

从零开始手写 VIO - 作业 4

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1. (a) 系统的信息矩阵如 Fig.1所示

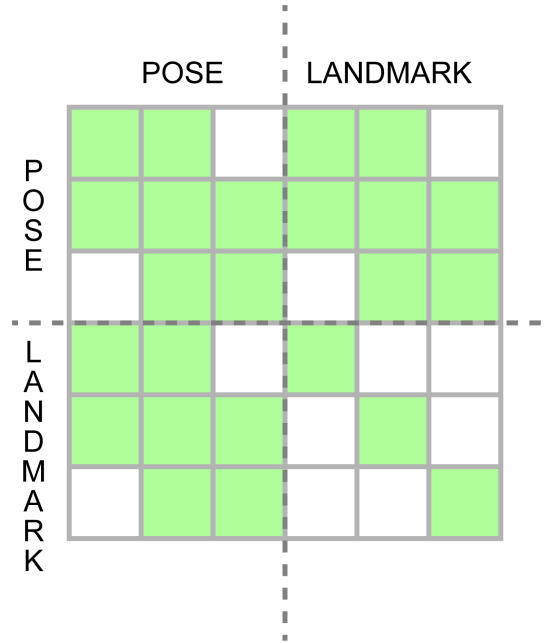


Figure 1: 系统信息矩阵

- (b) 边缘化后的系统信息矩阵如 Fig.2所示

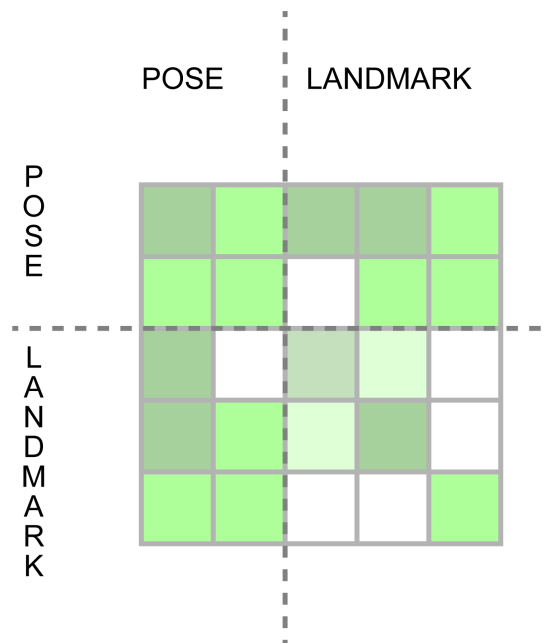


Figure 2: 边缘化

2. 假设随机变量 θ 服从多元正态分布 $\theta \sim N(\theta^*, \Sigma)$, 则其概率密度函数为

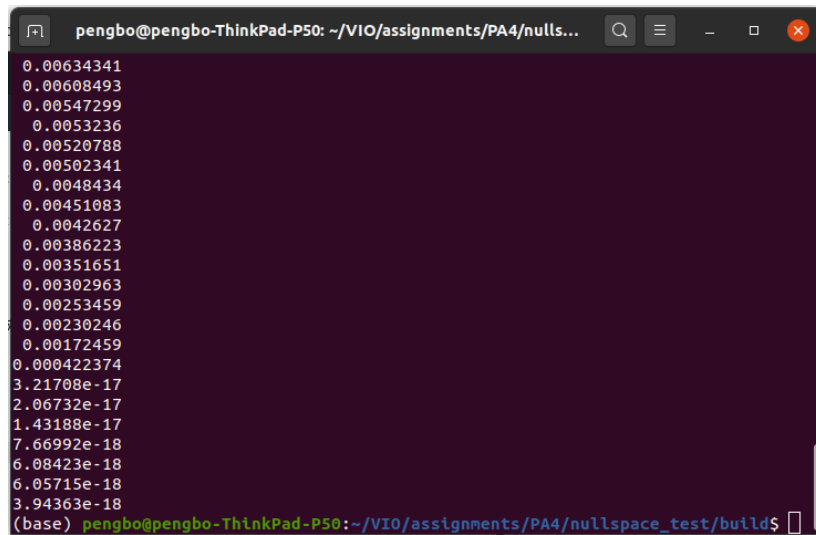
$$p(\theta) = (2\pi)^{-\frac{N_\theta}{2}} |\Sigma|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\theta - \theta^*)^T \Sigma^{-1} (\theta - \theta^*) \right\} \quad (1)$$

取负对数后则为

$$\begin{aligned} -\ln p(\theta) &= \frac{N_\theta}{2} \ln(2\pi) + \frac{1}{2} |\Sigma| + \frac{1}{2} (\theta - \theta^*)^T \Sigma^{-1} (\theta - \theta^*) \\ &= \frac{1}{2} (\theta - \theta^*)^T \Sigma^{-1} (\theta - \theta^*) + C \end{aligned} \quad (2)$$

因此概率密度函数的负对数为 $\theta - \theta^*$ 的二次型加上一个常数 C , 且二次型的系数矩阵为协方差矩阵的逆 Σ^{-1} 。此时最大化概率等价于求解最小二乘问题, 由此可知 θ 的信息矩阵即为其协方差矩阵的逆, 且信息矩阵也是二次型的二阶导数 (Hessian 矩阵)。

3. 补全代码后运行程序如 Fig.3所示，结果显示 Hessian 矩阵的奇异值最后 7 维接近于 0, 表明零空间的维度为 7

A terminal window with a dark purple background and white text. The window title is 'pengbo@pengbo-ThinkPad-P50: ~/VIO/assignments/PA4/nulls...'. It displays a list of 24 singular values of a Hessian matrix. The first 17 values are relatively large, ranging from approximately 0.000422374 to 0.00634341. The last 7 values are significantly smaller, in scientific notation, ranging from approximately 3.94363e-18 to 3.21708e-17. The prompt '(base) pengbo@pengbo-ThinkPad-P50:~/VIO/assignments/PA4/nullspace_test/build\$' is visible at the bottom.

```
0.00634341
0.00608493
0.00547299
0.0053236
0.00520788
0.00502341
0.0048434
0.00451083
0.0042627
0.00386223
0.00351651
0.00302963
0.00253459
0.00230246
0.00172459
0.000422374
3.21708e-17
2.06732e-17
1.43188e-17
7.66992e-18
6.08423e-18
6.05715e-18
3.94363e-18
(base) pengbo@pengbo-ThinkPad-P50:~/VIO/assignments/PA4/nullspace_test/build$
```

Figure 3: Hessian 矩阵奇异值