

Let A be the event that Al will find a taxi waiting or will be picked up by the bus after 5 minutes. Note that the probability of boarding the next bus, given that Al has to wait, is

$$\mathbf{P}(\text{a taxi will take more than 5 minutes to arrive}) = \frac{1}{2}.$$

Al's waiting time, call it X , is a mixed random variable. With probability

$$\mathbf{P}(A) = \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{2} = \frac{5}{6},$$

it is equal to its discrete component Y (corresponding to either finding a taxi waiting, or boarding the bus), which has PMF

$$\begin{aligned} p_Y(y) &= \begin{cases} \frac{2}{3\mathbf{P}(A)}, & \text{if } y = 0, \\ \frac{1}{6\mathbf{P}(A)}, & \text{if } y = 5, \end{cases} \\ &= \begin{cases} \frac{12}{15}, & \text{if } y = 0, \\ \frac{3}{15}, & \text{if } y = 5. \end{cases} \end{aligned}$$

[This equation follows from the calculation

$$p_Y(0) = \mathbf{P}(Y = 0 \mid A) = \frac{\mathbf{P}(Y = 0, A)}{\mathbf{P}(A)} = \frac{2}{3\mathbf{P}(A)}.$$

The calculation for $p_Y(5)$ is similar.] With the complementary probability $1 - \mathbf{P}(A)$, the waiting time is equal to its continuous component Z (corresponding to boarding a taxi after having to wait for some time less than 5 minutes), which has PDF

$$f_Z(z) = \begin{cases} 1/5, & \text{if } 0 \leq z \leq 5, \\ 0, & \text{otherwise.} \end{cases}$$

The CDF is given by $F_X(x) = \mathbf{P}(A)F_Y(x) + (1 - \mathbf{P}(A))F_Z(x)$, from which

$$F_X(x) = \begin{cases} 0, & \text{if } x < 0, \\ \frac{5}{6} \cdot \frac{12}{15} + \frac{1}{6} \cdot \frac{x}{5}, & \text{if } 0 \leq x < 5, \\ 1, & \text{if } 5 \leq x. \end{cases}$$

The expected value of the waiting time is

$$\mathbf{E}[X] = \mathbf{P}(A)\mathbf{E}[Y] + (1 - \mathbf{P}(A))\mathbf{E}[Z] = \frac{5}{6} \cdot \frac{3}{15} \cdot 5 + \frac{1}{6} \cdot \frac{5}{2} = \frac{15}{12}.$$