(a) We have

$$\mathbf{E}[X] = \int_1^3 \frac{x^2}{4}\,dx = \frac{x^3}{12}\Big|_1^3 = \frac{27}{12} - \frac{1}{12} = \frac{26}{12} = \frac{13}{6},$$

$$\mathbf{P}(A) = \int_2^3 \frac{x}{4}\,dx = \frac{x^2}{8}\Big|_2^3 = \frac{9}{8} - \frac{4}{8} = \frac{5}{8}.$$

We also have

$$f_{X|A}(x) = \begin{cases} \dfrac{f_X(x)}{\mathbf{P}(A)}, & \text{if} \quad x \in A \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \dfrac{2x}{5}, & \text{if} \quad 2 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

from which we obtain

$$\mathbf{E}[X \mid A] = \int_2^3 x \cdot \frac{2x}{5}\,dx = \frac{2x^3}{15}\Big|_2^3 = \frac{54}{15} - \frac{16}{15} = \frac{38}{15}.$$

(b) We have

$$\mathbf{E}[Y] = \mathbf{E}[X^2] = \int_1^3 \frac{x^3}{4}\,dx = 5,$$

and

$$\mathbf{E}[Y^2] = \mathbf{E}[X^4] = \int_1^3 \frac{x^5}{4}\,dx = \frac{91}{3}.$$

Thus,

$$\mathrm{var}(Y) = \mathbf{E}[Y^2] - \big(\mathbf{E}[Y]\big)^2 = \frac{91}{3} - 5^2 = \frac{16}{3}.$$