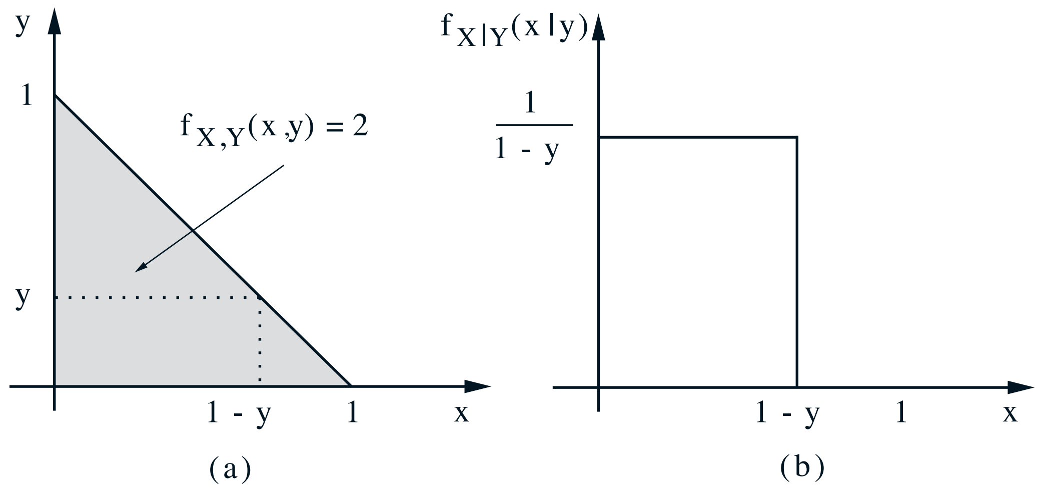

a) The area of the triangle is 1/2, so that $f_{X,Y}(x, y) = 1/2$, on the triangle indicated in Fig. (a), and zero everywhere else.

(b) We have

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)\, dx = \int_0^{1-y} 2\, dx = 2(1 - y), \qquad 0 \le y \le 1.$$

(c) We have

$$f_{X|Y}(x \mid y) = \frac{f_{X,Y}(x, y)}{f_Y(y)} = \frac{1}{1 - y}, \qquad 0 \le x \le 1 - y.$$

The conditional density is shown in the figure.

Intuitively, since the joint PDF is constant, the conditional PDF (which is a "slice" of the joint, at some fixed y) is also constant. Therefore, the conditional PDF must be a uniform distribution. Given that $Y = y$, X ranges from 0 to $1 - y$. Therefore, for the PDF to integrate to 1, its height must be equal to $1/(1 - y)$, in agreement with the figure.

(d) For $y > 1$ or $y < 0$, the conditional PDF is undefined, since these values of y are impossible. For $0 \le y < 1$, the conditional mean $\mathbf{E}[X \mid Y = y]$ is obtained using the uniform PDF in Fig. (b), and we have

$$\mathbf{E}[X \mid Y = y] = \frac{1 - y}{2}, \qquad 0 \le y < 1.$$

For $y = 1$, X must be equal to 0, with certainty, so $\mathbf{E}[X \mid Y = 1] = 0$. Thus, the above formula is also valid when $y = 1$. The conditional expectation is undefined when y is outside $[0, 1]$.

The total expectation theorem yields

$$\mathbf{E}[X] = \int_0^1 \frac{1-y}{2} f_Y(y)\, dy = \frac{1}{2} - \frac{1}{2}\int_0^1 y f_Y(y)\, dy = \frac{1 - \mathbf{E}[Y]}{2}.$$

(e) Because of symmetry, we must have $\mathbf{E}[X] = \mathbf{E}[Y]$. Therefore, $\mathbf{E}[X] = \big((1 - \mathbf{E}[X])\big)/2$, which yields $\mathbf{E}[X] = 1/3$.