

1. The sum of the values of the PMF of a random variable over all values that it takes with positive probability must be equal to 1. Hence, we have

$$\begin{aligned}
 1 &= \sum_{x=-3}^3 p_X(x) \\
 &= \frac{9}{a} + \frac{4}{a} + \frac{1}{a} + \frac{1}{a} + \frac{4}{a} + \frac{9}{a} \\
 &= \frac{28}{a},
 \end{aligned}$$

which implies that $a = 28$.

2. The following table shows the value of Z for a given value of X and the probability of that event.

x	-3	-2	-1	1	2	3
$p_X(x)$	9/28	1/7	1/28	1/28	1/7	9/28
$Z X=x$	9	4	1	1	4	9

We see that Z can take only three possible values with non-zero probability, namely 1, 4, and 9. In addition, for each value, there correspond two values of X . So we have, for example, $p_Z(9) = \mathbf{P}(Z = 9) = \mathbf{P}(X = -3) + \mathbf{P}(X = 3) = p_X(-3) + p_X(3)$. Hence the PMF of Z is given by

$$p_Z(z) = \begin{cases} 1/14, & \text{if } z = 1, \\ 2/7, & \text{if } z = 4, \\ 9/14, & \text{if } z = 9, \\ 0, & \text{otherwise.} \end{cases}$$