# MAT121 Weekly Exam 02

This quiz focuses on the application of normal distribution and the sampling distribution of sample means. Except for a few questions related to the concepts, all other questions are categorized into one of the two basic types of questions: finding probability and finding percentile. You are strongly encouraged to draw at least one density curve for each of the questions and then use the standard normal distribution table in the textbook.

**Problem 1.**
Suppose X follows a normal distribution with mean 5 and variance 9. After z-score transformation, Probability $P(X \leq 8)$ is equal to

A  $P(Z \leq 1)$
B  $P(Z \leq -1)$
C  $P(Z \leq 0.6)$
D  $P(Z \leq -0.6)$
E  $P(Z \leq 1.67)$

**Correct Answer A.**

**Problem 2**

Suppose X is normally distributed with a mean of 5 and a standard deviation of 0.4. Using the z-score transformation $Z = (X - \mu)/\sigma$ we find $P(X \leq X_0) = P(Z \leq 1.3)$. What is the value of $X_0$? Exactly one option must be correct)

A.  6.9
B.  4.48
C.  2.00
D.  5.52
E.  3.75

**Correct Answer: D**

**Problem 3**

A company issues a general knowledge test to its potential employees as part of its recruitment procedures. The company **does not** invite the lowest scoring 20% back for a second interview. If the scores on this test are normally distributed with a mean of 60 and a standard deviation of 17. What mark must a candidate achieve to secure a second interview?

A.  55

B. 79
C. 46
D. 75
E. 65

**Correct Answer: C**

## Problem 4.

Find the z-score in the standard normal distribution such that the area to the right of z is 0.12.

A. – 1.17
B. – 0.88
C. – 0.88
D. 1.17
E. 0.12

**Correct Answer: D**

**Problem 5.**

The height of an adult male is known to be normally distributed with a mean of 175 cm and a standard deviation of 6 cm.  We are interested in the 75th percentile in this distribution of heights. Let Q3 be denoted as the 75th percentile. Then Q3 can be solved from which of the following equation?

**A.**

$$\frac{Q3 - 175}{6} = 0.675$$

B.
$$\frac{Q3 - 175}{36} = 0.675$$

C.

$$\frac{Q3 - 175}{6} = 0.75$$

D.
$$\frac{Q3 - 175}{6} = -0.75$$

E.
$$\frac{175 - Q3}{6} = 0.675$$

**Correct Answer: A**

**Problem 6.**

The time that it takes a WCU student to find parking once they have arrived on campus is approximately normally distributed with a mean of 25 minutes and a standard deviation of 5 minutes. If a student arrives on campus at 8:15 AM, we want to find the probability that the student will find parking before 8:45 AM. Which of the calculation is correct?

A.

$$P(X < 45) = P\left(\frac{X - 25}{5} < \frac{45 - 25}{5}\right) = P(Z < 4) \approx 1$$

B.

$$P(X > 15) = P\left(\frac{X - 25}{5} > \frac{15 - 25}{5}\right) = P(Z > -2) \approx 0.9772$$

**C.**

$$P(X < 30) = P\left(\frac{X - 25}{5} < \frac{30 - 25}{5}\right) = P(Z < 1) \approx 0.8413$$

D.

$$P(X > 30) = P\left(\frac{X - 25}{5} > \frac{30 - 25}{5}\right) = P(Z > 1) \approx 0.1587$$

E.

$$P(X > 30) = P\left(\frac{25 - X}{5} > \frac{25 - 30}{5}\right) = P(Z > -1) \approx 0.8413$$

**Correct Answer: C.**

**Problem 7.**

A machine that cuts corks for wine bottles operates in such a way that the distribution of the diameter of the corks produced is well approximated by a normal distribution with a mean of 3 cm and a standard deviation of 0.1 cm. The specifications call for corks with diameters between 2.9 and 3.1 cm. A cork **not** meeting specifications is considered defective. We need to use the z-score transformation before using the standard normal distribution table.

Which of the following calculation for the proportion of defective corks is correct?

A.

$$P(2.9 < T < 3.1) = P(-1 < Z < 1) = 0.8413$$

B.

$$P(3.1 < T < 2.9) = P(1 < Z < -1) = 0.5$$

C.

$P(2.9 < T \text{ or } T < 3.1) = P(-1 < Z \text{ or } Z < 1) = .3413$

D.

$P(3.1 < T \text{ or } T < 2.9) = P(1 < Z \text{ or } Z < -1) = 0.3174$

E.

$P(3.1 < T \text{ and } T < 2.9) = P(1 < Z \text{ and } Z < -1) = 0.3413$

**Correct Answer: D**.


**Problem 8.**

The heights of American men aged 18 to 24 are approximately normal with a mean of 68 inches and a standard deviation of 2.5 inches. About 20% of these men are taller than
   A.   66 inches
   B.   68 inches
   C.   70 inches
   D.   72 inches
   E.   74 inches

**Correct Answer: C**

**Problem 9**

Suppose a normally distributed random variable x has a mean of 100 and $P(x < 90) = 0.20$. What is the probability that x is between 90 and 110? Ie. What is $P(90 < x < 110)$? [Draw a density curve and label the values, then you can see the region you want to find the area.]

A 0.2
B 0.3
C 0.4
D 0.5
E 0.6

**Answer: D**
Both tail areas are equal to 0.2. The middle area is 1- 0.4 = 0.6. The correct answer is E.

**Problem 10**

If you got a 79 on a test in a class with a mean score of 85 and a standard deviation of 4.8, the z-score of your test score would be
A 1.25
B -2.50
C -1.25
D 2.5
E 0

**Problem 11**

For a normal distribution with a mean of 480 and a standard deviation of 32, find the value of Q2 (2nd quartile, or median).

A 458

B 480

C 500

D It depends on the size of the population.

**Problem 12**

The scoring of modern IQ is such that Intelligence Quotients (IQs) have a normal distribution of μ= 100 and **σ** = 15. Mensa International is a non-profit organization that accepts only people with IQ scores within the top 2%. What level of IQ qualifies one to be a member of Mensa?

A). 115

B). 130.75

C). 145

D). 125

**Problem 13**

Suppose you are working with a data set that is normally distributed with a mean of 70 and a standard deviation of 10. Determine the value of x such that 65% of the values are greater than x

A). 60

B). 66

C).  80

D).  74

**Problem 14.**

The standard deviation of SAT scores is 100 points. A researcher decides to take a sample of 500 students' scores to estimate the mean score of students in your state. What is the standard deviation of the sample mean?

A). 0.2

B). 5

C). 4.47

D). 100

**Problem 15.**
The average number of pushups a United States Marine does daily is 300, with a standard deviation of 50. A random sample of 36 marines is selected. What is the probability that the sample mean is at most 320 pushups?
A. 0.0082
B. 0.3446
C. 0.6554
D. 0.8767
E. 0.9918
**Correct Answer: E**
Note that $\bar{x} \to N(300, 50/\sqrt{36})$, $P(\bar{x} < 320) \approx 0.9918025$

**Problem 16**

A factory produces plate glass with a mean thickness of 4 mm and a standard deviation of 1.1 mm. A simple random sample of 100 sheets of glass is to be measured, and the sample mean thickness of the 100 sheets is to be computed. The probability that the average thickness of the 100 sheets of glass is less than 4.1 mm is closest to

A. 0.8183.

B. 0.5361.

C. 0.1814.

D. 0.6817.

E. 0.8413.

**Correct Answer: A**

**Problem 17**
A population has a normal distribution with a mean of 50 and a standard deviation of 20. If a random sample of size 49 is taken from the population, then what is the probability that this sample mean will be between 48 and 54?
A. 0.060
B. 0.228
C. 0.399
D. 0.611
**Correct Answer: D.**
Note that the sampling distribution: $\bar{X} \to N(50, 10/7)$, $P(48 < \bar{X} < 54) = 0.6106772$

**Problem 18.**

A random sample of size 36 is to be taken from a normal population having a means of 100 and variance 4. What is the 90$^{th}$ percentile of the sampling distribution of the sample mean $\bar{x}$?

A. 97.44
B. 97.44
C. 101.08
D. 100.43

**Correct Answer: D.**
**$\bar{X} \rightarrow N(100, 2/6)$, the 90$^{th}$ percentile is equal to 100.43.**

**Problem 19.**

Assuming that 6% of donors have type-O blood. For the sample of 300 donors, what is the probability that the sample proportion is greater than 0.10?

a) 0.0018
b) 0.181
c) 0.819
d) 0.991

**Answer: A**

$$P(\hat{p} > 0.1) = P\left(\frac{\hat{p} - 0.06}{\sqrt{\frac{0.06 \times 0.94}{300}}} > \frac{0.1 - 0.06}{\sqrt{\frac{0.06 \times 0.94}{300}}}\right) = P(Z > 2.91) = 0.0018$$

**Problem 20**

Recent studies have shown that 20% of Americans fit the medical definition of obese. A random sample of 100 Americans is selected and the number of obese in the sample is determined. What is the probability that the sample proportion is greater than 0.24?

a) 0.1841
b) 0.1587
c) 0.8413
d) 1.0

**Correct Answer C.**

$$P(\hat{p} < 0.24) = P\left(\frac{\hat{p} - 0.2}{\sqrt{\frac{0.2 \times 0.8}{100}}} < \frac{0.24 - 0.2}{\sqrt{\frac{0.2 \times 0.8}{100}}}\right) = P(Z < 1) = 0.8413$$