Hw7

Peng Zhang

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1 A-5-4

Minimum variance portfolio is to choose the weights w_t that are the solution to the following simple optimization problem:

$$\min_w w^T V_t w \text{ ,such that } \sum_{i=1}^{k} w_i = 1$$

The solution is

$$w_t = \frac{V_t^{-1} I}{I' V_t^{-1} I} \tag{1.1}$$

By using the GARCH models, we could get the covariance matrix V_t.

```
> da=read.table("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/d-a2a-0110.txt",header=T)
> aa=log(da$AA+1)
> axp=log(da$AXP+1)
> abt=log(da$ABT+1)
> lrtn=cbind(aa,axp,abt)
> source("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/GMVP.R")
> M2=GMVP(lrtn,start=2011)
> names(M2)
```

```
[1] "weights"     "minVariance" "variances"   "returns"     "det"
```

```
> wgt=M2$weights
> range(wgt)
```

```
[1] -0.3263187  1.2799164
```

```
> prtn=M2$returns
> mean(prtn)
```

```
[1] 5.926154e-06
```

```
> sqrt(var(prtn))
```

```
[1] 0.01353345
```

```
> Mean=apply(lrtn[2012:2515,],2,mean)
> Mean
```

```
          aa           axp           abt
 6.963823e-04  1.768064e-03 -8.356484e-05
```

```
> v1=sqrt(apply(lrtn[2012:2515,],2,var))
> print(v1)
```

```
        aa        axp        abt
0.03607875 0.03596523 0.01315420
```

```
> minV=sqrt(M2$minVariance)
> Vol=sqrt(M2$variances)
> range(minV,Vol)
```

```
[1] 0.007338741 0.095915252
```

```
> tdx=c(1:505)/2515+2009
```

```
> plot(tdx,wgt[1,],xlab='year',ylab='weights',type='l',ylim=c(-0.75,1.5))
> lines(tdx,wgt[2,],lty=2)
> lines(tdx,wgt[3,],lty=3)
```

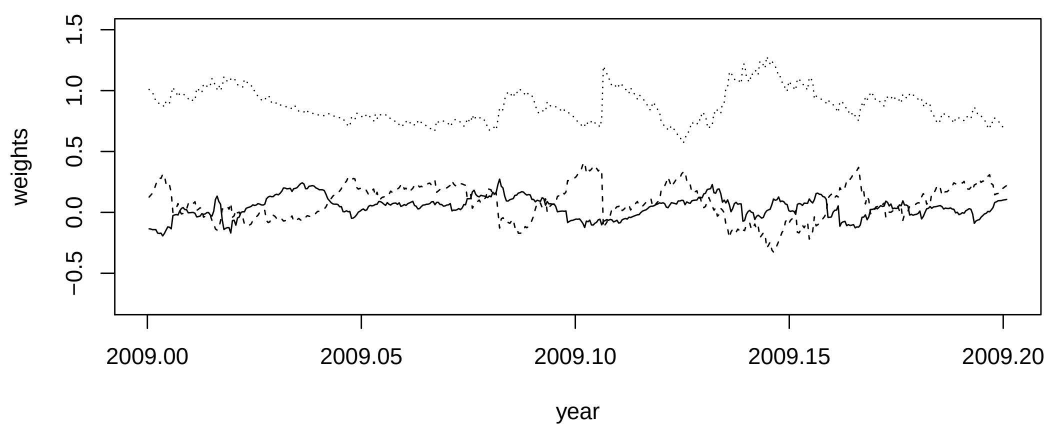

Figure 1: Weights of the minimum variance portfolio for three US stock returns (Alcoa,American Express,and Abbott Laboratories) from Dec 29 2008 to Dec 31,2010,The solid,dashed and dotted lines are three stocks respectively

2 A-5-5

By using GARCH(1,1) for the daily log returns, we get the model:

$$r_t = 2.415 * 10^{-3} + a_t, a_t = \sigma_t \epsilon_t, \epsilon \sim N(0,1) \tag{2.1}$$

$$\sigma_t^2 = 9.408 * 10^{-6} + 5.619 * 10^{-2} a_{t-1}^2 + 0.9319 \sigma_{t-1}^2 \tag{2.2}$$

```
> plot(tdx,Vol[,1],xlab='year',ylab='vol',type='l',ylim=c(0,0.1))
> lines(tdx,Vol[,2],lty=2)
> lines(tdx,Vol[,3],lty=3)
> lines(tdx,minV,lty=4)
```

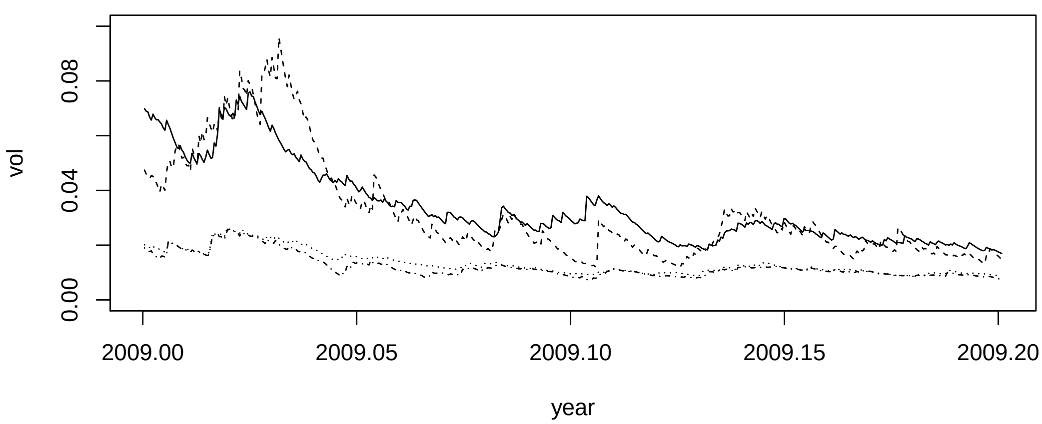

Figure 2: Volatility of log returns for stocks of Alcoa,American Express,and Abbott Laboratories from Dec 29 2008 to Dec 31,2010.The solid,dashed, dotted and dot-dashed lines are three stocks and portfolio respectively

```
> da=read.table("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/d-a2a-0110.txt",header=T)
> aapl=log(da$AAPL+1)
> library(fGarch)
> m1=garchFit(~1+garch(1,1),data=aapl,trace=F)
> summary(m1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(1, 1), data = aapl, trace = F)

Mean and Variance Equation:
 data ~ 1 + garch(1, 1)
<environment: 0x55f2453598a0>
 [data = aapl]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
2.4153e-03  9.4078e-06  5.6188e-02  9.3188e-01

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     2.415e-03   4.729e-04    5.107 3.27e-07 ***
omega  9.408e-06   3.583e-06    2.626  0.00865 **
alpha1 5.619e-02   9.745e-03    5.766 8.14e-09 ***
beta1  9.319e-01   1.268e-02   73.479  < 2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Log Likelihood:
 5644.98    normalized:  2.244525

Description:
 Fri Mar 11 16:59:44 2016 by user:


Standardised Residuals Tests:
                                Statistic p-Value
```

```
 Jarque-Bera Test   R    Chi^2  777.2411  0
 Shapiro-Wilk Test  R    W      0.9760997 0
 Ljung-Box Test     R    Q(10)  10.87873  0.3670442
 Ljung-Box Test     R    Q(15)  13.12118  0.5929383
 Ljung-Box Test     R    Q(20)  15.47371  0.7486906
 Ljung-Box Test     R^2  Q(10)  6.764498  0.747475
 Ljung-Box Test     R^2  Q(15)  8.135184  0.9182486
 Ljung-Box Test     R^2  Q(20)  12.65884  0.8915372
 LM Arch Test       R    TR^2   6.525836  0.8872945

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.485869 -4.476596 -4.485874 -4.482504
```

Then using the model to predict 10 values, we get corresponding log returns and variances.By using the formula,

$$S(i) = e^{r_i} * S(i-1), S(0) = 350 \tag{2.3}$$

we get the stock prices for each trading days.

```
> p1=predict(m1,10)
> r1=p1$meanForecast
> d1=p1$standardDeviation
> s=350
> S=rep(1,1,10)
> for (i in 1:10)
+   {
+   if(i==1)
+    {S[i]=s*exp(r1[i])}
+    else{S[i]=S[i-1]*exp(r1[i])}
+   }
> S

 [1] 350.8464 351.6948 352.5452 353.3978 354.2524 355.1090 355.9677 356.8285
 [9] 357.6914 358.5564
```

Through using the European call option formula, we get the price is 3.555. European call option price:

$$C = e^{-0.1/252} max(S(10) - 355) \tag{2.4}$$

$$S(i) = e^{r_i} * S(i-1), S(0) = 350 \tag{2.5}$$

```
> C1=exp(-0.1/252)*max(S[10]-355,0)
> C1

[1] 3.554954
```

Similary, we get the price of Asian call option is 0. Asian call option price:

$$C(10) = e^{-0.1/252} max(A(0,10) - 355, 0) \tag{2.6}$$

$$A(0,10) = \frac{1}{10}\sum_{i=1}^{10} S(i) \tag{2.7}$$

```
> C2=exp(-0.1/252)*max(mean(S)-355,0)
> C2
```

```
[1] 0
```

2.1 A-6-1

We know the mean and variance of r_t is 0.02 and 0.04.And the probability of a trade happen is 1/2.

$$r_t^0 = \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ r_t & \text{with probability } \frac{1}{4} \\ r_t & \text{with probability } \frac{1}{8} \\ \dots & \dots \\ \sum\limits_{i=0}^{k} & \text{with probability } \frac{1}{2^{k+2}} \\ \dots & \dots \end{cases} \tag{2.8}$$

Then we could get the mean and variance of r_t^0.

$$E(r_t^0) = 0.02; Var(r_t^0) = 0.0408. \tag{2.9}$$

Meanwhile using the formula in the book ,we could get r_t^0 is serially correlated. And the first three lags of autocorrelations of r_t^0 is $-\frac{1}{204}, -\frac{1}{408}, -\frac{1}{816}$.

$$Cov(r_t^0, r_{t-j}^0) = -0.02 * \frac{1}{2^j} \tag{2.10}$$

$$\rho_i(r_t^0) = -\frac{1}{102 * 2^j} \tag{2.11}$$

$$\rho_1 = -\frac{1}{204}; \rho_2 = -\frac{1}{408}; \rho_3 = -\frac{1}{816} \tag{2.12}$$

2.2 A-6-7

]

(a)

By using the R scripts hfanal.R, we could get the histrogram of the returns.

```
> da=read.table("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/taq-sbux-jul2011.txt",header=
> source("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/hfanal.R")
> m1=hfanal(da,5)
```

```
[1] 257343 256791 251943
[1] 32082
[1] 62761
[1] 108105
[1] 177542
[1] 251943
```

```
> lrt=m1$returns
```

```
> hist(lrt,breaks=50)
```

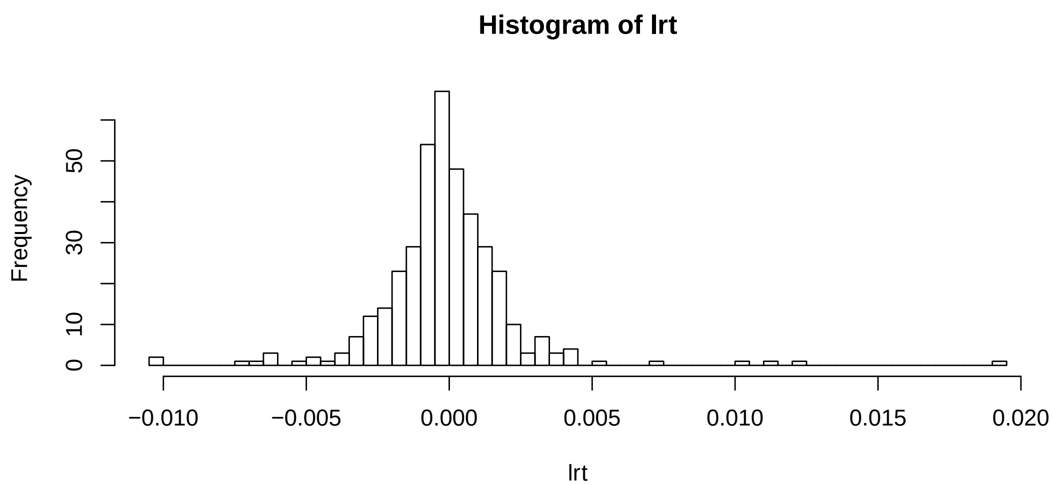

Figure 3: Histrogram of intraday 5-min log returns for Starbuscks from July 25 to July 29,2011

(b)

Meanwhile,we could get the daily realized volatility 0.148, 0.184, 0.268, 0.329, 0.573.

```
> drv=m1$realized
> drv
```

```
[1] 0.1477784 0.1843577 0.2679561 0.3285999 0.5733907
```

(c)

Through the R scripts hf2ts.R, we could easily get the realized volatility with the method of average estimator,0.174, 0.184, 0.260, 0.309, 0.510.

```
> source("http://faculty.chicagobooth.edu/ruey.tsay/teaching/introTS/hf2ts.R")
> m2=hf2ts(da,int=1)
```

```
[1] 257343 256791 251943
```

```
> mRv=m2$ave.RV
> mRv
```

```
[1] 0.1747410 0.1844607 0.2599831 0.3094573 0.5098162
```

(d)

Similarly, we could get the daily realized volatility of the stock with the method of two-scale estimator.The values is 0.173, 0.182, 0.254, 0.308, 0.504.

```
> Rv=m2$realized
> Rv
```

```
[1] 0.1728124 0.1822430 0.2538816 0.3080801 0.5042857
```