



中国科学技术大学
University of Science and Technology of China

GAMES 301: 第6讲

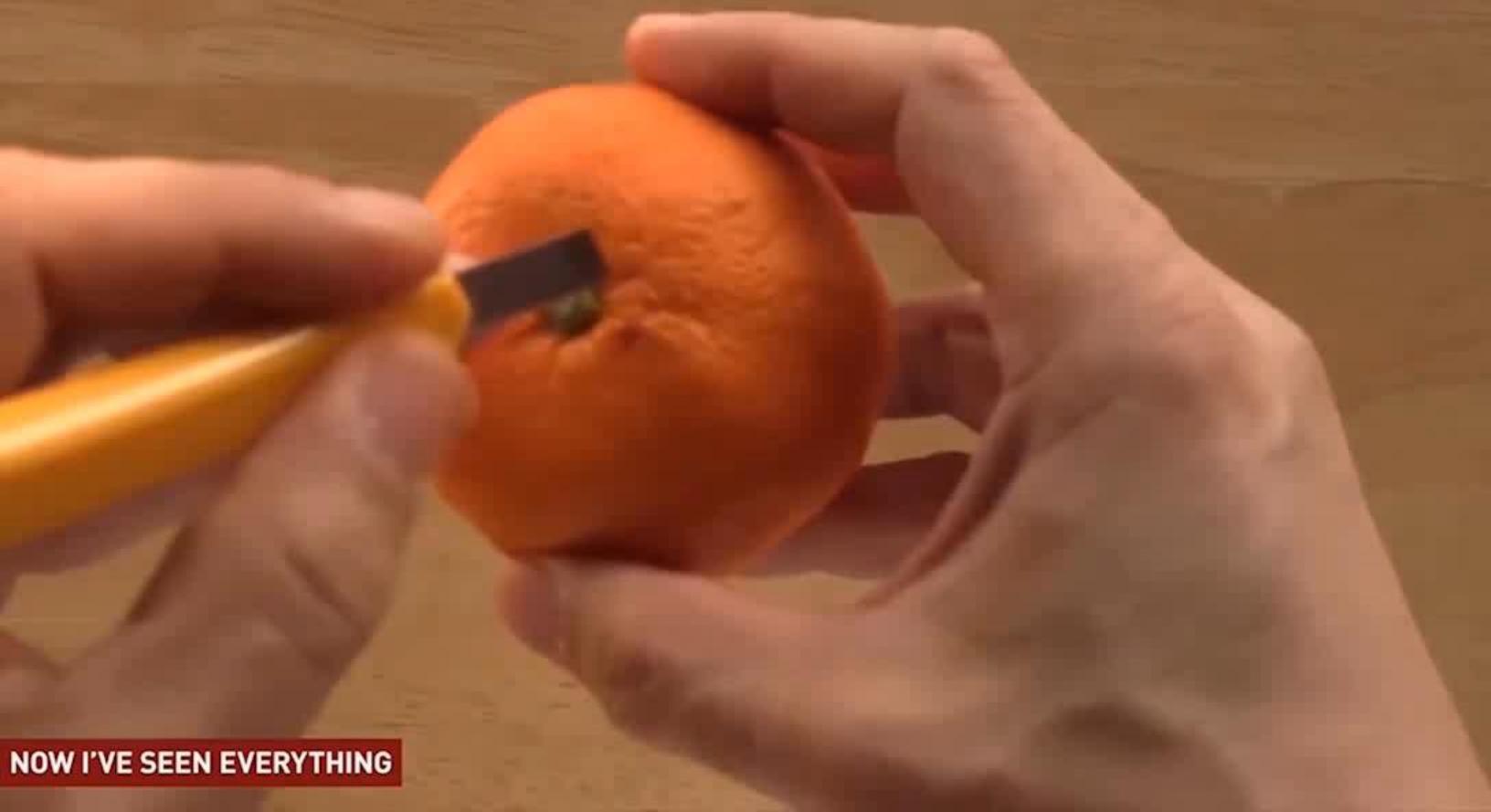
参数化应用1
- 艺术设计

傅孝明
中国科学技术大学

Art design

Peeling art

Computational Peeling Art Design



NOW I'VE SEEN EVERYTHING

Popular art form



Peeling art examples



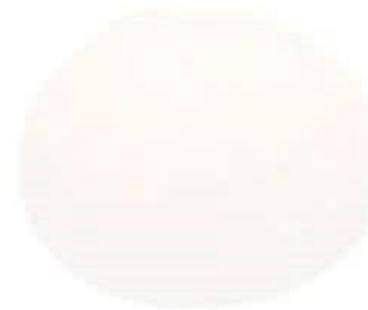
Problem: cut generation



complex, tedious, time consuming

Peeling art design problem

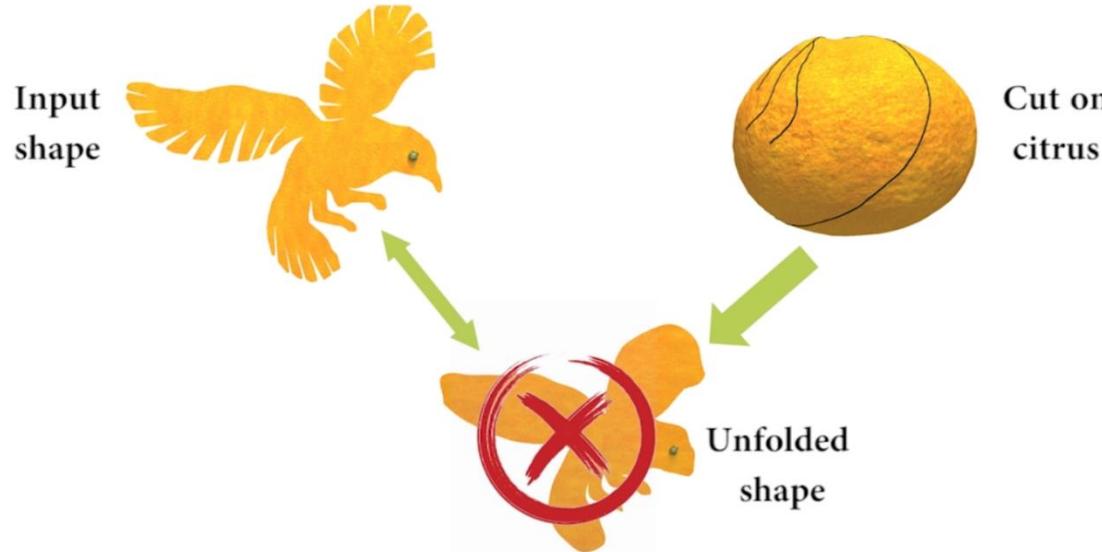
Input
shape



Challenges



- Non-trivial to optimize the similarity
- Unsuitable input shape



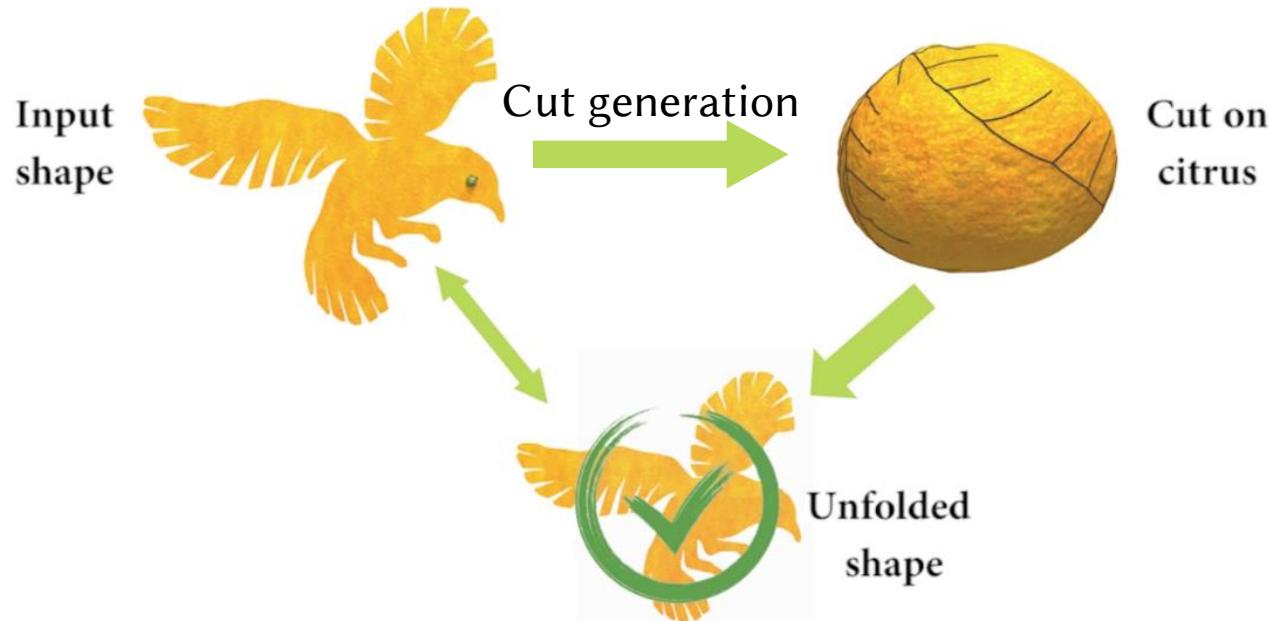
Existing work: cut generation



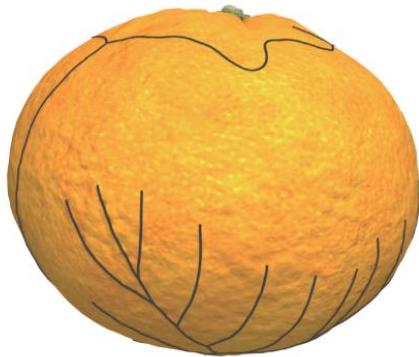
- Minimum spanning tree method [Chai et al. 2018; Sheffer 2002; Sheffer and Hart 2002]
- Mesh segmentation approaches [Julius et al. 2005; Lévy et al. 2002; Sander et al. 2002, 2003; Zhang et al. 2005; Zhou et al. 2004]
- Simultaneous optimization [Li et al. 2018; Poranne et al. 2017]
- Variational method [Sharp and Crane 2018]

unfolded shapes \neq input shapes

Our approach



Key idea



Cut generation

Difficult

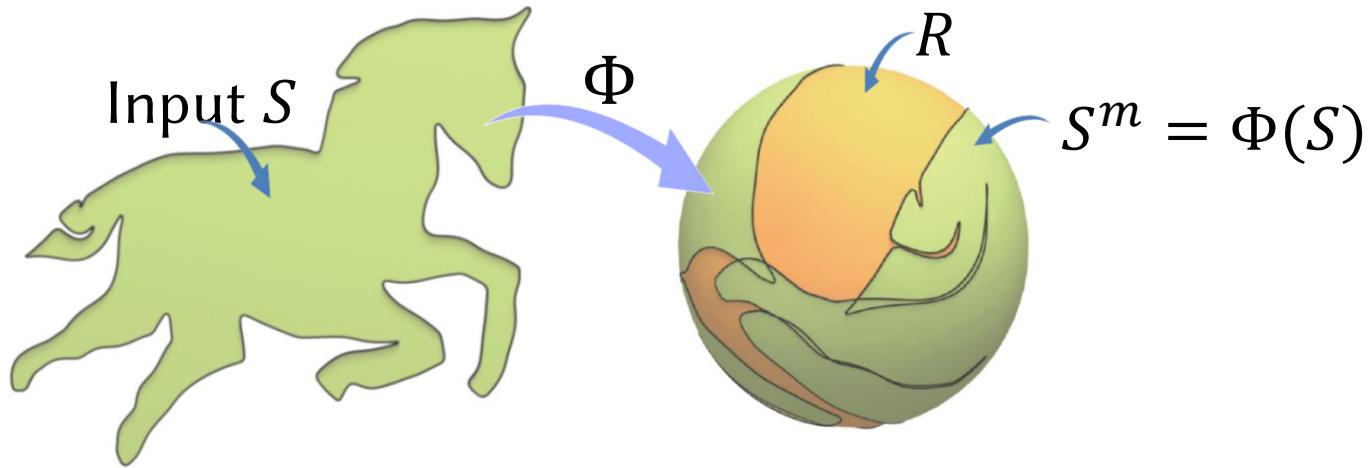


Mapping computation

Easy



Mapping computation



Two goals:

1. Low isometric distortion
2. Area of R approaches zero

$$\min E_{iso}(S^m, S) + wE_{shr}(R)$$

Isometric energy



- ARAP distortion metric [Liu et al. 2008]

$$E_{iso}(S^m, S) = \sum_{i=1}^{N_f} Area(f_i) \|J_i - R_i\|_F^2$$

R_i is an orthogonal matrix

Shrink energy



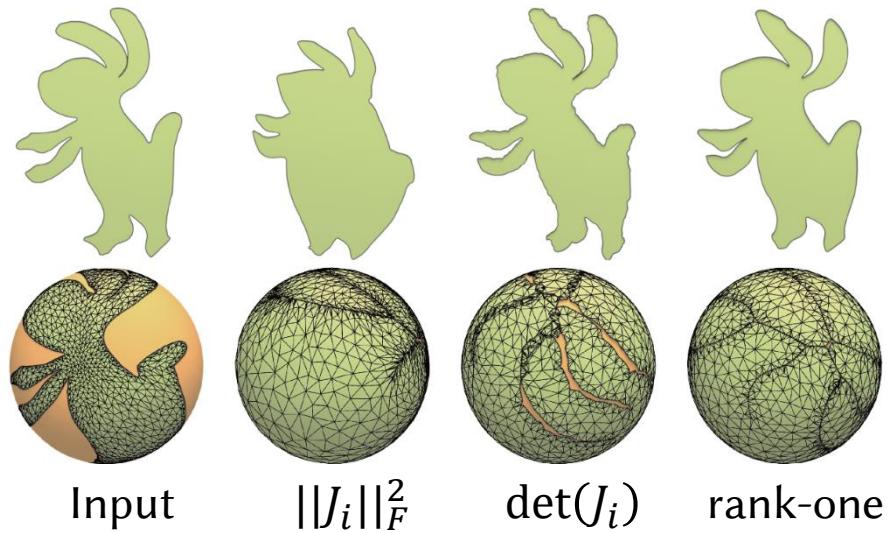
- Our novel rank-one energy

$$E_{shr}(R) = \sum_{i=1}^{N_{Rf}} Area(t_i) \|J_i - B_i\|_F^2$$

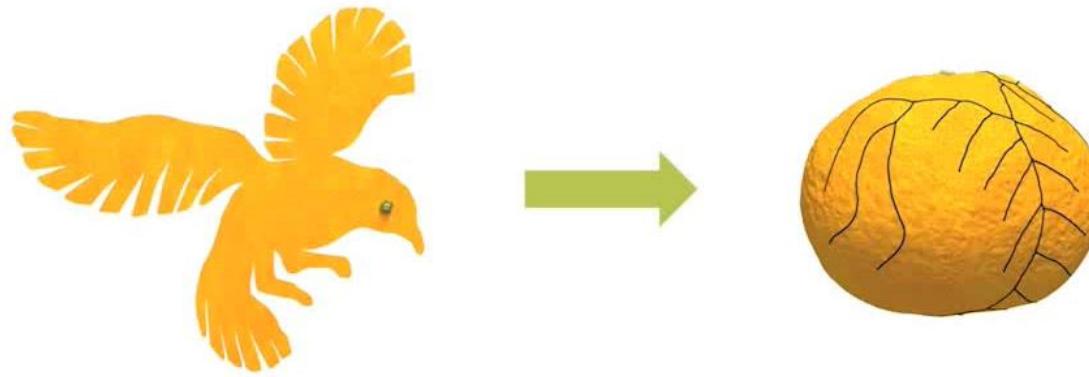
B_i is a rank one matrix

- Other choices

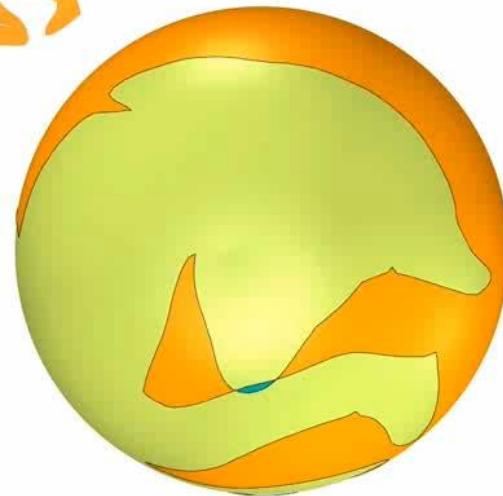
- Frobenius energy $\|J_i\|_F^2$
- Determinant energy $\det(J_i)$



Suitable input



Unsuitable input



Front view



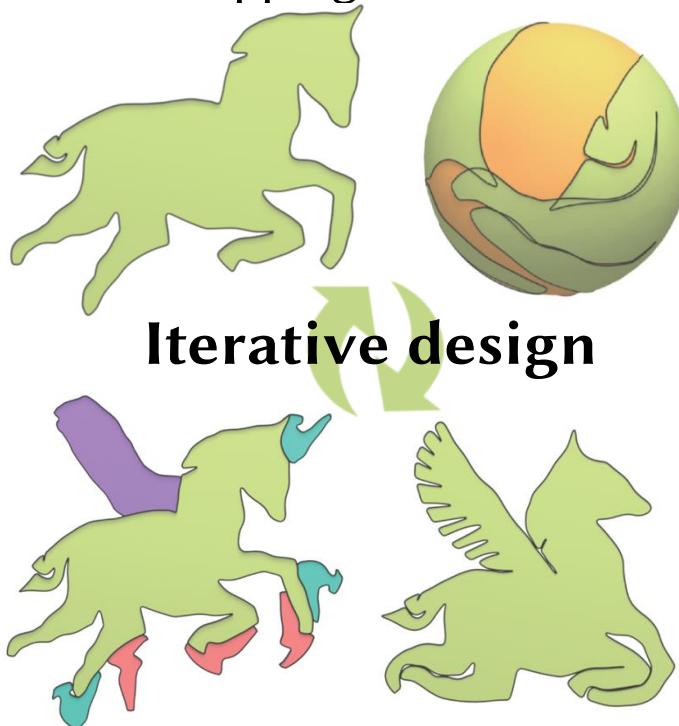
Back view

- Mapped input shape
- Overlap
- Citrus

Iterative interaction



Mapping Process



Interaction Process

Almost cover Final resulting cut



Cut generation

Interaction place



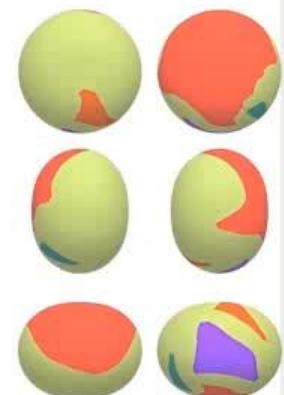
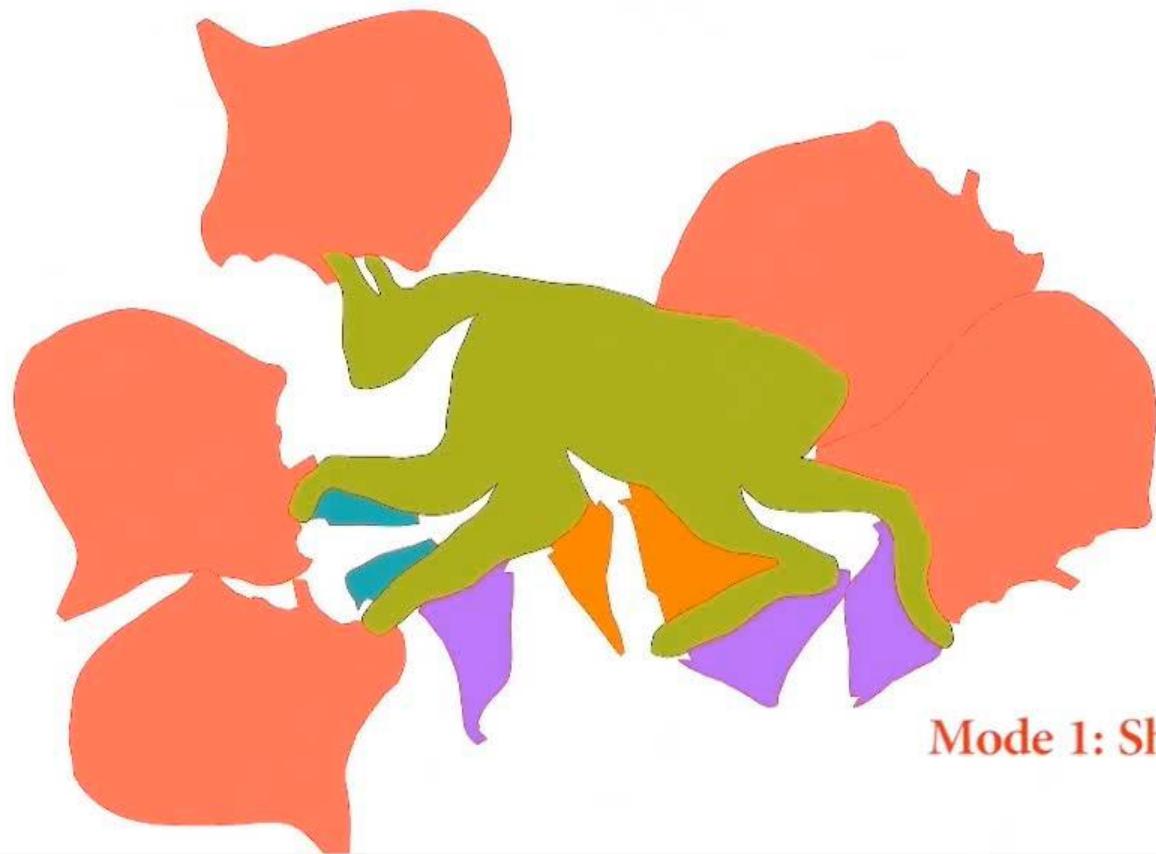
Prune and
Decompose



Unfold
 S^m and R

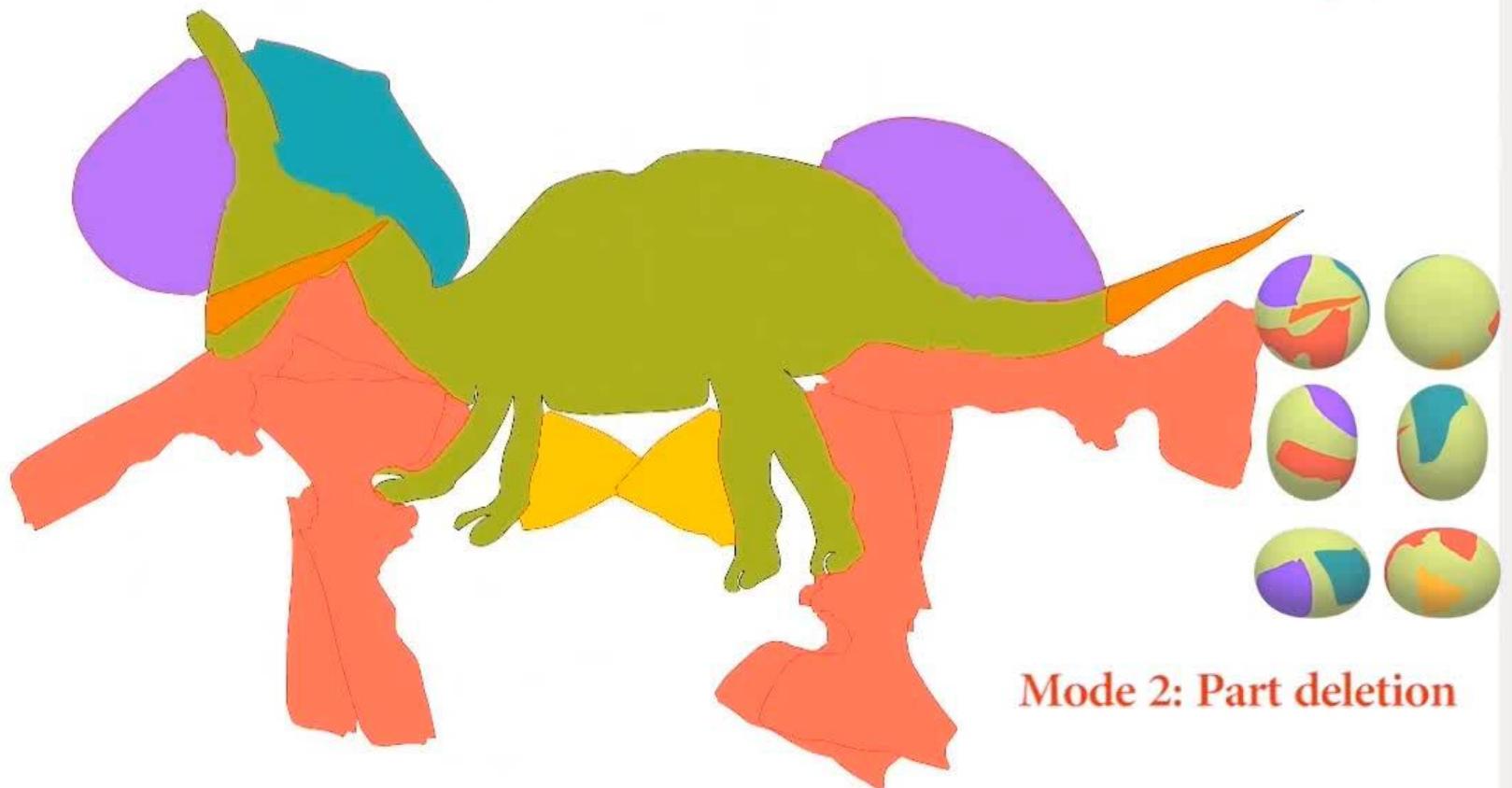


Interaction 1: shape augmentation

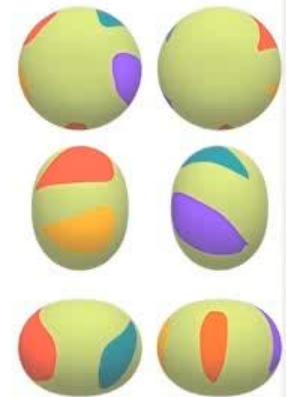
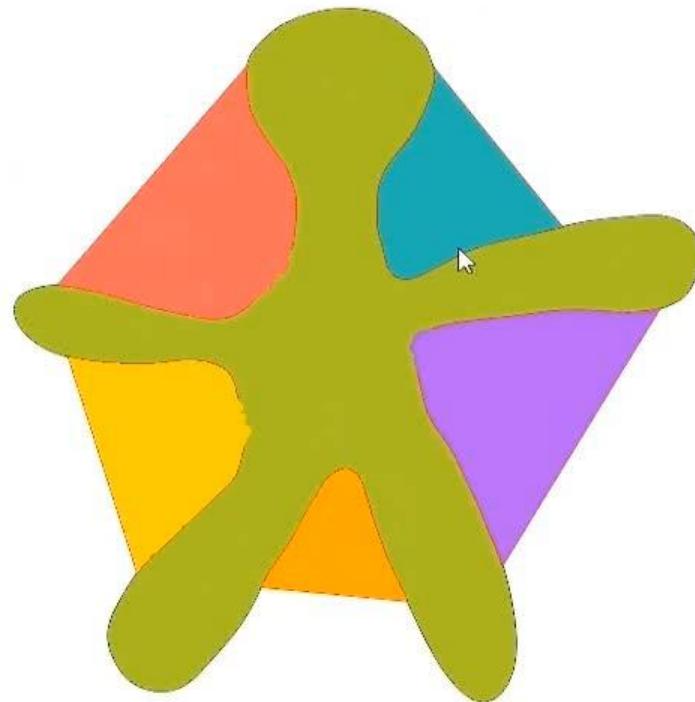


Mode 1: Shape augmentation

Interaction 2: part deletion

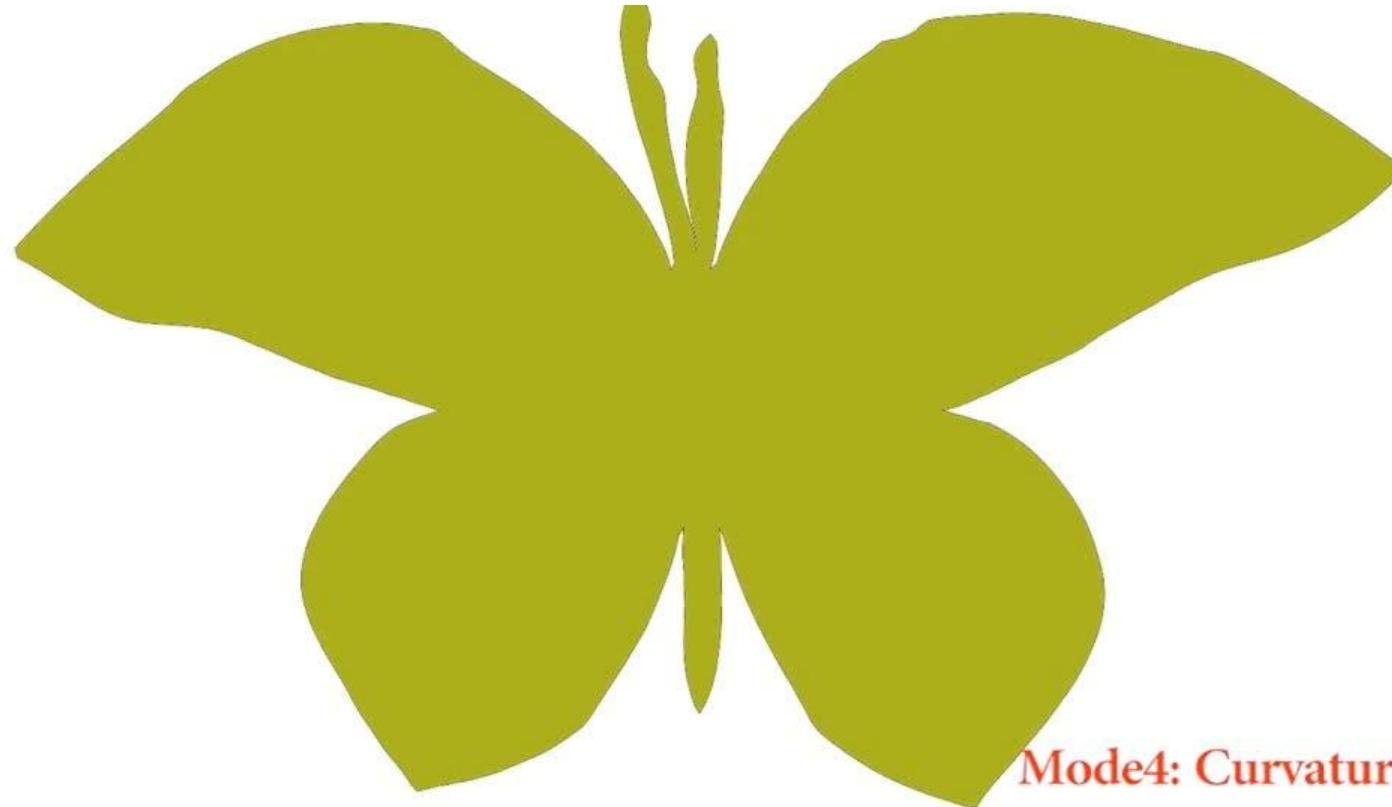


Interaction 3: angle augmentation



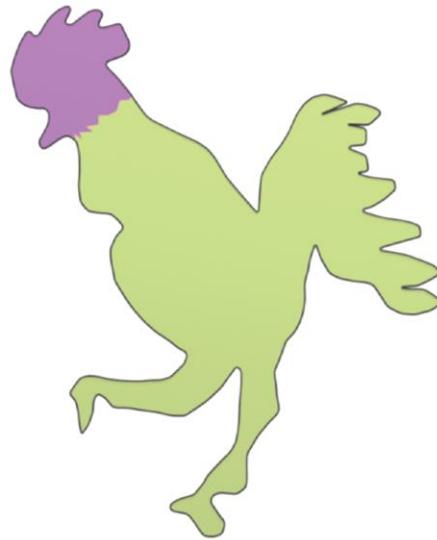
Mode 3: Angle augmentation

Interaction 4: curvature reduction

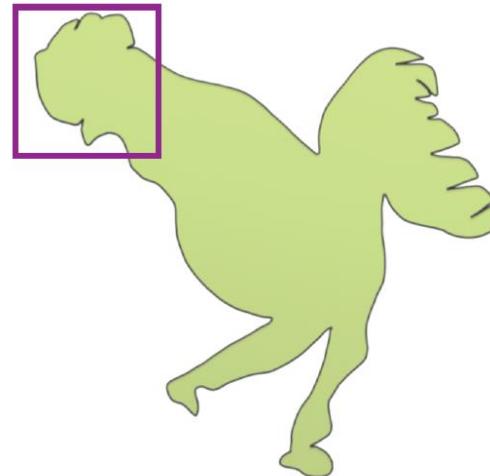


Mode4: Curvature reduction

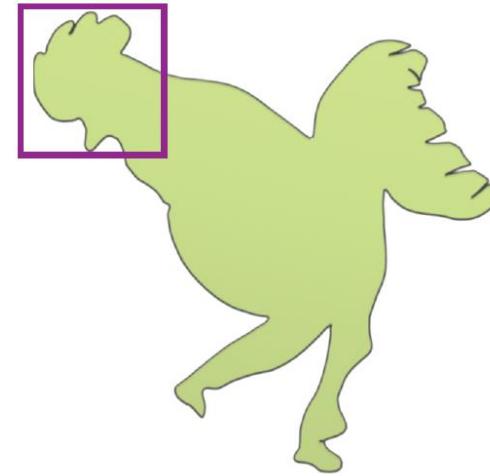
Interaction 5: pre-processing



Input
with specify area

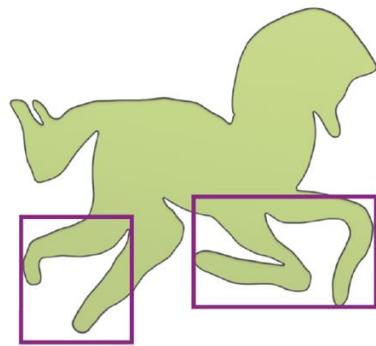


Unprocessed
high distortion

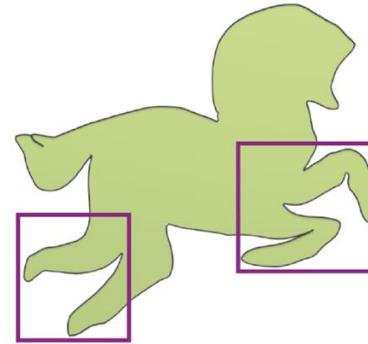


Processed
low distortion

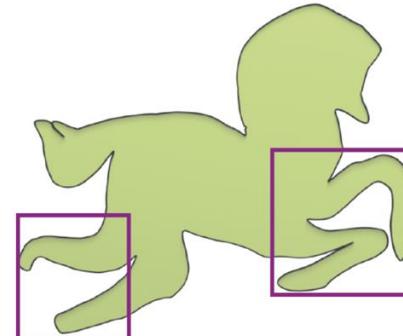
Interaction 5: pre-processing



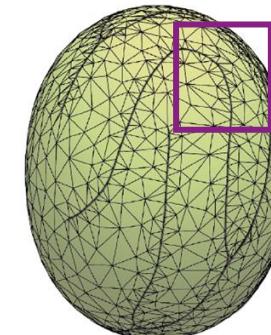
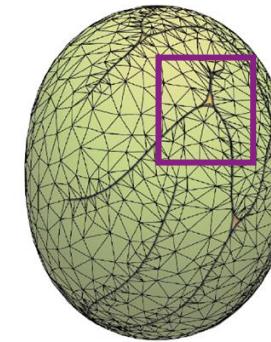
Input with specify area



Unprocessed: high distortion



Processed: low distortion



Cut generation



Mapped shape

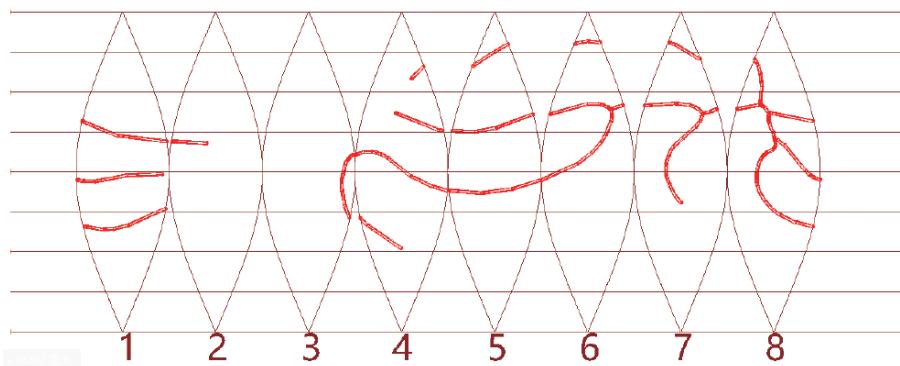
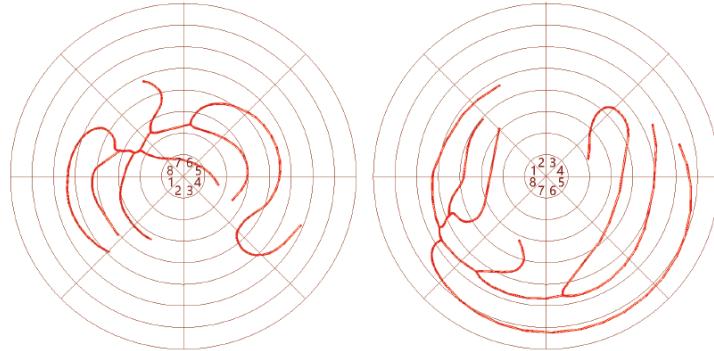


Resulting cut



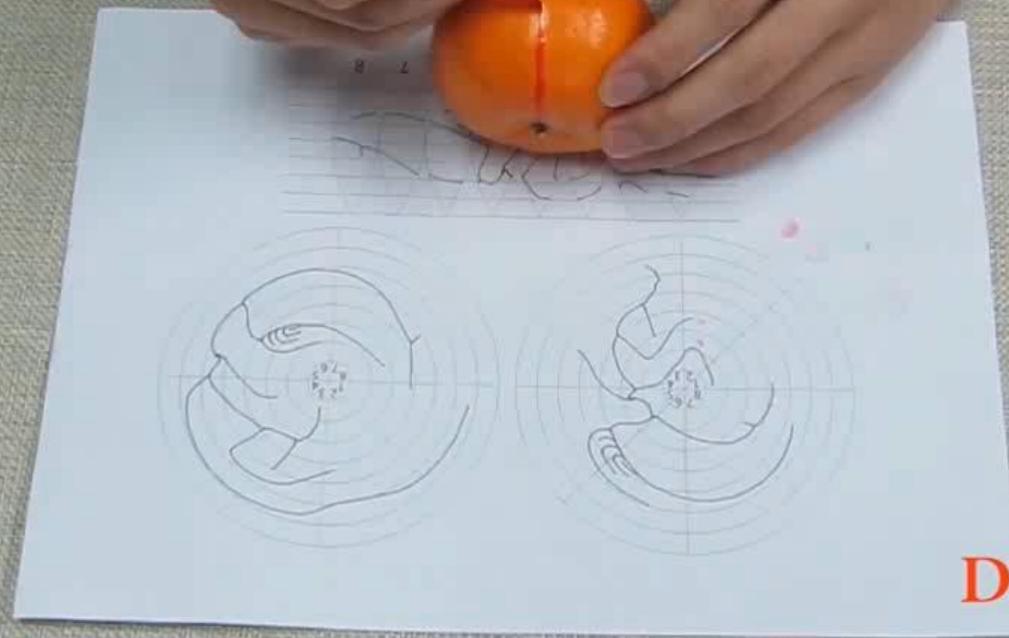
Simplify boundary

Real peeling



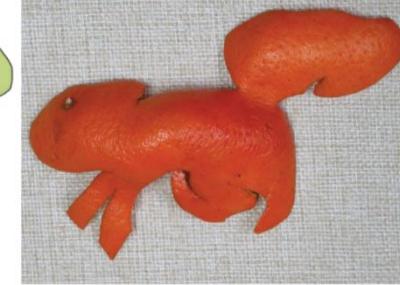
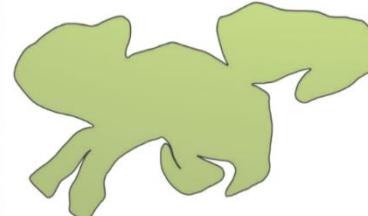
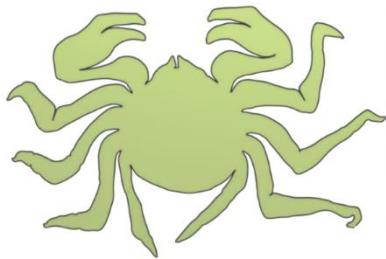
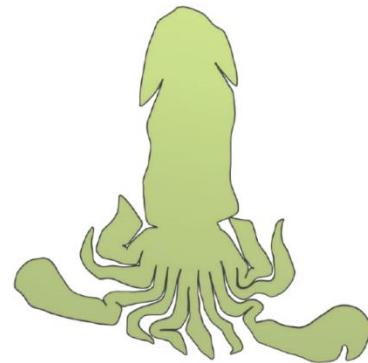
Real peeling

50 × playback



Drawing graticule

Shapes designed by Yoshihiro Okada



Comparison to Yoshihiro Okada



Okada's



Ours



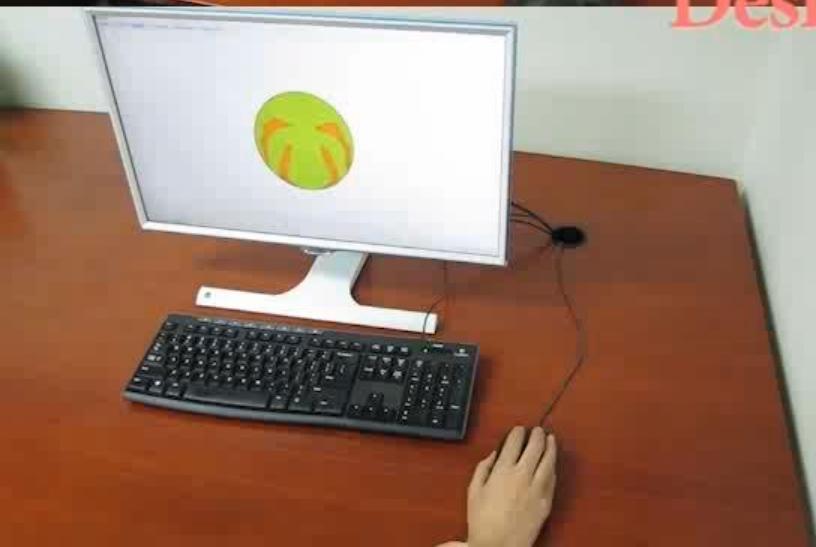
Dove

Eagle

Shrimp



Designing shapes



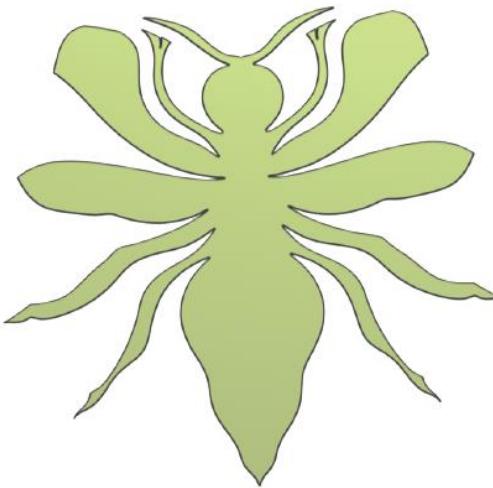
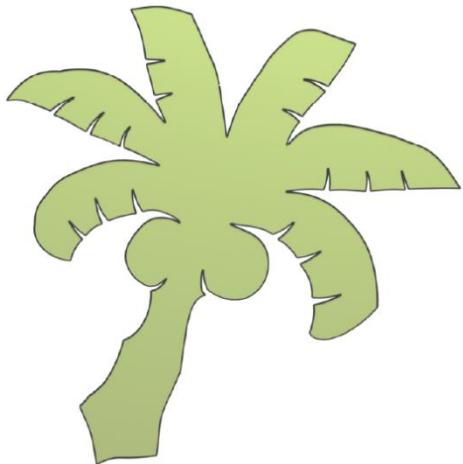
20 × playback

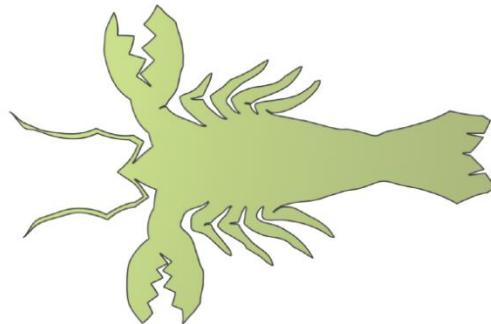
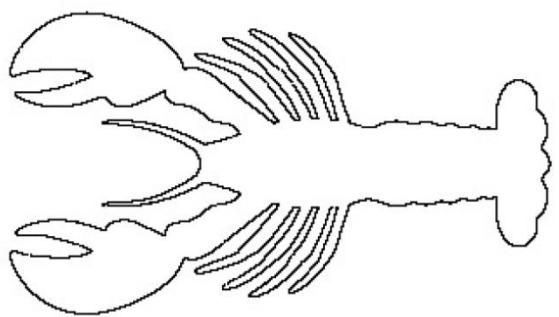
Our results

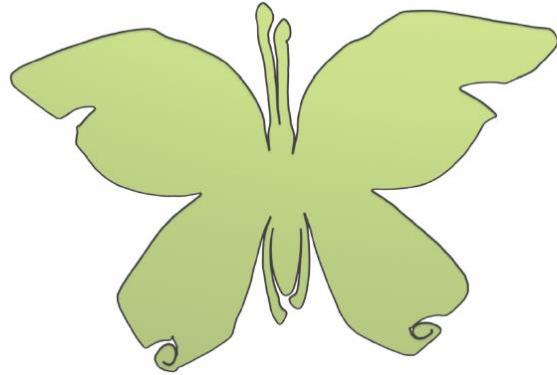


SIG NG











Mantis



Crane



Kangaroo



Pelican



Egret



Horse



Stegosaurus



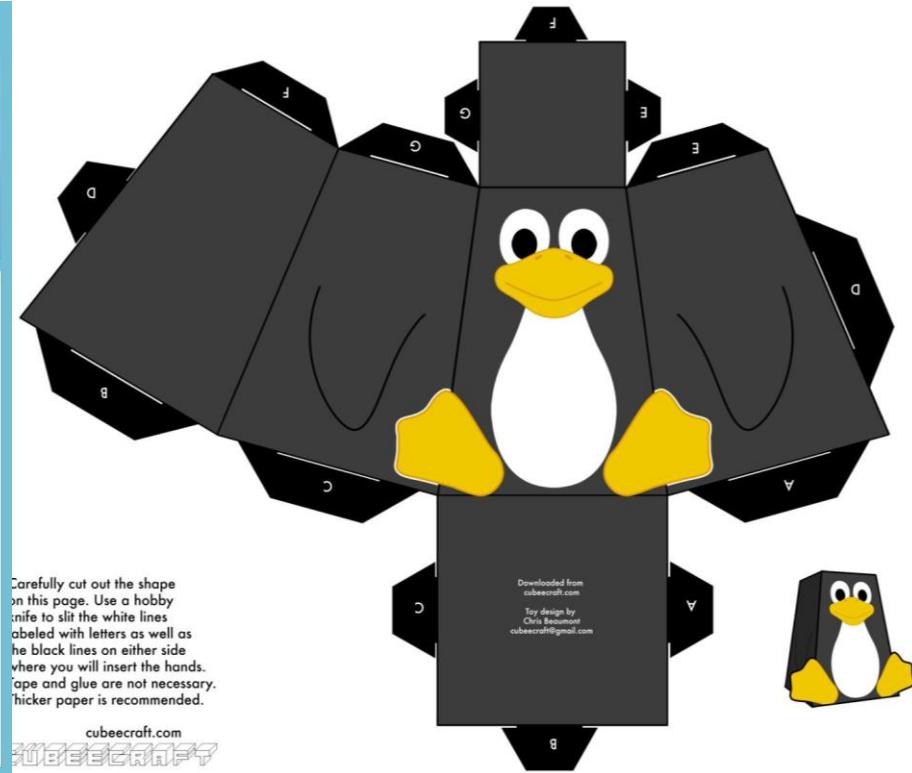
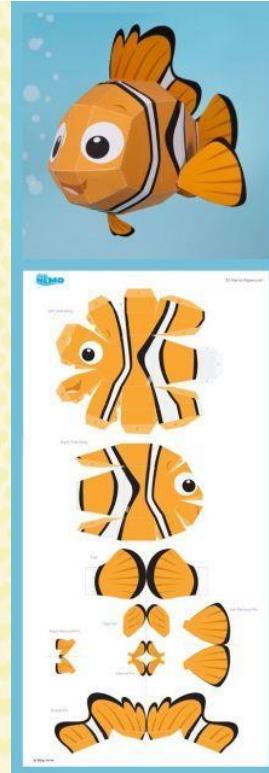
Tiger



Rat

Papercraft

Paper craft



Developable Surfaces



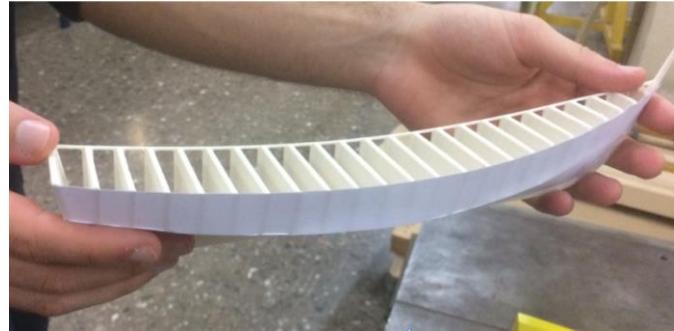
Twice differentiable
Zero Gaussian curvature



Developable Surfaces



Origami



Ship hulls

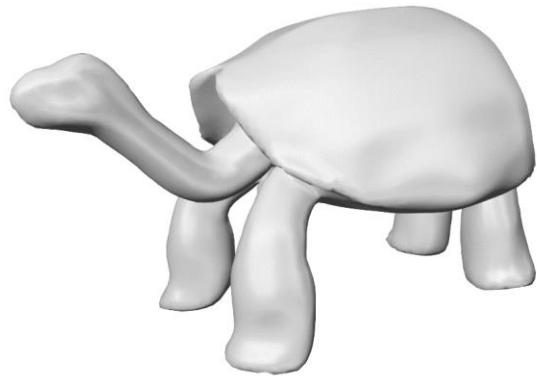


Clothing

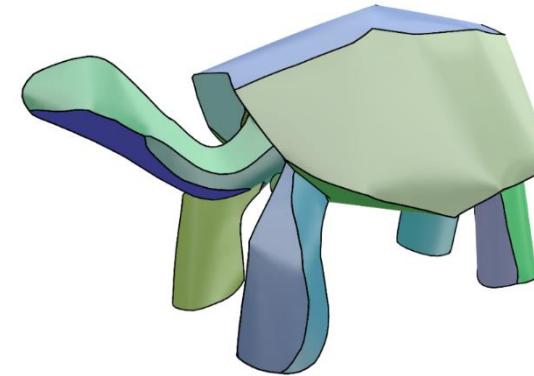


Architecture

Goal



Not globally developable



Piecewise developable approximation

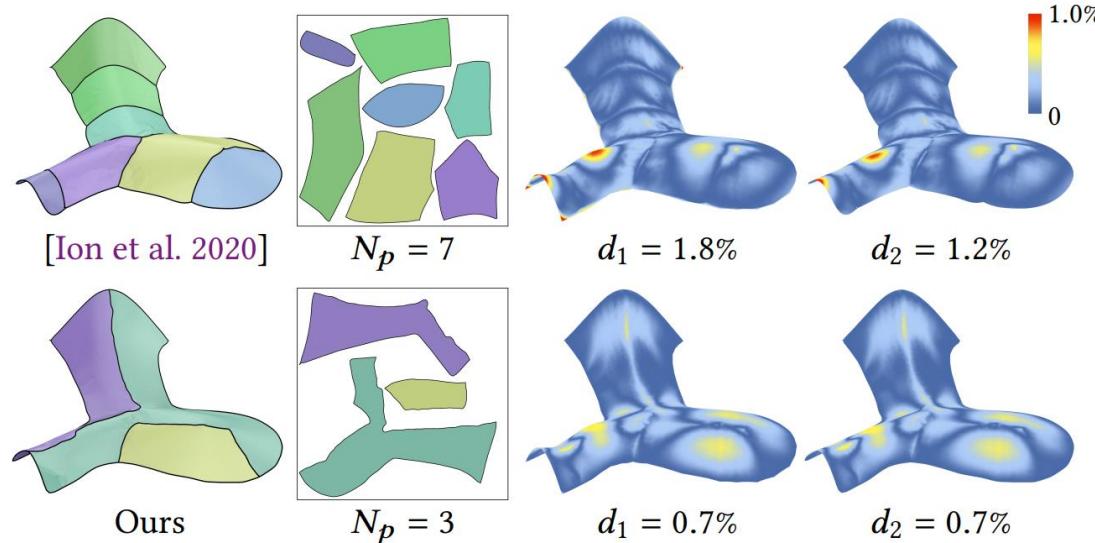
Small number of patches

High similarity

Challenges



- Determining the numbers, placements and shapes of patches under the developability and similarity constraints



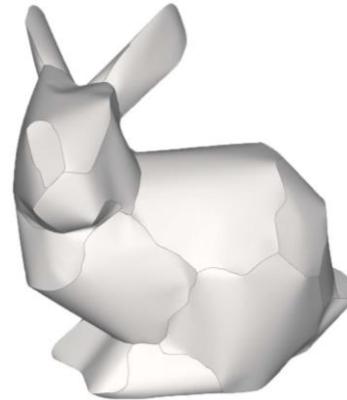
Related Work



Segmentation-based



[Shatz et al. 2006]



[Ion et al. 2020]

Pro: explicit patches

Con: large approximation error

Deformation-based



[Stein et al. 2018]

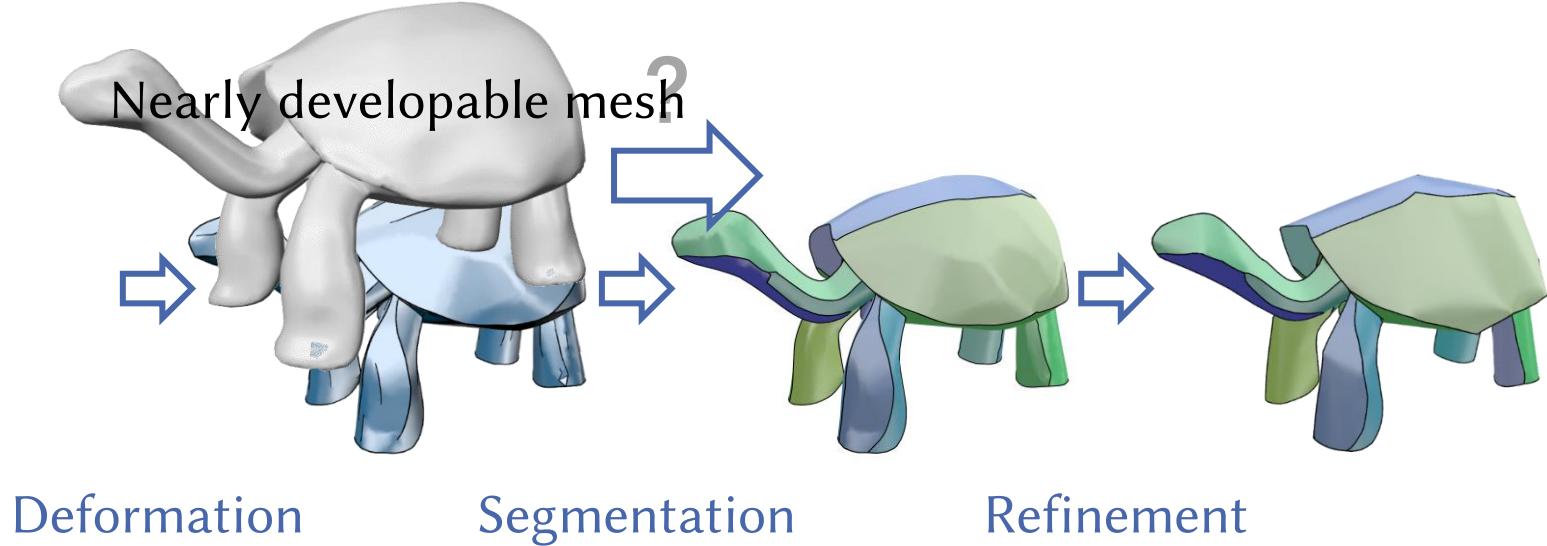


[Binninger et al. 2021]

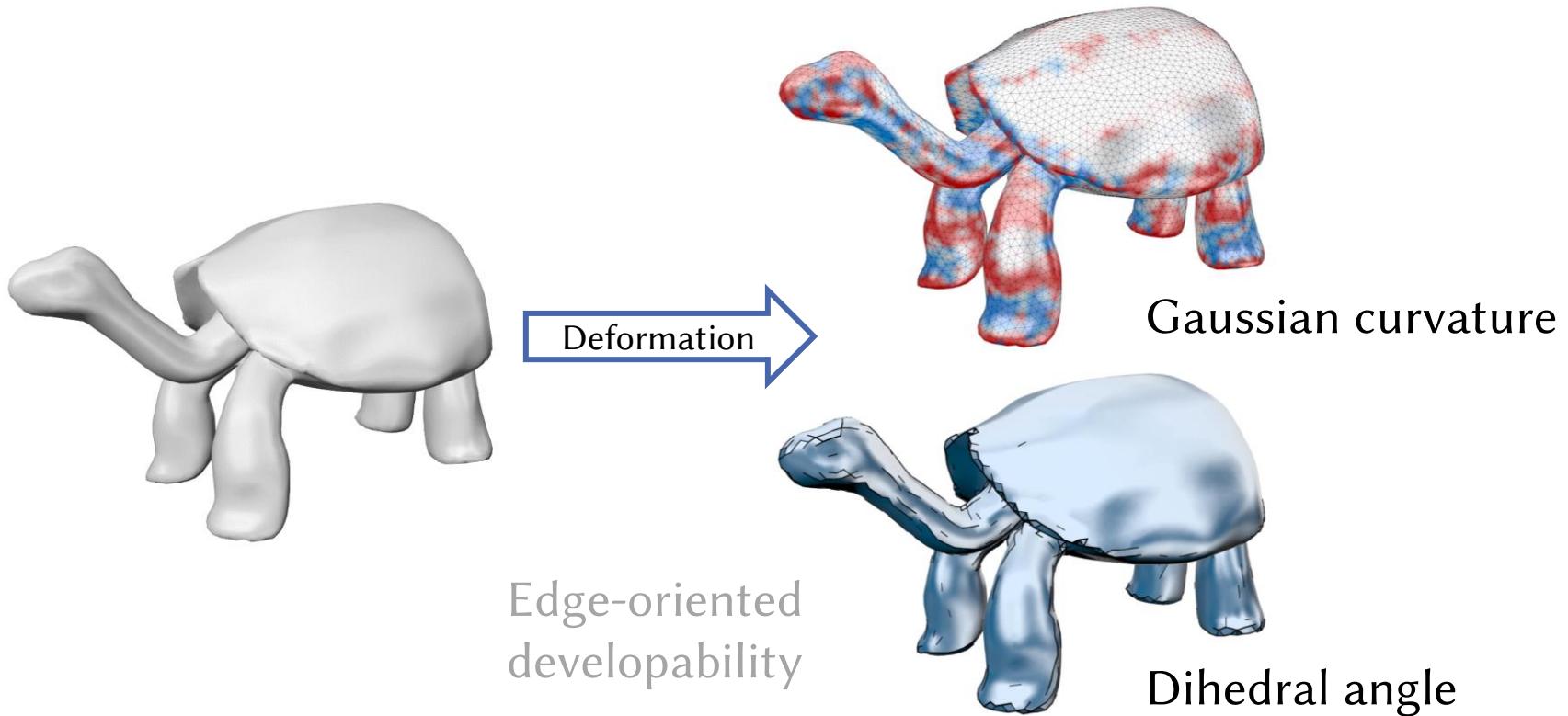
Pro: seam curves

Con: no explicit patch layouts

Key Idea & Method Overview



Method – Deformation



Edge-oriented Developability



Developable surface



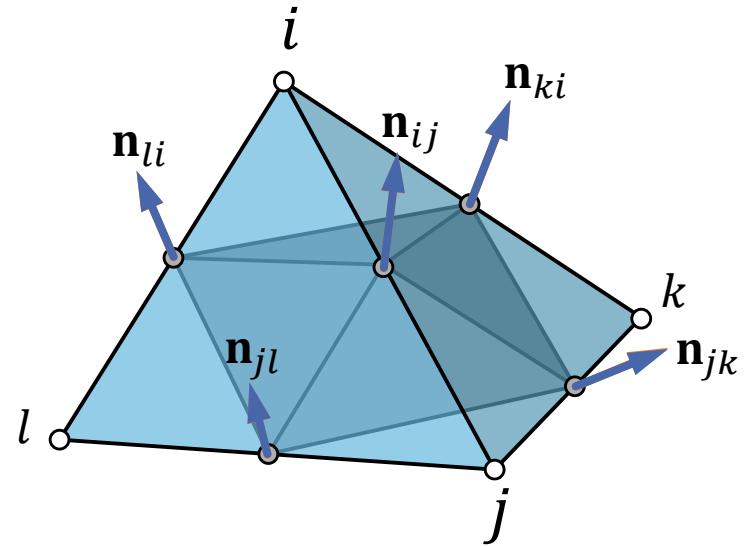
A set of curves

$$\det(\mathbf{n}_{ij}, \mathbf{n}_{jl}, \mathbf{n}_{jk}) = 0$$

$$\det(\mathbf{n}_{ij}, \mathbf{n}_{jk}, \mathbf{n}_{ki}) = 0$$

$$\det(\mathbf{n}_{ij}, \mathbf{n}_{ki}, \mathbf{n}_{li}) = 0$$

$$\det(\mathbf{n}_{ij}, \mathbf{n}_{li}, \mathbf{n}_{jl}) = 0$$

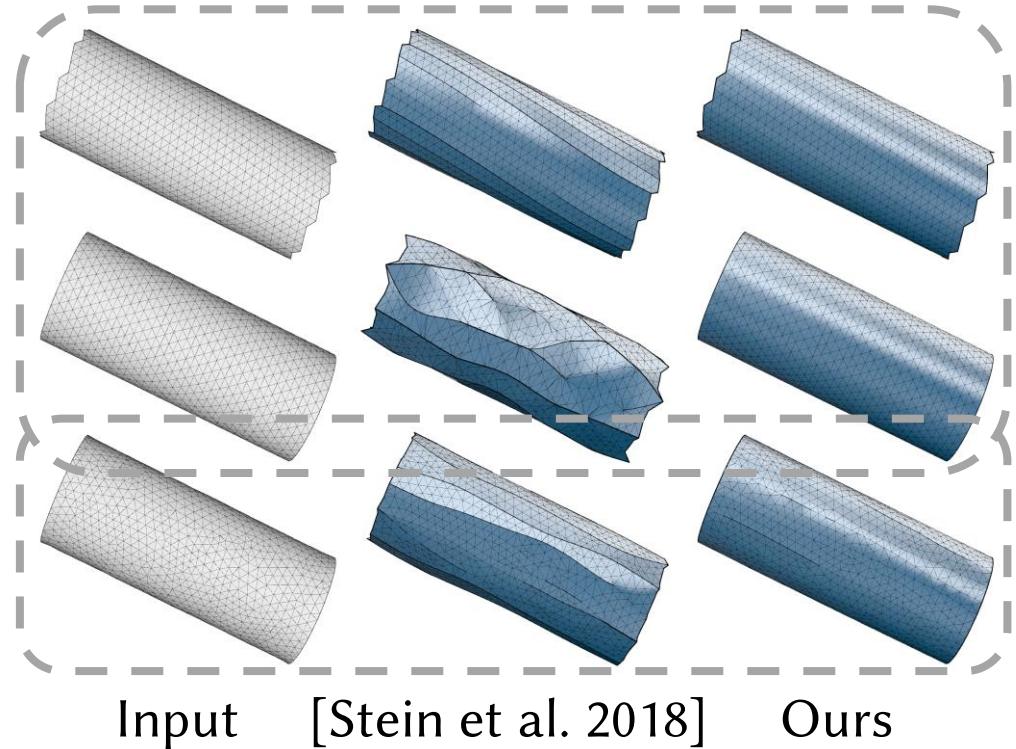


The Gauss map degenerates if the determinant condition is satisfied

Edge-oriented Developability



- Weaker than the definition in [Stein et al. 2018]
- Some special examples satisfy our edge-oriented definition directly



Deformation Optimization



$$\min_{\mathcal{V}_d} \quad \omega_{\text{dev}} E_{\text{dev}} + \omega_{\text{app}} E_{\text{app}} + \omega_{\text{dis}} E_{\text{dis}}$$

Developability Approximation Distortion

Challenges:

- Highly nonlinear
- Hard to minimize

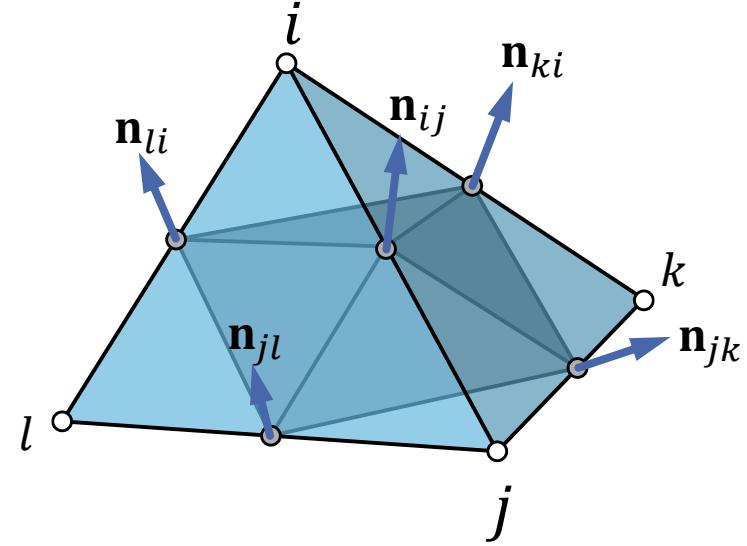


Auxiliary variables

Deformation Energy – Developability



$$E_{\text{dev}} = \sum_{ij \in \mathcal{E}} \left(\| [\mathbf{n}_{ij}, \mathbf{n}_{ki}, \mathbf{n}_{il}] - [P_r([\mathbf{n}_{ij}, \mathbf{n}_{ki}, \mathbf{n}_{il}])] \|_F^2 + \| [\mathbf{n}_{ij}, \mathbf{n}_{jk}, \mathbf{n}_{lj}] - [P_r([\mathbf{n}_{ij}, \mathbf{n}_{jk}, \mathbf{n}_{lj}])] \|_F^2 \right) + \sum_{ijk \in \mathcal{F}} \| [\mathbf{n}_{ij}, \mathbf{n}_{jk}, \mathbf{n}_{ki}] - [P_r([\mathbf{n}_{ij}, \mathbf{n}_{jk}, \mathbf{n}_{ki}])] \|_F^2$$

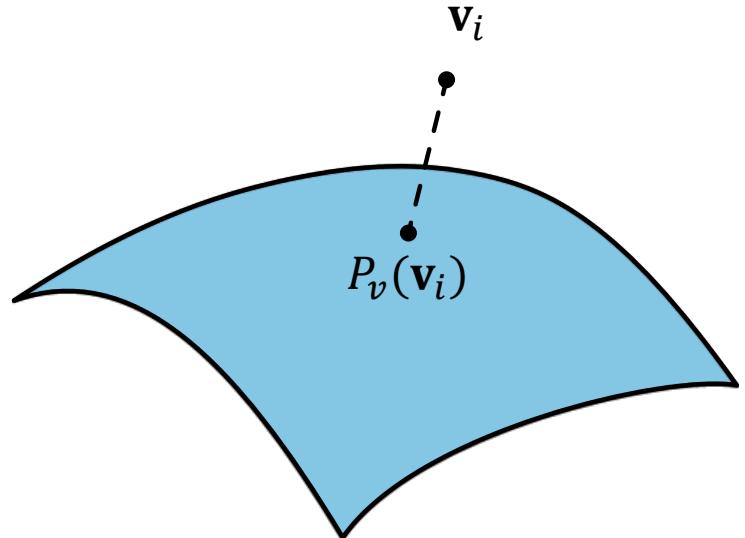


The projection to the
rank-two matrix space

Deformation Energy – Approximation

$$E_{\text{app}} = \sum_{i \in \mathcal{V}} \|v_i - \underbrace{P_v(v_i)}_{}^{}\|_2^2$$

The projection to
the input mesh



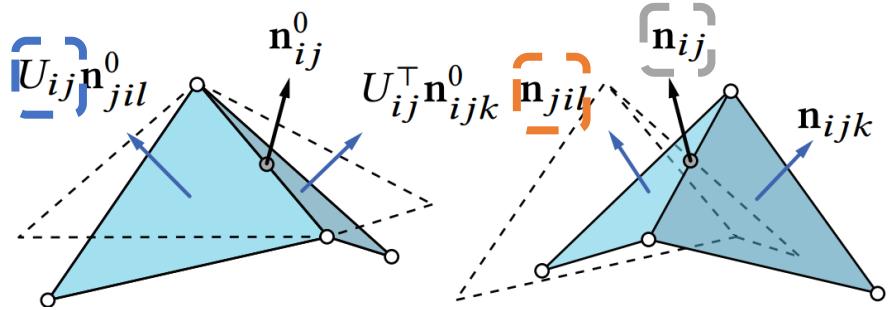
Deformation Energy – Distortion



$$E_{\text{dis}} = E_{\text{shape}} + \omega_{\text{sim}} E_{\text{sim}}$$

$$E_{\text{shape}} = \frac{1}{2} \sum_{ij \in \mathcal{E}} \| \mathbf{e}_{ij} - s_{ij} R_{ij} \mathbf{e}_{ij}^0 \|_2^2 + \\ \| \mathbf{e}_{ij}^0 \|_2^2 \left(\| \mathbf{n}_{ijk} - R_{ij} U_{ij} \mathbf{n}_{ijk}^0 \|_2^2 + \| \mathbf{n}_{jil} - R_{ij} U_{ij}^\top \mathbf{n}_{jil}^0 \|_2^2 \right)$$

$$E_{\text{sim}} = \frac{1}{2} \sum_{ijk \in \mathcal{F}} (s_{ij} - s_{jk})^2 + (s_{jk} - s_{ki})^2 + (s_{ki} - s_{ij})^2$$



$$\mathbf{n}_{ij} = \underline{R}_{ij} \mathbf{n}_{ij}^0$$

$$\underline{\mathbf{n}}_{ijk} = R_{ij} \underline{U}_{ij} \mathbf{n}_{ijk}^0$$

Scaling s_{ij}

Deformation optimization



repeat

$P_r(\cdot) \leftarrow \text{UpdatePr}(A), \forall A \in \mathcal{A}_{ij};$

$R_{ij} \leftarrow \text{UpdateR}(\mathbf{v}_i, s_{ij}, U_{ij}, \mathbf{n}_{ijk}, P_r(\cdot));$

$U_{ij} \leftarrow \text{UpdateU}(R_{ij}, \mathbf{n}_{ijk});$

$P_v(\cdot) \leftarrow \text{UpdatePv}(\mathbf{v}_i);$

$\mathbf{v}_i \leftarrow \text{UpdateV}(s_{ij}, R_{ij}, P_v(\cdot));$

$\mathbf{n}_{ijk} \leftarrow \text{UpdateN}(U_{ij}, R_{ij});$

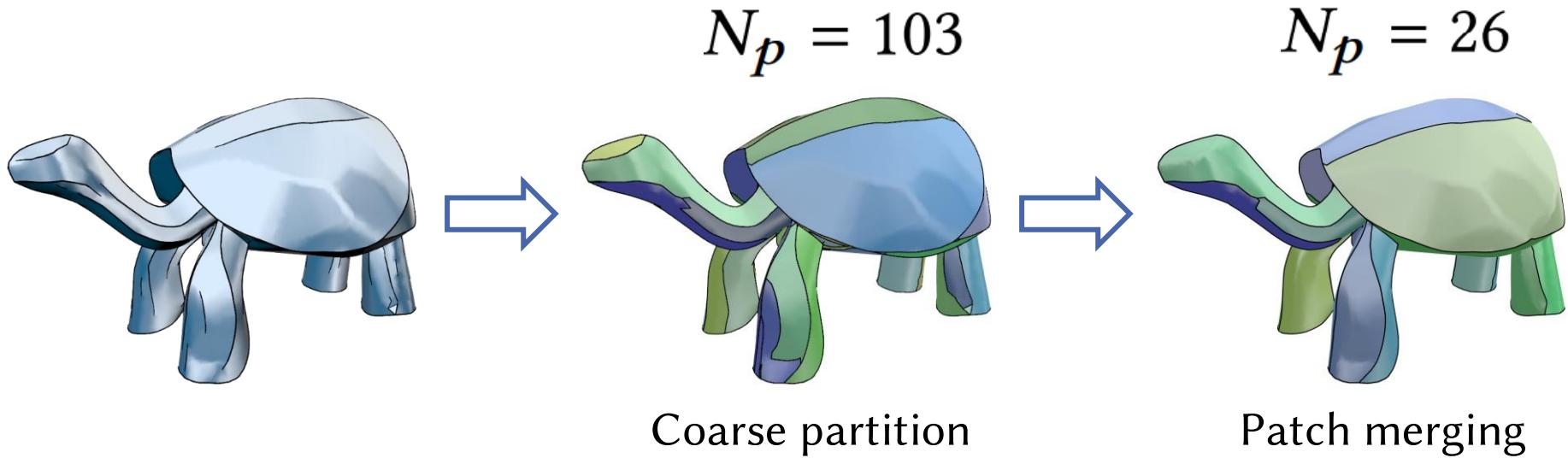
$s_{ij} \leftarrow \text{UpdateS}(\mathbf{v}_i, R_{ij});$

$k \leftarrow k + 1;$

until $k < N;$

Block nonlinear
Gauss–Seidel method

Method – Segmentation



Segmentation – Coarse partition



$$N_p = 2$$



Segmentation algorithm
Step-by-step
in [Tong et al. 2020]

Segmentation – Patch merging



Minimum area P_{\min}

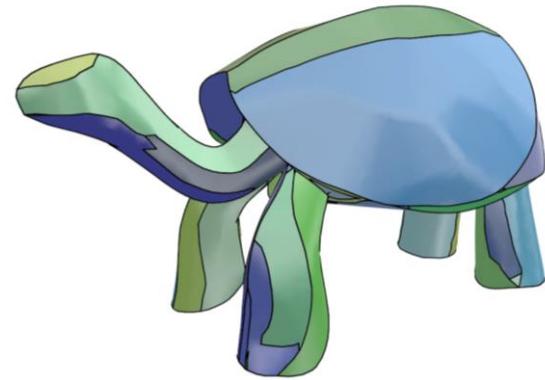
$$N_p = 103$$

$P = \arg \min_{P \in \mathcal{P}_{\min}} \text{Str}(P, P_{\min})$ Straightness

$\text{Len}(P, P_{\min}) > \epsilon_{\text{len}}$ Length

$\text{Cur}(P, P_{\min}) < \epsilon_{\text{cur}}$ Curvature

Iterative strategy



Method – Refinement



- Small approximation error
- Smooth patches
- Smooth seam curves



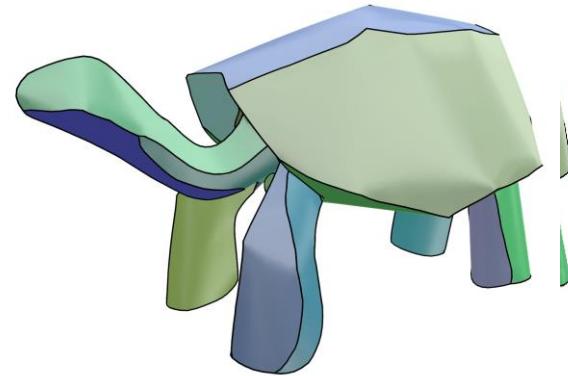
Method – Refinement



Method in [Ion et al. 2020]:

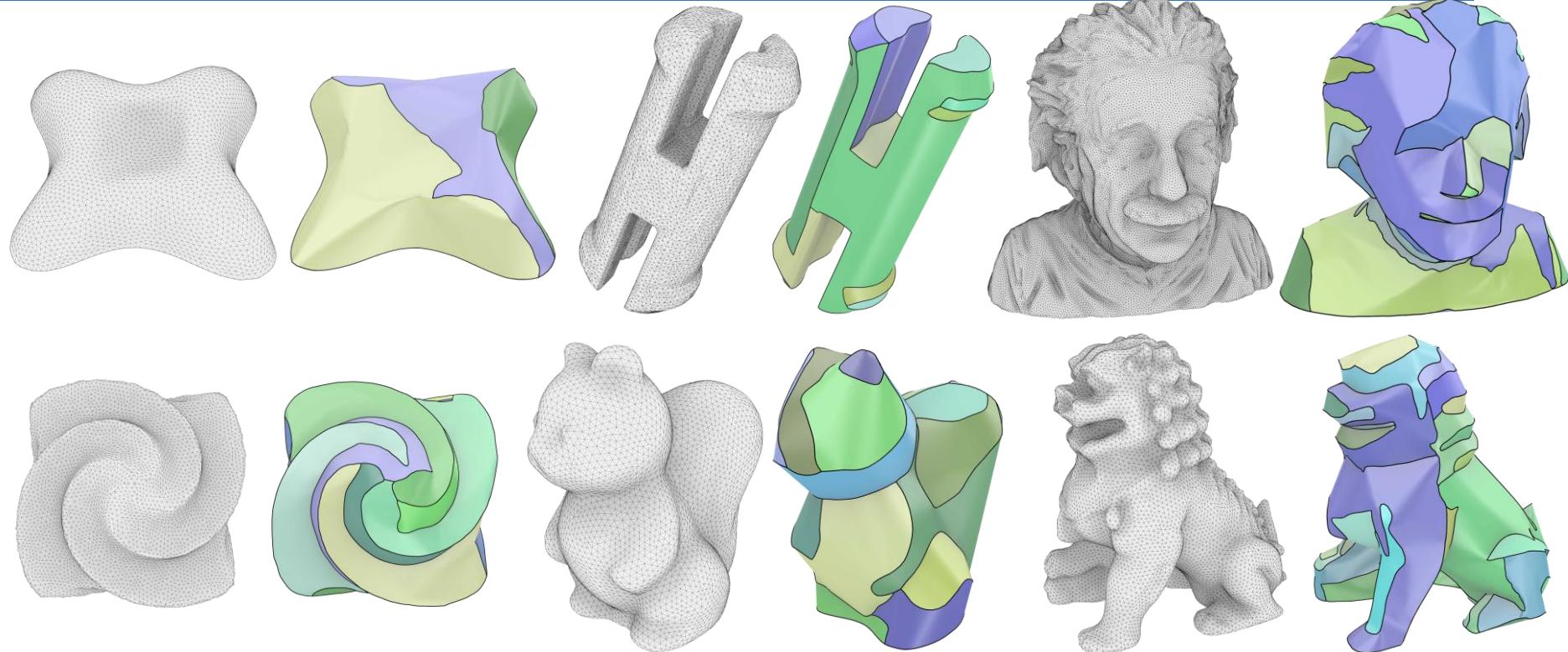
- DOG projection
- Developability
- Smoothness
- Seam smoothness
- Connectedness

The distance to \mathcal{M}_d

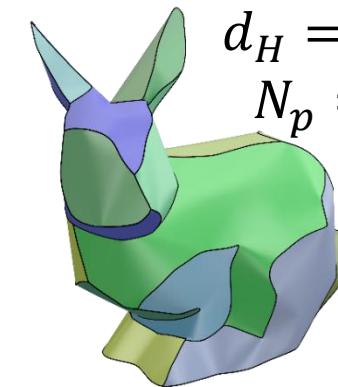
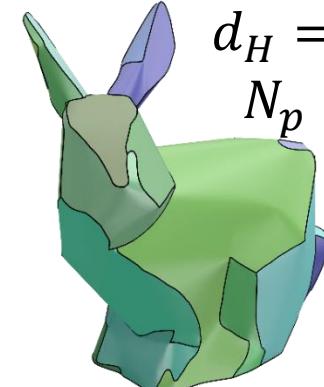
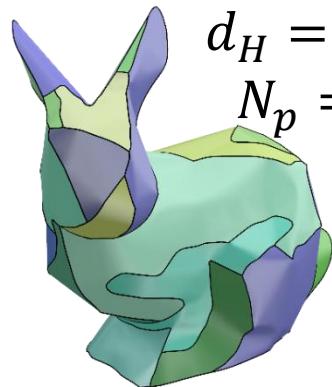
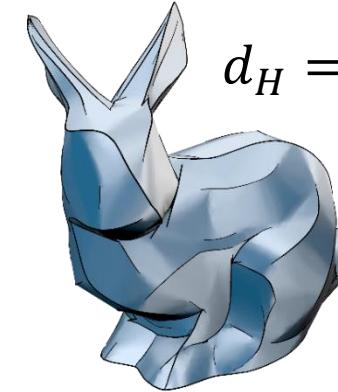
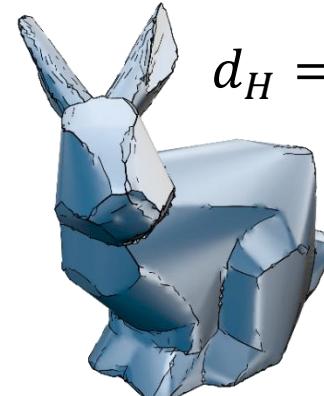
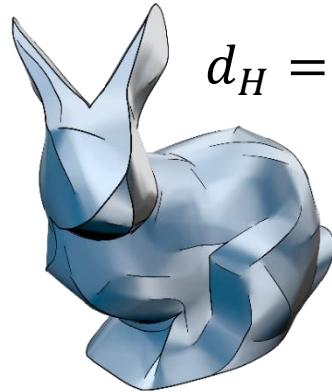


Cut overlaps

More Examples



Comparisons



[Stein et al. 2018]

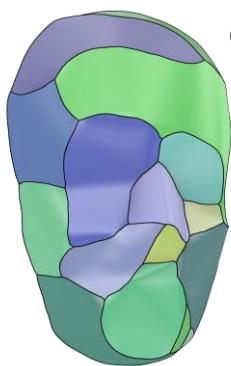
[Binninger et al. 2021]

Ours

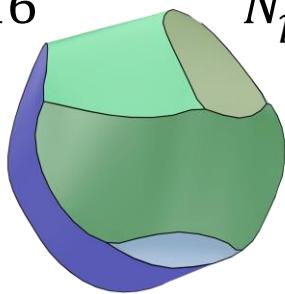
Comparisons



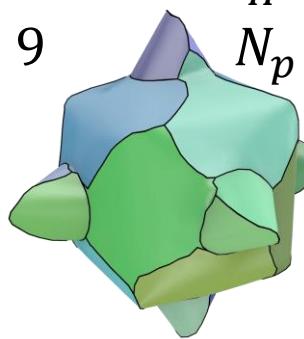
[Ion et al.
2020]



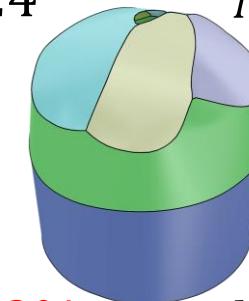
$$d_H = 2.2\% \\ N_p = 16$$



$$d_H = 4.0\% \\ N_p = 9$$

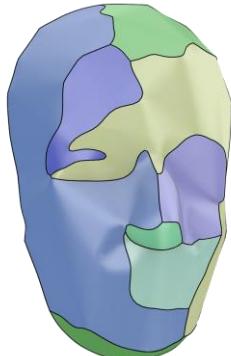


$$d_H = 1.8\% \\ N_p = 24$$

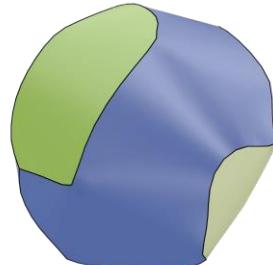


$$d_H = 1.8\% \\ N_p = 6$$

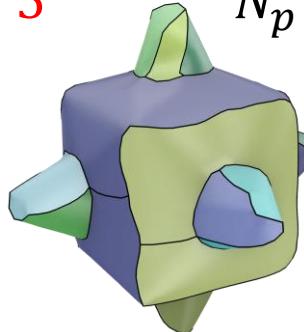
Ours



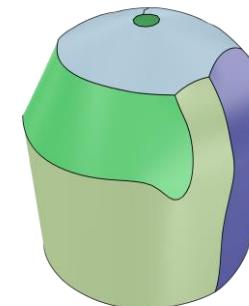
$$d_H = 1.4\% \\ N_p = 8$$



$$d_H = 2.6\% \\ N_p = 5$$



$$d_H = 1.2\% \\ N_p = 20$$



$$d_H = 1.5\% \\ N_p = 6$$

Fabrication



Fabrication



中国科学技术大学
University of Science and Technology of China

谢 谢 !

