

Week 09 Quiz: One-sample Normal Tests

Problem 1

The primary goal of a one-sample Z-test is to:

- a) Estimate the population mean.
- b) Test whether a sample mean is significantly different from a hypothesized population mean.
- c) Compare the means of two independent samples.
- d) Calculate the probability of observing the sample data.

Answer: B

Problem 2.

For a large one-sample Z-test with unknown population variance, the test statistic is calculated as:

- a) $(\bar{x} - \mu_0) / (s/\sqrt{n})$
- b) $(\bar{x} - \mu_0) / (\sigma/\sqrt{n})$
- c) $(\mu - \mu_0) / (\sigma/\sqrt{n})$
- d) $(\bar{x} - \mu) / (s/\sqrt{n})$

Answer: A

Problem 3

A sample of size $n=25$ taken from a normal population has a mean of 110. The hypothesized population mean is 100, and the population standard deviation is known to be 15. What is the Z-test statistic?

- a) 3.33
- b) 0.67
- c) 10.0
- d) 1.67

Answer: A

Problem 4

A factory process produces steel bars with a mean length of 120 cm and a known $\sigma=5$ cm. A new process is tested on 40 bars, yielding a sample mean

of 118.5 cm. What is the calculated z-test statistic for $H_0: \mu = 120$ vs. $H_1: \mu < 120$?

- a) 1.5
- b) -1.5
- c) -1.897
- d) -0.237

Answer: C

Data Source

- ☒ summarized statistics
- ☐ raw data

sample mean (\bar{x})

118.5

sample standard deviation (s)

5

sample size (n)

40

Claimed Value (μ_0)

120


Claim Type

less than

Significance level α

0.01 0.05 0.2

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



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Solution: This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

Given sample information: $n = 40$, $\bar{x} = 118.5$, $s = 5$.

Step 1: Identify the claim of the population mean (μ_0).
The given information indicates that the claim is: μ_0 is less than 120.

Step 2: Set up the null and alternative hypotheses.
Based on the claim, the null and alternative hypotheses are given by $H_0: \mu = 120$ and $H_1: \mu < 120$.

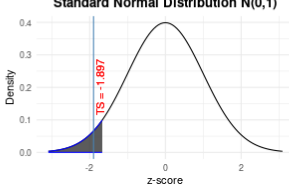
Step 3: Evaluate the test statistic.
The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{118.5 - 120}{5/\sqrt{40}} = -1.897$

Step 4: Find the critical value and calculate the p-value.
Based on the significance level, we found the critical values to be: $-z_\alpha = -z_{0.05} = -1.645$
The p-value is can be found as p-value ≈ 0.029 .

Step 5: Make a statistical decision on H_0 .
At the 5% significance level, we reject the null hypothesis. (p-value = 0.029).

Step 6: Draw conclusion [justify the claim in step 1].
At the 5% significance level, we conclude the alternative hypothesis.

Standard Normal Distribution $N(0,1)$



Problem 5

For a two-tailed one-sample z-test at $\alpha=0.05$, what are the critical values that define the rejection region?

- a) $z = \pm 1.96$
- b) $z = \pm 1.645$
- c) $z = +1.96$
- d) $z = +1.645$

Answer: A

Problem 6

The principal claims the mean IQ of students at a school is 110. A psychologist tests 36 random students, finding a mean of 107. The population standard deviation is known to be 15. If the p-value for the two-sided test is 0.27, what is the correct conclusion at $\alpha=0.05$?

- a) Reject H_0 ; the mean IQ is significantly different from 110.
- b) Fail to reject H_0 ; there is not enough evidence that the mean IQ is different from 110.
- c) Reject H_0 ; the mean IQ is less than 110.
- d) Accept H_0 ; the mean IQ is exactly 110.

Answer: D

Data Source

☒ summarized statistics

☐ raw data

sample mean (\bar{x})

107

sample standard deviation (s)

15

sample size (n)

36

Claimed Value (μ_0)

110


Claim Type

equal to

Significance level α

0.01 0.05 0.2

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



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Solution: This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

Given sample information: $n = 36$, $\bar{x} = 107$, $s = 15$.

Step 1: Identify the claim of the population mean (μ_0).

The given information indicates that the claim is: μ_0 is equal to 110.

Step 2: Set up the null and alternative hypotheses.

Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 110$ and $H_1 : \mu \neq 110$.

Step 3: Evaluate the test statistic.

The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{107 - 110}{15/\sqrt{36}} = -1.2$

Step 4: Find the critical value and calculate the p-value.

Based on the significance level, we found the critical values to be: $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$

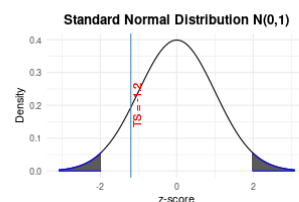
The p-value is can be found as p-value ≈ 0.23 .

Step 5: Make a statistical decision on H_0 .

At the 5% significance level, we do not reject the null hypothesis. (p -value = 0.23).

Step 6: Draw conclusion [justify the claim in step 1].

At the 5% significance level, we reject the alternative hypothesis .



Problem 7

A pizza restaurant advertises a mean delivery time of 25 minutes. A manager collects a sample of 50 deliveries, finding a mean time of 26.5 minutes with a standard deviation of 4 minutes. The test statistic is 2.65. What is the approximate p-value for this two-tailed test?

- a) 0.004
- b) 0.008
- c) 0.05
- d) 0.02

Answer: B

Data Source

☒ summarized statistics

☐ raw data

sample mean (\bar{x})

26.5

sample standard deviation (s)

4

sample size (n)

50

Claimed Value (μ_0)

25


Claim Type

equal to

Significance level α

0.01 0.05 0.2

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



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Solution: This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

Given sample information: $n = 50$, $\bar{x} = 26.5$, $s = 4$.

Step 1: Identify the claim of the population mean (μ_0).
The given information indicates that the claim is: μ_0 is equal to 25.

Step 2: Set up the null and alternative hypotheses.
Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 25$ and $H_1 : \mu \neq 25$.

Step 3: Evaluate the test statistic.
The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{26.5 - 25}{4/\sqrt{50}} = 2.652$

Step 4: Find the critical value and calculate the p-value.
Based on the significance level, we found the critical values to be: $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$
The p-value is can be found as p-value $\approx 0.008000000000000001$.

Step 5: Make a statistical decision on H_0 .
At the 5% significance level, we reject the null hypothesis. (p -value = 0.008).

Step 6: Draw conclusion [justify the claim in step 1].
At the 5% significance level, we conclude the alternative hypothesis.

Standard Normal Distribution $N(0,1)$

Problem 8

A bottling plant claims its 2-liter bottles contain a mean of 2.01 liters. A regulator measures 36 bottles, finding a mean volume of 2.005 liters with a standard deviation of 0.015 liters. For a two-tailed test at $\alpha=0.05$, what is the correct statistical decision?

- a) Reject H_0 because the sample mean is less than 2.01.
- b) Fail to reject H_0 because the test statistic is within the critical region.
- c) Reject H_0 because the p-value is less than 0.05.
- d) Fail to reject H_0 because the p-value is greater than 0.05.

Answer: C

Data Source

☒ summarized statistics
☐ raw data

sample mean (\bar{x})

2.005

sample standard deviation (s)

0.015

sample size (n)

36

Claimed Value (μ_0)

2.01


Claim Type

equal to

Significance level α

0.01 0.05 0.2

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



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Solution: This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

Given sample information: $n = 36$, $\bar{x} = 2.005$, $s = 0.015$.

Step 1: Identify the claim of the population mean (μ_0).
 The given information indicates that the claim is: μ_0 is equal to 2.01.

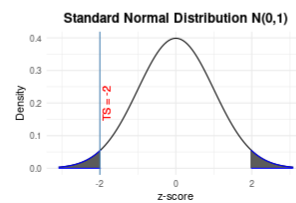
Step 2: Set up the null and alternative hypotheses.
 Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 2.01$ and $H_1 : \mu \neq 2.01$.

Step 3: Evaluate the test statistic.
 The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.005 - 2.01}{0.015/\sqrt{36}} = -2$

Step 4: Find the critical value and calculate the p-value.
 Based on the significance level, we found the critical values to be: $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$
 The p-value can be found as p-value ≈ 0.046 .

Step 5: Make a statistical decision on H_0 .
 At the 5% significance level, we reject the null hypothesis. (p-value = 0.046).

Step 6: Draw conclusion [justify the claim in step 1].
 At the 5% significance level, we conclude the alternative hypothesis.



Problem 9

An ISP guarantees mean network latency is below 50 ms. A user runs 35 speed tests, finding a mean latency of 52 ms with a known population standard deviation of 6 ms. What is the z-test statistic for $H_1: \mu > 50$?

- a) 1.75
- b) 1.97
- c) 2.36
- d) 2.08

Answer: B

$$(52-50)/(6/\sqrt{35})=1.972$$

Problem 10

A car model is rated at 30 mpg highway. A magazine tests 45 cars and finds a mean of 29.2 mpg with a standard deviation of 1.5 mpg. They perform a one-sample Z-test. What is the critical value for a two-tailed test at $\alpha=0.10$?

- a) ± 1.761

- b) ± 1.645
- c) ± 1.345
- d) ± 1.753

Answer: B

Problem 11

A hotel claims its average check-in time is under 2 minutes. A mystery guest records 40 check-ins. The sample mean is 2.1 minutes with a known population standard deviation of 0.4 minutes. For $H_1: \mu < 2$, what is the p-value?

- a) 0.9429
- b) 0.0571
- c) 0.1142
- d) 0.8858

Answer: A

Problem 12

A company requires data entry clerks to have a mean speed of 60 words per minute (wpm). A supervisor tests 38 new clerks, finding a mean of 58 wpm and a standard deviation of 5 wpm. For a one-tailed test ($H_1: \mu < 60$) at $\alpha=0.05$, what is the decision?

- a) Reject H_0 ; clerks are too slow.
- b) Fail to reject H_0 ; cannot prove clerks are too slow.
- c) Accept H_0 ; clerks are fast enough.
- d) The result is inconclusive.

Answer: A

Data Source
☒ summarized statistics
☐ raw data

sample mean (\bar{x})


sample standard deviation (s)

sample size (n)

Claimed Value (μ_0)

Claim Type

Significance level α



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Solution: This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

Given sample information: $n = 38$, $\bar{x} = 58$, $s = 5$.

Step 1: Identify the claim of the population mean (μ_0).

The given information indicates that the claim is: μ_0 is greater than or equal to 60.

Step 2: Set up the null and alternative hypotheses.

Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 60$ and $H_1 : \mu < 60$.

Step 3: Evaluate the test statistic.

The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{58 - 60}{5/\sqrt{38}} = -2.466$

Step 4: Find the critical value and calculate the p-value.

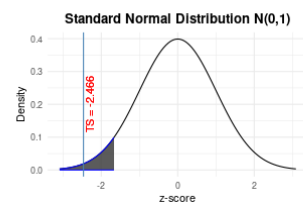
Based on the significance level, we found the critical values to be: $-z_\alpha = -z_{0.05} = -1.645$
 The p-value is can be found as p-value ≈ 0.007 .

Step 5: Make a statistical decision on H_0 .

At the 5% significance level, we reject the null hypothesis. (p -value = 0.007).

Step 6: Draw conclusion [justify the claim in step 1].

At the 5% significance level, we conclude the alternative hypothesis.



Problem 13

An environmental standard states a river's pH should be 7.0. A researcher takes 36 water samples, finding a mean pH of 6.9 with a standard deviation of 0.15. What is the standard error of the mean?

- a) 0.15
- b) 0.025
- c) 0.06
- d) 0.003

Answer: B

Problem 14

An assembly line is designed to produce one item every 45 seconds. A time-motion study on 50 items finds a mean time of 46 seconds with a known standard deviation of 3 seconds. What is the z-test statistic for $H_1: \mu \neq 45$?

- a) 1.67
- b) 2.36

- c) 3.12
d) 4.01

Answer: B

Data Source

☒ summarized statistics
☐ raw data

sample mean (\bar{x})


sample standard deviation (s)

sample size (n)

Claimed Value (μ_0)

Claim Type

Significance level α


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Solution: This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

Given sample information: $n = 50$, $\bar{x} = 46$, $s = 3$.

Step 1: Identify the claim of the population mean (μ_0).
The given information indicates that the claim is: μ_0 is equal to 45.

Step 2: Set up the null and alternative hypotheses.
Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 45$ and $H_1 : \mu \neq 45$.

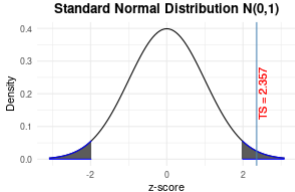
Step 3: Evaluate the test statistic.
The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{46 - 45}{3/\sqrt{50}} = 2.357$

Step 4: Find the critical value and calculate the p-value.
Based on the significance level, we found the critical values to be: $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$
The p-value is can be found as p-value ≈ 0.018 .

Step 5: Make a statistical decision on H_0 .
At the 5% significance level, we reject the null hypothesis. (p -value = 0.018).

Step 6: Draw conclusion [justify the claim in step 1].
At the 5% significance level, we conclude the alternative hypothesis.

Standard Normal Distribution N(0,1)



Problem 15

A coffee franchise standard requires milk to be steamed for 35 seconds. A regional manager times 36 baristas, finding a mean time of 33 seconds with a standard deviation of 4 seconds. The test is $H_1: \mu \neq 35$. What is the correct conclusion at $\alpha=0.10$? (The critical value is 1.96)

- a) Reject H_0 because $|t| > 196$.
b) Fail to reject H_0 because $|t| < 1.96$.
c) Reject H_0 because the sample mean is less than 35.
d) Accept H_0 because the p-value is large.

Answer: A

Data Source

☒ summarized statistics
☐ raw data

sample mean (\bar{x})

33

sample standard deviation (s)

4

sample size (n)

36

Claimed Value (μ_0)

35


Claim Type

equal to

Significance level α

0.01 0.05 0.2

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



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Solution: This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

Given sample information: $n = 36$, $\bar{x} = 33$, $s = 4$.

Step 1: Identify the claim of the population mean (μ_0).

The given information indicates that the claim is: μ_0 is equal to 35.

Step 2: Set up the null and alternative hypotheses.

Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 35$ and $H_1 : \mu \neq 35$.

Step 3: Evaluate the test statistic.

The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{33 - 35}{4/\sqrt{36}} = -3$

Step 4: Find the critical value and calculate the p-value.

Based on the significance level, we found the critical values to be: $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$

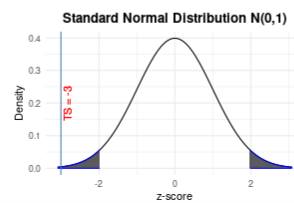
The p-value is can be found as p-value ≈ 0.002 .

Step 5: Make a statistical decision on H_0 .

At the 5% significance level, we reject the null hypothesis. (p -value = 0.002).

Step 6: Draw conclusion [justify the claim in step 1].

At the 5% significance level, we conclude the alternative hypothesis.



Summary of Weekly Quiz #9

1. Five Number Summary :

The five-number summary is used to describe the shape of the distribution of a given numerical data. It consists of five numbers: minimum data value, first quartile, median, the third quartile, and the maximum data value.

The five-number summary of this given data set is:

stats	value
Min.	60.00
1st Qu.	80.00
Median	85.00
3rd Qu.	95.00
Max.	95.00

2. Boxplot :

The boxplot is a geometric representation of the five-number summary. The boxplot of the given data set is given below.

