# hw6

peng zhang

March 11, 2016

## 1 A-5-2

Considering the daily log return of Aboratories stock,we use out-of-sample forecasts to compute $\sigma_t^2(l)$ for h=1,5,...,40 and t from 2011 to 2515.

$$\sigma_{t,h}^2 = \sum_{l=1}^{h} \sigma_t^2(l) \tag{1.1}$$

where $\sigma_{t,h}^2$ denote the conditional variance of the h-period log return $r_{t,h}$ at the forecast origin t. The annualized h-period volatility

$$\sigma_{t,h,a} = \sqrt{\frac{252}{h}}\sigma_{t,h} \tag{1.2}$$

where the subscript "a" is used to signigy that the volatility is annualized. We are trying to use it to study the volatility term structure.

```
> temp <- tempfile()
> download.file("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/ch5data.zip",temp)
> da <- read.table(unz(temp, "d-a2a-0110.txt"),header=T)
> unlink(temp)
> lrt=log(da$ABT+1)
> length(lrt)
```

```
[1] 2515
```

```
> tdx=c(1:2515)/252+2001
> tdx1=c(1:2010)/252+2001
> tdx2=c(2011:2515)/252+2001
> library(fGarch)
> m1=garchFit(~1+garch(1,1),data=lrt,trace=F)
> summary(m1)
```

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(1, 1), data = lrt, trace = F)
```

```
Mean and Variance Equation:
 data ~ 1 + garch(1, 1)
<environment: 0x562f9793dc90>
 [data = lrt]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
3.3041e-04  1.7978e-06  4.4518e-02  9.4806e-01

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     3.304e-04   2.548e-04    1.297 0.194748
omega  1.798e-06   5.355e-07    3.357 0.000787 ***
alpha1 4.452e-02   6.736e-03    6.609 3.86e-11 ***
beta1  9.481e-01   7.527e-03  125.953  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 7150.29    normalized:  2.843058

Description:
 Fri Mar 11 13:43:53 2016 by user:


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  2918.297  0
 Shapiro-Wilk Test  R    W      0.9599487 0
 Ljung-Box Test     R    Q(10)  12.16173  0.2743786
 Ljung-Box Test     R    Q(15)  27.43711  0.02537026
 Ljung-Box Test     R    Q(20)  29.45983  0.07909205
 Ljung-Box Test     R^2  Q(10)  5.026799  0.8893807
 Ljung-Box Test     R^2  Q(15)  8.338571  0.9094928
 Ljung-Box Test     R^2  Q(20)  93.80681  1.589129e-11
 LM Arch Test       R    TR^2   5.724373  0.9293338
```

```
Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.682935 -5.673662 -5.682940 -5.679569
```

```
> vall=sqrt(252)*volatility(m1)
> t1=matrix(0,nrow=505,ncol=8)
> for(i in 1:505){
+                 mm1=garchFit(~1+garch(1,1),data=lrt[1:2009+i],trace=F)
+                 t1[i,1]=sqrt(252)*predict(mm1,1)$standardDeviation
+
+                 for (j in 2:8)
+                 {
+                   t1[i,j]=sqrt(252/(5*(j-1))*sum((predict(mm1,5*(j-1))$standardDeviation)^2))
+                 }
+                 }

> plot(tdx,vall,xlab='year',ylab='volatility','l')
```

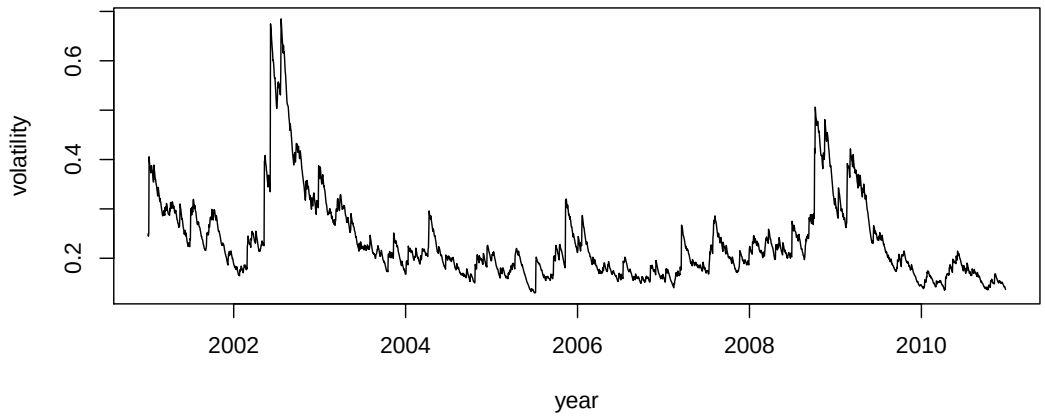


Figure 1: Volatility of log returns for stocks of Abbott Laboratories

```
> plot(tdx2,t1[,1],'l',ylim=c(0,0.5),xlab='year',ylab='Annualized Vol')
> par(new=TRUE)
> plot(tdx2,t1[,2],'l',ylim=c(0,0.5),xlab='year',ylab='Annualized Vol',lty=2,lwd=2)
> par(new=TRUE)
> plot(tdx2,t1[,3],'l',ylim=c(0,0.5),xlab='year',ylab='Annualized Vol',lty=3,lwd=2)
> par(new=TRUE)
> plot(tdx2,t1[,4],'l',ylim=c(0,0.5),xlab='year',ylab='Annualized Vol',lty=4,lwd=2)
> par(new=TRUE)
> plot(tdx2,t1[,5],'l',ylim=c(0,0.5),xlab='year',ylab='Annualized Vol',lty=5,lwd=2)
> par(new=TRUE)
> plot(tdx2,t1[,6],'l',ylim=c(0,0.5),xlab='year',ylab='Annualized Vol',lty=6,lwd=2)
> par(new=TRUE)
> plot(tdx2,t1[,7],'l',ylim=c(0,0.5),xlab='year',ylab='Annualized Vol',lty=7,lwd=2)
> par(new=TRUE)
> plot(tdx2,t1[,8],'l',ylim=c(0,0.5),xlab='year',ylab='Annualized Vol',lty=8,lwd=2)
```

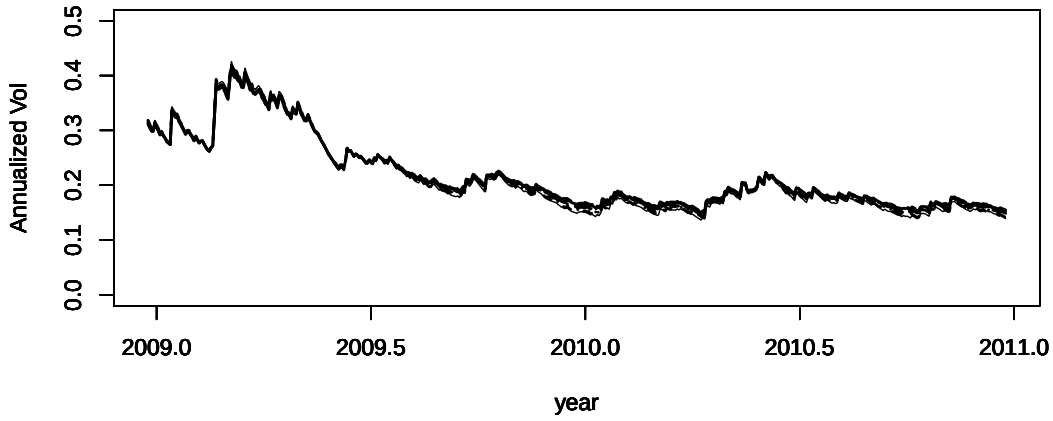


Figure 2: Volatility term structure for daily log returns of Abbott Laboratories stock from Jan 2,2001 to Dec 31,2010

## 2 A-5-3

The CAPM model can be written as

$$r_t = \alpha + \beta r_{m,t} + e_t \tag{2.1}$$

where $r_{m,t}$ denotes the return of the market and $r_t$ is the asset return of interest.

$$\beta = \frac{Cov(r_t, r_{m,t})}{Var(r_{m,t})} \tag{2.2}$$

And we have

$$Cov(r_t, r_{m,t}) = \frac{Var(r_t + r_{m,t}) - Var(r_t - r_{m,t})}{4} \tag{2.3}$$

To demonstrate this, we consider the daily log returns of Abbott Laboratories stock from Jan 2,2001 to Dec 31,2010. we use the daily log returns of the $S\&P$ 500 index as the market returns. The traditional CAPM model for the data is

$$r_t = 8.035 * 10^{-5} + 0.533 r_{m,t} + e_t \tag{2.4}$$

```
> da = read.table("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/d-abtsp-0110.txt",header=T)
> lrt=log(da$abt+1)
> sp5=da[,3]
> xp=lrt+sp5
> xm=lrt-sp5
> m1=garchFit(~1+garch(1,1),data=xp,trace=F)
> summary(m1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(1, 1), data = xp, trace = F)

Mean and Variance Equation:
 data ~ 1 + garch(1, 1)
<environment: 0x562f972fbe60>
 [data = xp]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
6.9795e-04  5.3309e-06  6.8953e-02  9.2153e-01

Std. Errors:
 based on Hessian
```

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     6.980e-04   3.675e-04    1.899 0.057508 .
omega  5.331e-06   1.513e-06    3.524 0.000425 ***
alpha1 6.895e-02   8.967e-03    7.690 1.47e-14 ***
beta1  9.215e-01   9.662e-03   95.374  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 6117.732    normalized:  2.432498

Description:
 Fri Mar 11 13:45:42 2016 by user:


Standardised Residuals Tests:
                             Statistic p-Value
 Jarque-Bera Test   R    Chi^2  245.569   0
 Shapiro-Wilk Test  R    W      0.9895461 1.367729e-12
 Ljung-Box Test     R    Q(10)  17.6794   0.06061859
 Ljung-Box Test     R    Q(15)  34.35907  0.003031235
 Ljung-Box Test     R    Q(20)  35.16423  0.01924683
 Ljung-Box Test     R^2  Q(10)  5.806391  0.8312586
 Ljung-Box Test     R^2  Q(15)  9.285186  0.86214
 Ljung-Box Test     R^2  Q(20)  38.79151  0.007076735
 LM Arch Test       R    TR^2   7.068983  0.8530213

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.861815 -4.852542 -4.861820 -4.858449
```

```
> m2=garchFit(~1+garch(1,1),data=xm,trace=F)
> summary(m2)
```

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(1, 1), data = xm, trace = F)

Mean and Variance Equation:
 data ~ 1 + garch(1, 1)
<environment: 0x562f97994900>
```

```
 [data = xm]

Conditional Distribution:
 norm

Coefficient(s):
         mu        omega       alpha1        beta1
-7.2703e-05   1.9047e-06   4.9847e-02   9.4236e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -7.270e-05   2.449e-04   -0.297 0.766608
omega   1.905e-06   5.478e-07    3.477 0.000507 ***
alpha1  4.985e-02   8.485e-03    5.875 4.23e-09 ***
beta1   9.424e-01   9.317e-03  101.148  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 7238.223    normalized:  2.878021

Description:
 Fri Mar 11 13:45:42 2016 by user:


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  3590.587  0
 Shapiro-Wilk Test  R    W      0.9504783 0
 Ljung-Box Test     R    Q(10)  12.65645  0.2435194
 Ljung-Box Test     R    Q(15)  14.47432  0.4899017
 Ljung-Box Test     R    Q(20)  15.20882  0.7643365
 Ljung-Box Test     R^2  Q(10)  8.77148   0.5539197
 Ljung-Box Test     R^2  Q(15)  11.43152  0.7214484
 Ljung-Box Test     R^2  Q(20)  79.22102  5.321089e-09
 LM Arch Test       R    TR^2   7.050989  0.8542256

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.752861 -5.743589 -5.752866 -5.749496
```

```
> m3=garchFit(~1+garch(1,1),data=sp5,trace=F)
> summary(m3)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(1, 1), data = sp5, trace = F)

Mean and Variance Equation:
 data ~ 1 + garch(1, 1)
<environment: 0x562f9798afc8>
 [data = sp5]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
4.6691e-04  1.2457e-06  8.0141e-02  9.1132e-01

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     4.669e-04   1.753e-04    2.664  0.00773 **
omega  1.246e-06   3.082e-07    4.041 5.31e-05 ***
alpha1 8.014e-02   9.591e-03    8.356  < 2e-16 ***
beta1  9.113e-01   9.951e-03   91.585  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 7860.176    normalized:  3.125319

Description:
 Fri Mar 11 13:45:42 2016 by user:


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  174.1371  0
 Shapiro-Wilk Test  R    W      0.9901083 3.720934e-12
 Ljung-Box Test     R    Q(10)  16.21911  0.09352968
```

```
 Ljung-Box Test     R    Q(15)  25.32825  0.04570506
 Ljung-Box Test     R    Q(20)  28.91991  0.08934842
 Ljung-Box Test     R^2  Q(10)  22.20321  0.0141022
 Ljung-Box Test     R^2  Q(15)  23.94121  0.06609851
 Ljung-Box Test     R^2  Q(20)  24.73054  0.2119351
 LM Arch Test       R    TR^2   26.08833  0.01042916

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.247456 -6.238184 -6.247461 -6.244091
```

```
> vxp=volatility(m1)
> vxm=volatility(m2)
> vsp5=volatility(m3)
> beta=(vxp^2-vxm^2)/(4*vsp5^2)
> tdx=c(1:2515)/252+2001
> m4=lm(lrt~sp5)
> summary(m4)

Call:
lm(formula = lrt ~ sp5)

Residuals:
      Min        1Q    Median        3Q       Max
-0.167209 -0.006647 -0.000093  0.006725  0.120399

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 8.035e-05  2.769e-04    0.29    0.772
sp5         5.330e-01  2.013e-02   26.48   <2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Residual standard error: 0.01389 on 2513 degrees of freedom
Multiple R-squared:  0.2182,        Adjusted R-squared:  0.2179
F-statistic: 701.3 on 1 and 2513 DF,  p-value: < 2.2e-16
```

```
> plot(tdx,beta,xlab='year',ylab='beta ',type='l')
> abline(h=c(0.533))
```

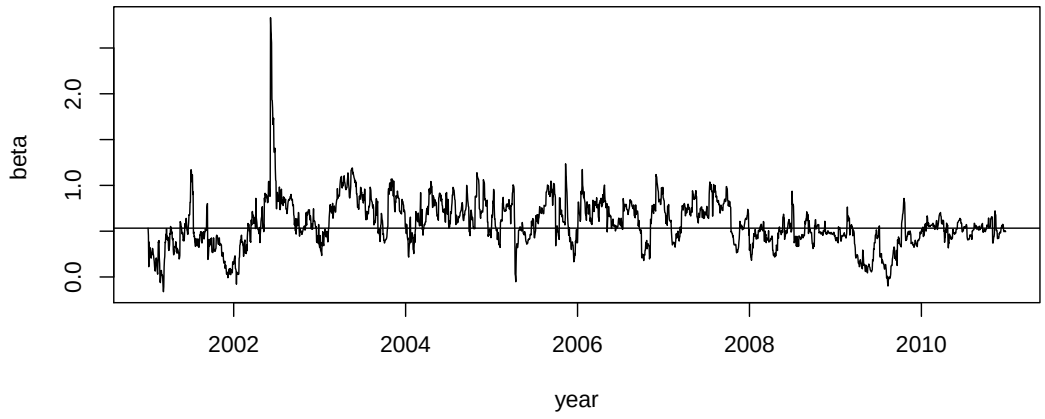


Figure 3: Time-varying betas for the daily log returns of Abbott Laboeatories stock from Jan 2,2001 to Dec 31,2010.The daily log returns of the SP 500 index is used as the market returns. The horizontal line denotes the constant beta at 0.533