

从零开始手写 VIO - 作业 6

peng00bo00

August 16, 2020

1. 由于矩阵 $D^T D$ 的奇异值向量构成一组正交基，可以将向量 y 分解为奇异值向量的线性组合：

$$y = k_1 u_1 + k_2 u_2 + k_3 u_3 + k_4 u_4 = U k \quad (1)$$

因此原优化问题可以转换为如下的优化问题：

$$\min (U k)^T D^T D (U k), \text{ s.t. } k^T k = 1 \quad (2)$$

其中

$$\begin{aligned} (U k)^T D^T D (U k) &= (U k)^T \sum_{i=1}^4 \sigma_i^2 u_i u_i^T (U k) \\ &= \sum_{i=1}^4 k_i^2 \sigma_i^2 \end{aligned} \quad (3)$$

故优化问题等价于如下的约束优化问题

$$\min \sum_{i=1}^4 k_i^2 \sigma_i^2, \text{ s.t. } \sum_{i=1}^4 k_i^2 = 1 \quad (4)$$

使用拉格朗日乘子法进行求解

$$J = \sum_{i=1}^4 k_i^2 \sigma_i^2 + \lambda (\sum_{i=1}^4 k_i^2 - 1) \quad (5)$$

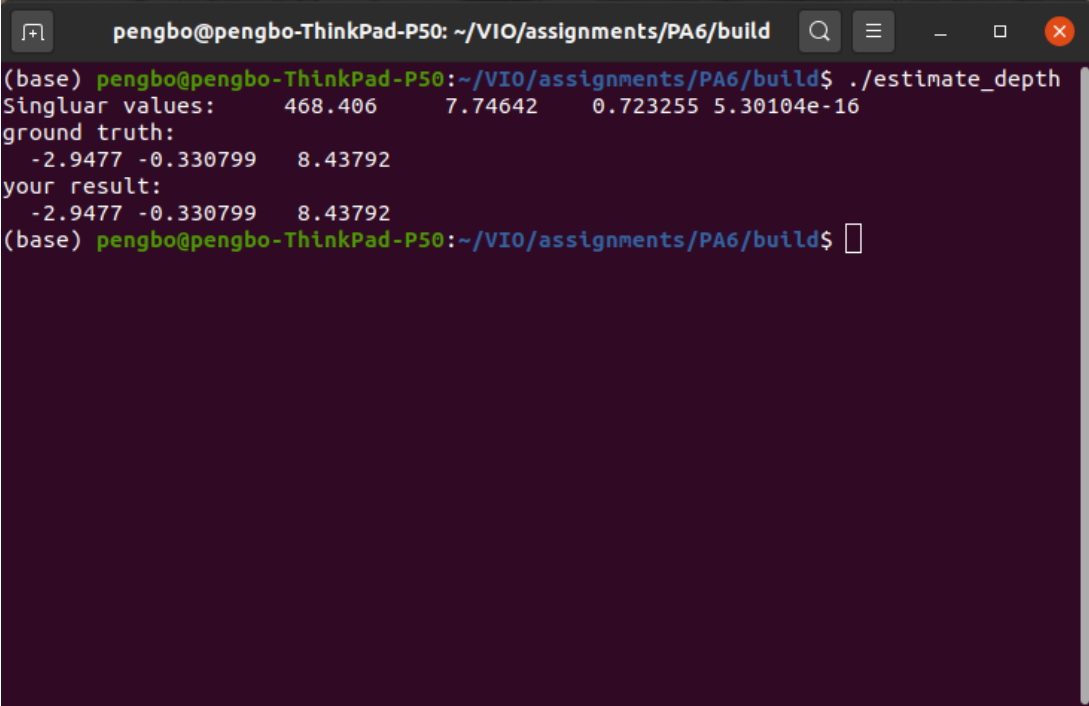
令

$$\begin{aligned} \frac{\partial J}{\partial k_i} &= 2k_i \sigma_i^2 + 2k_i \lambda = 0 \\ \frac{\partial J}{\partial \lambda} &= \sum_{i=1}^4 k_i^2 - 1 = 0 \end{aligned} \quad (6)$$

求解得到 4 个极值点，其中最小值点为

$$k_4 = 1, k_1 = k_2 = k_3 = 0 \quad (7)$$

2. 补全代码后得到三角化程序如 Fig.1所示，从中可以发现通过三角化可以求解到空间点的坐标。

A terminal window titled 'pengbo@pengbo-ThinkPad-P50: ~/VIO/assignments/PA6/build'. The prompt is '(base) pengbo@pengbo-ThinkPad-P50:~/VIO/assignments/PA6/build\$'. The user has entered './estimate_depth'. The output shows 'Singular values:' followed by four numbers: 468.406, 7.74642, 0.723255, and 5.30104e-16. Then it shows 'ground truth:' followed by three numbers: -2.9477, -0.330799, and 8.43792. Finally, it shows 'your result:' followed by the same three numbers: -2.9477, -0.330799, and 8.43792. The prompt is now '(base) pengbo@pengbo-ThinkPad-P50:~/VIO/assignments/PA6/build\$' with a cursor.

```
(base) pengbo@pengbo-ThinkPad-P50:~/VIO/assignments/PA6/build$ ./estimate_depth
Singular values:      468.406      7.74642      0.723255 5.30104e-16
ground truth:
-2.9477 -0.330799      8.43792
your result:
-2.9477 -0.330799      8.43792
(base) pengbo@pengbo-ThinkPad-P50:~/VIO/assignments/PA6/build$
```

Figure 1: 三角化程序截图

3. 给观测值添加正态分布噪声后绘制噪声方差与奇异值比值曲线如 Fig.2所示，可以发现随着噪声方差的增大奇异值的比值会增大，表明噪声的存在会影响到矩阵的零空间。

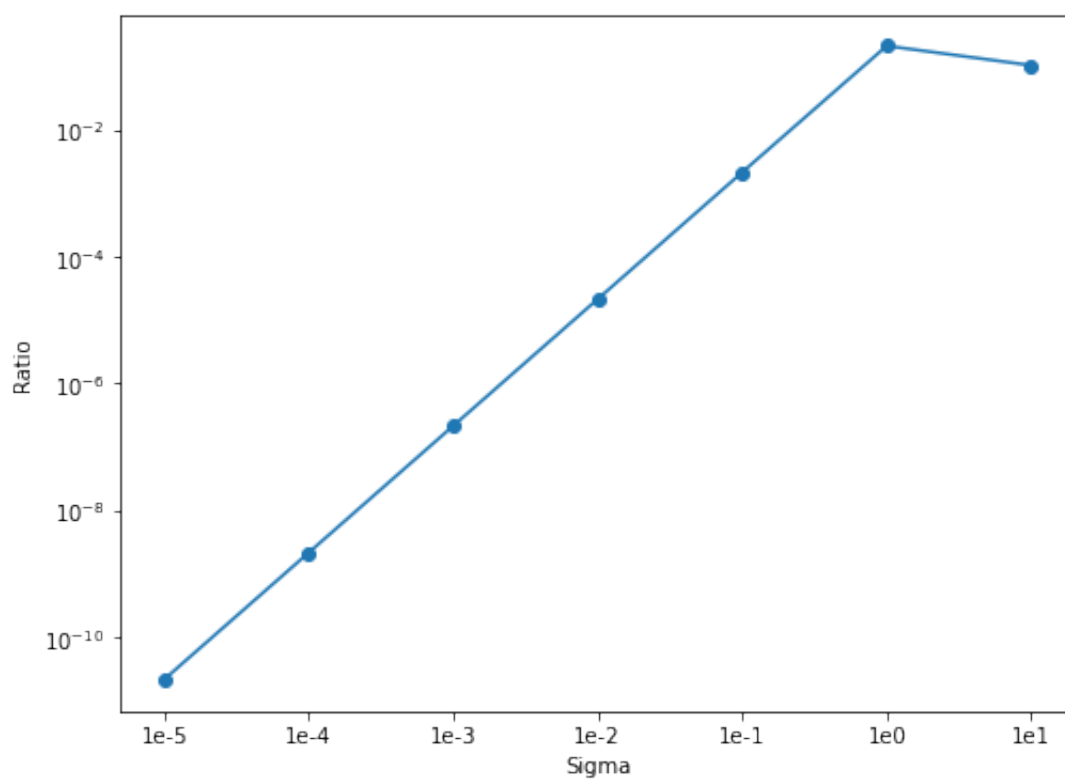


Figure 2: 方差-奇异值比值曲线

4. 固定噪声大小调整帧数后绘制帧数与奇异值比值曲线如 Fig.3所示，可以发现随着帧数的增大奇异值的比值会减小，表明增加观测数有利于减少观测噪声的影响。

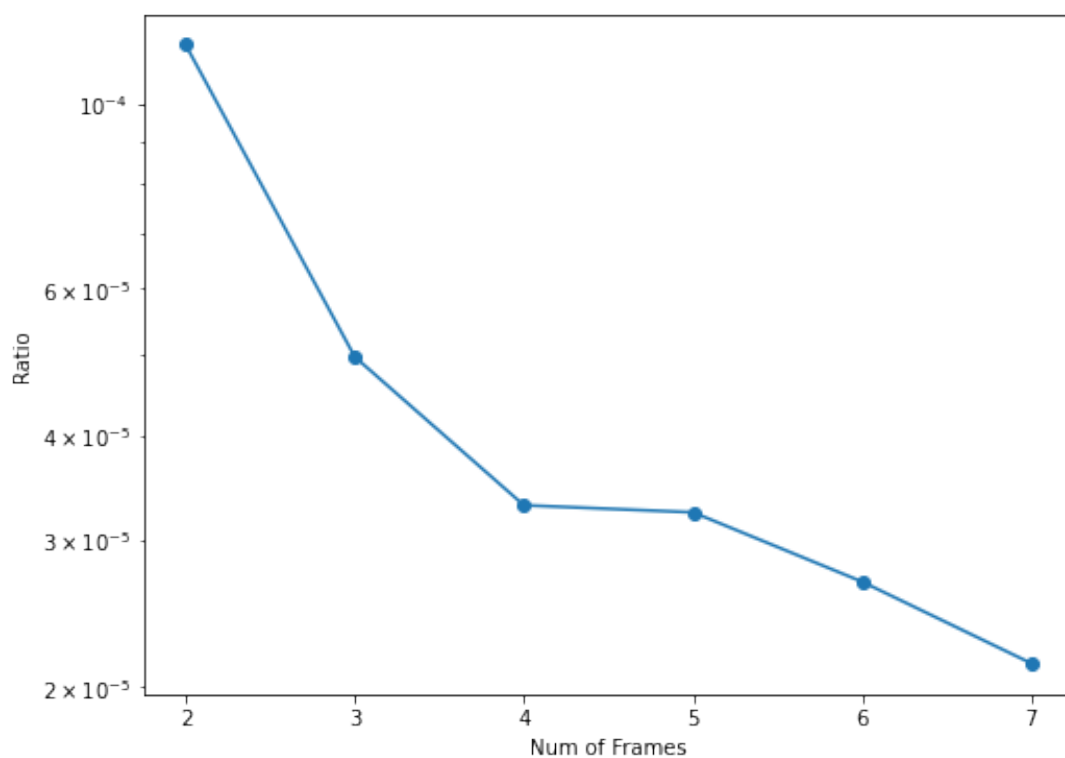


Figure 3: 帧数-奇异值比值曲线