

# hw5

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## 1 A-4-4

### 1.1 (a)

Considering the log returns of KO stock, set  $r_t = 100 \log(x_t + 1)$ , where  $x_t$  is the simple return. Using Tgarch11.R script, we obtain the fitted TGARCH(1,1) as

$$r_t = 1.1575 + a_t, a_t = \sigma_t \epsilon_t, \epsilon_t \sim N(0, 1) \quad (1.1)$$

$$\sigma_t^2 = 3.0349 + (0.0488 + 0.0805N_{t-1})a_{t-1}^2 + 0.8233\sigma_{t-1}^2 \quad (1.2)$$

where the standard error of the parameter for the mean equation is 0.2284 and the standard errors of the parameters in the volatility equation are 1.052, 0.030, 0.044, and 0.038. To check the fitted model, we have  $Q(10) = 10.08(0.4335)$ ,  $Q(20) = 21.528(0.3666)$  for  $\tilde{a}_t$  the standardized residual and  $Q(10) = 10.399(0.4062)$ ,  $Q(20) = 12.734(0.8885)$  for  $\tilde{a}_t^2$ . So the model is adequate. To check the leverage effect, the r-ratio for  $\gamma$  is 1.795 with p-value is 0.0363. So the leverage effect is significant.

```
> da=read.table("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/m-ko-6111.txt",header=T)
> lre=log(da$ko+1)*100
> source("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/Tgarch11.R")
> m1=Tgarch11(lre)
```

Log likelihood at MLEs:

```
[1] -1933.312
```

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t )
mu	1.1574960	0.2283502	5.06895	4.0001e-07 ***
omega	3.0348902	1.0524445	2.88366	0.0039309 **
alpha	0.0488288	0.0301262	1.62081	0.1050587
gam1	0.0804693	0.0448361	1.79474	0.0726948 .
beta	0.8233339	0.0379452	21.69799	< 2.22e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```
> names(m1)
```

```

[1] "residuals" "volatility" "par"

> at=m1$residuals
> sigt=m1$volatility
> resi=at/sigt
> Box.test(resi,lag=10,type='Ljung')

Box-Ljung test

data: resi
X-squared = 10.08, df = 10, p-value = 0.4335
> Box.test(resi,lag=20,type='Ljung')

Box-Ljung test

data: resi
X-squared = 21.528, df = 20, p-value = 0.3666
> Box.test(resi^2,lag=10,type='Ljung')

Box-Ljung test

data: resi^2
X-squared = 10.399, df = 10, p-value = 0.4062
> Box.test(resi^2,lag=20,type='Ljung')

Box-Ljung test

data: resi^2
X-squared = 12.734, df = 20, p-value = 0.8885

```

## 1.2 (b)

Similarly, we could use Ngarch.R script to get the NGARCH(1,1) model,

$$r_t = 1.464 + a_t, a_t = \sigma_t \epsilon_t, \epsilon_t \sim N(0, 1) \quad (1.3)$$

$$\sigma_t^2 = 1.1550 + 0.8684\sigma_{t-1}^2 + 0.0978(a_{t-1} - 0.1114\sigma_{t-1})^2 \quad (1.4)$$

The Ljung-Box statistics of the standardized residuals ( $\tilde{a}_t$ ) and their squared series fail to reject the model. Since  $Q(10) = 11.103(0.3496)$  for  $\tilde{a}_t$  and  $Q(10) = 11.052(0.3535)$  for  $\tilde{a}_t^2$ .

```

> source("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/Ngarch.R")
> m2=Ngarch(lre)

```

Estimation results of NGARCH(1,1) model:

```

estimates:  1.464186 1.154998 0.86844 0.09782551 0.1113541
std.errors:  0.2236933 0.4166718 0.02274459 0.02149025 0.1596848
t-ratio:    6.545508 2.771962 38.18226 4.552088 0.6973369

```

```
> res=m2$residuals
> vol=m2$volatility
> resi=res/vol
> Box.test(resi,lag=10,type='Ljung')
```

Box-Ljung test

```
data:  resi
X-squared = 11.103, df = 10, p-value = 0.3496

> Box.test(resi^2,lag=10,type='Ljung')
```

Box-Ljung test

```
data:  resi^2
X-squared = 11.052, df = 10, p-value = 0.3535
```

## 2 A-4-5

### 2.1 (a)

For the log return, there is serial correlation. One way, we could find it through the ACF graph. We could see  $n=1$  or  $2$ , the value is significant.

```
> da=read.table("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/d-pg-0111.txt",header=T)
> lrt=log(da$rtn+1)
```

The other way is trying to use the Box test. The result is  $Q(12) = 54.704$ , with  $P\text{-value} = 2.045e - 07$ . So we have to reject the null hypothesis to accept it has serial correlation.

```
> Box.test(lrt,lag=12,type='Ljung')
```

Box-Ljung test

```
data:  lrt
X-squared = 54.704, df = 12, p-value = 2.045e-07
```

### 2.2 (b)

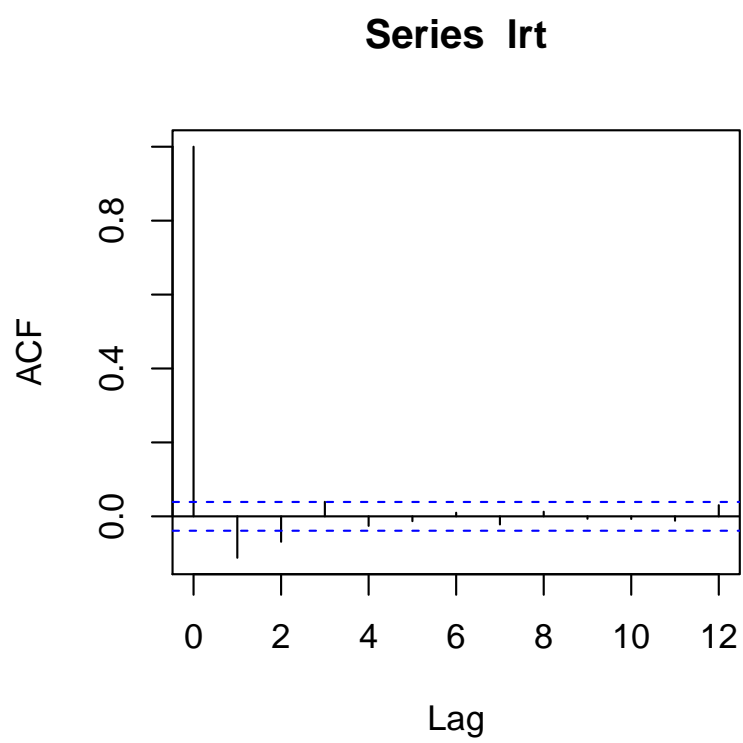
Through the Pacf graph, we are trying to AR(2) model. Setting  $y_t$  as the log return, the model is

$$(1 + 0.1205B + 0.0816B^2)y_t = a_t \quad (2.1)$$

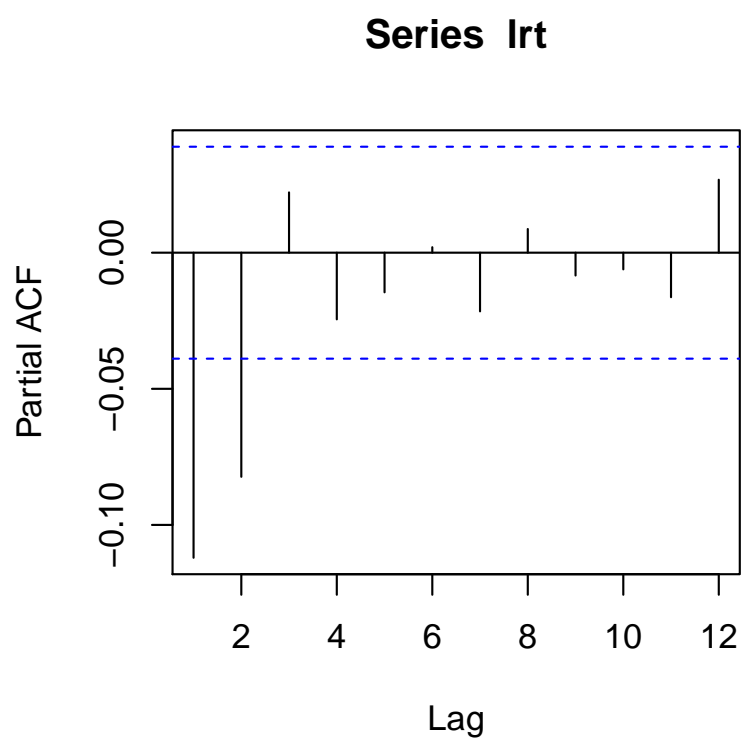
By checking the acf and pacf of residuals, we could make sure this model is adequate.

```
> m1=arima(lrt,order=c(2,0,0),include.mean=F)
> m1
```

```
> acf(lrt,lag=12)
```



```
> pacf(lrt,lag=12)
```



Call:

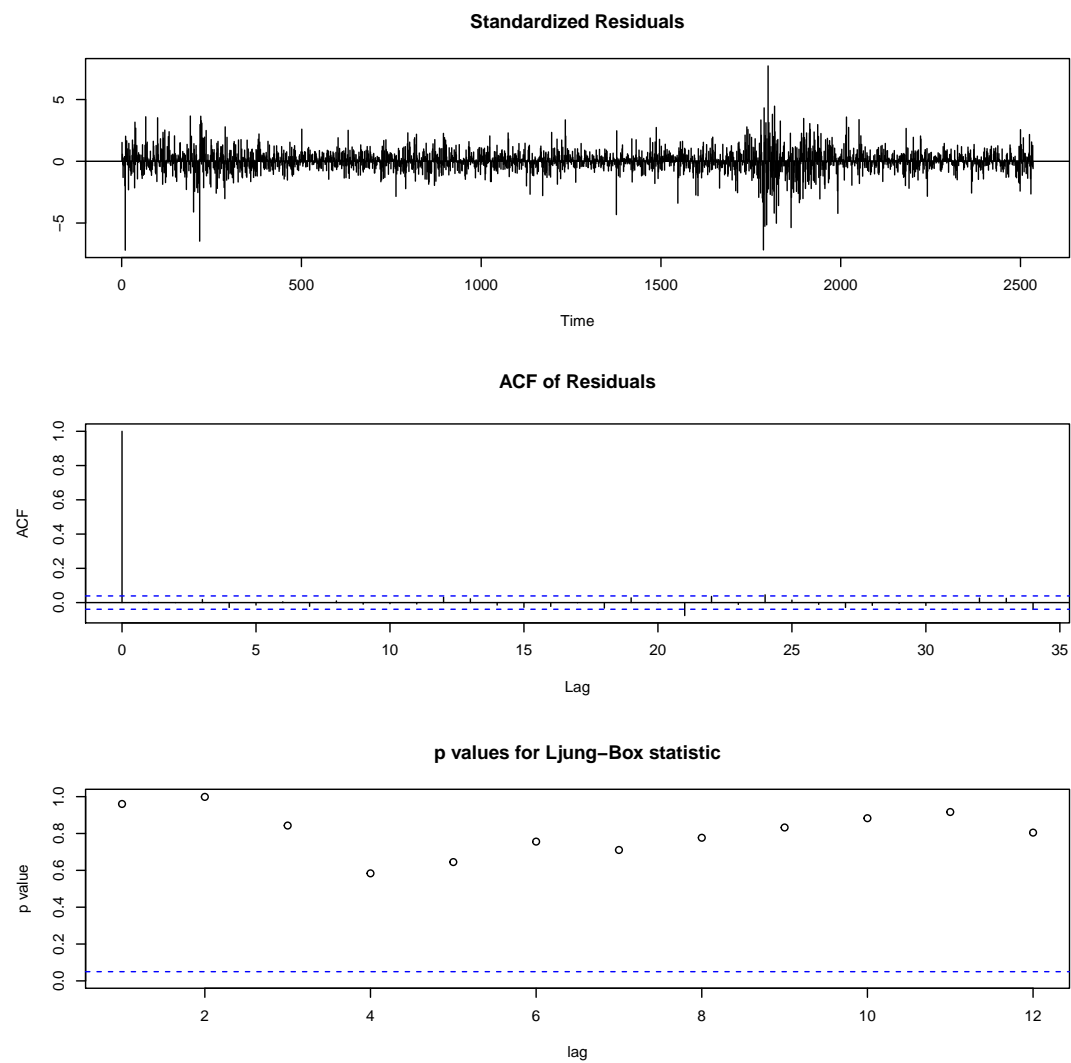
```
arima(x = lrt, order = c(2, 0, 0), include.mean = F)
```

Coefficients:

	ar1	ar2
	-0.1205	-0.0816
s.e.	0.0198	0.0198

sigma^2 estimated as 0.0001415: log likelihood = 7636.85, aic = -15267.7

```
> tsdiag(m1,gof=12)
```



### 2.3 (c)

The Ljung-Box statistics of  $z_t^2 = (x_t - \bar{x})^2$ , we could find  $Q(12) = 904$  with  $P\text{-value} < 2.2e-16$ . So the test confirm strong ARCH effects.

```
> r1=m1$residuals
> x1=100*r1
> y1=x1-mean(x1)
> Box.test(y1^2,lag=12,type='Ljung')
```

Box-Ljung test

```
data: y1^2
X-squared = 904.4, df = 12, p-value < 2.2e-16
```

### 2.4 (d)

By using EGARCH(1,1) model, we obtain

$$x_t = 0.0276 + a_t, a_t = \sigma_t \epsilon_t, \epsilon_t \sim N(0, 1) \quad (2.2)$$

$$\ln(\sigma_t^2) = -0.0622 + 0.0857(|\epsilon_t| - 0.7629\epsilon_t) + 0.9832 \ln(\sigma_{t-1}^2) \quad (2.3)$$

For the model checking, the Ljung-Box statistics give  $Q(12) = 6.4711(0.8905)$  and  $Q(24) = 23.266(0.5041)$  for the standardized residuals  $\tilde{a}_t = \frac{a_t}{\sigma_t}$  and  $Q(12) = 8.4412(0.7498)$  and  $Q(24) = 17.762(0.8144)$  for the squared series  $\tilde{a}_t^2$ , where the number in parentheses denotes p-value. The model fits the data reasonably well.

```
> source("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/Egarch.R")
> m2=Egarch(x1)
```

Estimation results of EGARCH(1,1) model:

```
estimates:  0.02756223 -0.06215496 0.08567267 -0.7628807 0.9831514
std.errors:  0.02075757 0.009255173 0.01264839 0.1436681 0.003870301
t-ratio:    1.327816 -6.715699 6.773408 -5.310022 254.0245
```

```
> stres1=m2$residuals/m2$volatility
> Box.test(stres1,lag=12,type='Ljung')
```

Box-Ljung test

```
data: stres1
X-squared = 6.4711, df = 12, p-value = 0.8905
```

```
> Box.test(stres1,lag=24,type='Ljung')
```

Box-Ljung test

```
data: stres1
X-squared = 23.266, df = 24, p-value = 0.5041
```

```
> Box.test(stresi^2,lag=12,type='Ljung')
```

Box-Ljung test

data: stresi^2

X-squared = 8.4412, df = 12, p-value = 0.7498

```
> Box.test(stresi^2,lag=24,type='Ljung')
```

Box-Ljung test

data: stresi^2

X-squared = 17.762, df = 24, p-value = 0.8144