Define coordinates such that the stick extends from position 0 (the left end) to position 1 (the right end). Denote the position of the first break by X and the position of the second break by Y. We have $X < Y$. We assume that $X < Y$ and we later account for the case $Y < X$ by using symmetry.

Under the assumption $X < Y$, the three pieces have lengths X, $Y - X$, and $1 - Y$. In order that they form a triangle, the sum of the lengths of any two pieces must exceed the length of the third piece. Thus they form a triangle if

$$X < (Y-X)+(1-Y), \qquad (Y-X) < X+(1-Y), \qquad (1-Y) < X+(Y-X).$$

These conditions simplify to

$$X < 0.5, \qquad Y > 0.5, \qquad Y - X < 0.5.$$

For X and Y to satisfy these conditions, the pair (X, Y) must lie within the triangle with vertices $(0, 0.5)$, $(0.5, 0.5)$, and $(0.5, 1)$. This triangle has area $1/8$. Thus the probability of the event that the three pieces form a triangle *and* $X < Y$ is $1/8$. By symmetry, the probability of the event that the three pieces form a triangle *and* $X > Y$ is $1/8$. Since there two events are disjoint and form a partition of the event that the three pieces form a triangle, the desired probability is $1/8 + 1/8 = 1/4$.