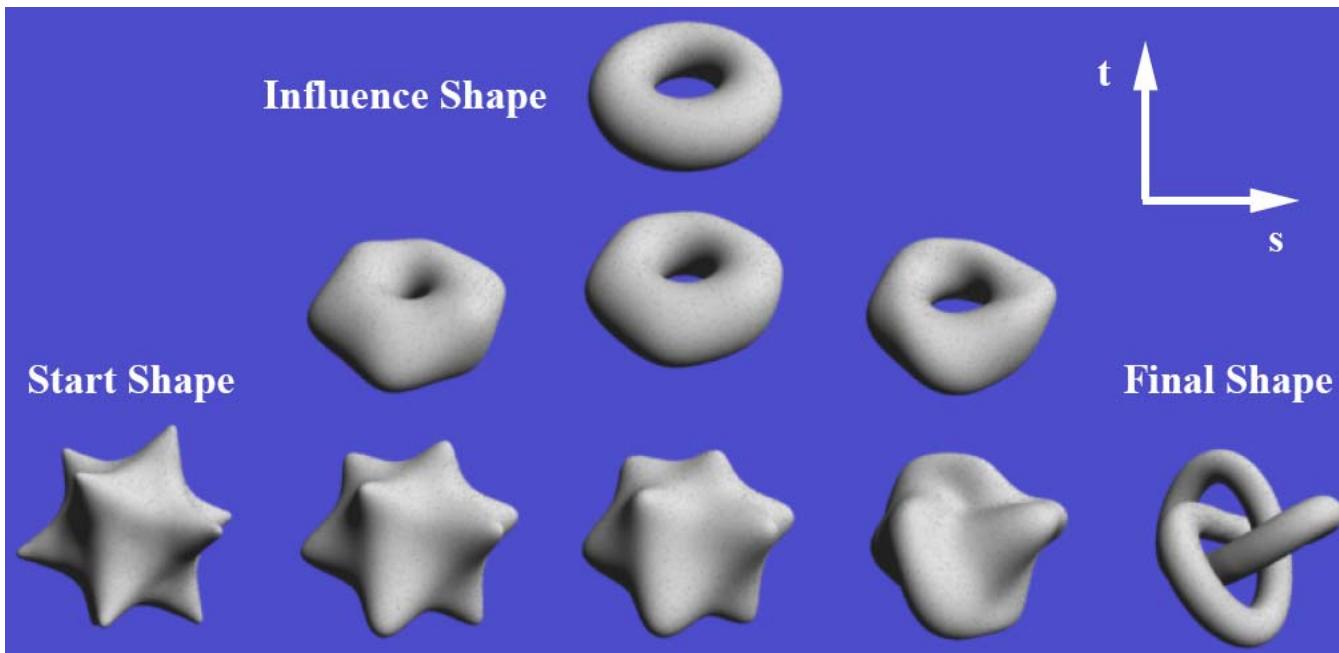
# Variational Implicit Function

[Turk et al. 1999]



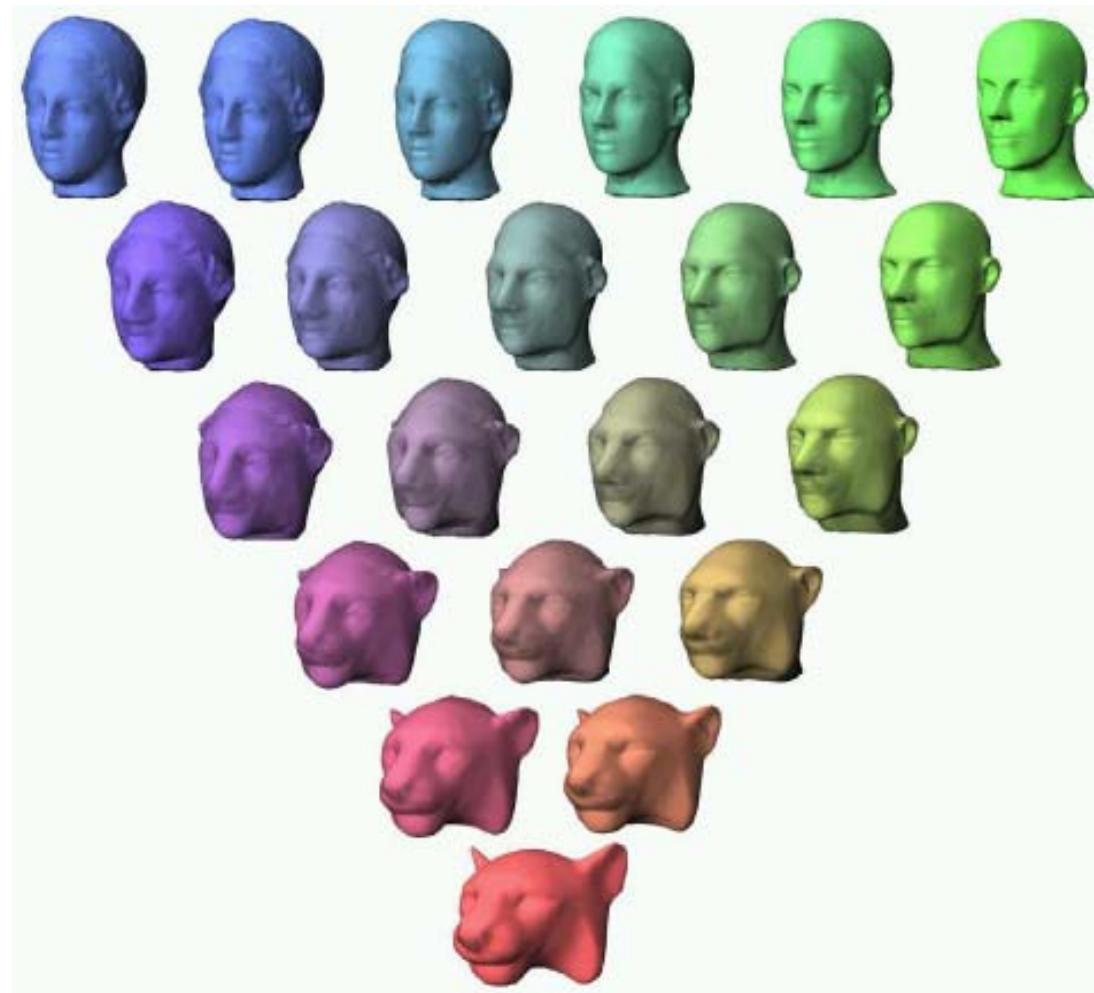
$$f(\mathbf{x}) = \sum_{j=1}^n d_j \phi(\mathbf{x} - \mathbf{c}_j) + P(\mathbf{x})$$

# Examples



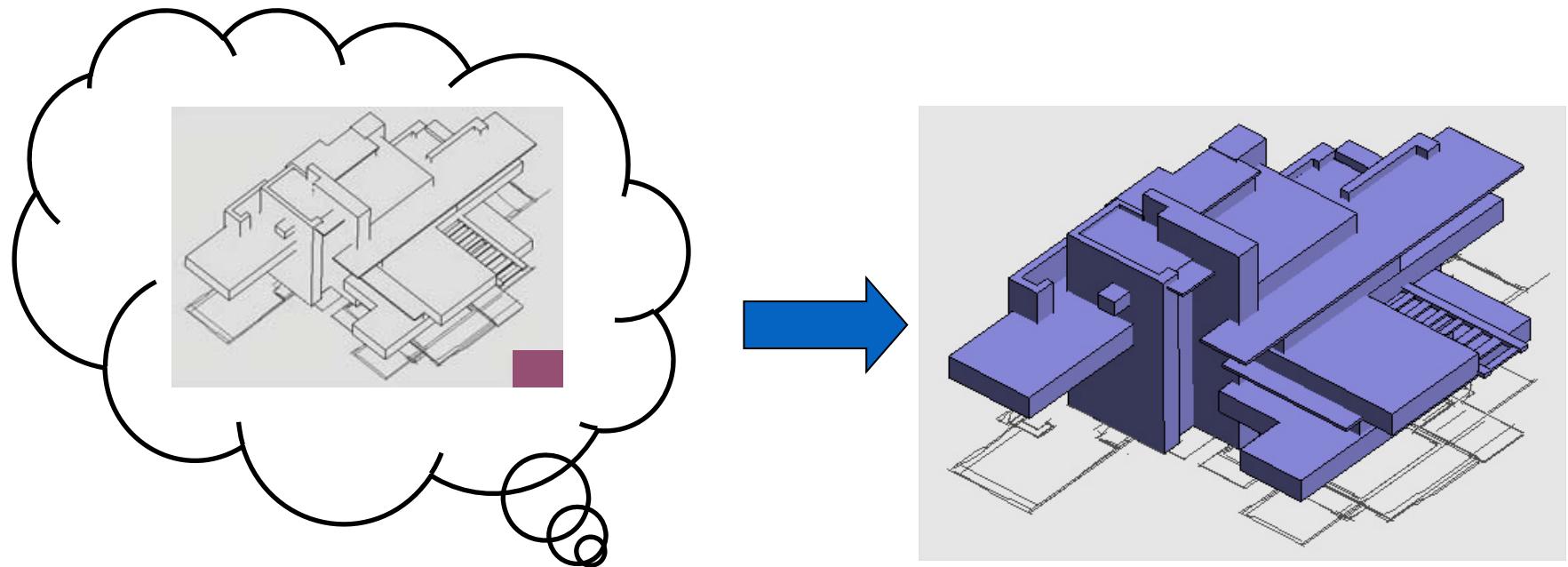
# Polymorph:

## Morphing between multiple shapes



# Summary

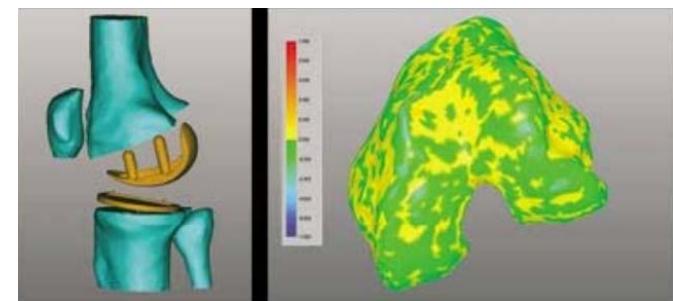
# 3D content creation is still hard



- One of the most central and challenging problems in graphics
  - (Comparing to simulation and rendering)

# Challenges to Geometric Modeling

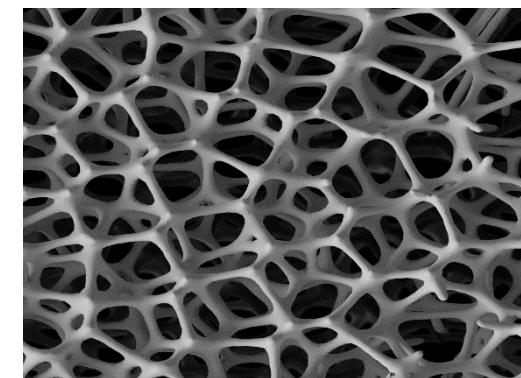
- Modeling porous structure
- Modeling non-homogeneous materials
  - varying density
  - changing composition
  - multiple phases (solid, liquid)
  - ...



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谢谢！