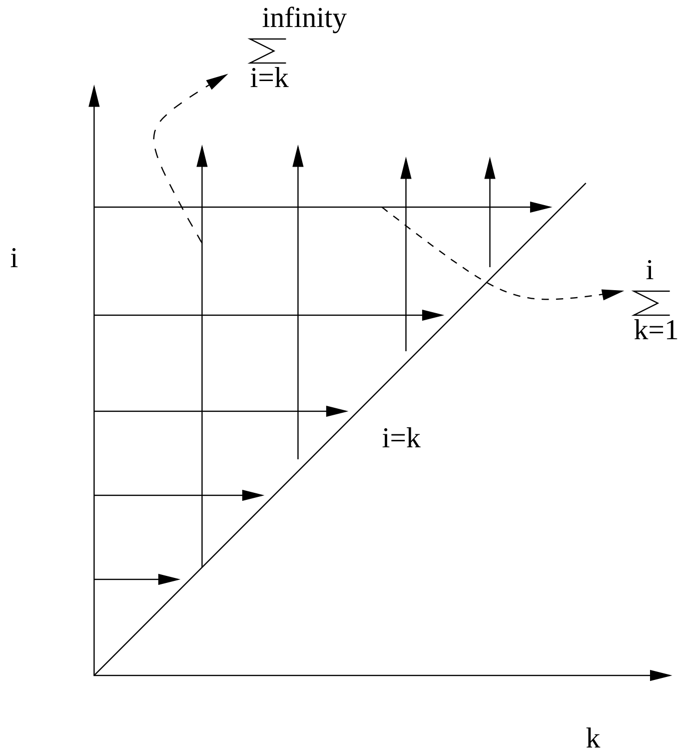
1. The picture below illustrates the double sum needed to prove the statement of this problem:



We first note that

$$\mathbf{P}(X \geq k) = \sum_{i=k}^{\infty} p_X(i)$$

and proceed as follows:

$$\sum_{k=1}^{\infty} \mathbf{P}(X \geq k) = \sum_{k=1}^{\infty} \sum_{i=k}^{\infty} p_X(i) = \sum_{i=1}^{\infty} \sum_{k=1}^{i} p_X(i) = \sum_{i=1}^{\infty} i\, p_X(i) = \mathbf{E}[X].$$

2. We first compute

$$\mathbf{P}(Y \geq k) = \begin{cases} 1, & k \leq a, \\ \frac{b-k+1}{b-a+1}, & a+1 \leq k \leq b, \\ 0, & k \geq b+1. \end{cases}$$

So

$$
\begin{aligned}
\sum_{k=1}^{\infty} \mathbf{P}(Y \geq k) &= \sum_{k=1}^{a} 1 + \sum_{k=a+1}^{b} \frac{b-k+1}{b-a+1} \\
&= a + \frac{1}{b-a+1} \sum_{k=1}^{b-a} k \\
&= a + \frac{1}{b-a+1} \frac{(b-a+1)(b-a)}{2} \\
&= a + \frac{b-a}{2} \\
&= \frac{b+a}{2}.
\end{aligned}
$$

Therefore $\mathbf{E}[Y] = \frac{b+a}{2}$.