根据FVM有

$$\begin{aligned}\mathbf{P} &= \mathbf{F}\frac{\partial W}{\partial \mathbf{G}} \\ &= \mathbf{FS} \\ &= \mathbf{UDV}^T\mathbf{S} \\ &= \mathbf{UDV}^T(2\mu\mathbf{G} + \lambda\mathrm{tr}(\mathbf{G})\mathbf{I})\end{aligned}$$

其中

$$\begin{aligned}\mathbf{G} &= \frac{1}{2}(\mathbf{VD}^2\mathbf{V}^T - \mathbf{I}) \\ &= \frac{1}{2}\mathbf{V}(\mathbf{D}^2 - \mathbf{I})\mathbf{V}^T\end{aligned}$$

把$\mathbf{G}$带入$\mathbf{P}$有

$$\begin{aligned}\mathbf{P} &= \mathbf{UDV}^T(2\mu\mathbf{G} + \lambda\mathrm{tr}(\mathbf{G})\mathbf{I}) \\ &= 2\mu\mathbf{UDV}^T\mathbf{G} + \lambda\mathrm{tr}(\mathbf{G})\mathbf{UDV}^T \\ &= \mu\mathbf{UD}(\mathbf{D}^2 - \mathbf{I})\mathbf{V}^T + \frac{\lambda}{2}\mathrm{tr}(\mathbf{D}^2 - \mathbf{I})\mathbf{UDV}^T \\ &= \mathbf{U}\left(\mu\mathbf{D}(\mathbf{D}^2 - \mathbf{I}) + \frac{\lambda}{2}\mathrm{tr}(\mathbf{D}^2 - \mathbf{I})\mathbf{D}\right)\mathbf{V}^T\end{aligned}$$

另一方面，根据SVD有

$$\mathbf{P} = \mathbf{U}\ \mathrm{diag}\left(\frac{\partial W}{\partial \lambda_i}\right)\ \mathbf{V}^T$$

因此只需证明

$$\mu\mathbf{D}(\mathbf{D}^2 - \mathbf{I}) + \frac{\lambda}{2}\mathrm{tr}(\mathbf{D}^2 - \mathbf{I})\mathbf{D} = \mathrm{diag}\left(\frac{\partial W}{\partial \lambda_i}\right)$$

以对角矩阵第一项λ_0为例，等式左边有

$$\mu\lambda_0(\lambda_0^2 - 1) + \frac{\lambda}{2}\lambda_0(\lambda_0^2 + \lambda_1^2 + \lambda_2^2 - 3)$$

而右边有

$$\frac{\partial W}{\partial \lambda_0} = \begin{bmatrix}\frac{\partial I}{\partial \lambda_0} & \frac{\partial II}{\partial \lambda_0} & \frac{\partial III}{\partial \lambda_0}\end{bmatrix}\begin{bmatrix}\frac{\partial W}{\partial I} \\ \frac{\partial W}{\partial II} \\ \frac{\partial W}{\partial III}\end{bmatrix}$$

对于StVK模型有

$$\begin{bmatrix}\frac{\partial I}{\partial \lambda_0} & \frac{\partial II}{\partial \lambda_0} & \frac{\partial III}{\partial \lambda_0}\end{bmatrix} = \begin{bmatrix}2\lambda_0 & 4\lambda_0^3 & *\end{bmatrix}$$

$$\begin{bmatrix}\frac{\partial W}{\partial I} \\ \frac{\partial W}{\partial II} \\ \frac{\partial W}{\partial III}\end{bmatrix} = \begin{bmatrix}s_0(I-3) - \frac{s_1}{2} \\ \frac{s_1}{4} \\ 0\end{bmatrix}$$

因此

$$
\begin{aligned}
\frac{\partial W}{\partial \lambda_0} &= 2\lambda_0[s_0(I-3)-\frac{s_1}{2}]+s_1\lambda_0^3 \\
&= 2s_0\lambda_0(I-3)+s_1\lambda_0(\lambda_0^2-1) \\
&= 2s_0\lambda_0(\lambda_0^2+\lambda_1^2+\lambda_2^2-3)+s_1\lambda_0(\lambda_0^2-1)
\end{aligned}
$$

因此只需令拉梅参数$\lambda=4s_0$, $\mu=s_1$即可使左右两边相等。对于对角矩阵的另外两项可以类似地证明。