中国科学技术大学
University of Science and Technology of China

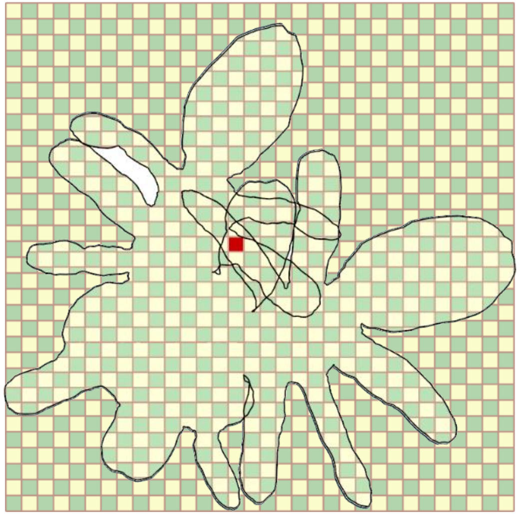
GAMES 301: 第5讲

无翻转参数化方法
初始无翻转

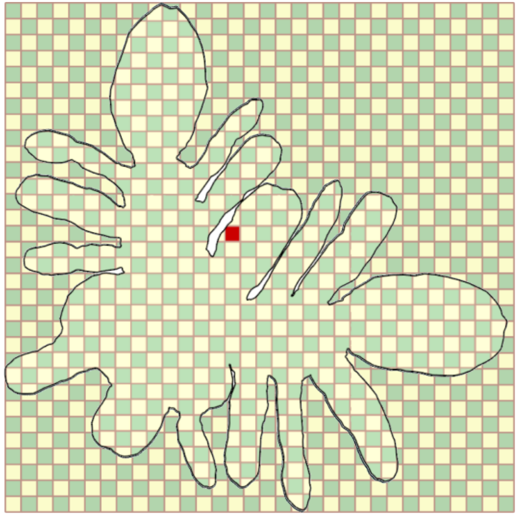
傅孝明
中国科学技术大学

Globally injective mappings

Overlap-free

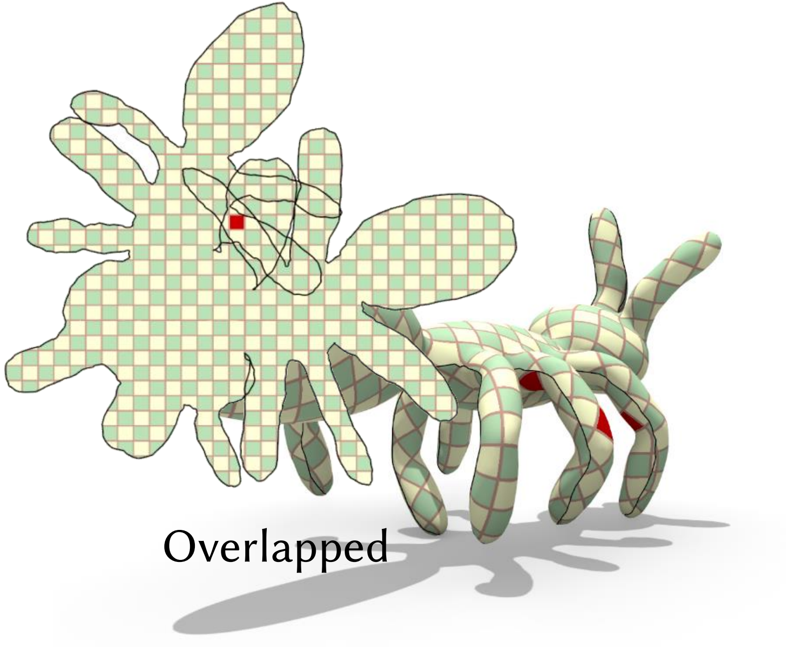


Overlapped

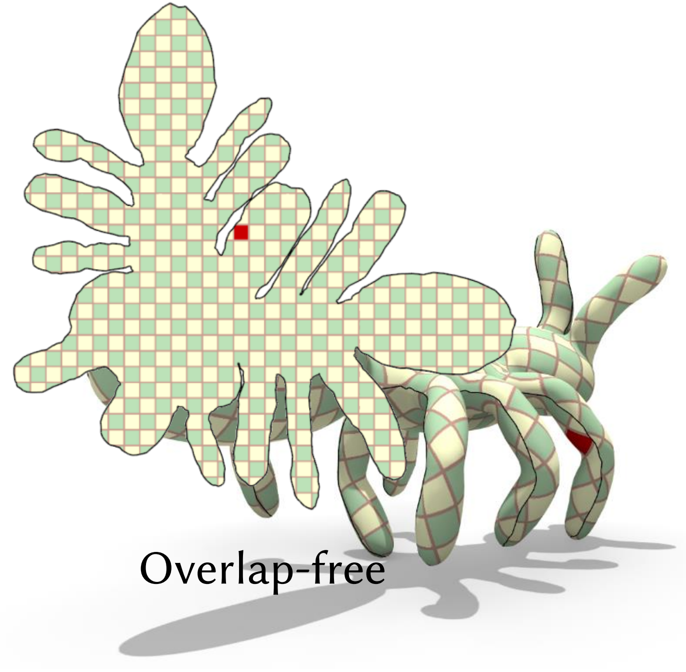


Overlap-free

Overlap-free

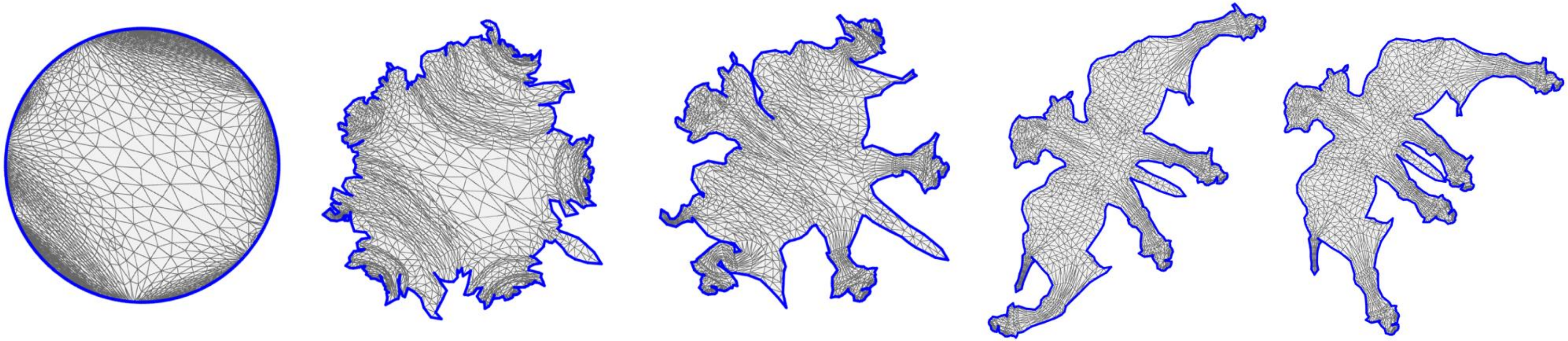


Overlapped



Overlap-free

Pipeline



Tutte's embedding

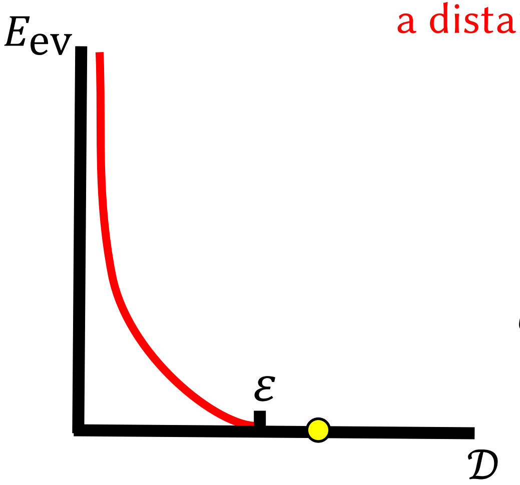
Barriers

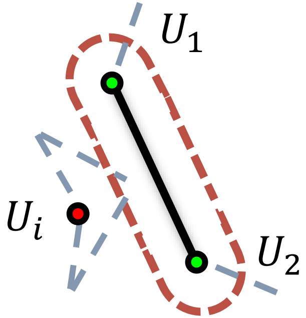
Barriers

- Boundary barrier function

$$E_{\text{ev}}(\mathbf{b}_j, \mathbf{x}_i) = \max\left(0, \frac{\varepsilon}{\mathcal{D}(\mathbf{b}_j, \mathbf{x}_i)} - 1\right)^2$$

a distance from $\mathbf{x}_i$ to $\mathbf{b}_j$





Smith J, Schaefer S. Bijective parameterization with free boundaries[J]. ACM Transactions on Graphics (TOG), 2015, 34(4): 1-9.

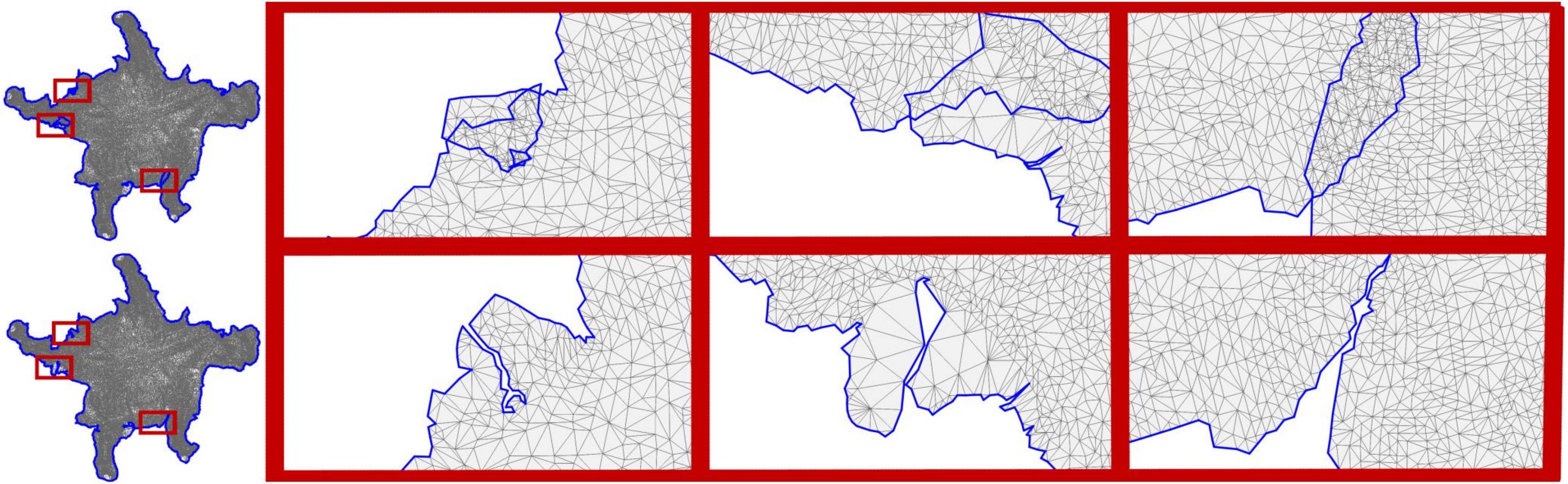
Formulation

$$\min_{\widehat{\mathcal{M}}} \quad E_{\mathrm{d}}(\mathcal{M}, \widehat{\mathcal{M}}) + \lambda E_{\mathrm{b}}(\mathcal{B})$$

$$E_{\mathrm{d}}\left(\mathcal{M}, \widehat{\mathcal{M}}\right) = \frac{1}{4}\sum_{i=1}^{N_f}\left(\|J_i\|_F^2 + \left\|J_i^{-1}\right\|_F^2\right)$$

$$E_{\mathrm{b}}(\mathcal{B}) = \sum_{\mathbf{b}_j \in \mathcal{E}_{\mathbf{b}}} \sum_{\mathbf{x}_i \in \mathcal{V}_{\mathbf{b}}} \max\left(0, \frac{\varepsilon}{\mathcal{D}\left(\mathbf{b}_j, \mathbf{x}_i\right)} - 1\right)^2$$

Results

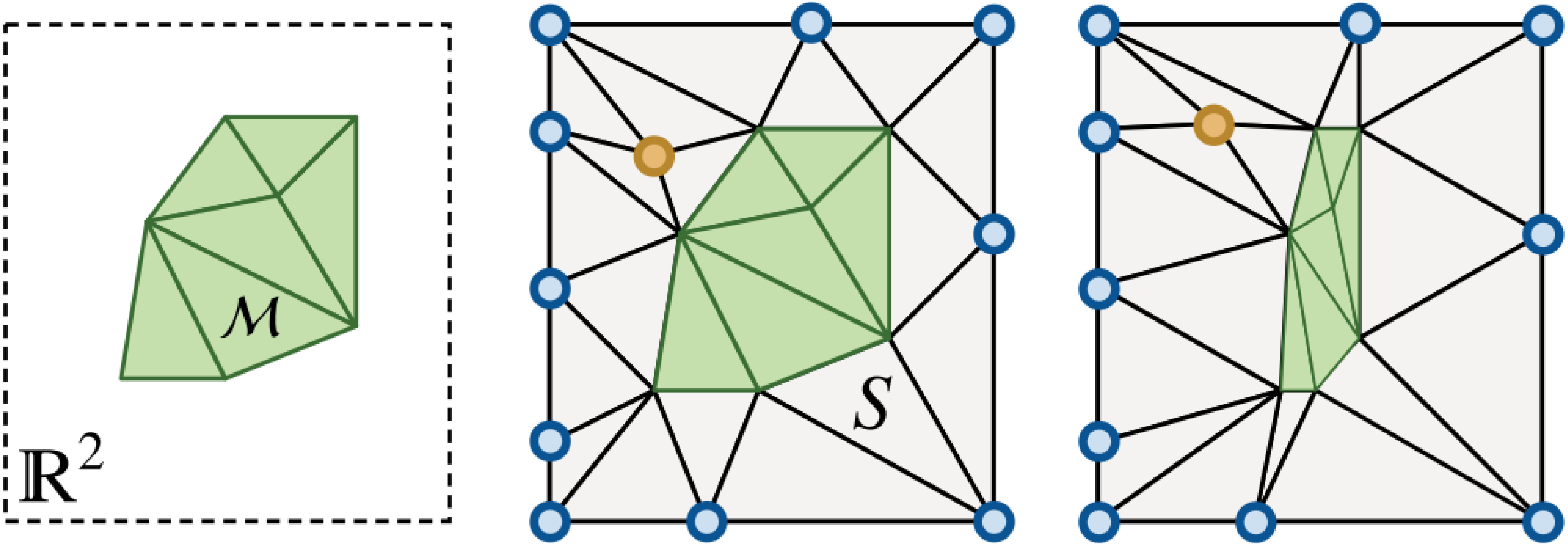


IPC

Scaffold

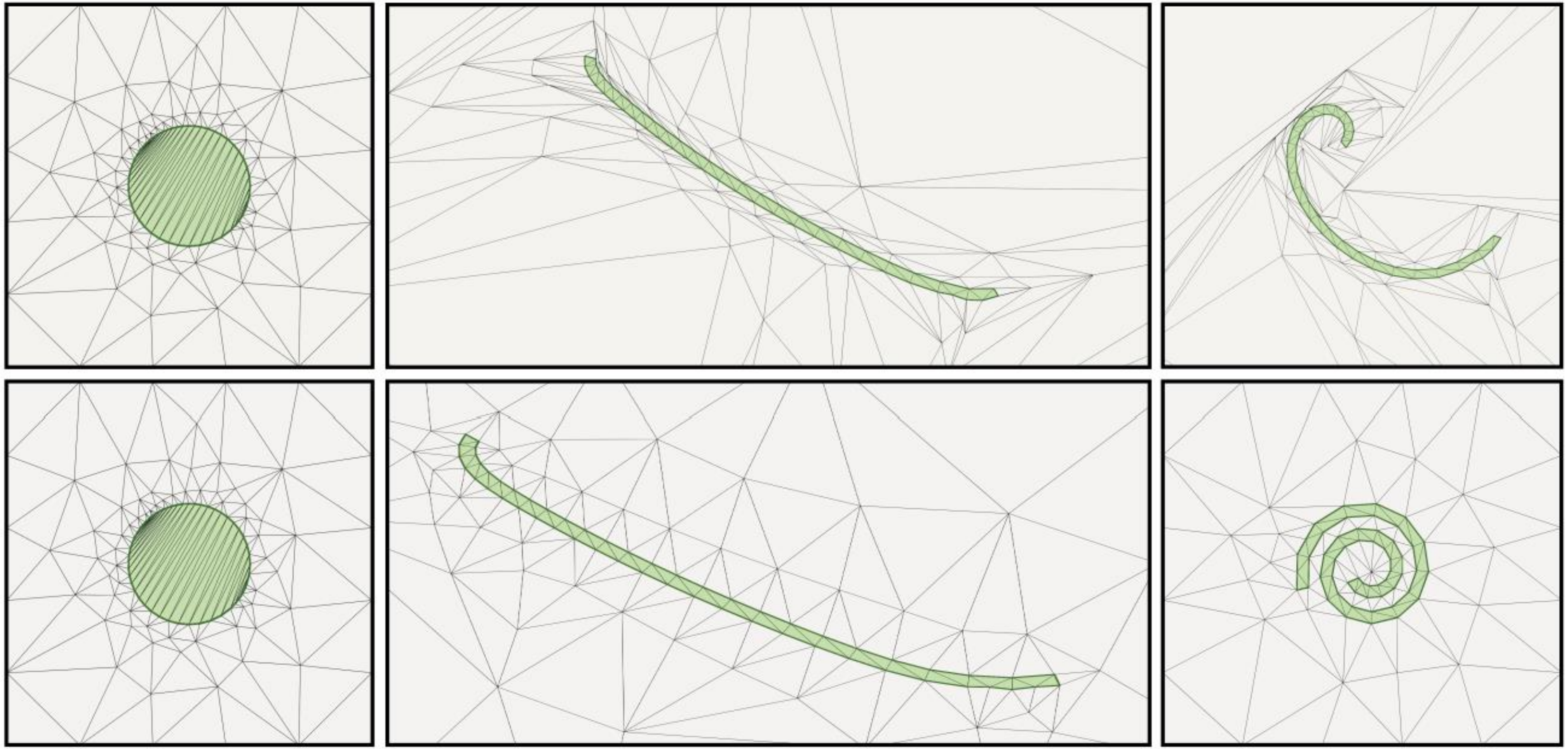
Scaffold

- Overlap-free $\Longrightarrow$ flip-free

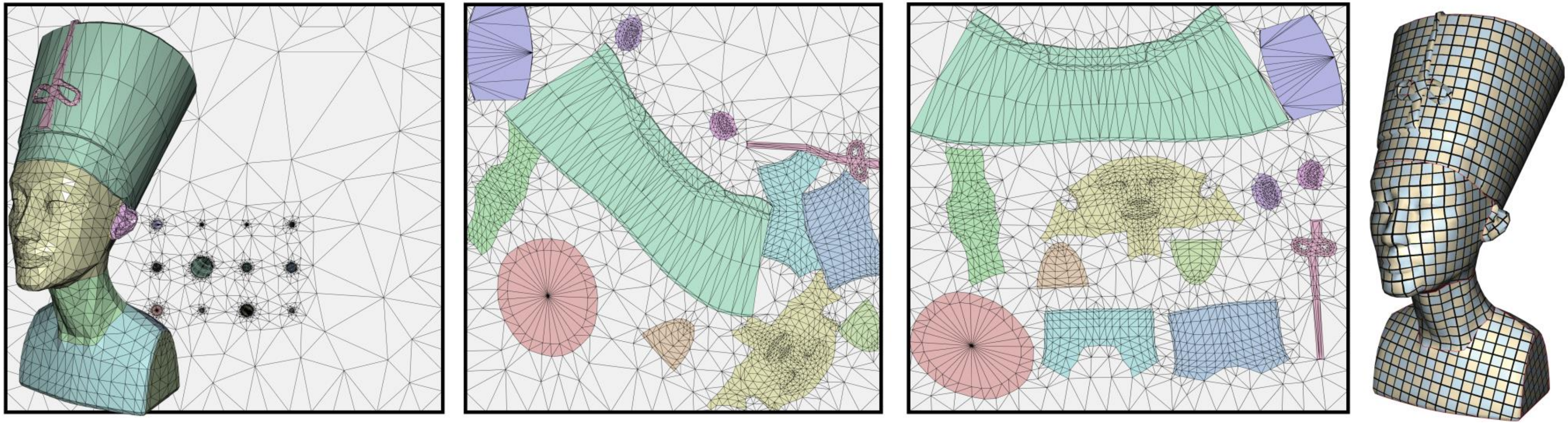


Jiang Z, Schaefer S, Panozzo D. Simplicial complex augmentation framework for bijective maps[J]. ACM Transactions on Graphics, 2017, 36(6).

Connectivity updating



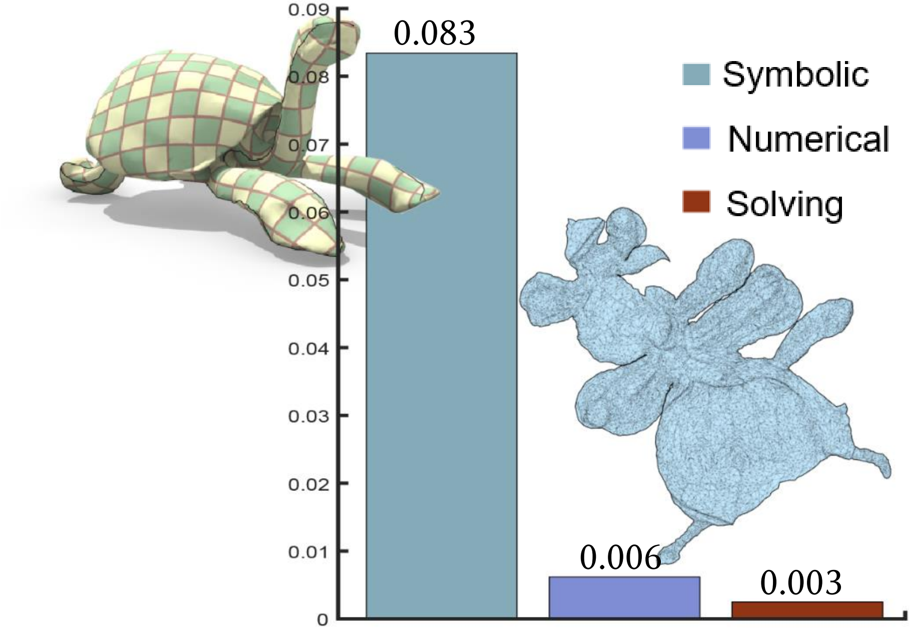
Results



Combined

Second-order solver

- Symbolic phase
  - Depend on the nonzero
- Numerical phase
  - Produce the factorizatio
- Solving phase
  - Use the factorization to



Hessian matrix with fixed nonzero structure.

Su J P, Ye C, Liu L, et al. Efficient bijective parameterizations[J]. ACM Transactions on Graphics (TOG), 2020, 39(4): 111: 1-111: 8.

Nonzero structure

$$\min_{\widehat{\mathcal{M}}} \quad E_{\mathrm{d}}(\mathcal{M}, \widehat{\mathcal{M}}) + \lambda E_{\mathrm{b}}(\mathcal{B})$$

$$E_{\mathrm{d}}\left(\mathcal{M}, \widehat{\mathcal{M}}\right) = \frac{1}{4}\sum_{i=1}^{N_f}\left(\|J_i\|_F^2 + \left\|J_i^{-1}\right\|_F^2\right)$$

$$E_{\mathrm{b}}(\mathcal{B}) = \sum_{\mathbf{b}_j \in \mathcal{E}_{\mathbf{b}}} \sum_{\mathbf{x}_i \in \mathcal{V}_{\mathbf{b}}} \max\left(0, \frac{\varepsilon}{\mathcal{D}\left(\mathbf{b}_j, \mathbf{x}_i\right)} - 1\right)^2$$

Nonzero structure

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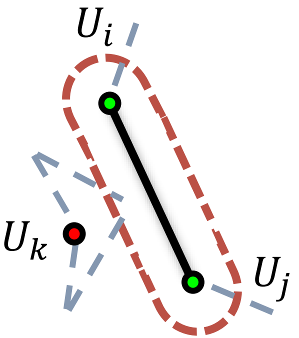
$$H = \begin{bmatrix} H_{II} & H_{IB} \\ H_{BI} & H_{BB} \end{bmatrix}$$

I: Internal vertices

B: Boundary vertices

$$\begin{array}{ccc|c} h_{ii} & h_{ij} & h_{ik} & i \\ h_{ji} & h_{jj} & h_{jk} & j \\ h_{ki} & h_{kj} & h_{kk} & k \\ \hline i & j & k & \end{array}$$



Nonzero structure

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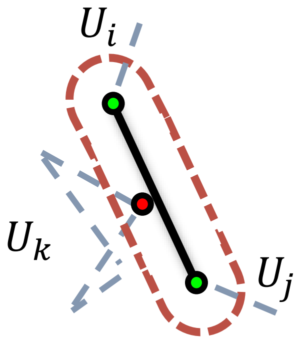
B: Boundary vertices

$$\begin{array}{cc} \begin{bmatrix} h_{ii} & h_{ij} & h_{ik} \\ h_{ji} & h_{jj} & h_{jk} \\ h_{ki} & h_{kj} & h_{kk} \end{bmatrix} & \begin{array}{c} i \\ j \\ k \end{array} \\ \begin{array}{ccc} i & j & k \end{array} & \end{array}$$



Updated nonzero structure

Nonzero structure

$$\min_{\widehat{\mathcal{M}}} \quad E_{\mathrm{d}}(\mathcal{M}, \widehat{\mathcal{M}}) + \lambda E_{\mathrm{b}}(\mathcal{B})$$

$$E_{\mathrm{d}}(\mathcal{M}, \widehat{\mathcal{M}}) = \frac{1}{4}\sum_{i=1}^{N_f}\left(\|J_i\|_F^2 + \|J_i^{-1}\|_F^2\right)$$

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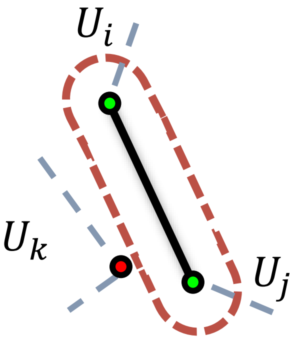
$$H = \begin{bmatrix} H_{II} & H_{IB} \\ H_{BI} & H_{BB} \end{bmatrix}$$

I: Internal vertices
B: Boundary vertices

$$\begin{array}{cc} \begin{bmatrix} 0 & 0 & & 0 \\ 0 & 0 & & 0 \\ & & & \\ 0 & 0 & & 0 \\ & & & \end{bmatrix} & \begin{array}{c} i \\ j \\ \\ k \\ \\ \end{array} \\ \begin{array}{cccc} i & j & & k \end{array} & \end{array}$$



Consider all potential collisions

Nonzero structure

$$\min_{\widehat{\mathcal{M}}} \quad E_{\mathrm{d}}(\mathcal{M}, \widehat{\mathcal{M}}) + \lambda E_{\mathrm{b}}(\mathcal{B})$$

$$E_{\mathrm{d}}(\mathcal{M}, \widehat{\mathcal{M}}) = \frac{1}{4} \sum_{i=1}^{N_f} \left(\|J_i\|_F^2 + \|J_i^{-1}\|_F^2 \right)$$

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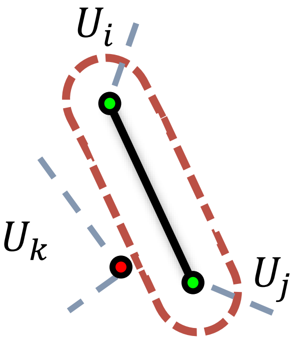
$$H = \begin{bmatrix} H_{II} & H_{IB} \\ H_{BI} & H_{BB} \end{bmatrix}$$

I: Internal vertices

B: Boundary vertices

$$H_{BB} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1b} \\ h_{21} & h_{22} & \cdots & h_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ h_{b1} & h_{b2} & \cdots & h_{bb} \end{bmatrix}$$



Consider all potential collisions $\Longrightarrow$ Fixed nonzero structure

Density

$$\min_{\widehat{\mathcal{M}}} \quad E_{\mathrm{d}}(\mathcal{M}, \widehat{\mathcal{M}}) + \lambda E_{\mathrm{b}}(\mathcal{B})$$

$$E_{\mathrm{d}}(\mathcal{M}, \widehat{\mathcal{M}}) = \frac{1}{4}\sum_{i=1}^{N_f}\left(\|J_i\|_F^2 + \|J_i^{-1}\|_F^2\right)$$

$$E_{\mathrm{b}}(\mathcal{B}) = \sum_{\mathbf{b}_j \in \mathcal{E}_{\mathbf{b}}} \sum_{\mathbf{x}_i \in \mathcal{V}_{\mathbf{b}}} \max\left(0, \frac{\varepsilon}{\mathcal{D}(\mathbf{b}_j, \mathbf{x}_i)} - 1\right)^2$$

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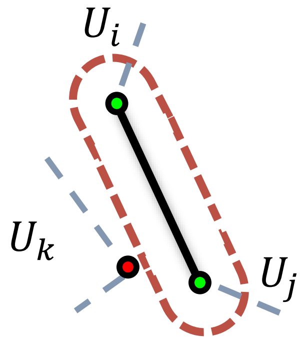
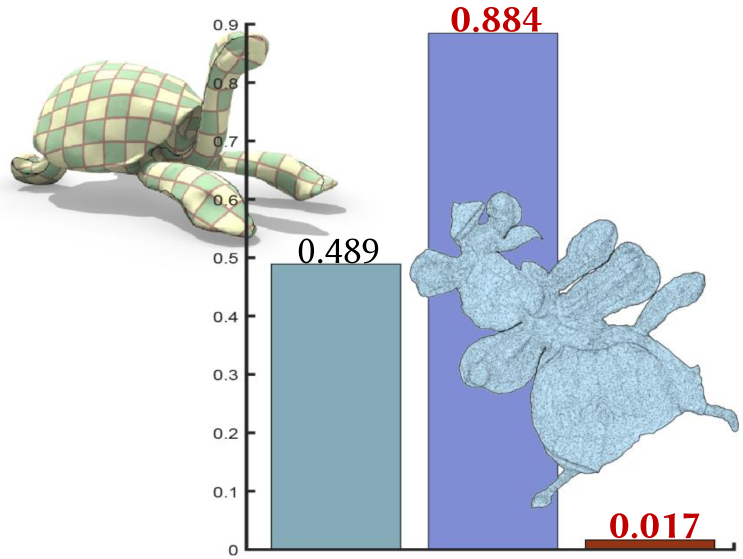


Consider all potential collisions $\Rightarrow$ Fixed nonzero structure

Too many non-zeros (b^2) $\Rightarrow$ High density

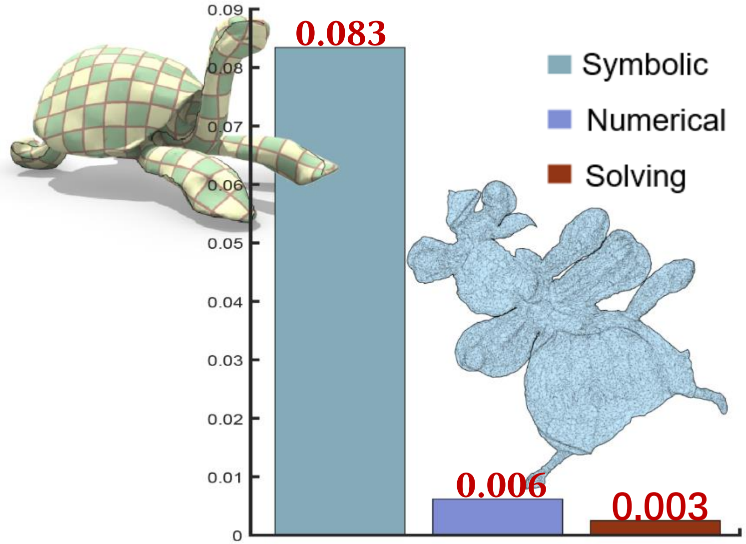
Density

Per iteration time: **0.901**



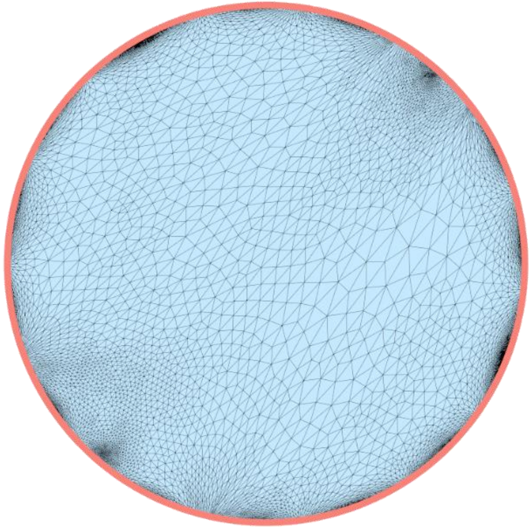
Fixed nonzero structure
High density

Per iteration time: **0.092**



Updated nonzero structure
Low density

Density



$$H = \begin{bmatrix} H_{II} & H_{IB} \\ H_{BI} & H_{BB} \end{bmatrix}$$

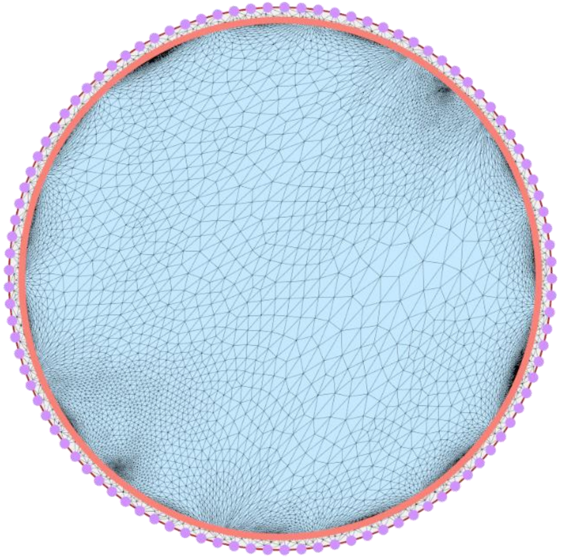
I: Internal vertices
B: Boundary vertices

$$\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1b} \\ h_{21} & h_{22} & \cdots & h_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ h_{b1} & h_{b2} & \cdots & h_{bb} \end{bmatrix}$$

b boundary vertices $\Rightarrow$ b^2 non-zeros $\Rightarrow$ high density

Density

Coarse shell mesh



$$H = \begin{bmatrix} H_{II} & H_{IB} \\ H_{BI} & H_{BB} \end{bmatrix}$$

I: Internal vertices
B: Boundary vertices

$$H_{BB} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1b} \\ h_{21} & h_{22} & \cdots & h_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ h_{b1} & h_{b2} & \cdots & h_{bb} \end{bmatrix}, \quad \begin{bmatrix} h_{11} & \cdots \\ & \ddots & \cdots \\ & & h_{ss} \end{bmatrix} \ll$$

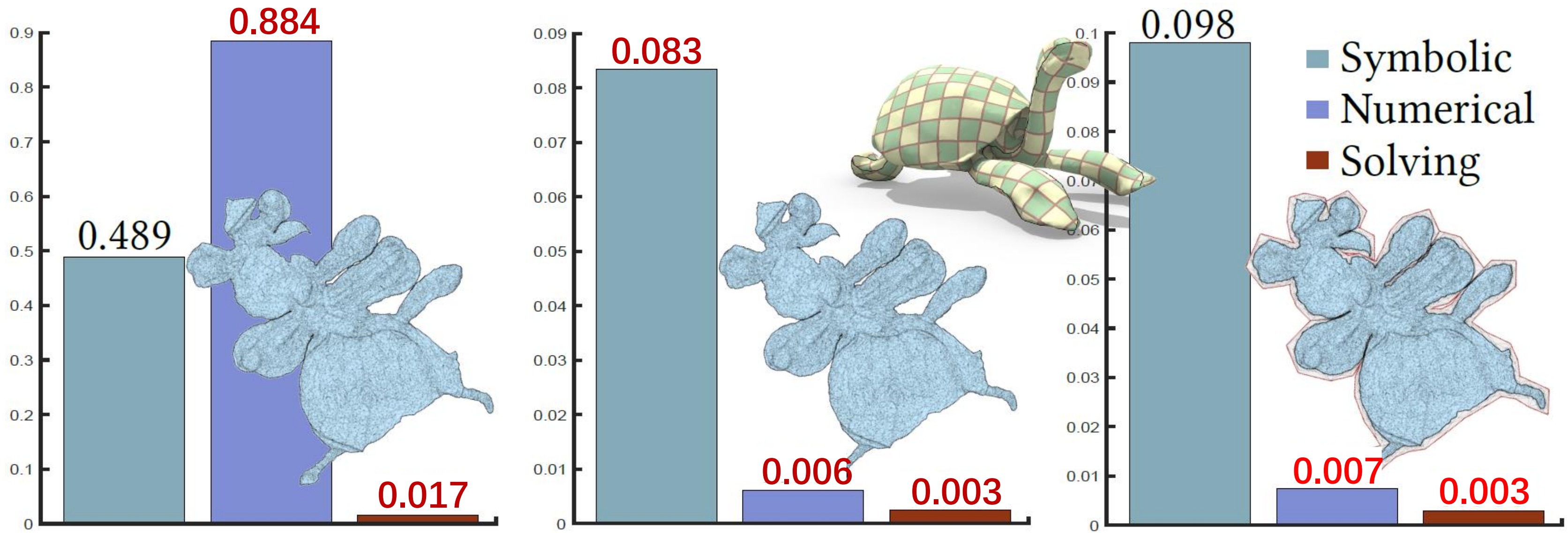
b **boundary vertices** $\Rightarrow$ $\boldsymbol{b^2}$ **non-zeros** $\Rightarrow$ **high density**

Density

Per iteration time: **0.901**

Per iteration time: **0.092**

Per iteration time: **0.010**



Fixed nonzero structure
High density

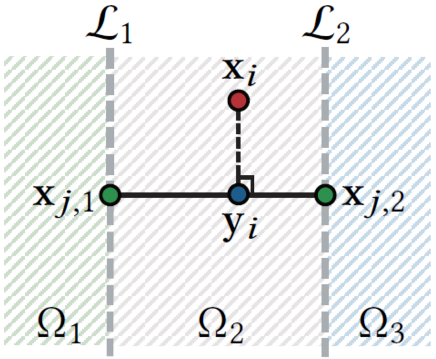
Updated nonzero structure
Low density

Fixed nonzero structure
Low density

Convex approximation

- Distance in [Smith et al. 2015]

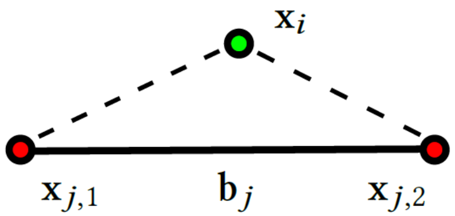
$$\mathcal{D}(\mathbf{b}_j, \mathbf{x}_i) = \begin{cases} \|\mathbf{x}_{j,1} - \mathbf{x}_i\|_2, & \text{if } \mathbf{x}_i \in \Omega_1 \\ \|\mathbf{y}_i - \mathbf{x}_i\|_2, & \text{if } \mathbf{x}_i \in \Omega_2 \\ \|\mathbf{x}_{j,2} - \mathbf{x}_i\|_2, & \text{if } \mathbf{x}_i \in \Omega_3 \end{cases}$$



Distance is not C^2.

- Triangle inequality-based distance

$$\mathcal{D}(\mathbf{b}_j, \mathbf{x}_i) = \frac{1}{2}\left(\|\mathbf{x}_{j,1} - \mathbf{x}_i\|_2 + \|\mathbf{x}_{j,2} - \mathbf{x}_i\|_2 - \|\mathbf{x}_{j,1} - \mathbf{x}_{j,2}\|_2\right)$$



Distance is C$^\infty$.

Convex approximation

$$E_{\text{ev}}(\mathbf{b}_j, \mathbf{x}_i) = \left(\frac{\varepsilon}{\mathcal{D}(\mathbf{b}_j, \mathbf{x}_i)} - 1\right)^2$$

Convex

$$f(g) = \left(\frac{\varepsilon}{g} - 1\right)^2, g = \mathcal{D}(\mathbf{b}_j, \mathbf{x}_i)$$

Convex Concave

$$g = \frac{1}{2}\left(\|\mathbf{x}_{j,1} - \mathbf{x}_i\|_2 + \|\mathbf{x}_{j,2} - \mathbf{x}_i\|_2 - \|\mathbf{x}_{j,1} - \mathbf{x}_{j,2}\|_2\right)$$

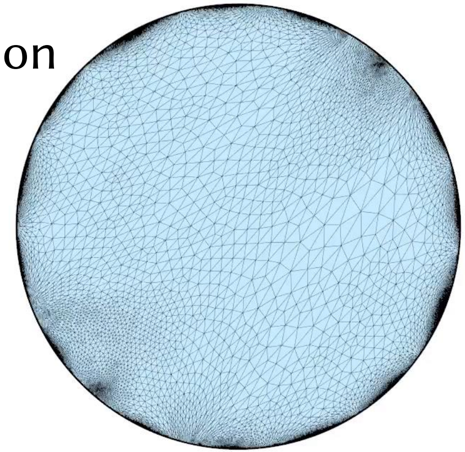
$$H_{\text{ev}}(\mathbf{b}_j, \mathbf{x}_i) = f''(g)g'(\mathbf{x})^T g'(\mathbf{x}) + f'(g)\nabla^2 g(\mathbf{x})$$

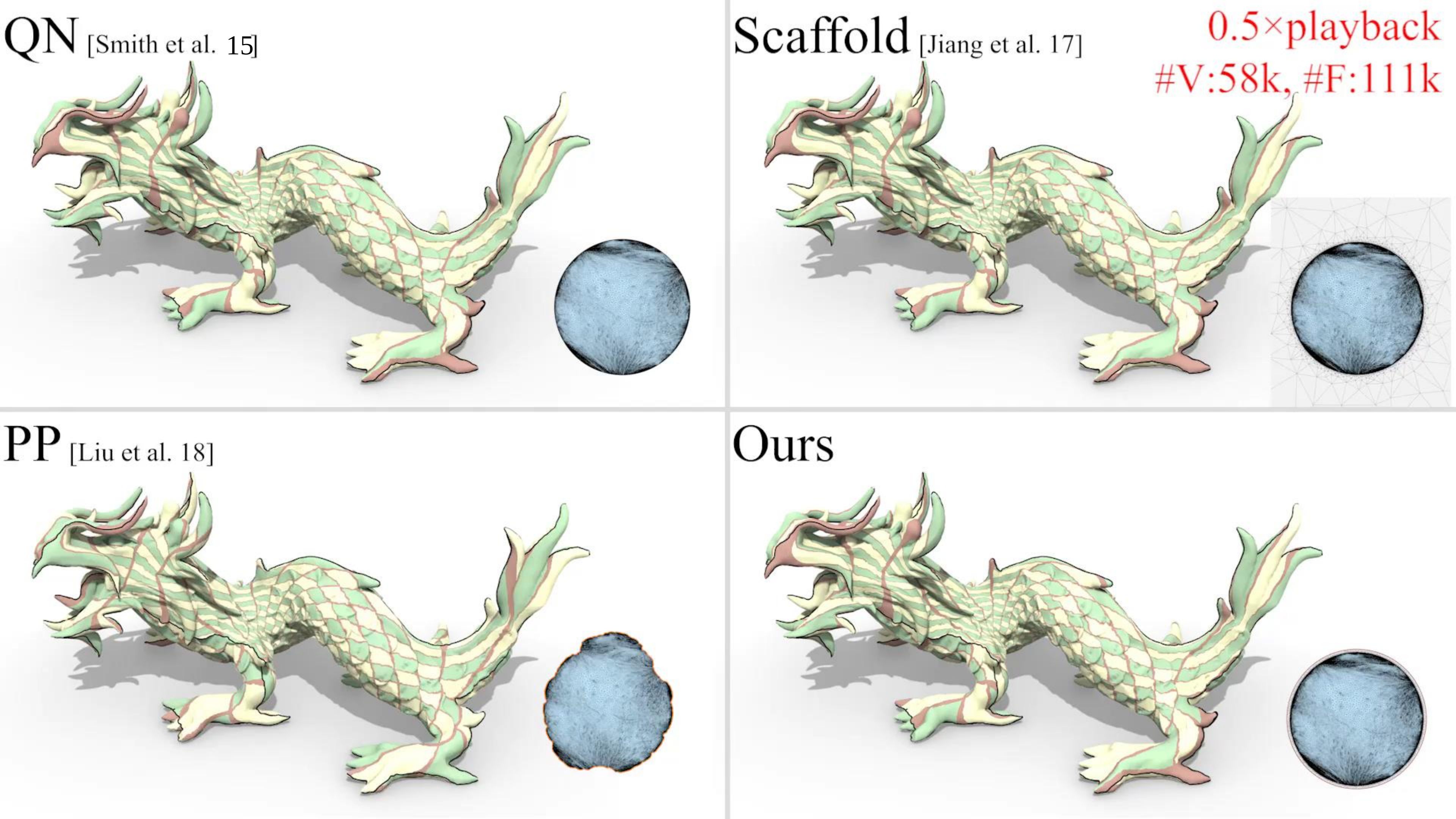
[Shtengel et al. 2017]

$$H_{\text{ev}}^{+}(\mathbf{b}_j, \mathbf{x}_i) = f''(g)g'(\mathbf{x})^T g'(\mathbf{x}) + f'(g)\left(-\|\mathbf{x}_{j,1} - \mathbf{x}_{j,2}\|_2\right)$$

High efficiency

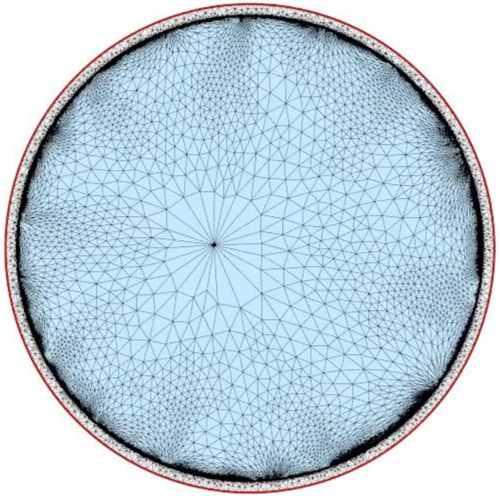
- Fast second-order solver
  - Fixed nonzero structure of the Hessian matrix
  - Low density of the Hessian matrix
  - An easily obtained convex approximation

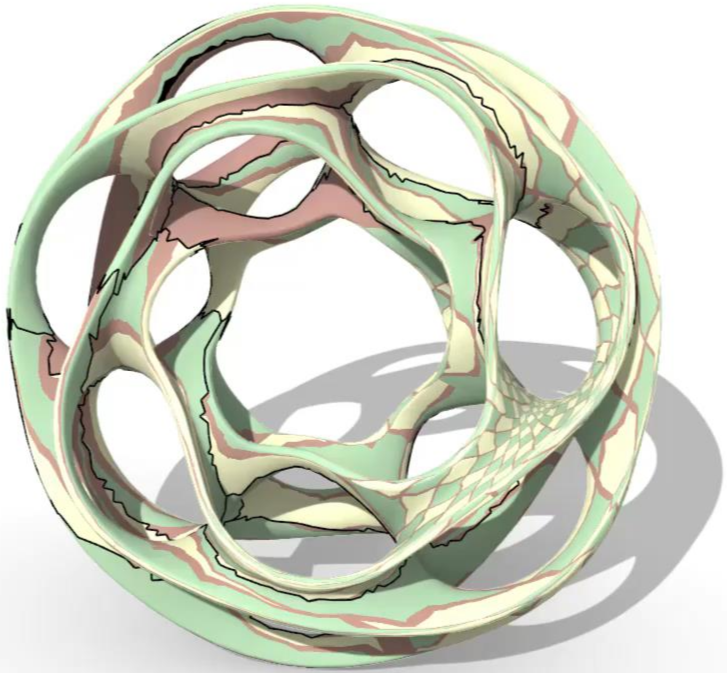



QN [Smith et al. 15]
Scaffold [Jiang et al. 17]
0.5×playback
#V:58k, #F:111k
PP [Liu et al. 18]
Ours

Heptoroid surface

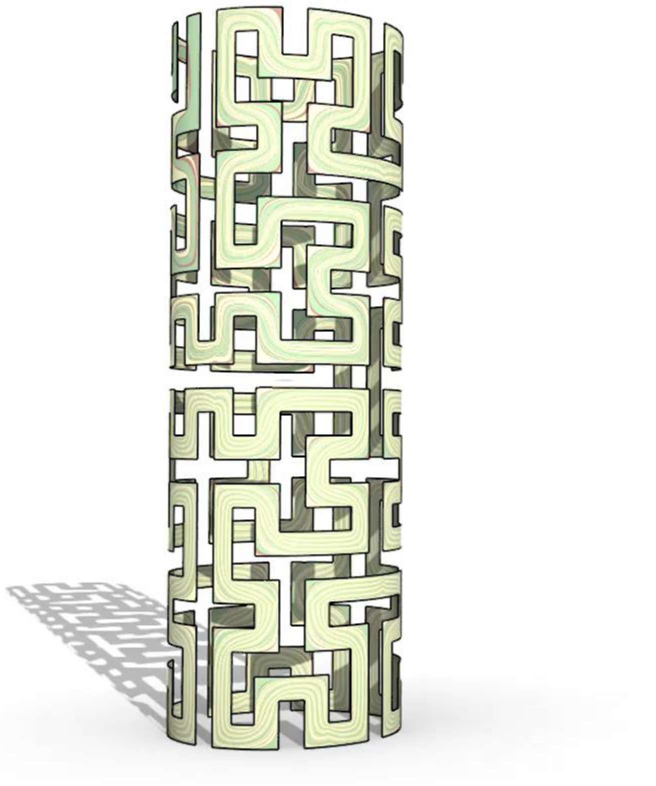
0.25×playback
#V:15k, #F:26k

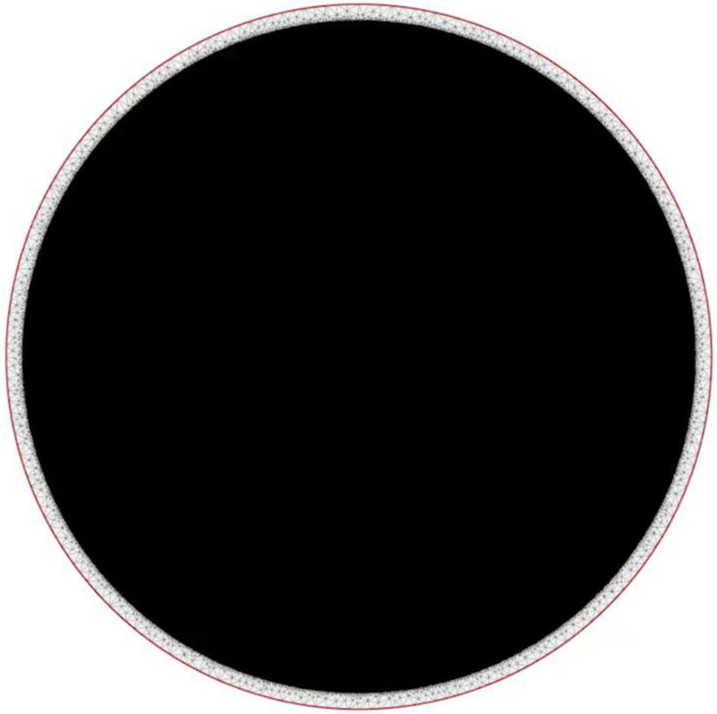




Hilbert-curve-shaped developable surface

5×playback
#V:80k, #F:147k

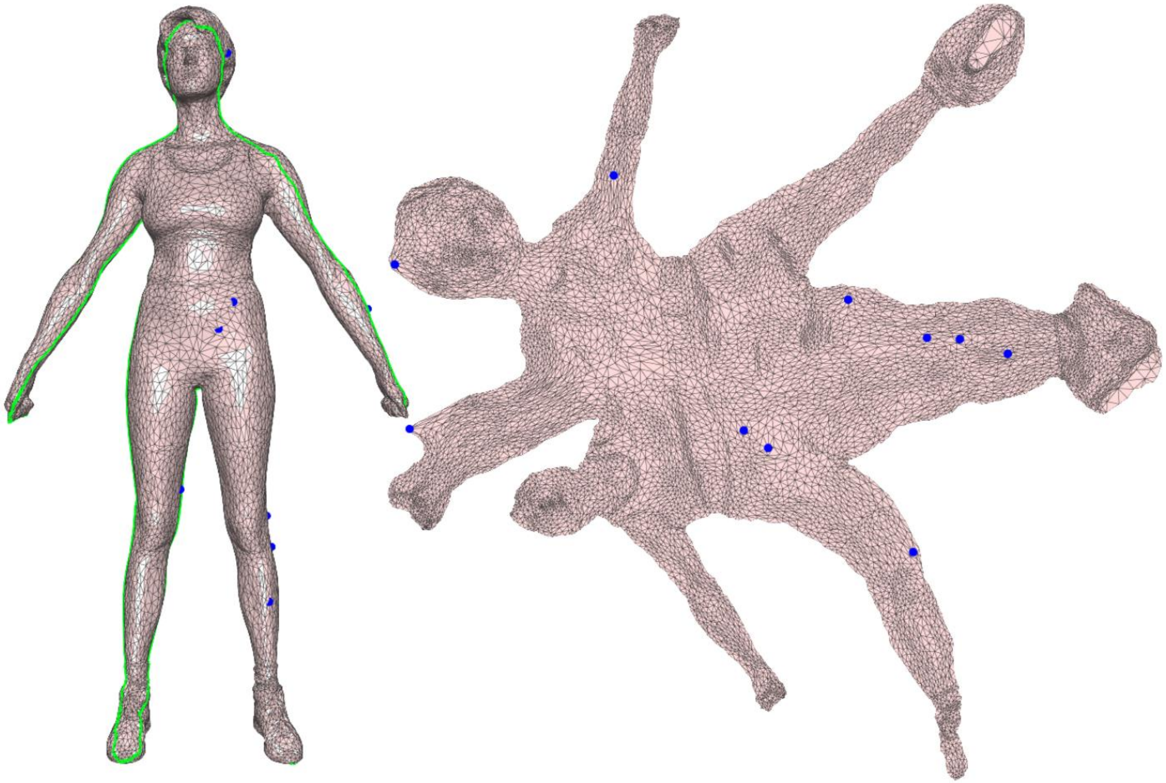




With positional constraints

Positional constraints

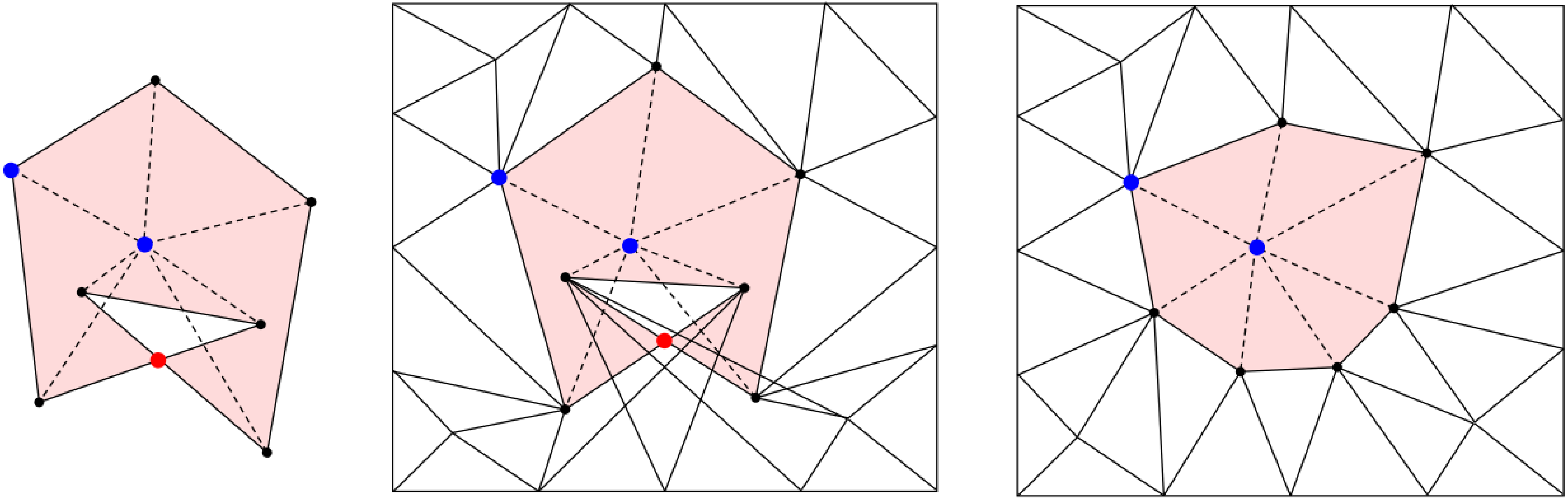
- Constrain a set of vertices to the target positions.
- Tutte's embedding is not applicable.
- Soft constraints:
  - self-locking issues



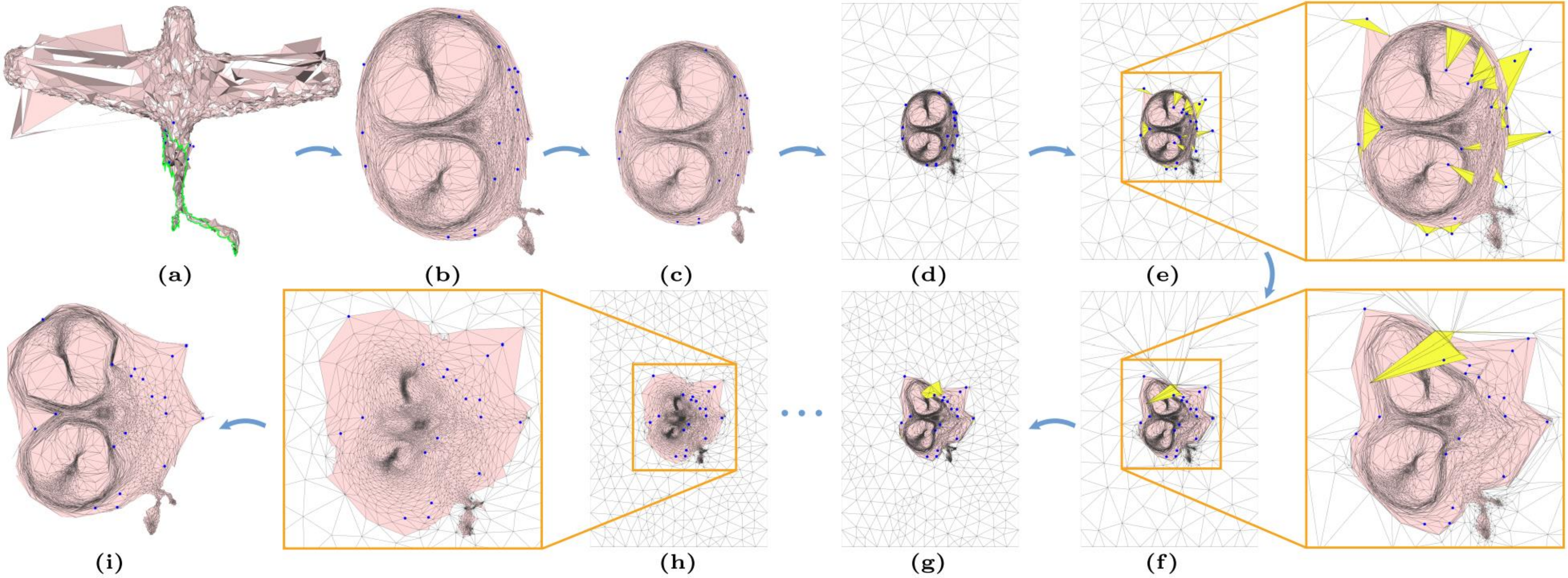
Practical Construction of Globally Injective Parameterizations with Positional Constraints, accepted to CVM

Key idea

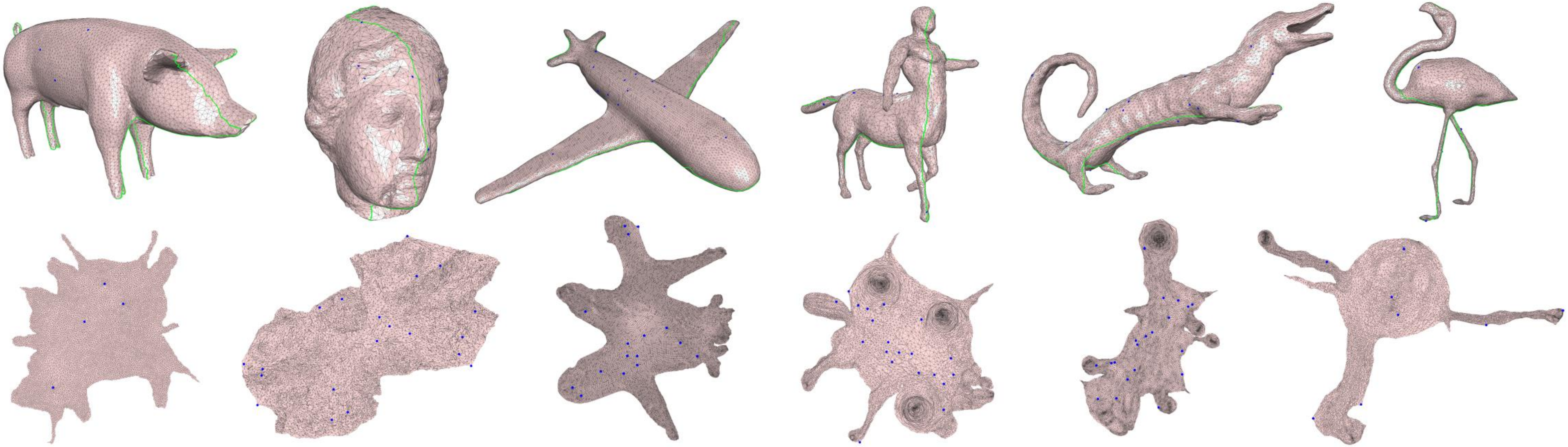
- With scaffold, we convert the problem to computing flip-free parameterizations.



Pipeline



Results



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谢 谢 ！

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