中国科学技术大学
University of Science and Technology of China

GAMES 301：第15讲

参数化在产业中的应用 (2)

刘利刚
中国科学技术大学

提纲

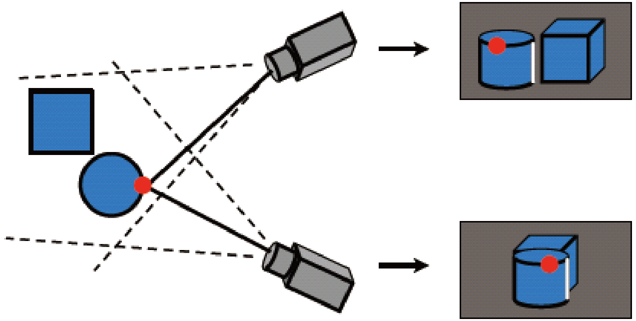
1. 参数化在三维扫描工业的应用
2. 参数化在工业软件中的应用
3. 其他类型的参数化
4. 高维数据的参数化
5. 课程总结

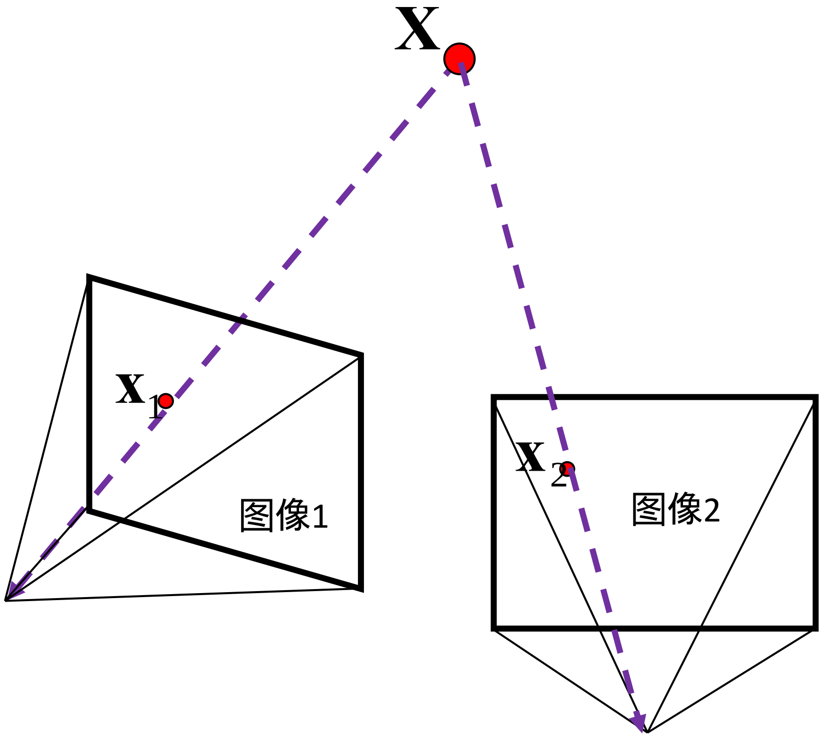
1

参数化在三维扫描工业的应用

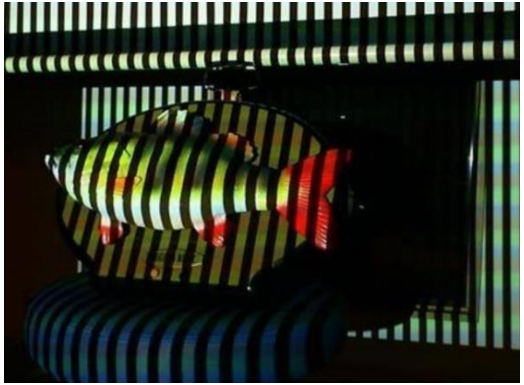
3D重建：从多视点图像重建3D模型

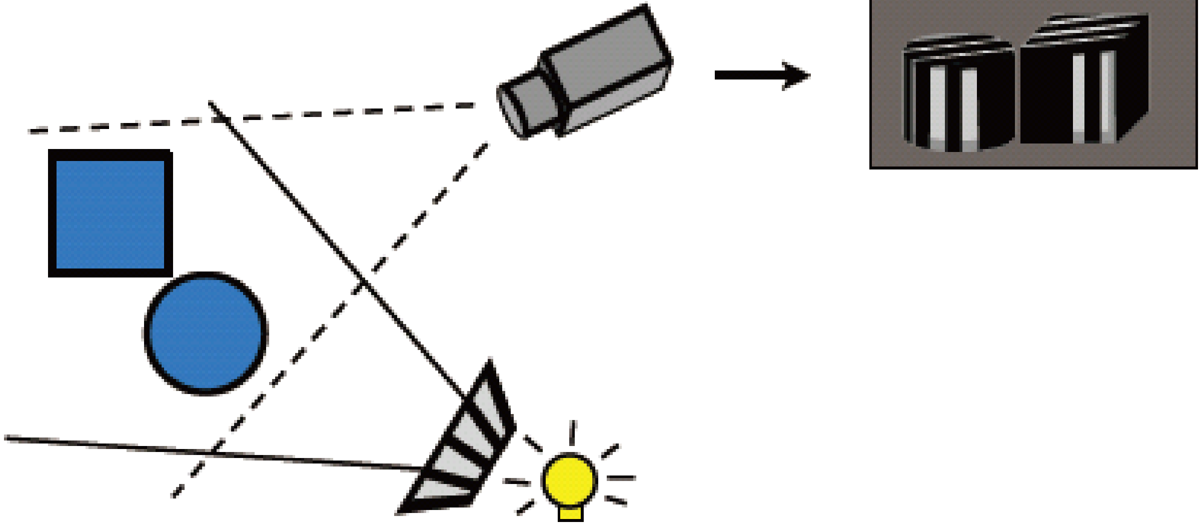
多视点几何





3D重建（主动式）：结构光扫描





产业应用2：

参数化在三维扫描工业的应用

贾颜铭
先临三维科技股份有限公司

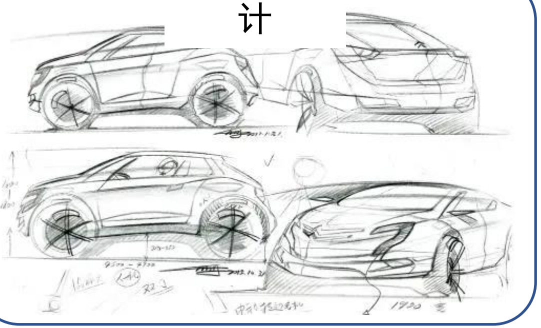
2

参数化在工业软件中的应用

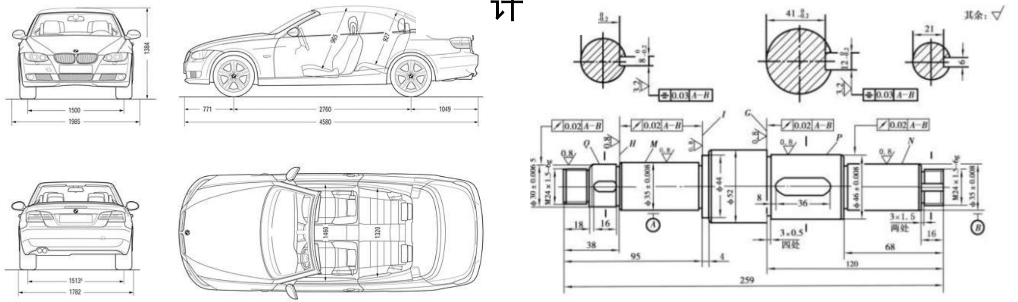
（等几何分析中的区域参数化）

工业产品制造：主要流程

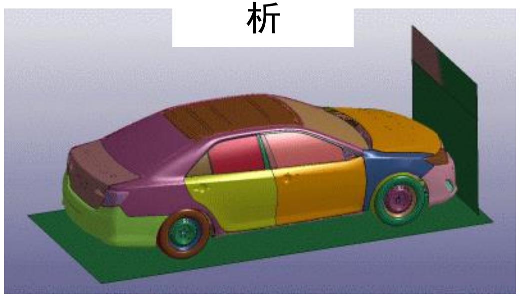
概念设计



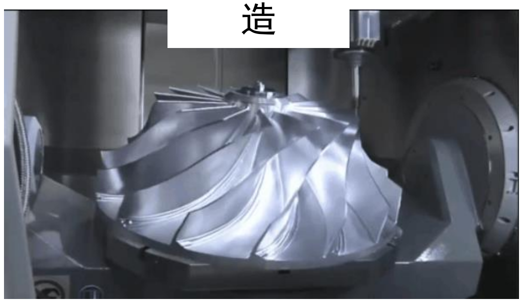
几何设计



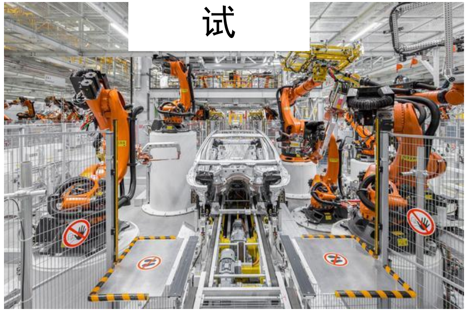
仿真分析



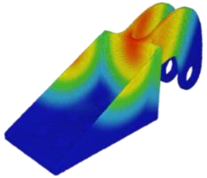
加工制造



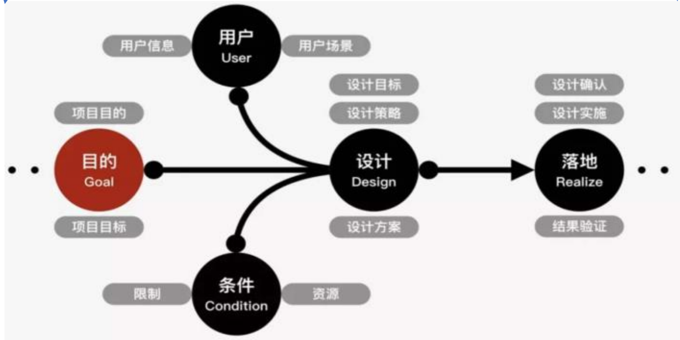
装配调试

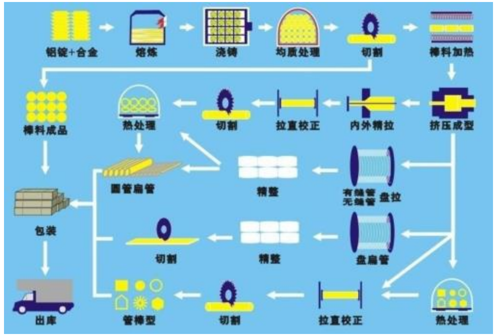


工业产品制造：复杂的流程

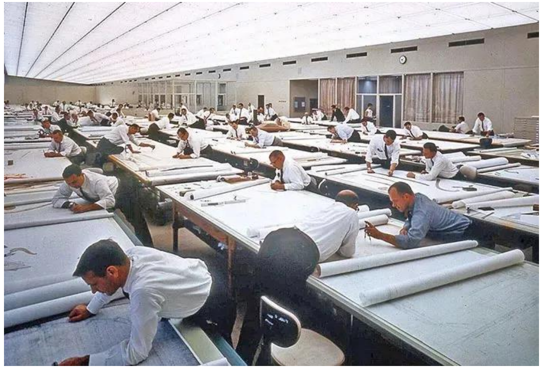


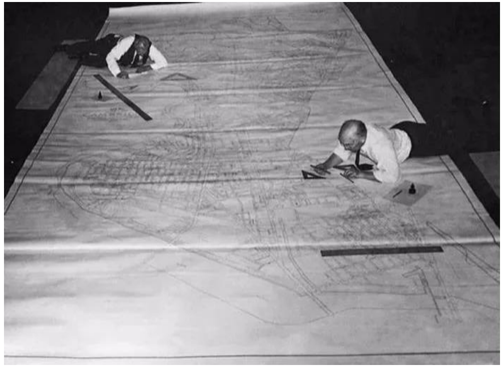
市场调研 → 外观设计 → 结构设计 → 生产工艺 → 出模试装





历史：手工制图设计

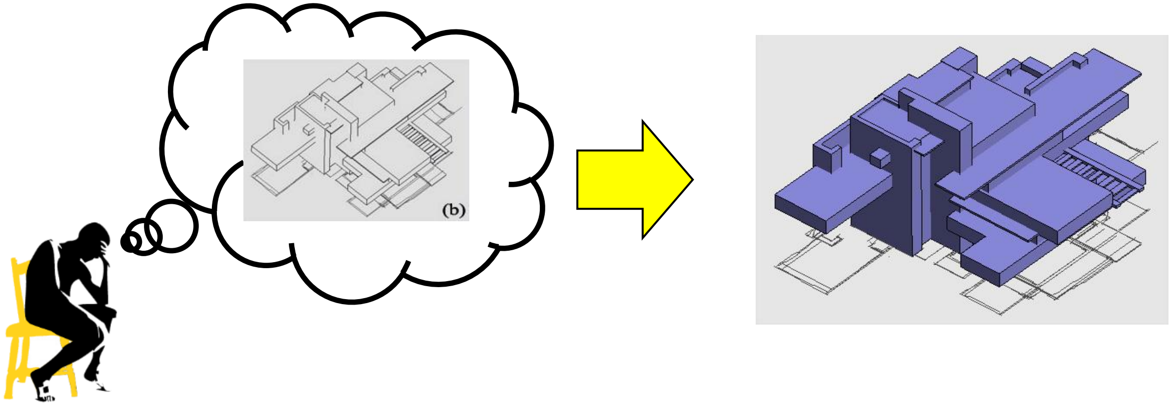




工业软件：系列的系统软件

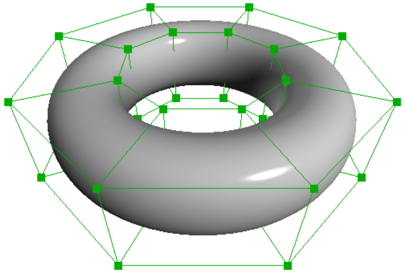
CAD (Computer Aided Design)
计算机辅助设计

- 【百度百科】利用计算机及其图形设备帮助设计人员进行设计工作（几何造型）
- **数学**：曲线曲面、样条、光滑、光顺…
- CAD软件：AutoCAD, SolidWorks, 3DMax, …

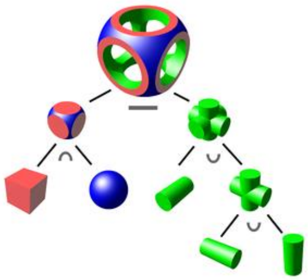


几何内核引擎

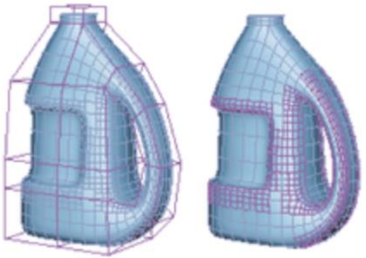
几何造型引擎



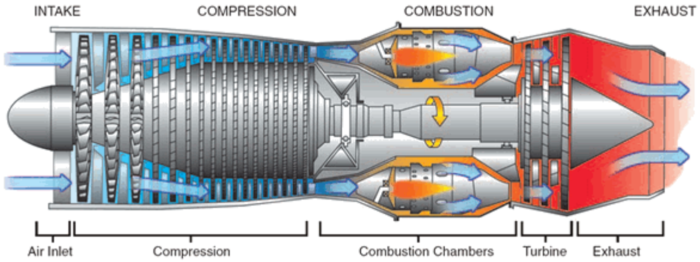
NURBS



CSG

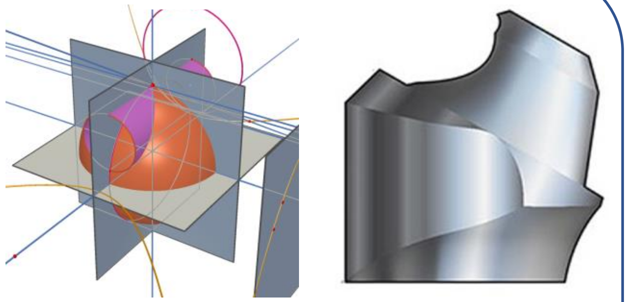


Subdivision

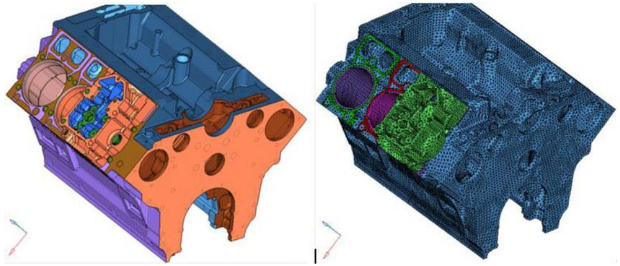


特征造型与参数化设计

几何计算引擎

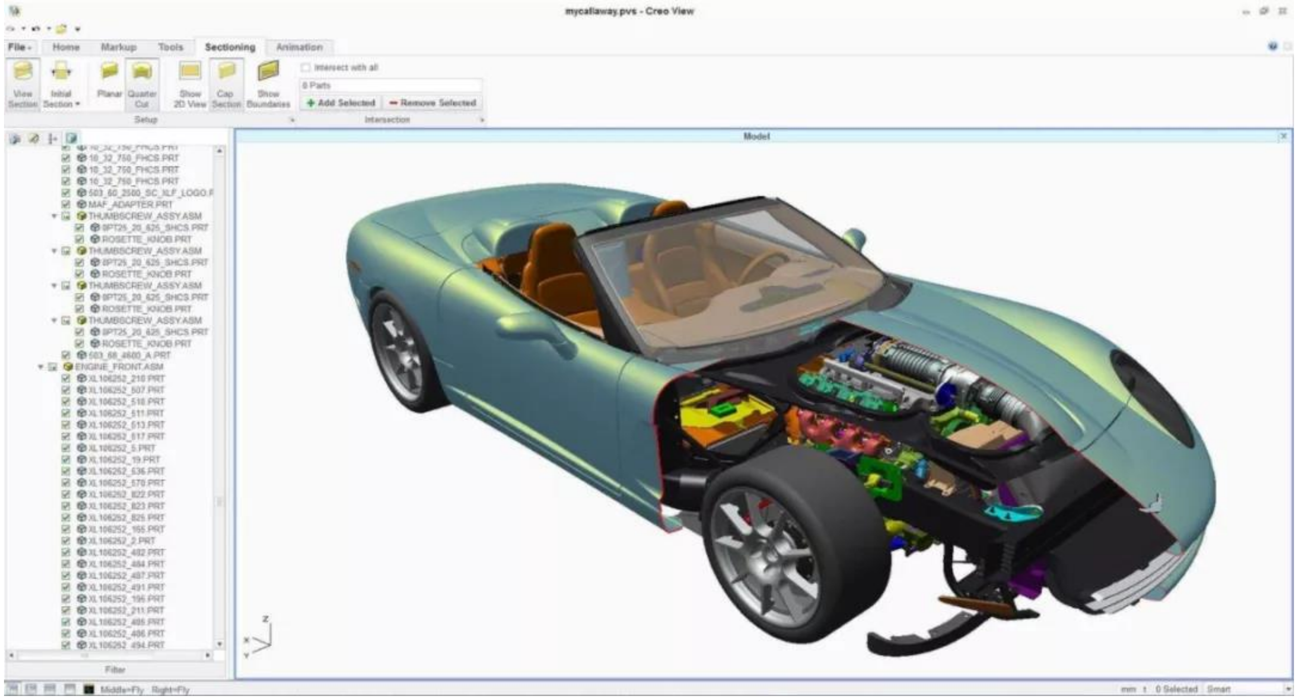


布尔操作



约束求解

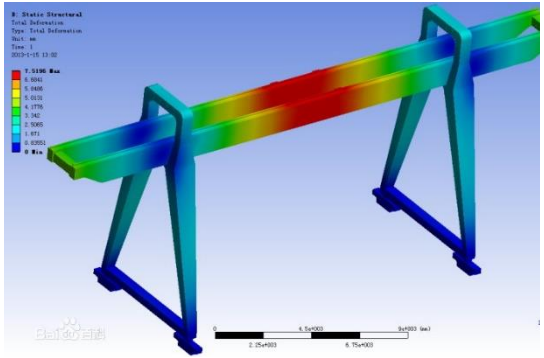
一辆汽车的CAD模型：数万构件



CAE (Computer Aided Engineering)
计算机辅助分析（工程）

- 【百度百科】用计算机辅助分析产品的结构力学性能，以及优化结构性能等
- **力学**：理论力学，材料力学，流体力学，机构力学，弹性力学（有限元分析）
- CAE软件：UG, Pro/E, CATIA, Ansys, Abaqus, …

几何体 → 材料属性 → 相互作用 → 约束载荷 → **网格划分** → 计算分析



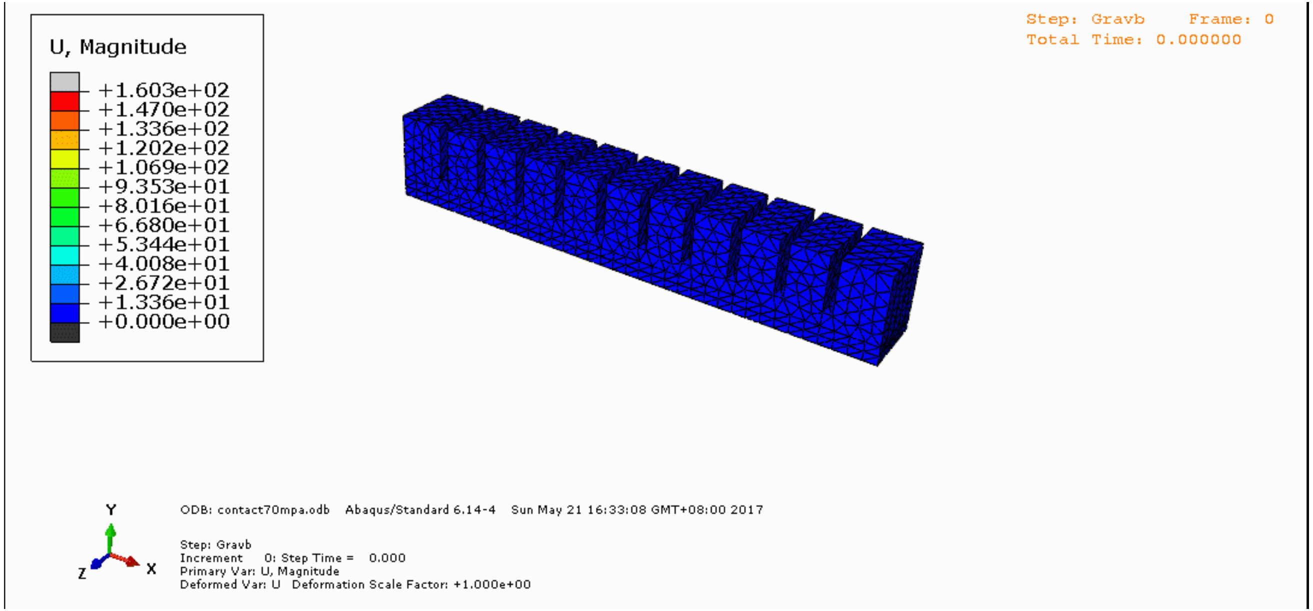
单元

杨氏模量

泊松比

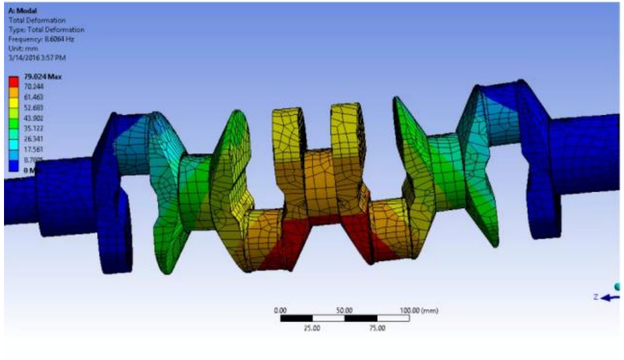
刚度方程

有限元仿真计算结果

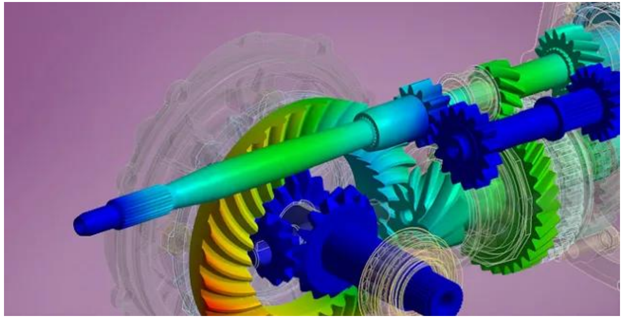


模拟计算：多腔体式驱动器的弯曲致动变形

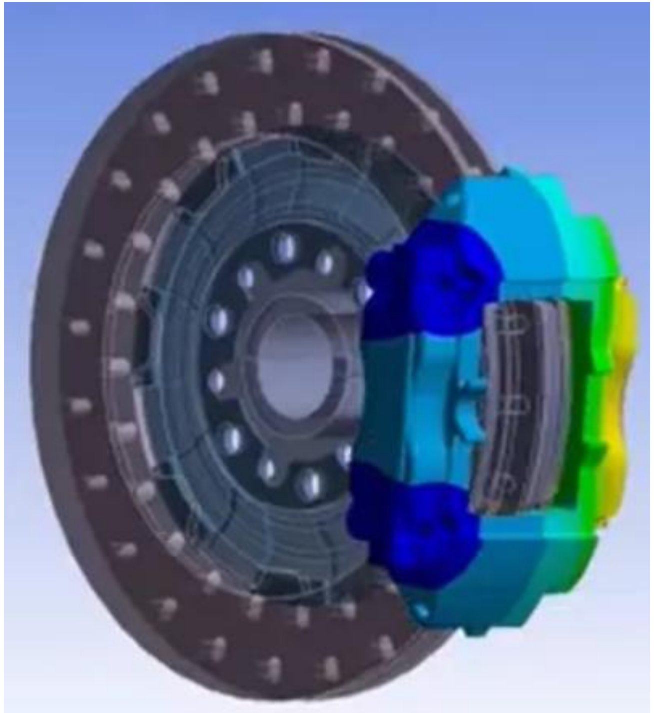
汽车各部件的仿真分析



曲轴的变应力分析

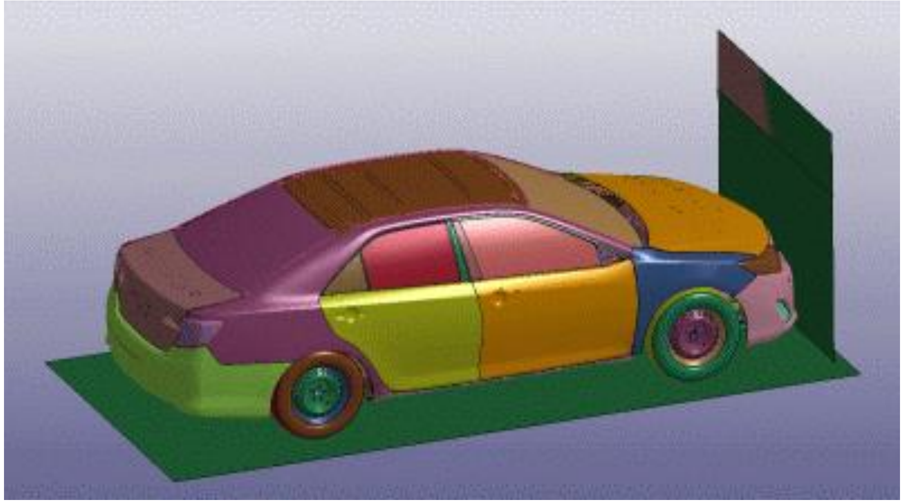


差速器的散热分析

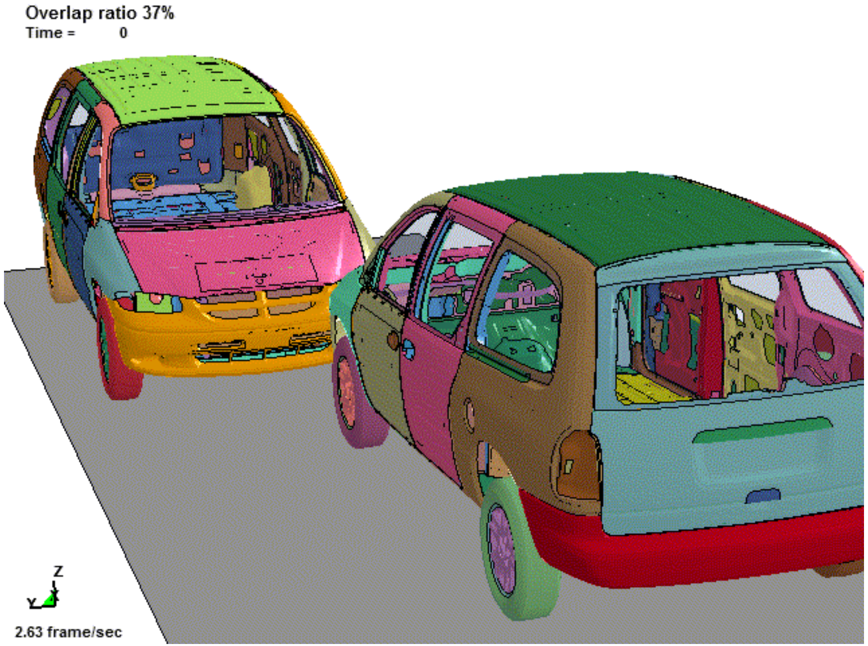


刹车片的摩擦与损耗分析

整车的碰撞分析



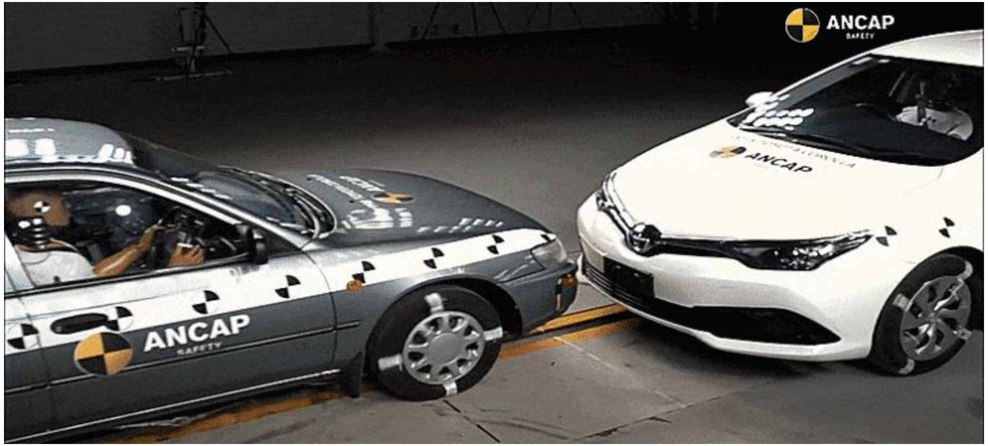
汽车撞击试验



两车相撞试验

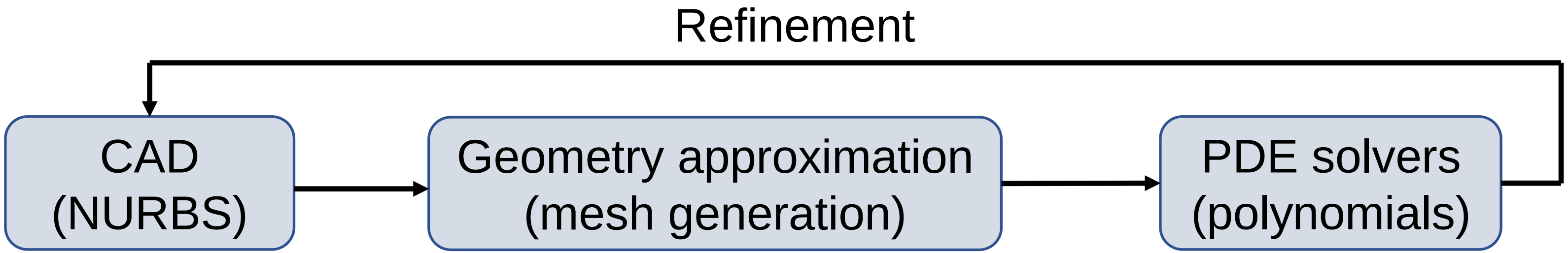
真车验证试验

汽车撞击试验



两车相撞试验

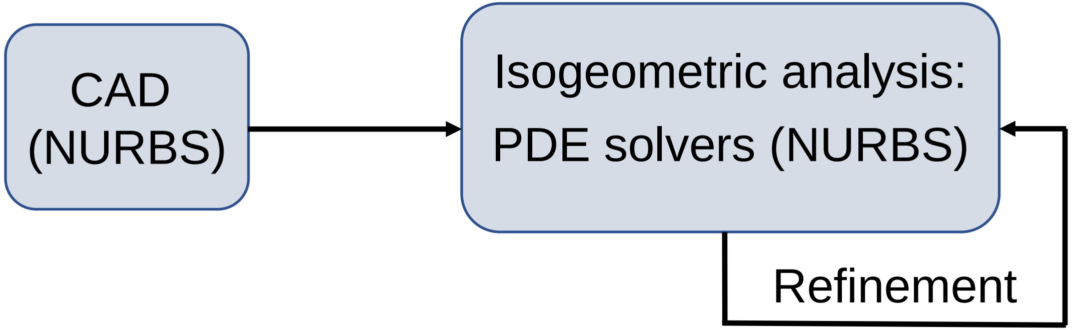
A revisit to FEM



Drawbacks:

- Mesh generation accounts for about 80% of overall analysis time
- CAD geometry is approximated by FEM geometry
- CAD and FEM use different descriptions for the geometry
- Mesh refinement requires interaction with CAD geometry

Isogeometric analysis (IGA)



Advantages:

- Avoid mesh generation
- Employ the same description (NURBS) for geometry and analysis
- Maintain the geometric representation given by CAD
- Superior approximation properties

Hughes et al. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. CMAME 2005.

FEM

- Elliptic BVP:

$$\begin{aligned} -\mathrm{div}(D\nabla u) + \lambda u = f \quad & \text{in } \Omega \\ u = 0 \quad & \text{on } \partial\Omega \end{aligned}$$

- Weak form:

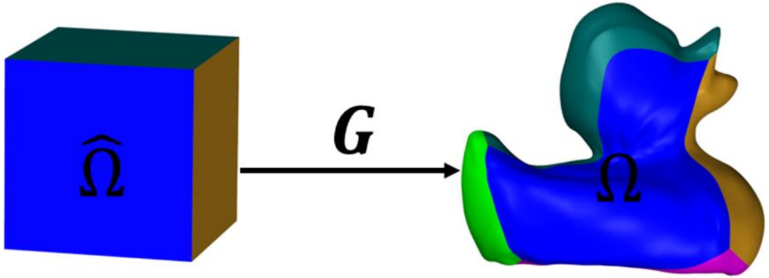
$$\text{Finding } u \in H_0^1(\Omega) \text{ such that } a(u,v) = f(v),\ \forall v \in H_0^1(\Omega)$$

- Numerical solution:

$$u_h(\boldsymbol{x}) = \sum_{i=1}^{n} u_i \phi_i(\boldsymbol{x})$$

IGA-FEM Framework

- Let $\boldsymbol{G}$ be a map from the unit cube to a computational domain:



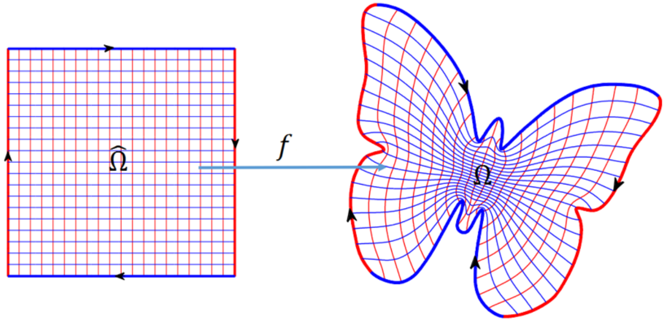
$$\boldsymbol{x} = \boldsymbol{G}(\hat{\boldsymbol{x}}) = \sum_{i=1}^{n} \boldsymbol{c}_i R_i(\hat{\boldsymbol{x}})$$

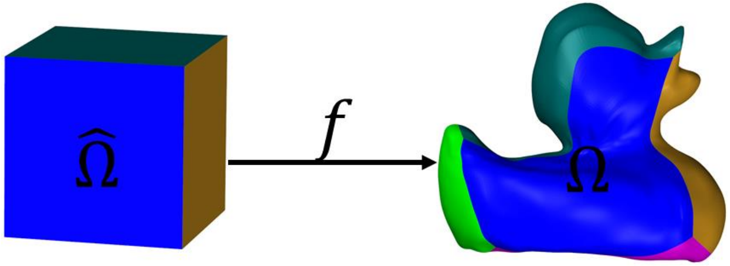
- Numerical solution:

$$u_h(\boldsymbol{x}) = \sum_{i=1}^{n} u_i(R_i \circ \boldsymbol{G}^{-1}(\boldsymbol{x}))$$

Domain parameterization （区域参数化）

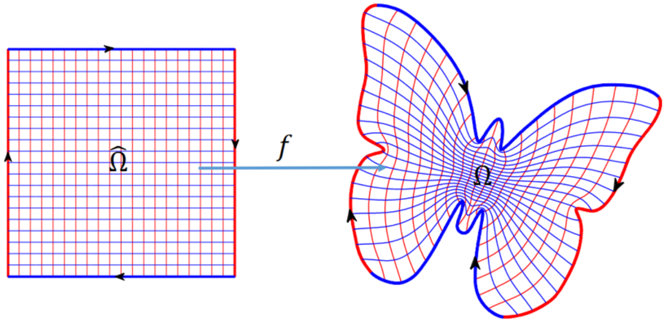
- A significantly challenging problem in isogeometric analysis.
- Given the boundary of a planar (volumetric) domain Ω, compute a parametric representation $f(u,v)$ or $f(u,v,w)$ of the domain Ω, i.e., a map f from a square (cube) to a planar (volumetric) domain.

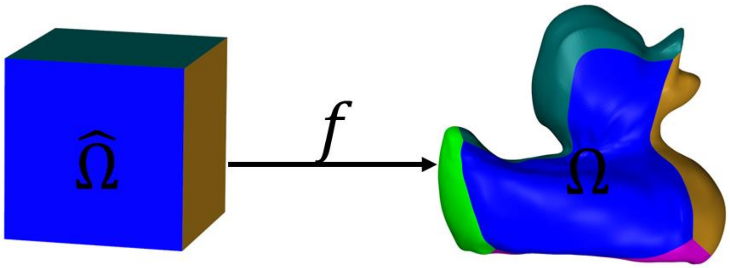




Domain parameterization

- Requirement of the map f:
  - Bijective
  - Low (angular and area, volume) distortion
    - Orthogonal
    - Uniform





区域参数化的主要方法

- Coons surfaces
  - [Farin and Hansford 1999]
- Harmonic map-based methods
  - [Martin et al. 2009; Nguyen and Jüttler 2010; Xu et al. 2013]
- Nonlinear optimization-based methods
  - Injectivity condition constrained methods [Xu et al. 2013; Wang and Qian 2014; Ji et al. 2022]
  - Collocation-based method [Pan and Chen 2020]
  - Quasi-conformal mapping [Nian and Chen 2016; Pan and Chen 2018; Pan and Chen 2022]
- Parameterizations with locally refinable splines
  - T-splines [Escobar et al. 2011; Zhang et al. 2012; Wang et al. 2013]
  - PHT-splines [Chan et al. 2017]
  - THB-splines [Falini and Jüttler 2015; Zheng and Chen 2022]
- Low-rank parameterizations
  - [Jüttler and Mokriš 2017; Pan and Chen 2018; Pan and Chen 2019]
- Multi-patch parameterizations
  - Skeleton-based methods [Xu et al., 2015; Bastl and Slabá 2021]
  - Polycube-based methods [Chen et al. 2018; Wang et al. 2022]
  - Graph-based methods [Buchegger and Jüttler, 2017; Falini and Jüttler 2019]

致谢：潘茂东（南京航空航天大学）

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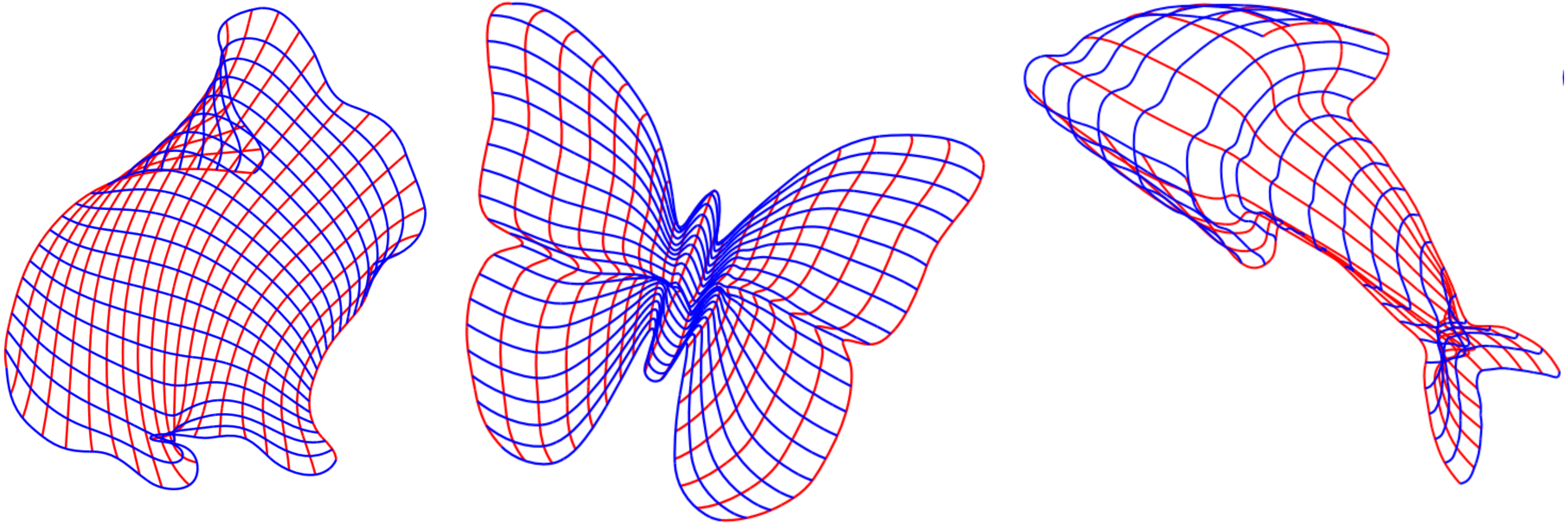
Coons surfaces

- The interior control points are constructed as linear combination of boundary control points
- Simple but not injective



Farin and Hansford. Discrete Coons patches. CAGD 1999.

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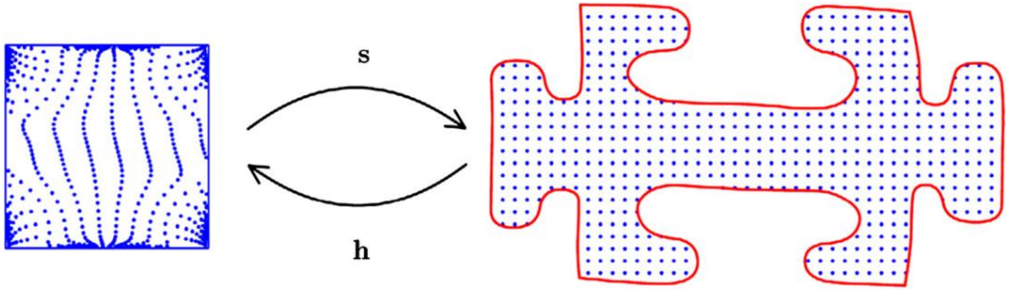
Harmonic map-based methods

- Compute a harmonic map $\boldsymbol{h}$ from Ω to $\widehat{\Omega}$ by solving the Laplace equation

$$\begin{cases} \Delta \boldsymbol{h} \; = \; 0 & \text{in } \Omega \\ \boldsymbol{h} \text{ is given} & \text{on } \partial\Omega \end{cases}$$

- The desired parameterization $\boldsymbol{s}$ is obtained by approximating the inverse map $\boldsymbol{h}^{-1}$ using splines



- Not injective in 3D case

Martin et al. Volumetric Parameterization and Trivariate B-spline Fitting using Harmonic Functions. CAGD 2009.

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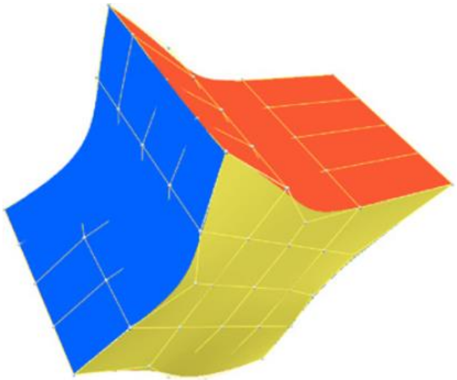
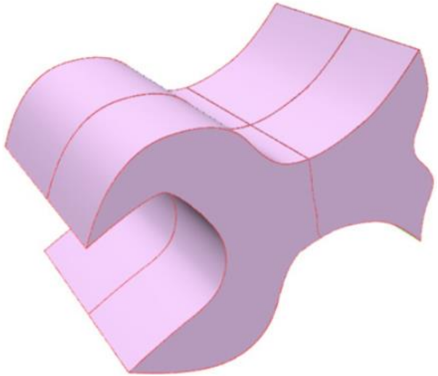
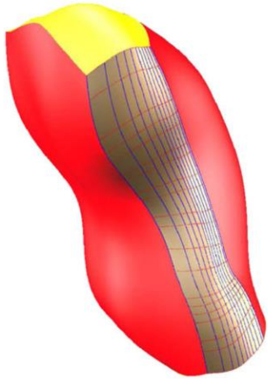
Injectivity condition constrained methods

- 目标函数：映射的某种扭曲（共形扭曲、体积扭曲等）
- 约束：保证映射单射性的充分条件（映射的雅可比是一个高阶样条，只要该样条的组合系数大于零即可）
- 这类方法的约束太严格，对于形状较为复杂的区域，通常没有可行解，只适用于一些简单区域

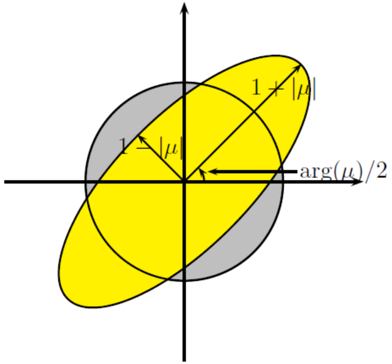
Xu et al. Optimal analysis-aware parameterization of computational domain in 3D isogeometric analysis. CAD 2013.

Quasi-conformal Map

- $f : \widehat{\Omega} \to \Omega$ is a quasi-conformal map provided it satisfies

$$\frac{\partial f}{\partial \bar{z}} = \mu(f)\frac{\partial f}{\partial z} \text{ with } \sup|\mu(f)| < 1$$

- Quasi-conformal map is locally injective.
- A quasi-conformal map maps an infinitely small circle to an ellipse.
- **Dilatation:** $K(z) = \frac{1+|\mu(z)|}{1-|\mu(z)|}$.



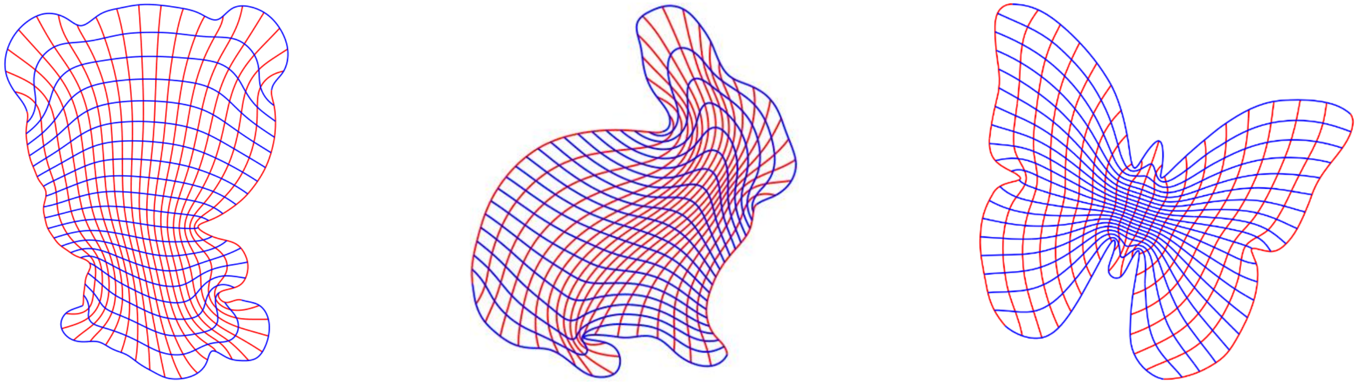
Compute A Quasi-conformal Map

Mathematical model:

$$\min_{f} \int_{\widehat{\Omega}} |\mu(f)|^2 \, dz + \lambda \int_{\widehat{\Omega}} |\nabla \mu(f)|^2 \, dz$$

$$\text{s.t.} \quad \|\mu(f)\|_\infty < 1$$

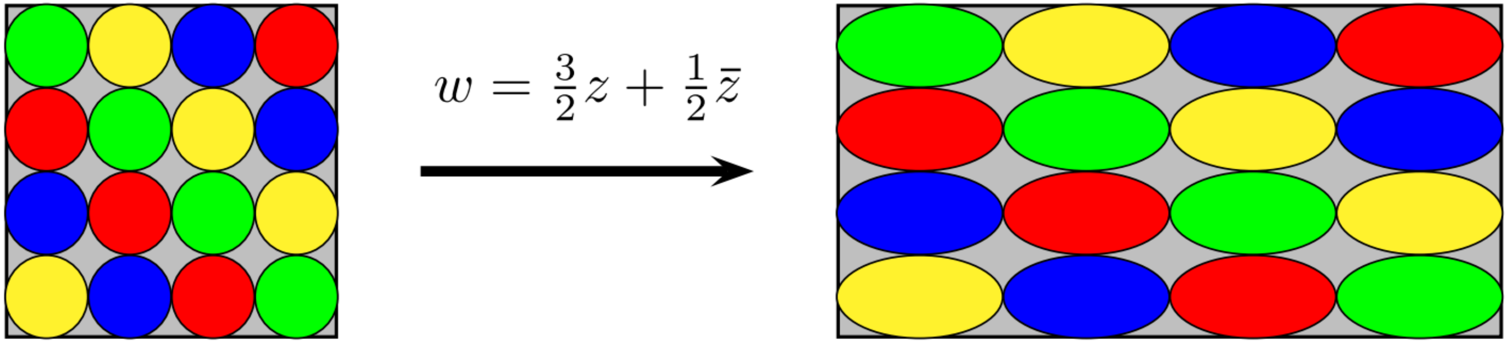
$f|_{\partial\widehat{\Omega}}$ is given.



Pan, Chen and Tong. Low-rank parameterization of planar domains for isogeometric analysis. CAGD 2018.

Teichmüller map

- $f: \widehat{\Omega} \to \Omega$ is a Teichmüller map provided it satisfies $\mu(f) = k\frac{\bar{\phi}}{|\phi|}$ for constant $0 < k < 1$ and ϕ is a holomorphic function.
- A special case of quasi-conformal map.

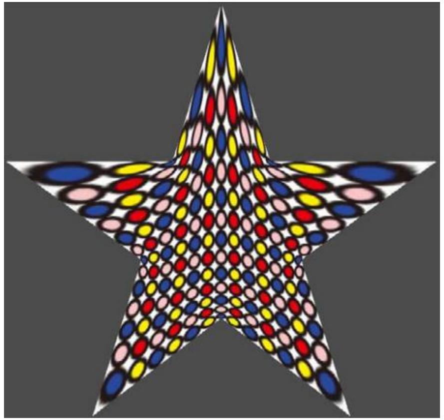


Compute A Teichmüller Map

Since

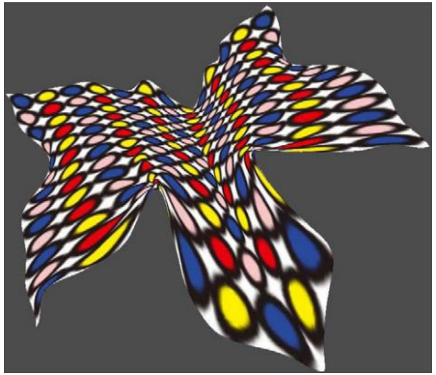
$$|\mu(f)| = \left|\frac{f_{\bar{z}}}{f_z}\right| = \left|k\frac{\bar{\phi}}{|\phi|}\right| = k$$

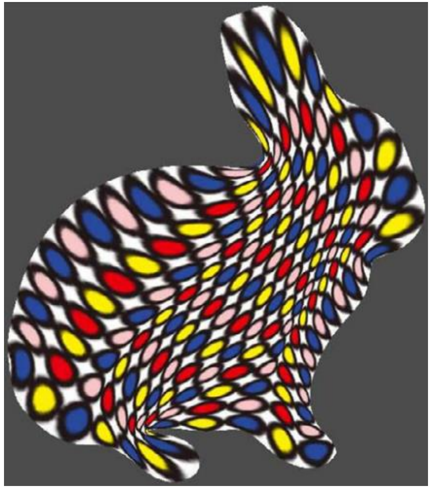
Mathematical model:



$$\min_f \int_{\hat{\Omega}} \left(\frac{|f_{\bar{z}}|^2}{|f_z|^2} - k^2\right)^2 dz$$

$\mathrm{s.t.}\ f|_{\partial\hat{\Omega}}$ is given.

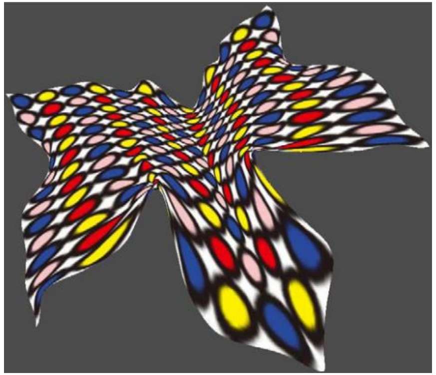




Nian and Chen. Planar domain parameterization for isogeometric analysis based on Teichmüller mapping. CMAME 2016.

Compute A Teichmüller Map

Quasi-conformal map is only for 2D case!



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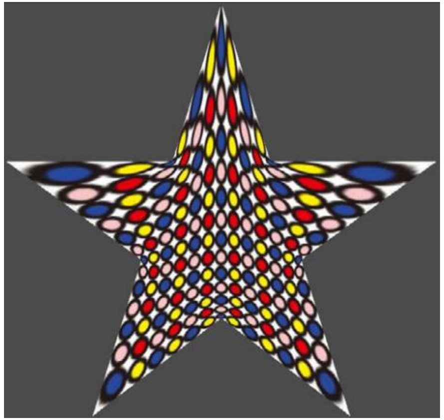

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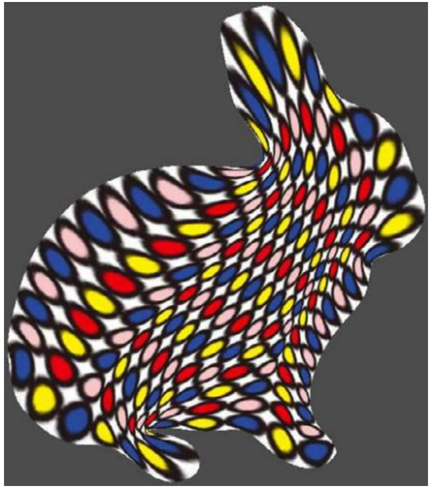


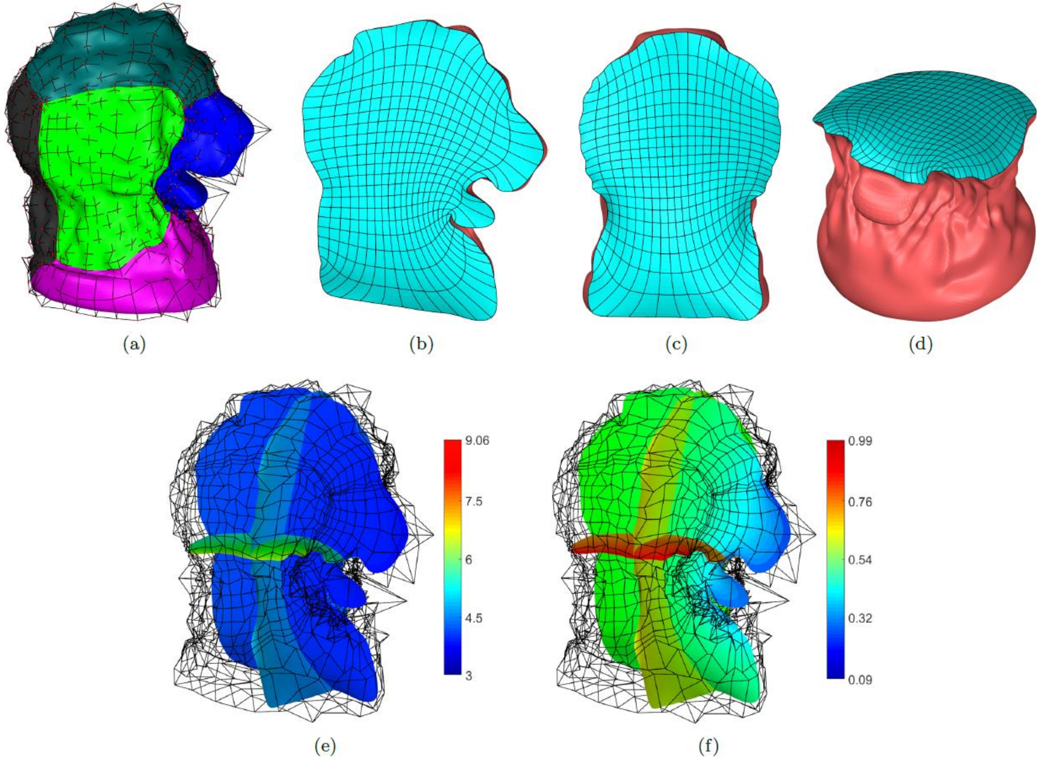
Nian and Chen. Planar domain parameterization for isogeometric analysis based on Teichmuller mapping. CMAME 2016.

Collocation-based method

- Three stages:
  - Computing an initial map
  - Construction of a bijective parameterization by solving a max-min optimization problem
  - Improving parameterization quality using MIPS

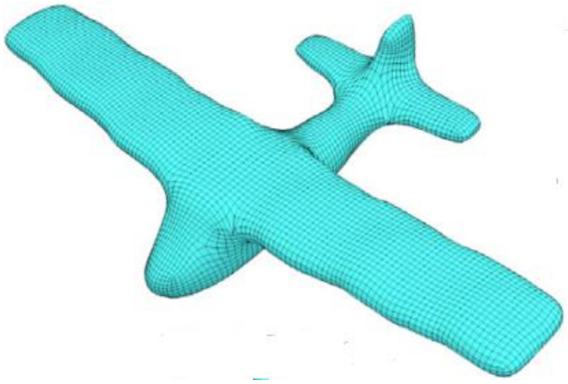
Pan, Chen and Tong. Volumetric spline parameterization for isogeometric analysis. CMAME 2020.

Parameterization Results



Limitations of Collocation-based Method

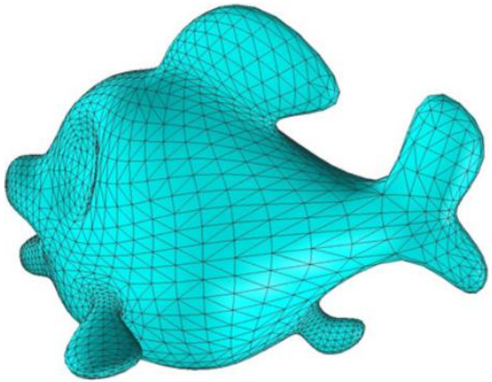
- Solving the sophisticated optimization problem is really computationally expensive.
- For some complicated geometries, the collocation-based method can not produce satisfactory results.



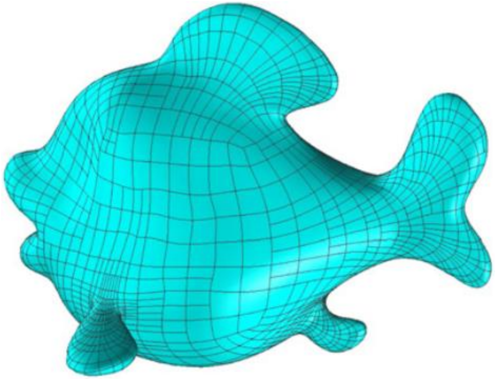
区域参数化的主要方法

- Coons surfaces
  - [Farin and Hansford 1999]
- Harmonic map-based methods
  - [Martin et al. 2009; Nguyen and Jüttler 2010; Xu et al. 2013]
- Nonlinear optimization-based methods
  - Injectivity condition constrained methods [Xu et al. 2013; Wang and Qian 2014; Ji et al. 2022]
  - Collocation-based method [Pan and Chen 2020]
  - Quasi-conformal mapping [Nian and Chen 2016; Pan and Chen 2018; Pan and Chen 2022]
- Parameterizations with locally refinable splines
  - T-splines [Escobar et al. 2011; Zhang et al. 2012; Wang et al. 2013]
  - PHT-splines [Chan et al. 2017]
  - THB-splines [Falini and Jüttler 2015; Zheng and Chen 2022]
- Low-rank parameterizations
  - [Jüttler and Mokriš 2017; Pan and Chen 2018; Pan and Chen 2019]
- Multi-patch parameterizations
  - Skeleton-based methods [Xu et al., 2015; Bastl and Slabá 2021]
  - Polycube-based methods [Chen et al. 2018; Wang et al. 2022]
  - Graph-based methods [Buchegger and Jüttler, 2017; Falini and Jüttler 2019]

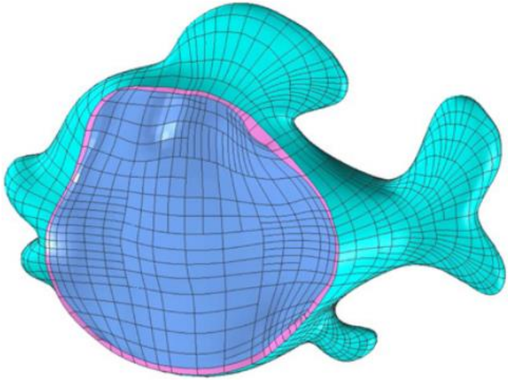
Parameterization With T-splines



The input boundary



The constructed solid T-spline



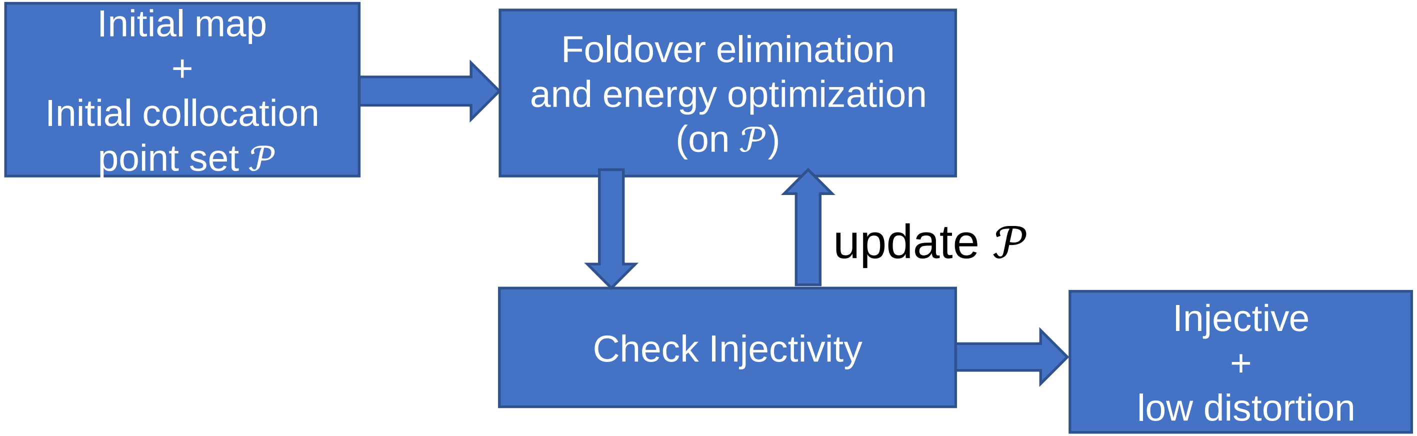
The interior information

- No guarantee on the injectivity of the map

Zhang and Hughes. Solid T-spline construction from boundary representations for genus-zero geometry. CMAME 2012.

Parameterization With THB-splines

- Goal:
  - Construct a strictly bijective mapping
  - Accelerate the collocation-based method [Pan et al. CMAME 2020]
- Framework:



Zheng and Chen. Volumetric parameterization with truncated hierarchical B-Splines for IGA. CMAME 2022.

Foldover Elimination

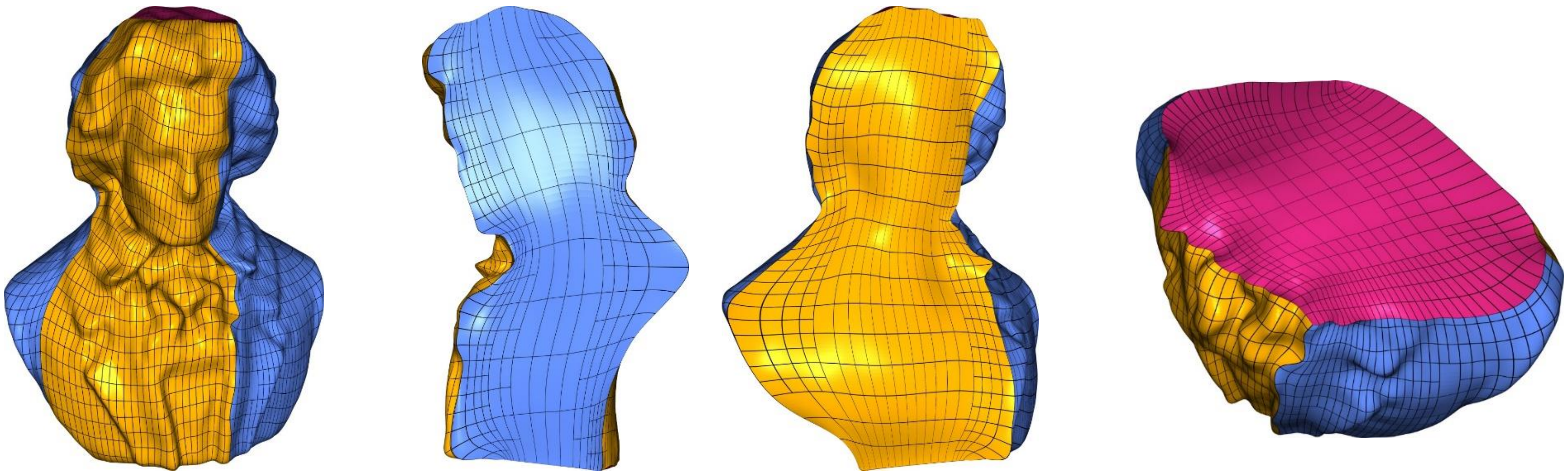
- Given the collocation point set P, the foldovers are eliminated by solving the problem:

$$\min_{f,H_j} \sum_{\mathbf{p}_j \in P} \|J_f(\mathbf{p}_j) - H_j\|_F^2$$

$$s.t. \quad H_j \in H_K$$

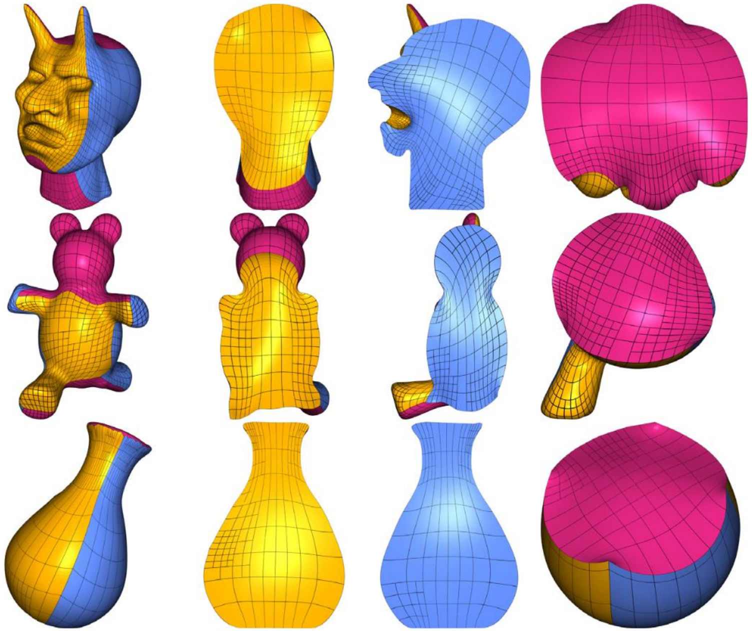
- H_K: K-bounded distortion space
- Local-global solver:
  - Update THB coefficients: quadratic problem
  - Update H_K: explicit formula
  - When falling in stagnation: enlarge K or refine the THB bases

Results: Bust model



Results on a dataset

- Bijective results: 116/215
- Strongly influenced by the boundary correspondence!



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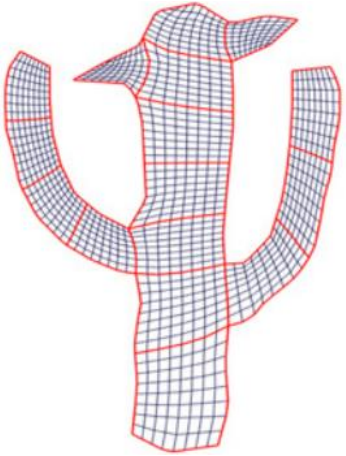
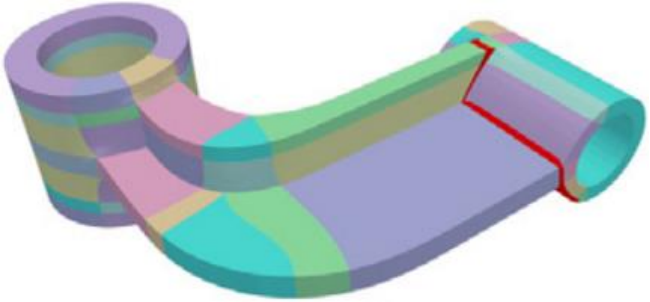
  - Graph-based methods [Buchegger and Jüttler, 2017; Falini and Jüttler 2019]

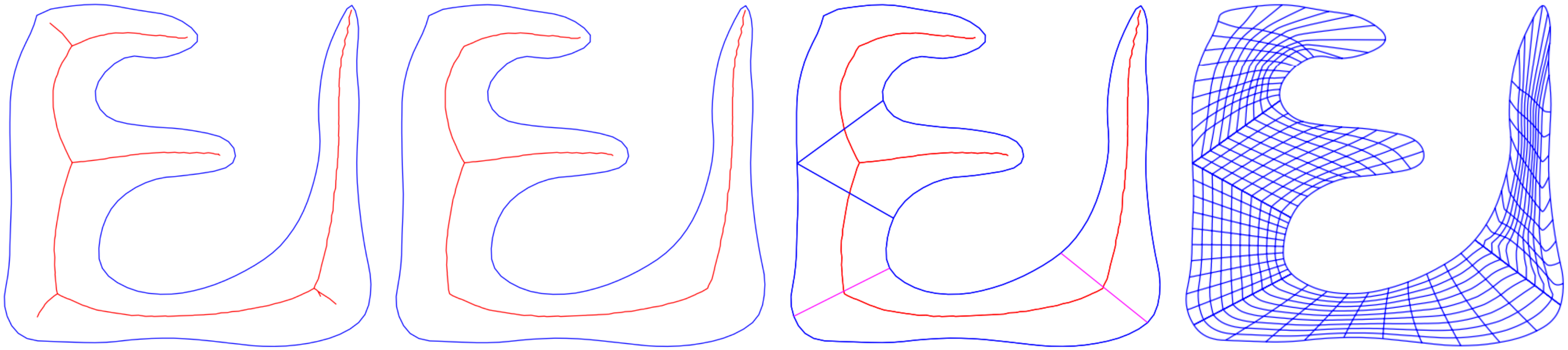
Multi-patch Parameterizations

- When dealing with complicated geometries, the flexibility of single-patch parameterization is usually insufficient.
- The procedure of multi-patch parameterizations includes
  - Decompose a given physical domain into several simple regions.
  - Each simple region is parameterized by previous methods.





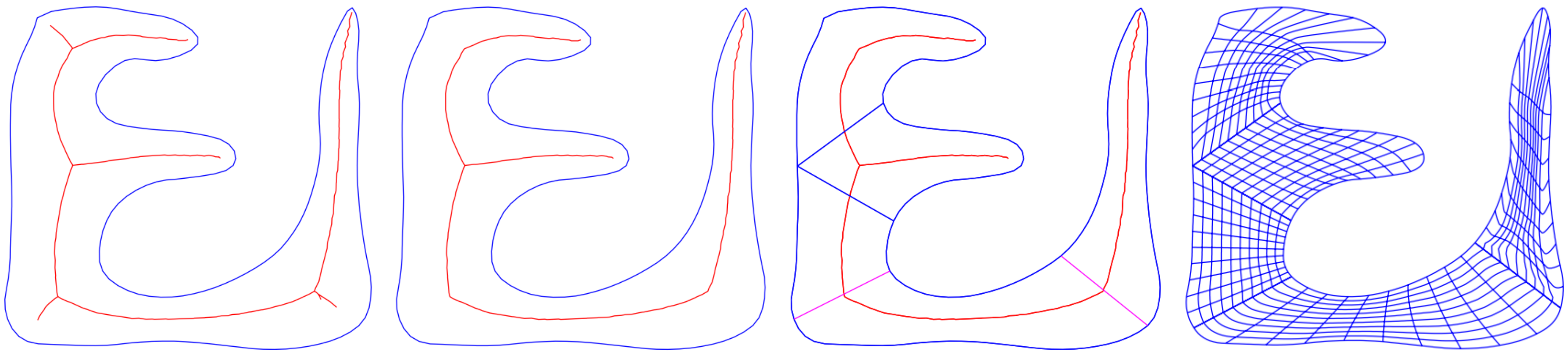
Skeleton-based methods



Original skeleton | Simplified skeleton | Decomposition result | Parameterization result

Bastl and Slabá. Planar multi-patch domain parameterization for isogeometric analysis based on evolution of fat skeleton. CMAME 2021.

Skeleton-based methods

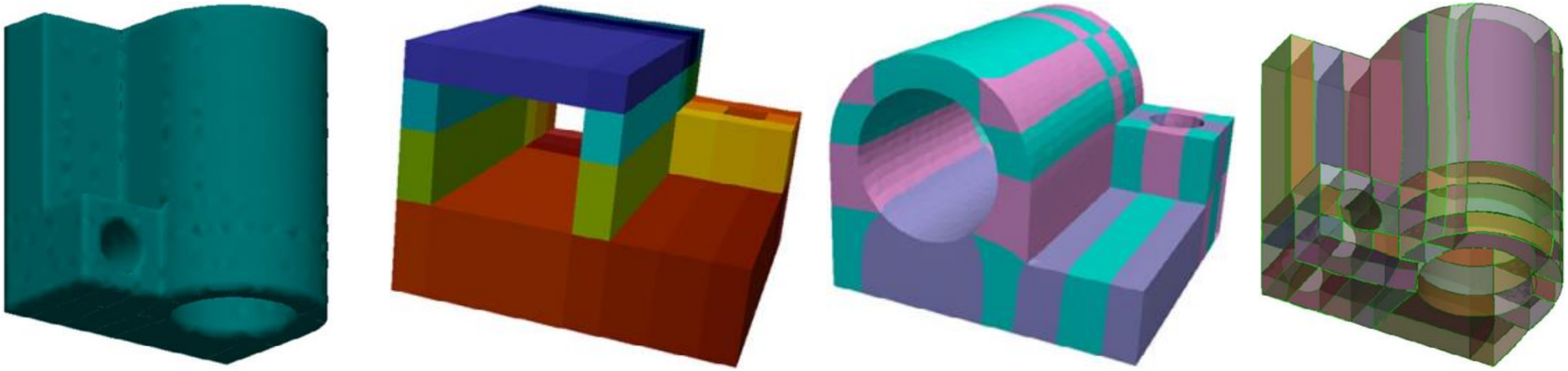


Original skeleton | Simplified skeleton | Decomposition result | Parameterization result

Only C^0 continuity!

Bastl and Slabá. Planar multi-patch domain parameterization for isogeometric analysis based on evolution of fat skeleton. CMAME 2021.

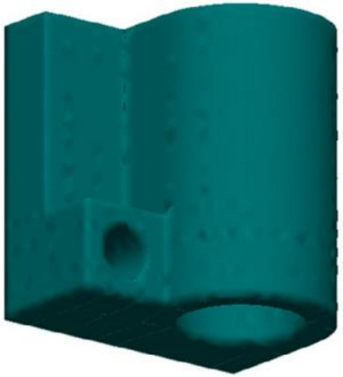
Polycube-based methods

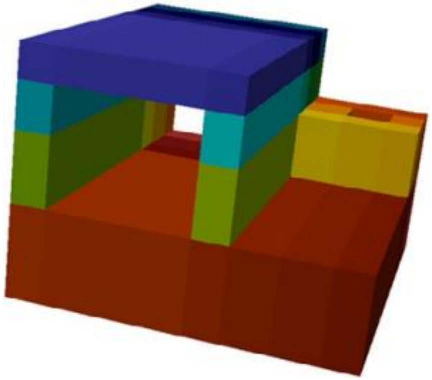


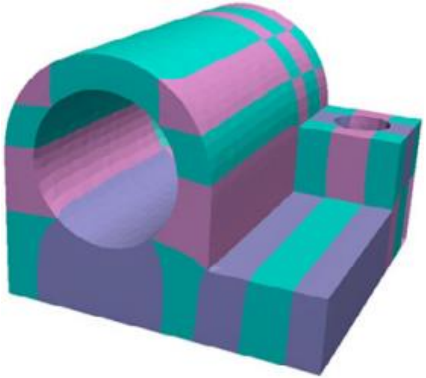
The input domain | Polycube structure | Decomposition result | Parameterization result

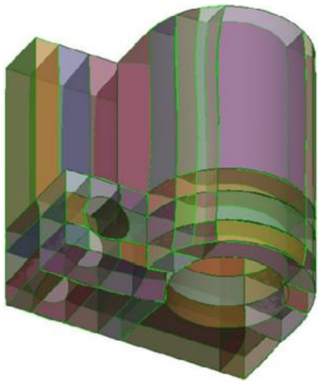
Chen et al. Constructing volumetric parameterization based on directed graph simplification of polycube structure from complex shapes. CMAME 2019.

Polycube-based methods









The input domain | Polycube structure | Decomposition result | Parameterization result

Only C^0 continuity!
Polycube structure induces internal singularities!

Chen et al. Constructing volumetric parameterization based on directed graph simplification of polycube structure from complex shapes. CMAME 2019.

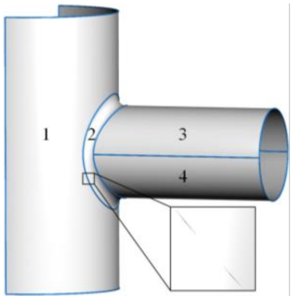
Summary

Method		Injective	Work for complicated domains	C^1/C^2 continuity	Work for 3D case
1	Coons surfaces	×	×	✓	✓
2	Harmonic map-based methods	×	×	✓	✓
3	Injectivity condition constrained methods	✓	×	✓	✓
	Collocation-based method	✓	×	✓	✓
	Quasi-conformal mapping	✓	×	✓	×
4	Parameterizations with T-splines	×	×	✓	✓
	Parameterizations with THB-splines	✓	×	✓	✓
5	Skeleton-based methods	✓	✓	×	✓
	Polycube-based methods	✓	✓	×	✓

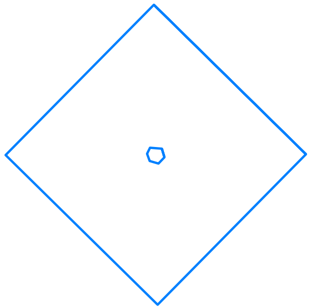
一种新的样条表达：

Triangle configuration B-splines (TCB-splines)

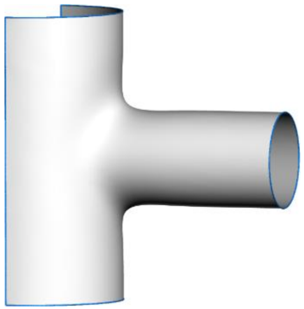
- TCB-splines retain many attractive theoretic properties of classical B-splines
- TCB-splines support local refinement
- TCB-splines are more flexible to accommodate general parametric domain



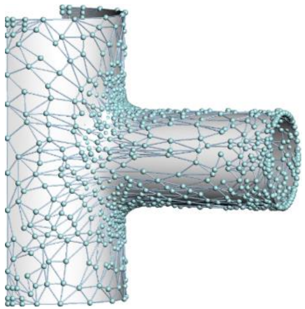
(a) Given CAD model (4 patches)



(b) Parameter domain



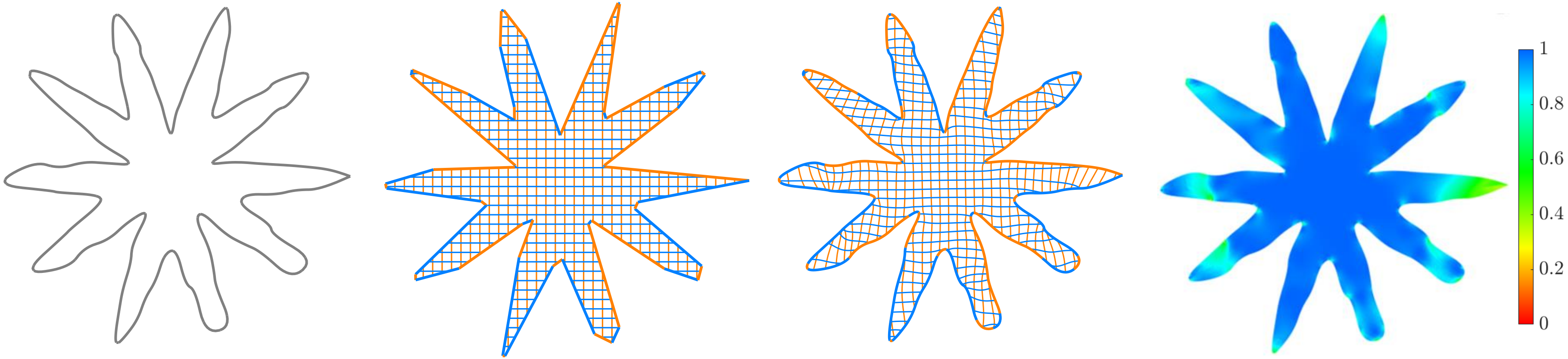
(c) TCB-spline surface (1 patch)



(d) Control net of (c)

致谢：曹娟、陈中贵（厦门大学）

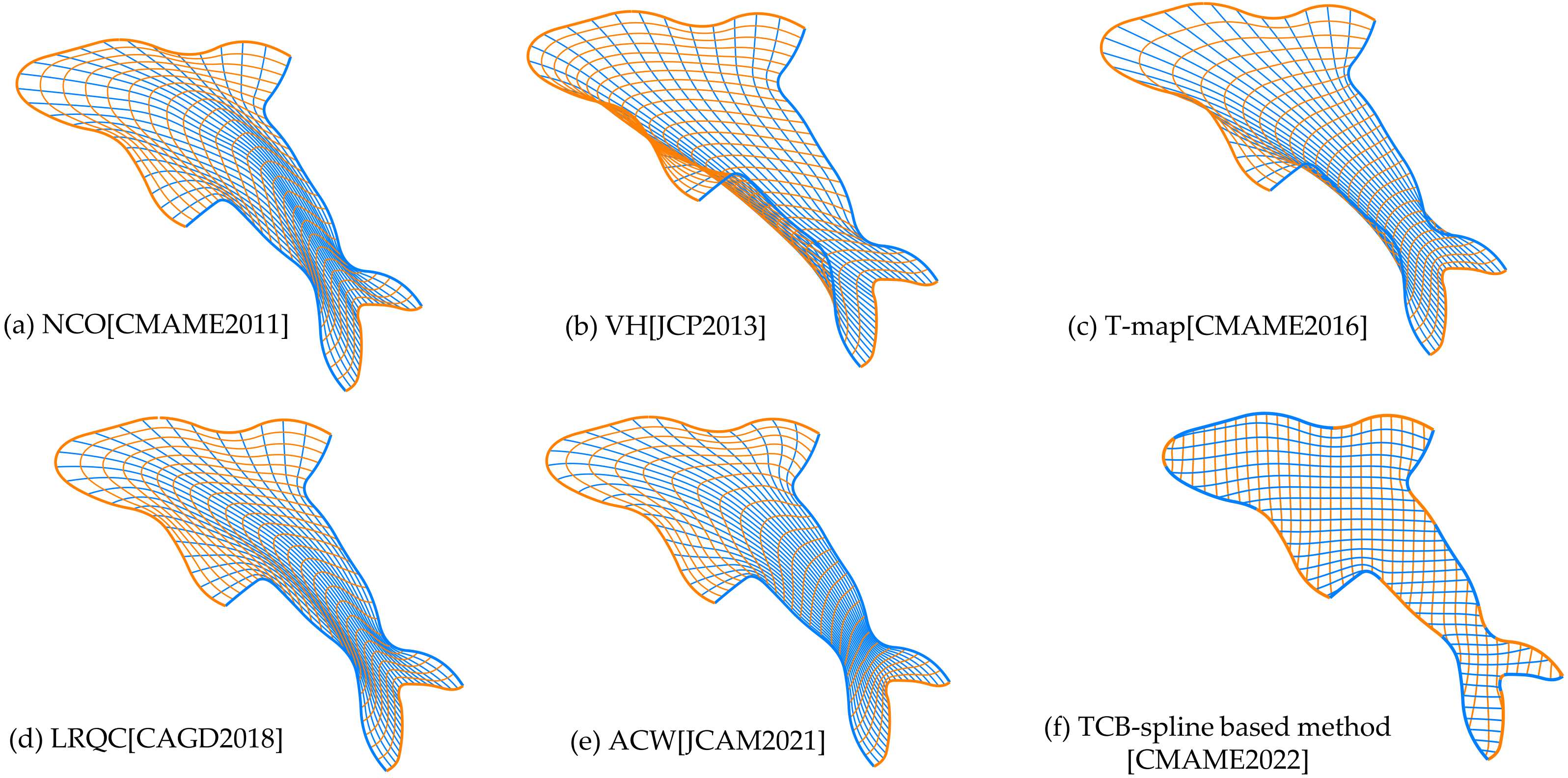
TCB-spline based high-quality parameterizations



(a) Physical domain (b) Parameter domain (c) Parameterization (d) Color-coded Jacobians

TCB-spline-based parameterization method [CMAME 2022]

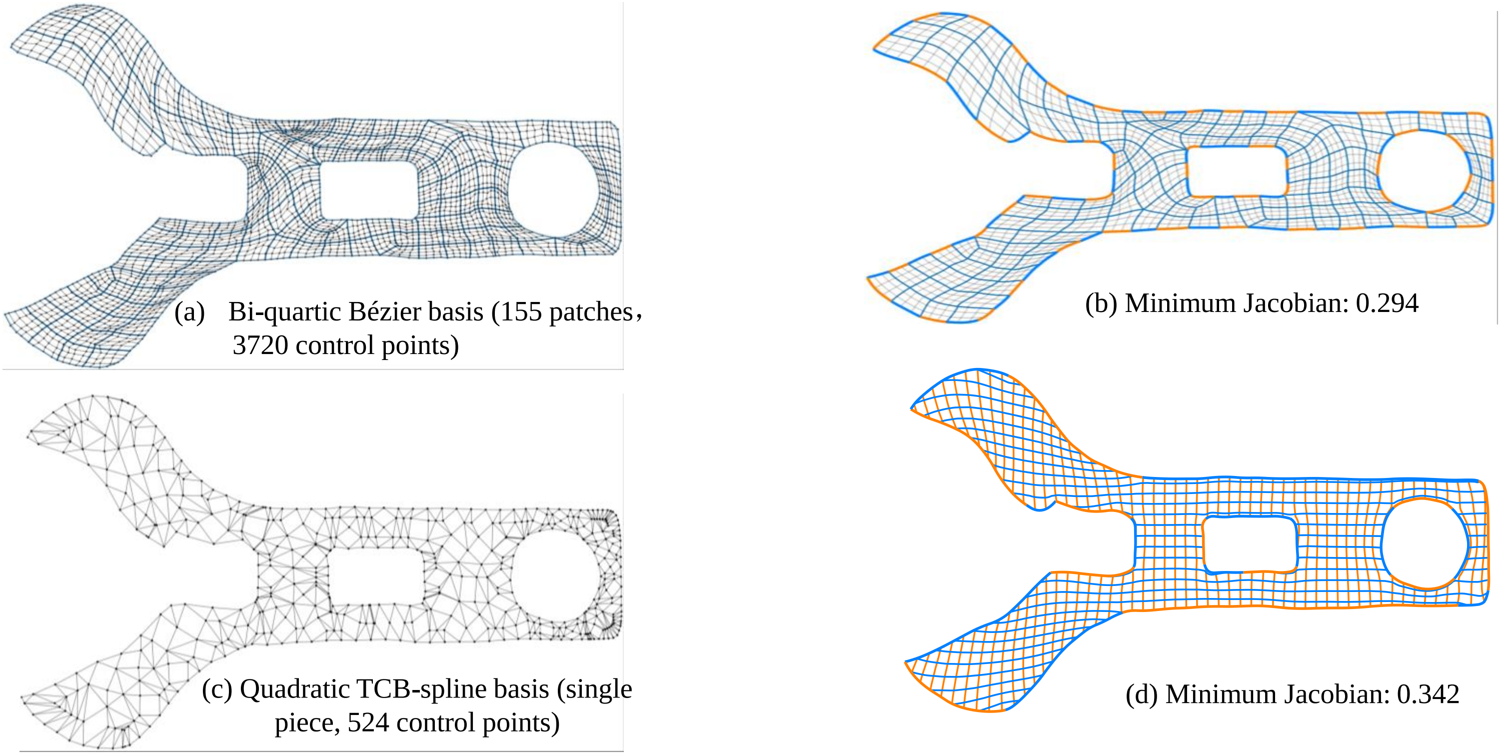
TCB-spline based high-quality parameterizations



(a) NCO[CMAME2011]
(b) VH[JCP2013]
(c) T-map[CMAME2016]
(d) LRQC[CAGD2018]
(e) ACW[JCAM2021]
(f) TCB-spline based method [CMAME2022]

Comparison between different parameterization results for the dolphin model

TCB-spline based high-quality parameterizations



(a) Bi-quartic Bézier basis (155 patches, 3720 control points)

(b) Minimum Jacobian: 0.294

(c) Quadratic TCB-spline basis (single piece, 524 control points)

(d) Minimum Jacobian: 0.342

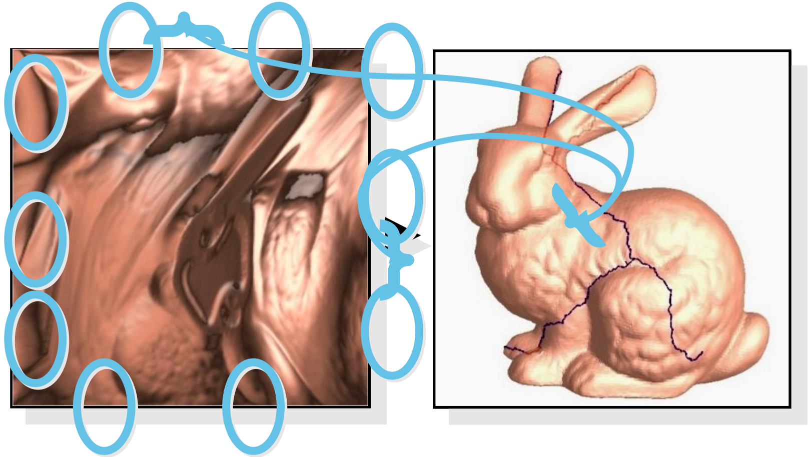
Comparison between DPO [CMAME, 2018] and TCB-spline based method [CMAME2022] on the spanner model

3 其他类型的参数化

封闭曲面的参数化

- 需要将封闭曲面切开成拓扑拓扑同胚于圆盘 (disk-like topology)

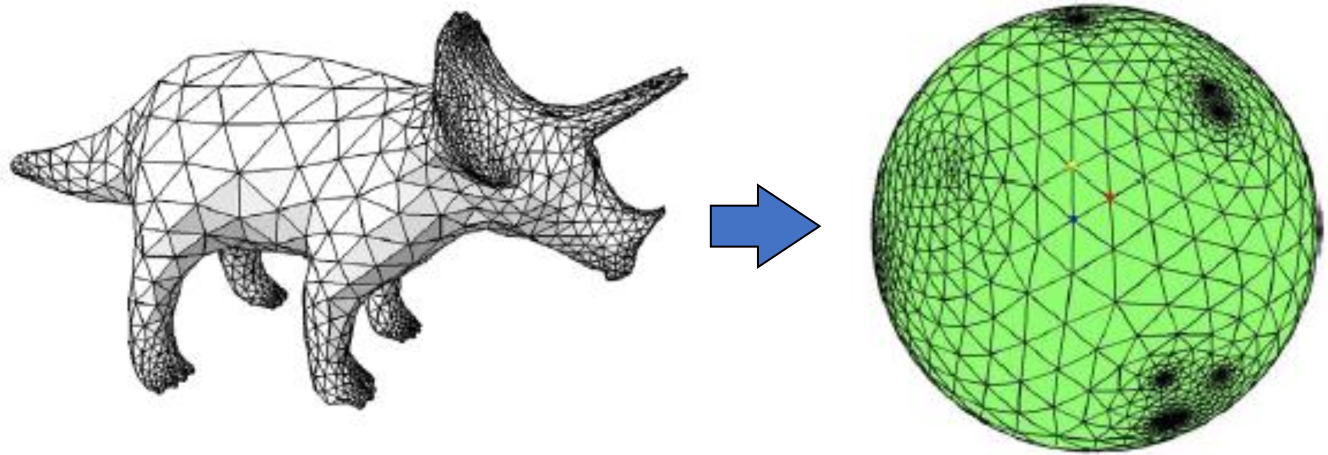
- 割缝生成算法
  - 亏格为0的封闭曲面
  - 高亏格封闭曲面



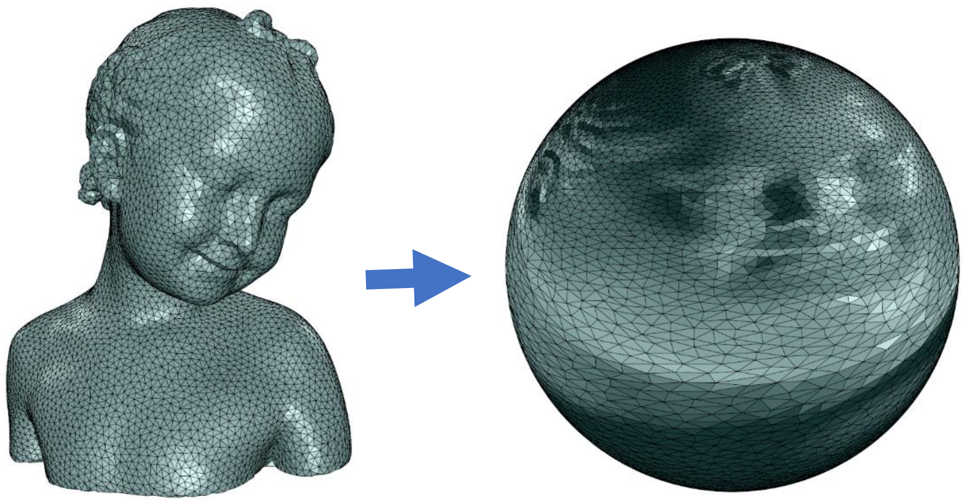
Geometry Image
[Gu et al. 2002]

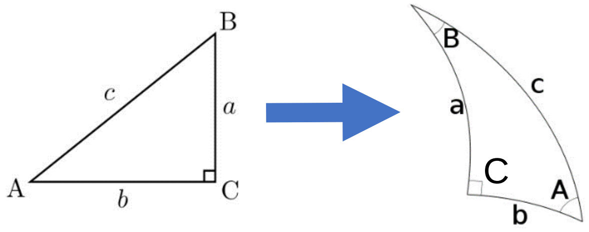
球面参数化

- 亏格为0的封闭曲面：拓扑同胚于球面
- 球面参数化：将曲面映射到球面上
  - 不需要割开曲面



球面参数化





球面参数化的主要方法

- Direct methods
  - [Kent et al. 1992], [Kobbelt et al. 99], [Gu et al. 03]
- Optimization methods
  - [Sheffer et al. 04], [Li et al.06&07], [Zayer et al.06], [Friedel et al., 07], [Kazhdan et al. 2012], [Wan et al. 12&13], [Wang et al., 14&16]
- Coarse-to-fine methods
  - [Praun and Hoppe 04], [Tang et al. 16], [Hu et al. 17]

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Projection Method

[Kent et al. 1992]

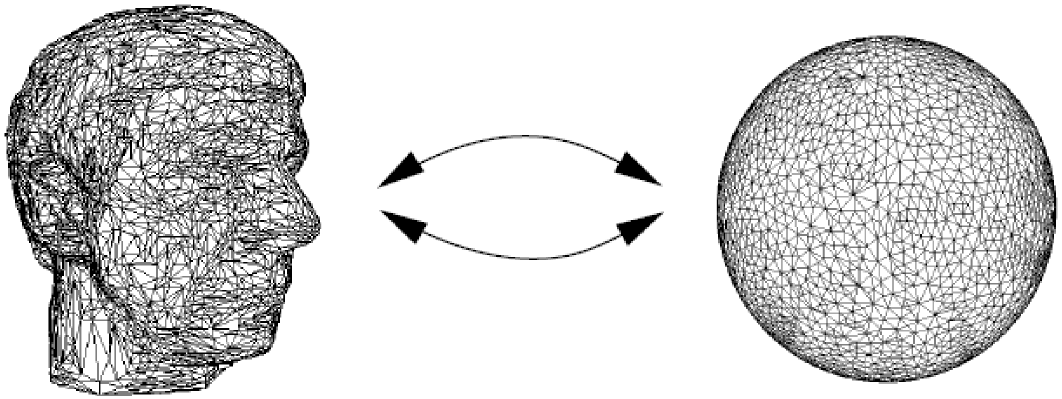
- Convex and star-shaped objects
  - Projection to sphere from star point
- Physically-based simulation
  - Simulating balloon inflation process

- Limitations
  - Difficult to optimize, due to greedy nature
  - Lack desirable mathematical properties

Shrink-wrapping Approach

[Kobbelt et al. 99]

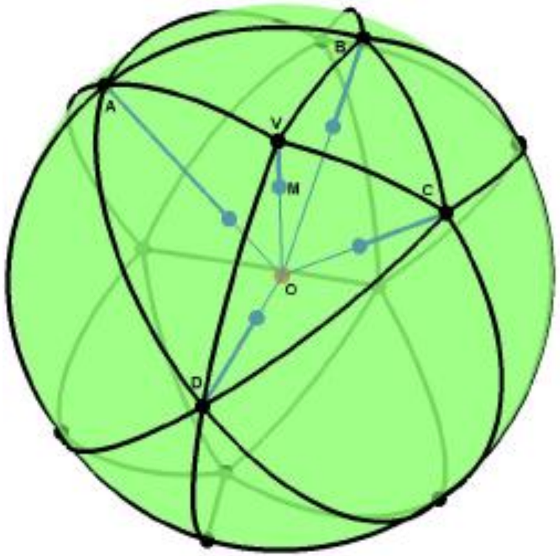
- Simulate the shrink wrapping process
- Converting a triangle mesh into one having subdivision connectivity
- Iterative optimization process



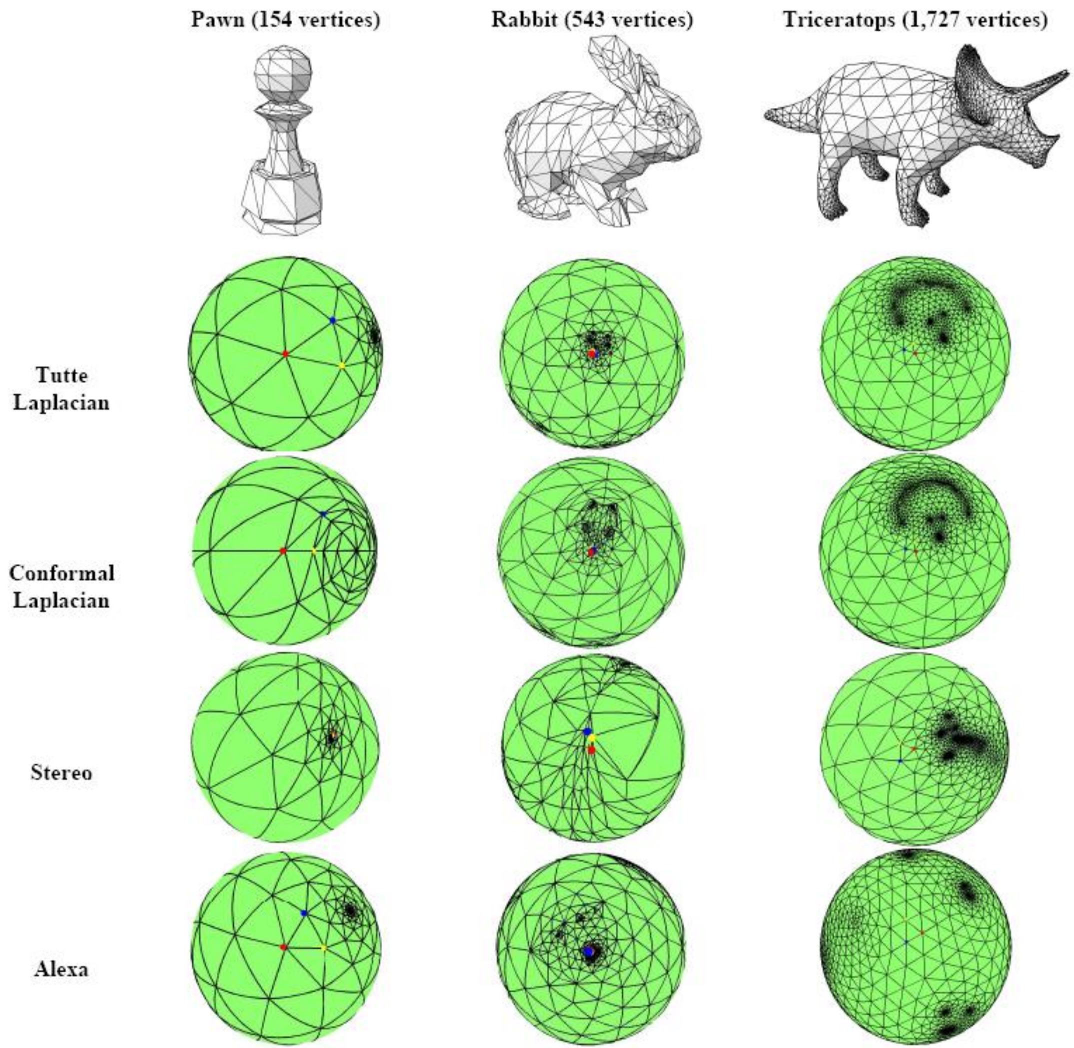
Floater-like Method

[Gotsman et al. 03]

- Nonlinear extension of the linear theory barycentric coordinates
  - Each vertex is some convex combination of its neighbors, projected the on sphere
- Spectral Graph Theory



Comparisons



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- Coarse-to-fine methods
  - [Praun and Hoppe 04], [Tang et al. 16], [Hu et al. 17]

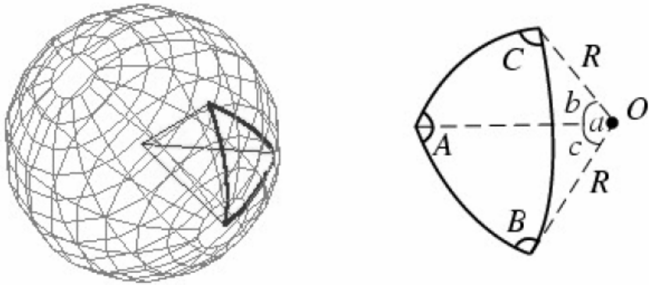
Angle-Based Method

[Sheffer et al. 04]

- Spherical geometry
- Non-linear optimization

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a,$$
$$\cos B = -\cos C \cos A + \sin C \sin A \cos b,$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c.$$



$$\alpha_i^j > 0 \quad i = 1, \ldots, t, \quad j = 0, 1, 2$$
$$e_i > 0 \quad i = 1, \ldots, t$$
$$e_i < 2\alpha_i^j \quad i = 1, \ldots, t, \quad j = 0, 1, 2$$
$$\alpha_i^0 + \alpha_i^1 + \alpha_i^2 - e_i - \pi = 0 \quad i = 1, \ldots, t$$
$$\sum_{j=0}^{2} \sum_{i \in V_j(k)} \alpha_i^j - 2\pi = 0 \quad k = 1, \ldots, n.$$

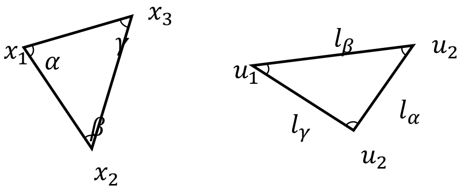
Unconstrained spherical parameterization

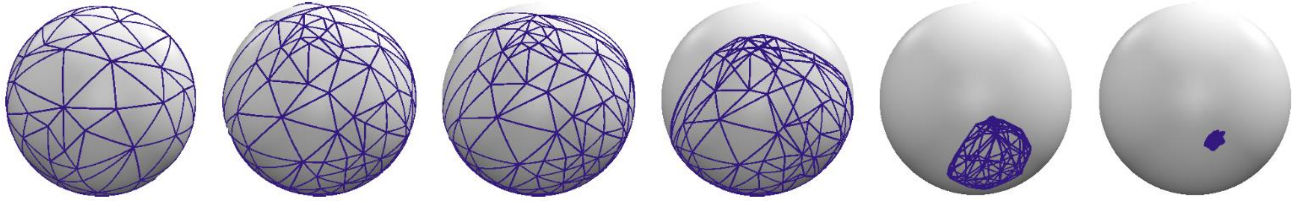
[Friedel et al., 2007]

- Initial guess is taken as the central projection around the center of gravity of the input mesh
- Dirichlet Energy:

$$E_{Dirichlet}(f) = \int \boldsymbol{trace}(Df^T Df)\,, \quad f: R \to S$$

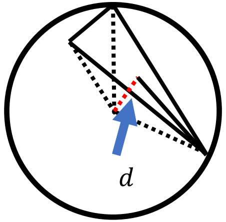
$$\approx \sum(\cot(\alpha) \cdot l_\alpha^2 + \cot(\beta) \cdot l_\beta^2 + \cot(\gamma) \cdot l_\gamma^2)$$

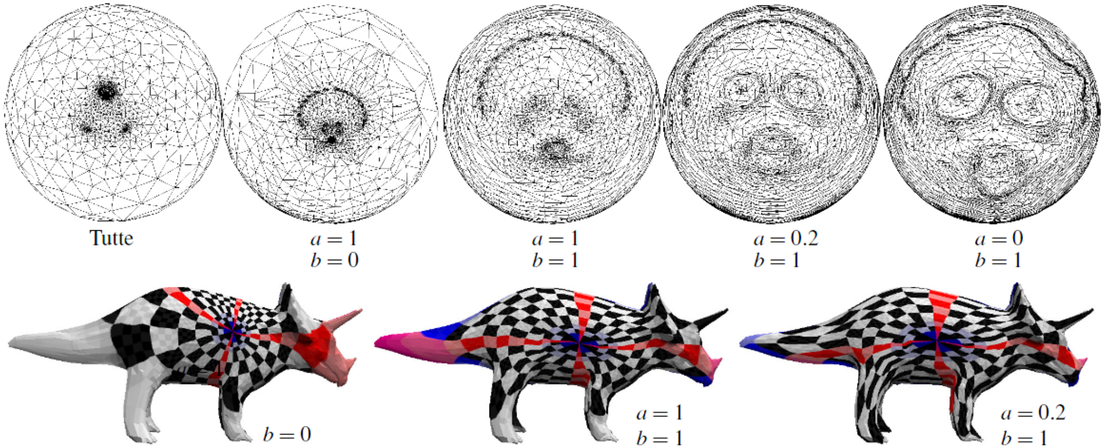




Improved Energy

- $E_{Dirichlet} = d_{min}^{-2} \cdot \left(\cot(\alpha) \cdot l_\alpha^2 + \cot(\beta) \cdot l_\beta^2 + \cot(\gamma) \cdot l_\gamma^2\right)$
- $E_{Area} = d_{min}^{-2} \cdot Area(A, B, C)^2 / InputArea$
- $E_{combined} = a \cdot E_{Dirichlet} + b \cdot E_{Area}$

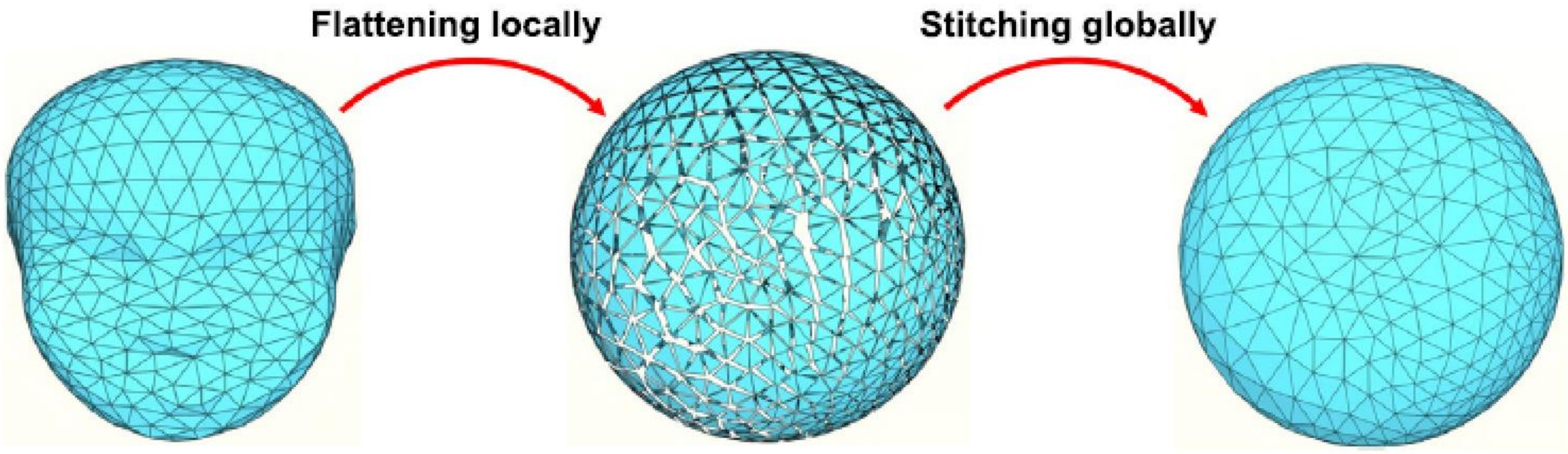




ARAP Spherical Parameterization

[Wang et al., 2014]

- 平面ARAP参数化方法[2008]的推广



AMIPS based Method

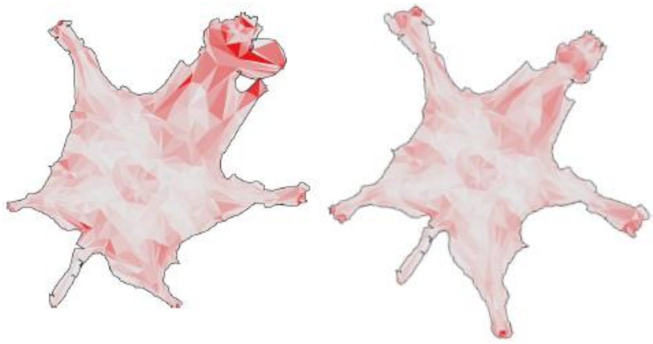
[Wang et al., 2016]

$$E^*_{mips} = \sum_t \exp(s \cdot E_{mips,t})$$

$$E^*_{iso} = \sum_t \exp(s \cdot E_{iso,t})$$

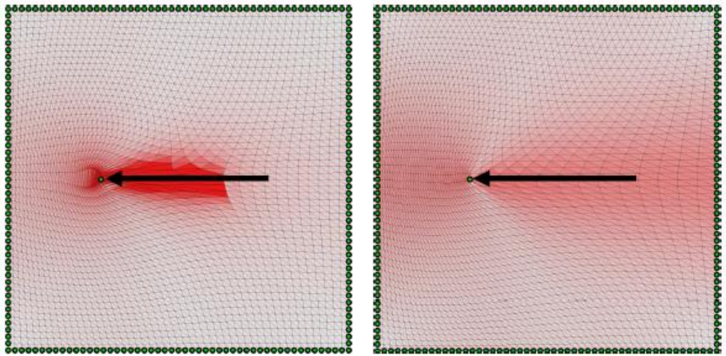
level of penalty

[Fu et al., 2015]



$\delta^{con}_{max} = 5.81$
without EXP

$\delta^{con}_{max} = 3.96$
with EXP



$\delta^{iso}_{max} = 21.47$
without EXP

$\delta^{iso}_{max} = 3.25$
with EXP

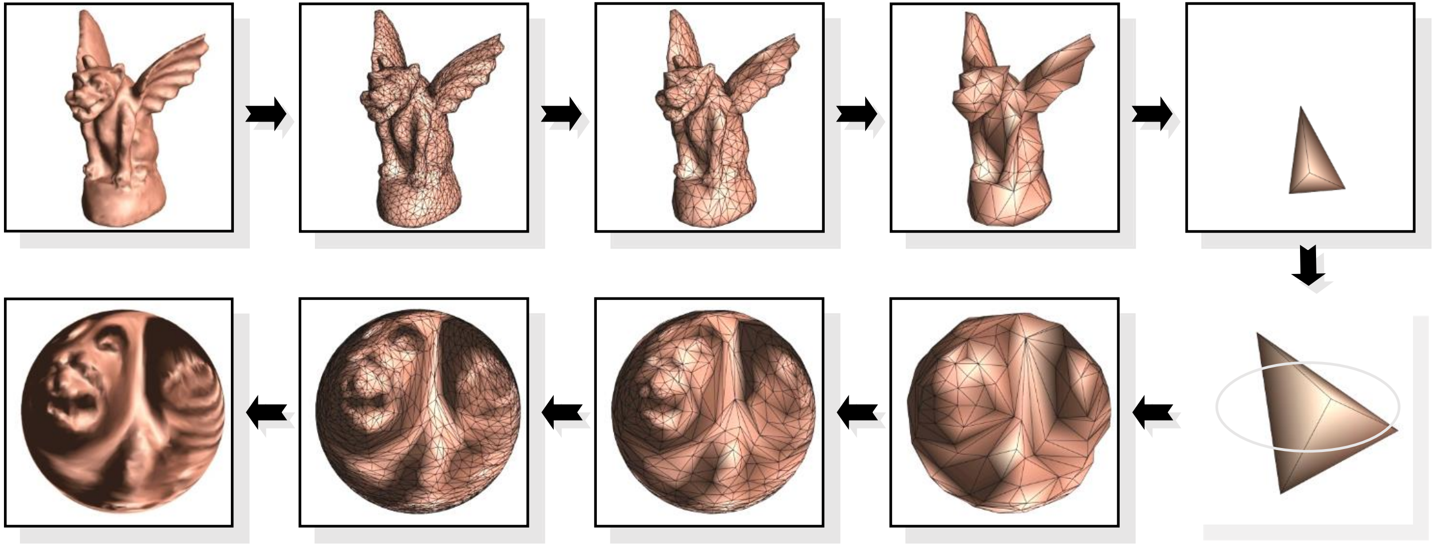
球面参数化的主要方法

- Direct methods
  - [Kent et al. 1992], [Kobbelt et al. 99], [Gu et al. 03]
- Optimization methods
  - [Sheffer et al. 04], [Li et al.06&07], [Zayer et al.06], [Friedel et al., 07], [Kazhdan et al. 2012], [Wan et al. 12&13], [Wang et al., 14&16]
- Coarse-to-fine methods
  - [Praun and Hoppe 04], [Tang et al. 16], [Hu et al. 17]

Coarse-to-Fine Algorithm

[Praun and Hoppe 04]

Convert to progressive mesh

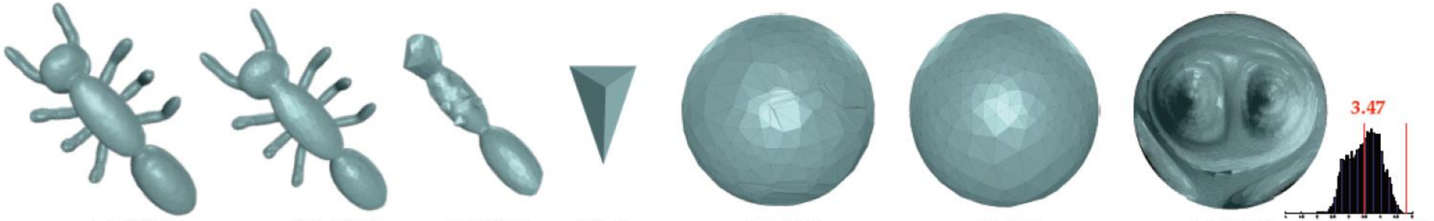


Parametrize coarse-to-fine

Maintain embedding & minimize stretch

Advanced Hierarchical Spherical Parameterizations

[Hu et al., 2017]



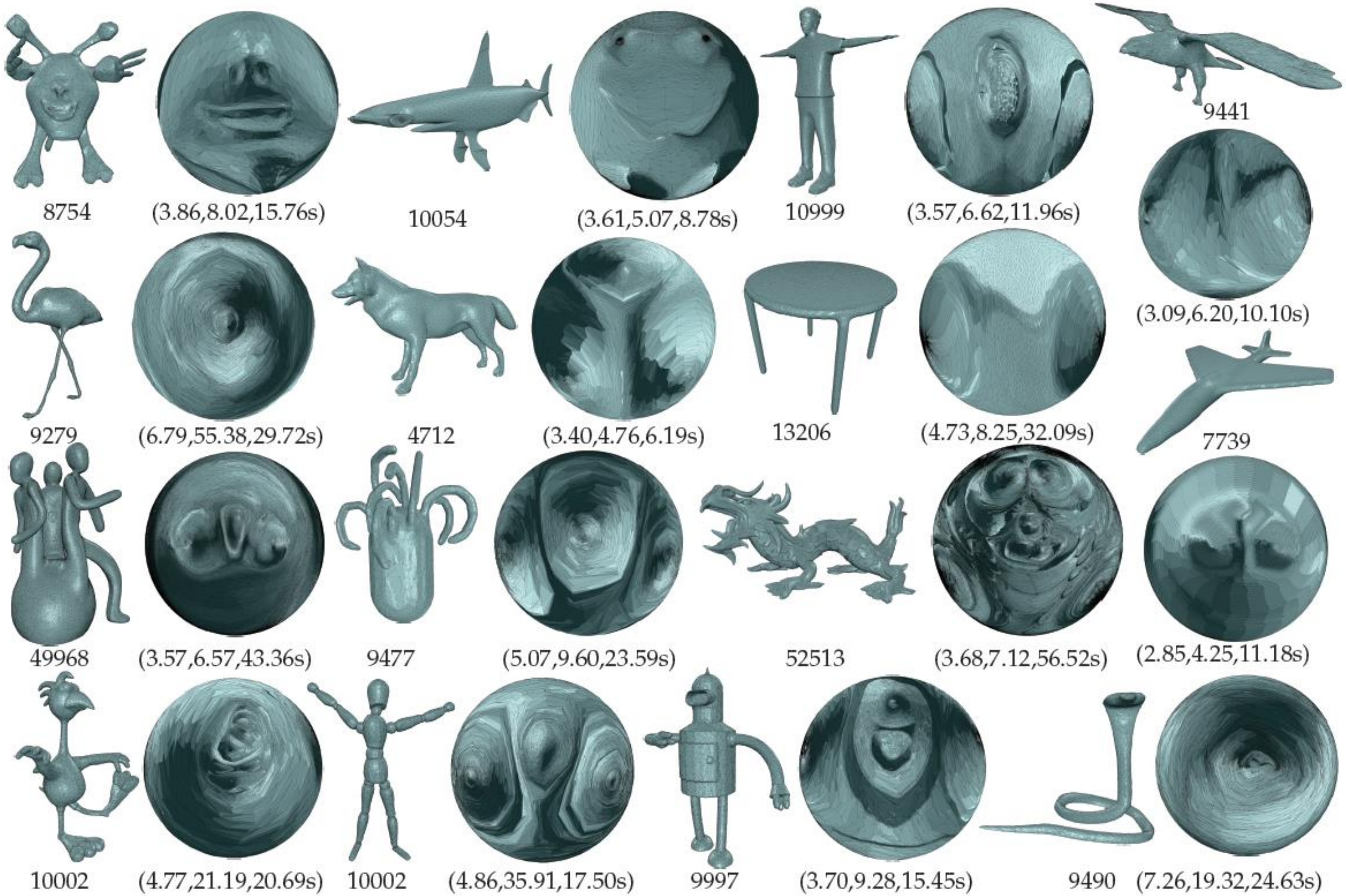
(a) 9979 (b) 4158 (c) 1000 (d) 4 (e) 620 (f) 620 (g) 9979

A combined Decimation scheme: **QEM + CEM**

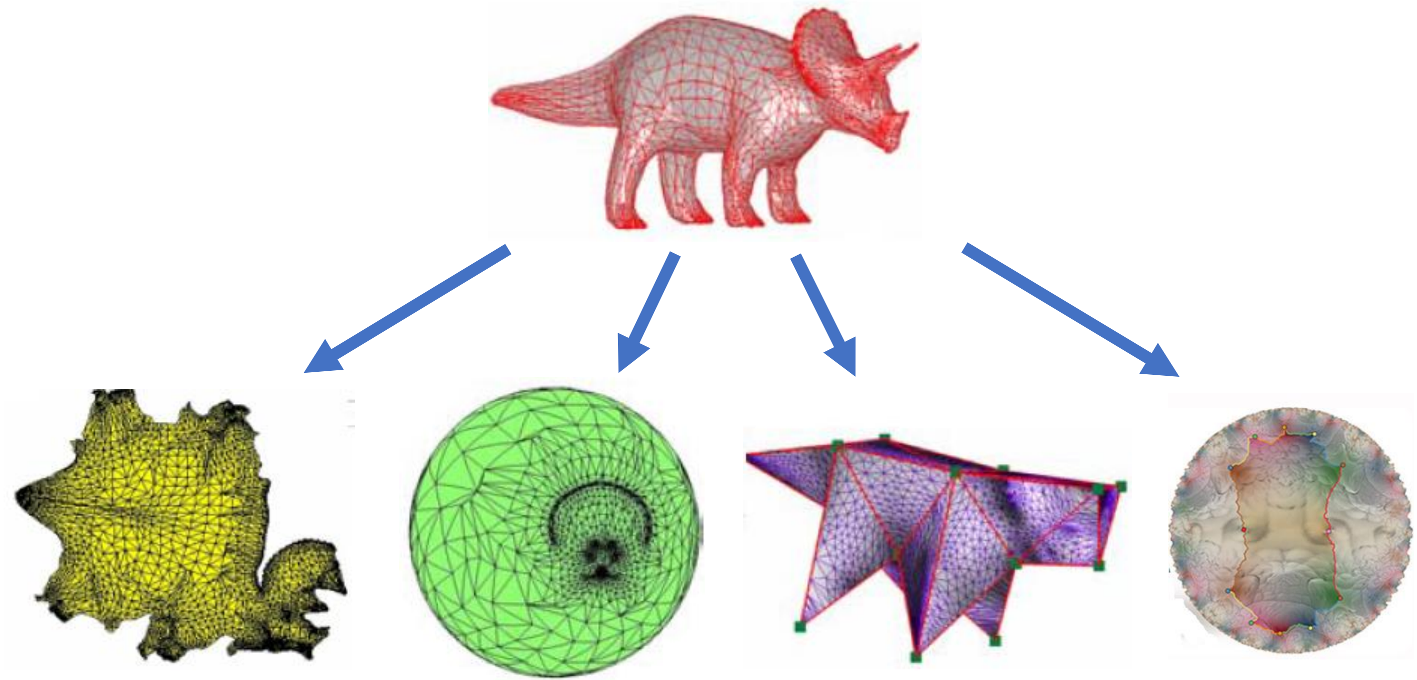
Group refinement with **distortion control** and **global optimization**

Practically robust + Efficient + No need initialization + Low distortion + bijective

Results


8754
(3.86,8.02,15.76s)
10054
(3.61,5.07,8.78s)
10999
(3.57,6.62,11.96s)
9441
(3.09,6.20,10.10s)
9279
(6.79,55.38,29.72s)
4712
(3.40,4.76,6.19s)
13206
(4.73,8.25,32.09s)
7739
49968
(3.57,6.57,43.36s)
9477
(5.07,9.60,23.59s)
52513
(3.68,7.12,56.52s)
(2.85,4.25,11.18s)
10002
(4.77,21.19,20.69s)
10002
(4.86,35.91,17.50s)
9997
(3.70,9.28,15.45s)
9490
(7.26,19.32,24.63s)

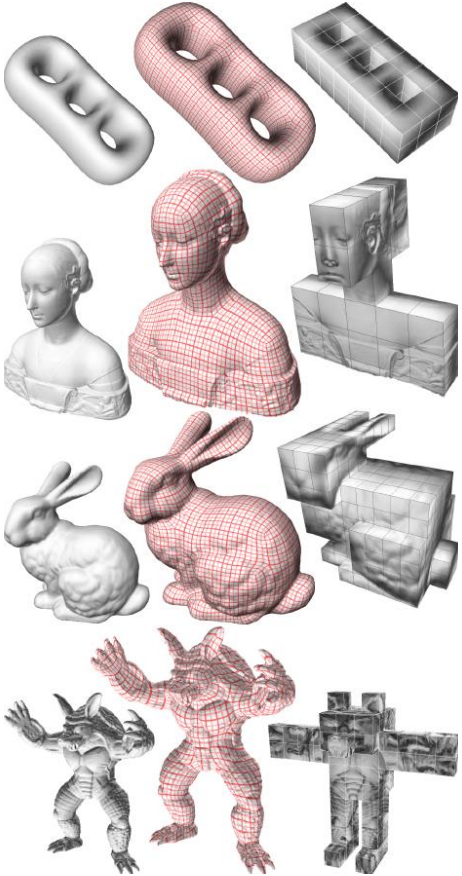
不同的参数化定义域（参数域）



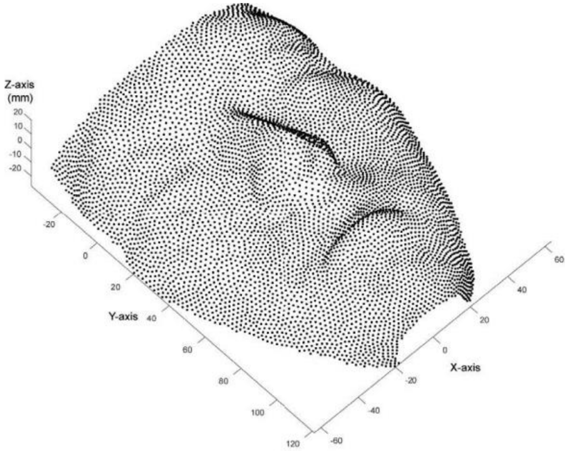
平面参数化　球面参数化　流形参数化　双曲轨道嵌入

其他类型的参数化

- PolyCube 参数化



- 点云参数化

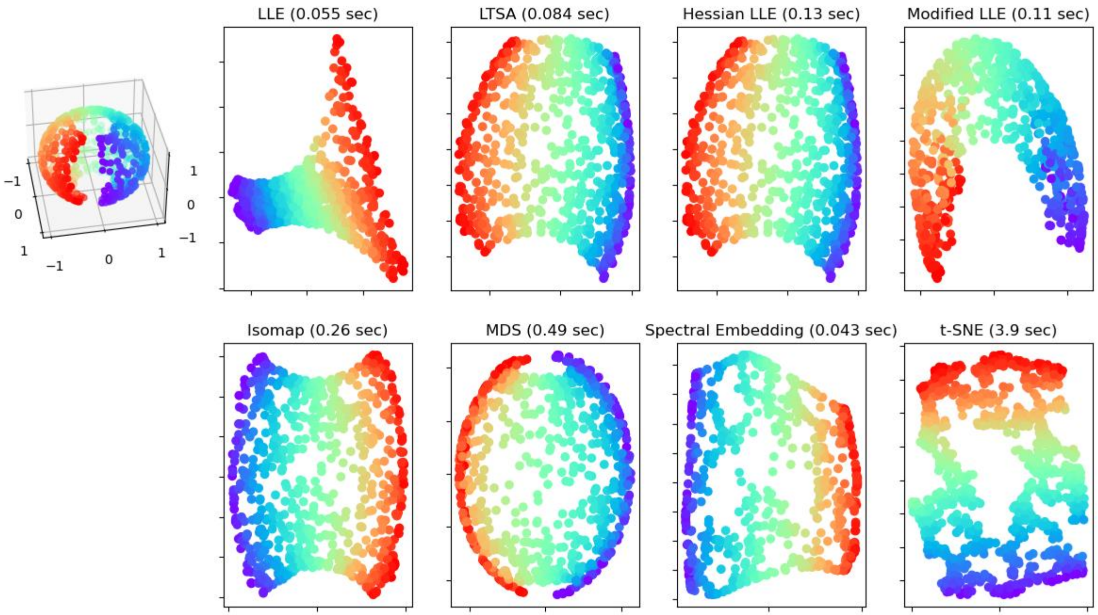


4

高维数据的参数化

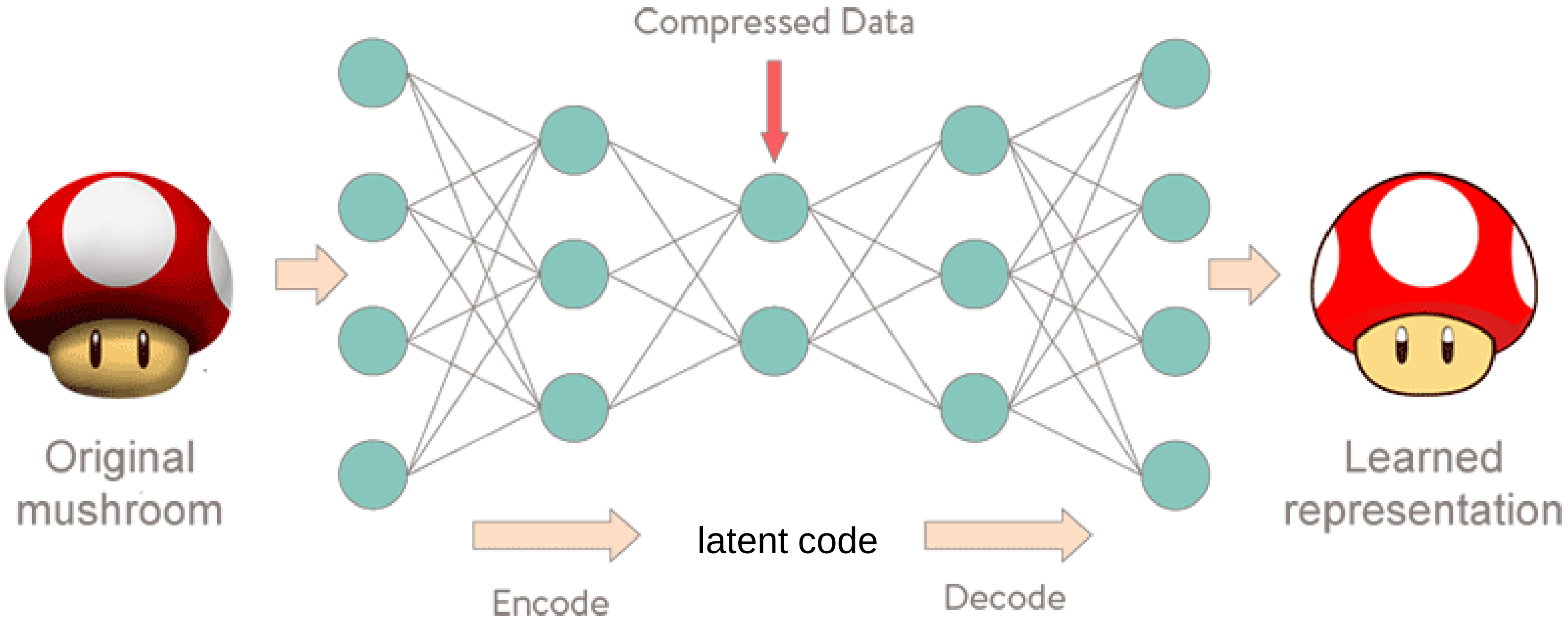
高维数据的参数化

- 流形学习 (Manifold learning) / 降维 (Dimension reduction)
- 本质：高维数据所在流形的低维嵌入，即嵌入后的坐标系的各个基向量为相互无关（解耦）的特征



高维数据的参数化

自编码神经网络 (AutoEncoder)



输入数据　　参数化　　参数　　拟合　　输出表达

5

课程总结

课程总结

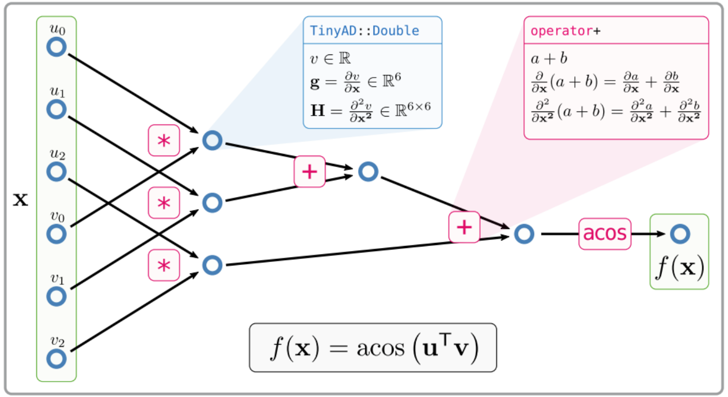
时间			授课老师	课程题目 (点击可下载课件)
第一周	第1讲	10月7日, 10:00-11:30	刘利刚	曲面参数化介绍
	第2讲	10月9日, 10:00-11:30	傅孝明	面向离散网格的参数化概述 + 传统方法介绍
第二周	第3讲	10月15日, 10:00-11:30	傅孝明	无翻转参数化方法 – 初始存在翻转
	第4讲	10月16日, 10:00-11:30	傅孝明	无翻转参数化方法 – 初始无翻转
第三周	第5讲	10月22日, 10:00-11:30	傅孝明	全局单射参数化方法
	第6讲	10月23日, 10:00-11:30	傅孝明	参数化应用1 – Atlas生成、艺术设计
第四周	第7讲	10月29日, 10:00-11:30	傅孝明	参数化应用2 – 网格生成
	第8讲	10月30日, 10:00-11:30	陈仁杰	无翻转光滑映射
第五周	第9讲	11月5日, 10:00-11:30	陈仁杰	基于调和映射的高质量形变
	第10讲	11月6日, 10:00-11:30	方清	共形参数化1 – Circle填充、柯西黎曼方程
第六周	第11讲	11月12日, 10:00-11:30	方清	共形参数化2 – 离散共形等价类、曲率流
	第12讲	11月13日, 10:00-11:30	方清	锥奇异点参数化应用
第七周	第13讲	11月19日, 10:00-11:30	傅孝明	参数化应用3 – 高阶网格生成、曲面对应
	第14讲	11月19日, 15:00-16:30	刘利刚	参数化在产业中的应用(1)
	第15讲	11月20日, 10:00-11:30	刘利刚	参数化在产业中的应用(2)及课程总结

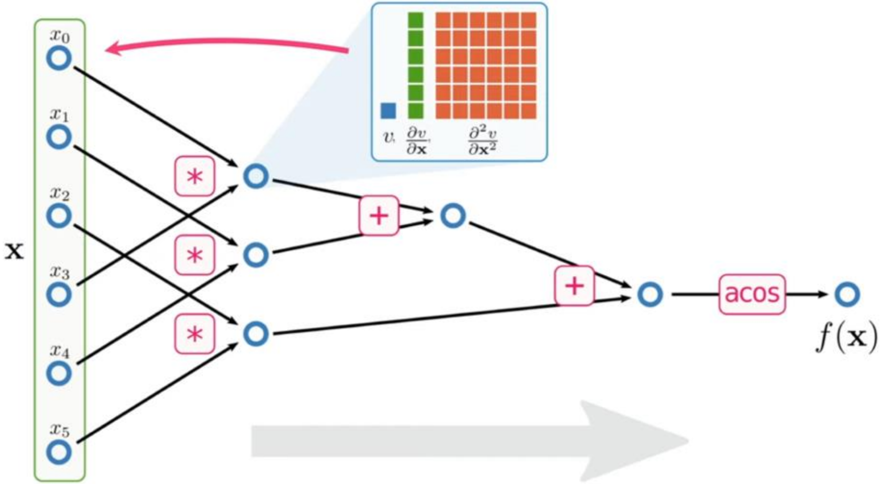
课程资料

- 课程主页：
  - http://staff.ustc.edu.cn/~renjiec/GAMES301/index.html
- 课件下载：
  - http://staff.ustc.edu.cn/~renjiec/GAMES301/program.html
- 课程录屏（B站）：
  - https://www.bilibili.com/video/BV18T411P7hT
- 课程作业：
  - http://staff.ustc.edu.cn/~renjiec/GAMES301/assignment.html

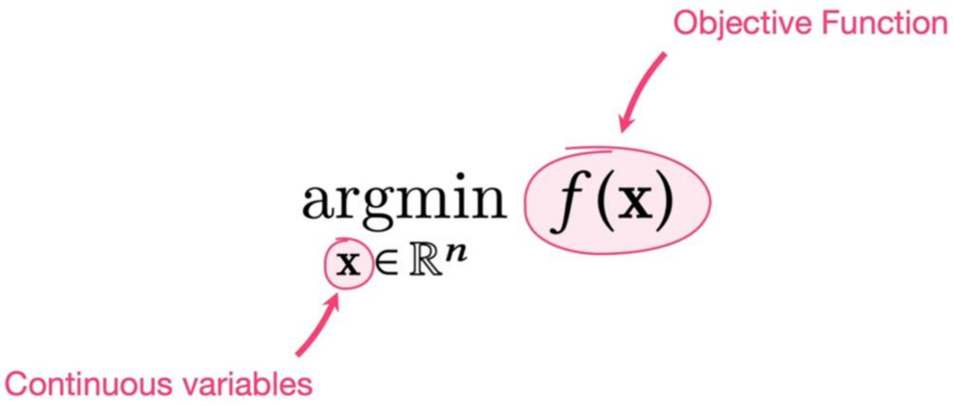
TinyAD

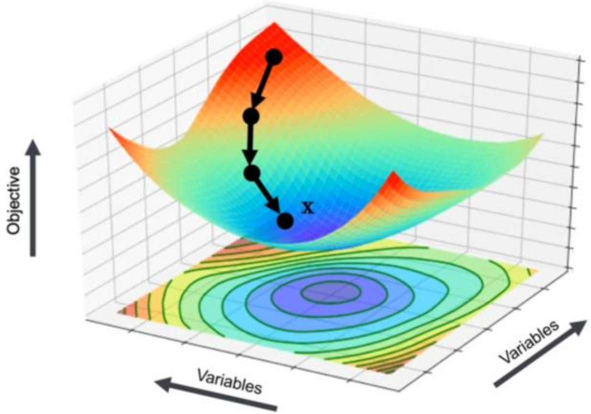
- Computation graph for geometry processing





TinyAD





- C++ (head only) AD library computing gradients and Hessians
- Decompose the problem by the sparsity of the mesh

App: Parameterization

```
// Read disk-topology mesh using OpenMesh
OpenMesh::TriMesh mesh = read_mesh("armadillo_disk.obj");

// Set up function with 2D vertex positions as variables.
auto func = TinyAD::scalar_function<2>(mesh.vertices());

// Add objective term per triangle. Each connecting 3 vertices.
func.add_elements<3>(mesh.faces(), [&] (auto& element)
{
    // Element is evaluated with either double or TinyAD::Double<6>
    using T = TINYAD_SCALAR_TYPE(element);

    // Get variable 2D vertex positions of triangle t
    OpenMesh::SmartFaceHandle t = element.handle;
    Eigen::Vector2<T> a = element.variables(t.halfedge().to());
    Eigen::Vector2<T> b = element.variables(t.halfedge().next().to());
    Eigen::Vector2<T> c = element.variables(t.halfedge().from());

    // Triangle flipped?
    Eigen::Matrix2<T> M = col_mat(b - a, c - a);
    if (M.determinant() <= 0.0)
        return (T)INFINITY;

    // Get constant 2D rest shape and area of triangle t
    Eigen::Matrix2d Mr = mesh.property(rest_shapes, t);
    double A = 0.5 * Mr.determinant();

    // Compute symmetric Dirichlet energy
    Eigen::Matrix2<T> J = M * Mr.inverse();
    return A * (J.squaredNorm() + J.inverse().squaredNorm());
});

// Projected Newton
Eigen::VectorXd x = tutte_embedding(mesh);
for (int i = 0; i < max_iters; ++i)
{
    auto [f, g, H_proj] = func.eval_with_hessian_proj(x);
    Eigen::VectorXd d = TinyAD::newton_direction(g, H_proj);
    if (TinyAD::newton_decrement(d, g) < eps)
        break;
    x = TinyAD::line_search(x, d, f, g, func);
}
```

Dirichlet Energy

Get derivatives

致谢：课程授课团队

刘利刚

陈仁杰

傅孝明

方清

中国科学技术大学 图形与几何计算实验室
http://gcl.ustc.edu.cn/

致谢：黄舒怀（腾讯）、贾颜铭（先临）、潘茂东（南京航空航天大学）、曹娟（厦门大学），及其他同仁！

GAMES 在线课程

http://games-cn.org

闫令琪
现代计算机图形学入门
2020.2

胡鸣渊
高级物理引擎实战指南
2020.6

刘利刚
几何建模与处理
2020.10

闫令琪
高质量实时渲染
2021.3

黄其兴
三维视觉与理解
2021.7

王华民
基于物理的计算机动画入
2021.11

王希
现代游戏引擎
2022.3

孙启霖、彭祎帆
计算成像
2022.7

基础课程 (1**)	GAMES101　现代计算机图形学入门	闫令琪	高级课程 (2**)	GAMES201　高级物理引擎实战指南	胡渊鸣
	GAMES102　几何建模与处理	刘利刚		GAMES202　高质量实时渲染	闫令琪
	GAMES103　基于物理的计算机动画入门	王华民		GAMES203　三维重建和理解	黄其兴
	GAMES104　现代游戏引擎	王　希		GAMES204　计算成像	孙启霖
	GAMES105　计算机角色动画	刘利斌			彭祎帆
专题课程 (3**)	New　GAMES 301 （已完成），GAMES 302、303（规划中）				
开发课程 (4**)	New　GAMES 401、402（规划中）				

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谢 谢 ！

GAMES 301