(a) For $x \geq 0$,

$$F_X(x) \;=\; \int_{-\infty}^{x} f_X(t)\,dt \;=\; \int_0^x \lambda e^{-\lambda t}\,dt \;=\; \left[-e^{-\lambda t}\right]_0^x \;=\; 1 - e^{-\lambda x}.$$

For $x < 0$, we have $F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt = 0$. Thus we conclude

$$F_X(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1 - e^{-\lambda x}, & \text{if } x \geq 0. \end{cases}$$

(b) The key step in the following computation uses integration by parts, whereby

$$\int_0^\infty u\,dv \;=\; uv\Big|_0^\infty - \int_0^\infty v\,du$$

is applied with $u = x$ and $v = -e^{-\lambda x}$:

$$\mathbf{E}[X] \;=\; \int_{-\infty}^{\infty} x f_X(x)\,dx \;=\; \int_0^\infty x\lambda e^{-\lambda x}\,dx \;=\; \left[-xe^{-\lambda x}\right]_0^\infty + \int_0^\infty e^{-\lambda x}\,dx \;=\; \frac{1}{\lambda}.$$

(c) Integrating by parts with $u = x^2$ and $v = -e^{-\lambda x}$ in the second line below gives

$$\begin{aligned} \mathbf{E}[X^2] \;&=\; \int_{-\infty}^{\infty} x^2 f_X(x)\,dx \;=\; \int_0^\infty x^2 \lambda e^{-\lambda x}\,dx \\ &=\; \left[-x^2 e^{-\lambda x}\right]_0^\infty + 2\int_0^\infty x e^{-\lambda x}\,dx \;=\; \frac{2}{\lambda}\mathbf{E}[X] \;=\; \frac{2}{\lambda^2}. \end{aligned}$$

Combining with the previous computation, we obtain

$$\mathrm{var}(X) \;=\; \mathbf{E}[X^2] - (\mathbf{E}[X])^2 \;=\; \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 \;=\; \frac{1}{\lambda^2}.$$

(d) The maximum of a set is upper bounded by z when each element of the set is upper bounded by z. Thus for any positive z,

$$\begin{aligned} \mathbf{P}(Z \leq z) \;&=\; \mathbf{P}(\max\{X_1, X_2, X_3\} \leq z) \;=\; \mathbf{P}(X_1 \leq z,\, X_2 \leq z,\, X_3 \leq z) \\ &=\; \mathbf{P}(X_1 \leq z)\,\mathbf{P}(X_2 \leq z)\,\mathbf{P}(X_3 \leq z) \\ &=\; (1 - e^{-\lambda z})^3, \end{aligned}$$

where the third equality uses the independence of X_1, X_2, and X_3. Thus,

$$F_Z(z) = \begin{cases} 0, & \text{if } z < 0, \\ (1 - e^{-\lambda z})^3, & \text{if } z \geq 0. \end{cases}$$

Differentiating the CDF gives the desired PDF:

$$f_Z(z) = \begin{cases} 0, & \text{if } z < 0, \\ 3\lambda e^{-\lambda z}(1 - e^{-\lambda z})^2, & \text{if } z \geq 0. \end{cases}$$

(e) The minimum of a set is lower bounded by w when each element of the set is lower bounded by w. Thus for any positive w,

$$\begin{aligned} \mathbf{P}(W \geq w) &= \mathbf{P}(\min\{X_1, X_2\} \geq w) = \mathbf{P}(X_1 \geq w,\, X_2 \geq w) \\ &= \mathbf{P}(X_1 \geq w)\, \mathbf{P}(X_2 \geq w) \\ &= (e^{-\lambda w})^2 = e^{-2\lambda w} \end{aligned}$$

where the third equality uses the independence of X_1 and X_2. Thus,

$$F_W(w) = \begin{cases} 0, & \text{if } w < 0, \\ 1 - e^{-2\lambda w}, & \text{if } w \geq 0. \end{cases}$$

We can recognize this as the CDF of an exponential random variable with parameter 2λ. The PDF is

$$f_W(w) = \begin{cases} 0, & \text{if } w < 0, \\ 2\lambda e^{-2\lambda w}, & \text{if } w \geq 0. \end{cases}$$