

# MAT 121 Statistics I

## Midterm Exam #1

09/27/2023

Time: 90 minutes

### Problem 1.

The following is a sample of ages (in months) of 18 children at a daycare:  
18 19 22 22 24 24 25 26 28 29 29 30 31 32 35 36 36 42  
What is the median age of the sample?

Answers \*

☐

29

☐

28.2

☐

30.5

☒

28.5

☐

31

## IntroStatsApps: Descriptive Statistics

**Types of Descriptive Statistics**  


Numerical Measures

**comma separated numerical data**  

18, 19, 22, 22, 24, 24, 25, 26, 28, 29, 29, 30, 31, 32, 35, 36, 36, 42

**Measure Types**  

measures of center

  
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### Measures of Center

The data values are:

18, 19, 22, 22, 24, 24, 25, 26, 28, 29, 29, 30, 31, 32, 35, 36, 36, 42

The sorted data values are:

18, 19, 22, 22, 24, 24, 25, 26, 28, 29, 29, 30, 31, 32, 35, 36, 36, 42

#### 1. Sample (population) mean

$\bar{x} = \sum_{i=1}^n \frac{x_i}{n} = 28.2$ , and  $\mu = \sum_{i=1}^n \frac{x_i}{n} = 28.2$  (if this data set is a population).

#### 2. Median

The median of a given data set is the middle number of **sorted** data set. Based on this definition, the median of the given data set is: 28.5

#### 3. Mode

A data value that appears most frequently (frequency  $> 1$ ) is called the mode of the data. Based on this definition, a set of data may have one mode, more than one mode, or no mode at all. Using the above definition, this data set has 4 modes: 22, 24, 29, 36.

Problem 2.

The following grouped frequency table of the income,  $x$ , of 30 employees at a local small business (in \$1000s).

Income	[26, 28]	(28, 30]	(30, 32]	(32, 34]	(34, 36]
Frequency	2	11	8	5	4

The relative cumulative frequency of the  $28 < x \leq 30$  class is

Answers \*

☐

11

☐

$\approx 0.43$

☐

$\approx 0.06$

☒

$\approx 0.37$

☐

$\approx 0.7$

$$\frac{11}{2+11+8+5+4} = \frac{11}{30} \approx 0.37.$$

Problem 3.

A study of 1106 college students asked about their preference for online resources. The following relative frequency distribution was determined based on the survey.

Resource	Relative Frequency
Google or Google Scholar	0.736
Library database or website	0.136
Wikipedia or online encyclopedia	0.094
Other	0.034

Of the 1106 students who participated in the survey, approximately how many chose Google or Google Scholar?

Answers \*

☐

34

☐

292

☐

736

☒

814

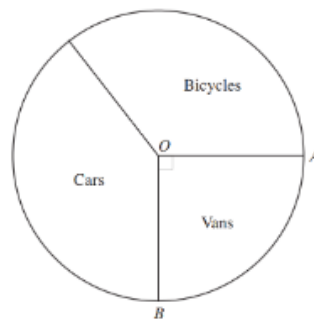
☐

921

**$1106 \times 0.736 = 84.046$**

**Problem 4.**

The pie chart above, not drawn to scale, shows the number of vehicles parked outside a supermarket. Angle AOB is a right angle. Given that there were 60 vehicles, how many vans were there?



Answers \*

☐

4

☐

6

☐

12

☒

15

☐

20

Based on the definition of the pie chart.

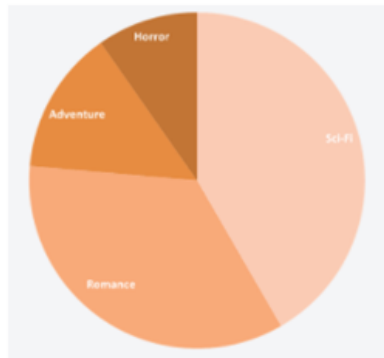
### Problem 5.

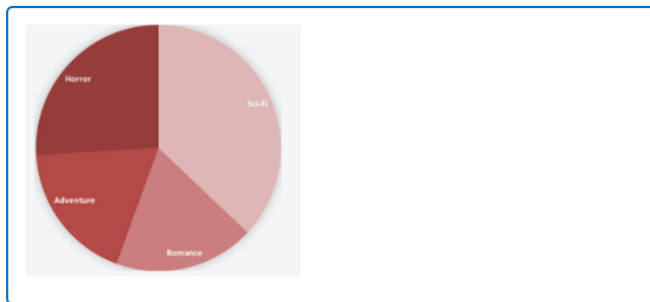
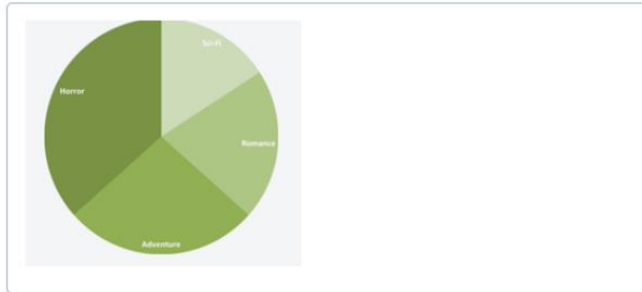
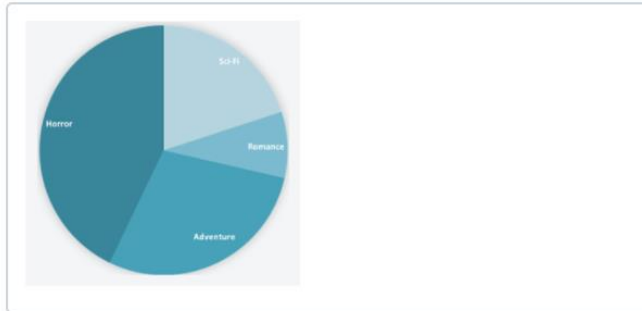
The table below represents 360360 books grouped by their category:

Book category	Frequency
Science-fiction	150150
Romance	125125
Adventure	5050
Horror	3535

Draw a pie chart representing this table

Answers \*





**Matching numbers with the corresponding slices.**

### Problem 6

Find the mean, median, and mode for the following data set.

4 7 9 11 11 11 13 17 22 26

Answers \*

☐

mode = 11, mean =12, median = 11

☐

mode = 11, mean =11, median = 11

☐

mode = 11, mean =13, median = 11.5

☒

mode = 11, mean =13, median = 11

☐

mode = 11, mean =14, median = 11

## IntroStatsApps: Descriptive Statistics

**Types of Descriptive Statistics**  


Numerical Measures

**comma separated numerical data**  

4,7,9,11,11,11,13,17,22,26

**Measure Types**  

measures of center

  
[Report bugs to C. Peng](#)

### Measures of Center

The data values are:

4, 7, 9, 11, 11, 11, 13, 17, 22, 26

The sorted data values are:

4, 7, 9, 11, 11, 11, 13, 17, 22, 26

#### 1. Sample (population) mean

$\bar{x} = \sum_{i=1}^n \frac{x_i}{n} = 13.1$  , and  $\mu = \sum_{i=1}^n \frac{x_i}{n} = 13.1$  (if this data set is a population).

#### 2. Median

The median of a given data set is the middle number of **sorted** data set. Based on this definition, the median of the given data set is: 11

#### 3. Mode

A data value that appears most frequently (frequency  $> 1$ ) is called the mode of the data. Based on this definition, a set of data may have one mode, more than one mode, or no mode at all. Using the above definition, this data set has 1 mode: 11 .

### Problem 7

The mean temperature in Glens Falls for the month of February is 23 degrees with a standard deviation of 4.2 degrees. What is the z-score for a temperature of 17 degrees (keeping 3 decimal places)?

Answers \*

☐

9.523

☒

-1.429

☐

1.429

☐

-2.928

☐

-0.340

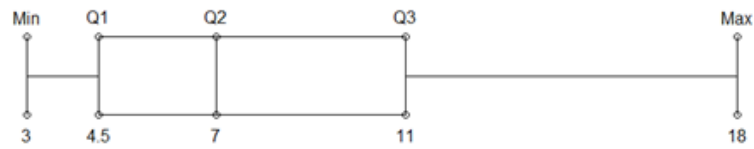
$$z_{17} = \frac{23 - 17}{4.2} \approx 1.428571$$

### Problem 8.



Construct a box-plot of data set: {3, 4, 4, 5, 5, 6, 8, 10, 10, 12, 15, 18}

Answers \*




## IntroStatsApps: Descriptive Statistics

**Types of Descriptive Statistics**  
Numerical Measures

comma separated numerical data  
3,4,4,5,5,6,8,10,10,12,15,18

**Measure Types**  
5-number summary and boxplot

  
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### Five Number Summary and Boxplot

The data values are:  
3, 4, 4, 5, 5, 6, 8, 10, 10, 12, 15, 18

#### 1. Five Number Summary :

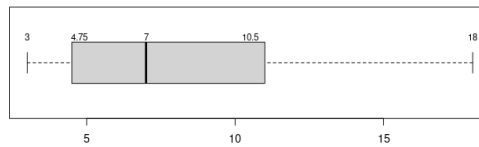
The five-number summary is use used to describe the shape of the distribution of a given numerical data. It consists of five numbers: minimum data value, first quartile, median, the third quartile, and the maximum data value.

The five-number summary of this given data set is:

stats	value
Min.	3.00
1st Qu.	4.75
Median	7.00
3rd Qu.	10.50
Max.	18.00

#### 2. Boxplot :

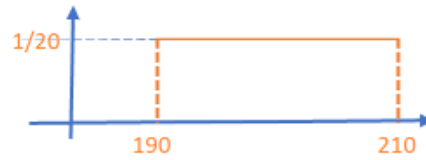
The boxplot is a geometric representation of the five-number summary. The boxplot of the given data set is given below.



**Choose a one that is close to the true one.**

### Problem 9

The cholesterol content of large chicken eggs is **uniformly** distributed between 190 and 210 milligrams. The density curve has a rectangular shape.



What proportion of these eggs has cholesterol content above 205 milligrams?

Answers \*



5/20



205/210



190/210



20/210

$$P(X > 205) = \frac{1}{20} (210 - 205) = \frac{1}{20} \times 5 = \frac{5}{20}$$

### Problem 10

Find the variance of the following **sample** data set taken from a population.

4 11 11 13 15 18

Answers \*

☐

112/6

☒

112/5

☐

72/6

☐

72/5

☐

976/5

## IntroStatsApps: Descriptive Statistics

**Types of Descriptive Statistics**


Numerical Measures

comma separated numerical data

4,11,11,13,15,18

**Measure Types**

measures of variation



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### Measures of Variation

The data values are:  
4, 11, 11, 13, 15, 18

The sorted data values are:  
4, 11, 11, 13, 15, 18

#### 1. Sample (population) variance

$$s^2 = \sum_{i=1}^n \frac{(x_i - \bar{x})^2}{n-1} = 22.4, \text{ and } \sigma^2 = \sum_{i=1}^n \frac{(x_i - \mu)^2}{n} = 18.7 \text{ (if this data set is a population)}$$

#### 2. Sample (population) standard deviation

The standard deviation is the square root of variance. Therefore, the both standard deviations are: 12

$$s = \sqrt{s^2} = \sqrt{\sum_{i=1}^n \frac{(x_i - \bar{x})^2}{n-1}} = 4.7, \text{ and } \sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^n \frac{(x_i - \mu)^2}{n}} = 4.3 \text{ (if this data set is a population)}$$

#### 3. Inter - quartile range (IQR)

The inter-quartile range is defined to the difference between the first and third quartiles. By the definition,  $IQR = P_{75} - P_{25} = 15 - 11 = 4$ .

$$\frac{112}{5} = 22.4$$

### Problem 11

A national achievement test is administered annually to 3rd graders. The test score is a continuous random variable that has a mean score of 100 and a standard deviation of 15. What is the probability that a randomly select student scored 95 in the test?

Answers \*



95/100



(100-95)/15



0



15/95



cannot be determined.

### Problem 12

Fifteen percent of the students in a school of Business Administration are majoring in Economics, 20% in Finance, 35% in Management, and 30% in Accounting. The graphical device(s) which can be used to present these data is (are)

Answers \*

☐

a line graph

☐

only a bar graph

☐

only a pie chart

☒

both a bar graph and a pie chart

☐

histogram

### Problem 13

A scientist obtained a normally distributed population of scores with a mean of 70 and a standard deviation of 10. What proportion of scores do you expect to find in the interval between 60 to 80?

Answers \*

☐

1.00

☐

0.50

☐

0.34

☒

0.68

☐

cannot be determined

☐ Probability ( $P_0$ )  
☐ Percentile ( $X_0$ )

---

2. Which Probability?

☐  $P[V_0 < X < V_1] = ?$   
☐  $P[X > V_0] = ?$   
☐  $P[X < V_0] = ?$

Given Value #1:  $V_0$

60

Given Value #2:  $V_1$

80

---

3. Input Information


Population Mean:  $\mu$

70

Population Standard Deviation:  $\sigma$

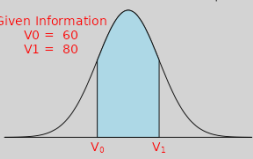
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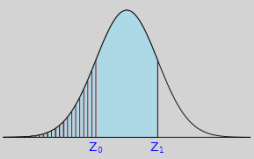
  
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General Normal Distribution (Fig. 9)

Given Information  
 $V_0 = 60$   
 $V_1 = 80$



Standard Normal Distribution



$Z = \frac{X - \mu}{\sigma}$

**Question:**  $P(60 < X < 80) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 70}{10}.$$

Step 2. Z-scores for  $V_0 = 60$  and  $V_1 = 80$  are given respectively by

$$Z_0 = \frac{60 - 70}{10} = -1 \text{ and } Z_1 = \frac{80 - 70}{10} = 1.$$

Step 3. The two left-tail Probabilities are

$$P(Z < 1) = 0.8413 \text{ and } P(Z < -1) = 0.1587.$$

Step 4. Note that

$$\begin{aligned}
 P(Z_0 < Z < Z_1) &= P(Z < Z_1) - P(Z < Z_0) \\
 &= P(Z < 1) - P(Z < -1) \\
 &= 0.8413 - 0.1587 = 0.6826.
 \end{aligned}$$

Step 5. Therefore,

$$P(60 < X < 80) = 0.6826.$$

## Problem 14



The scoring of modern IQ is such that Intelligence Quotients (IQs) have a normal distribution of  $\mu = 100$  and  $\sigma = 15$ .

Mensa International is a non-profit organization that accepts only people with IQ scores within the top 2%. What level of IQ qualifies one to be a member of Mensa?

Answers \*



115



130.8



145



120



cannot be determined

- Probability ( $P_0$ )
- Percentile ( $X_0$ )

### 2. $X_0$ in Which Probability?

- $P[X_0 < X < V] = P_0$
- $P[V < X < X_0] = P_0$
- $P[X > X_0] = P_0$
- $P[X < X_0] = P_0$

Given Probability:  $P_0$

0.02

### 3. Input Information

Population Mean:  $\mu$

100

Population Standard Deviation:  $\sigma$

15



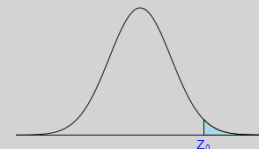
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General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $P_0 = 0.02$



Standard Normal Distribution



$$Z = \frac{X - \mu}{\sigma}$$

**Question:** Given  $P(X > X_0) = 0.02$ , what is  $X_0$ ?

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 100}{15}$$

Step 2. The Z-score corresponding to  $X_0$  is given by

$$Z_0 = \frac{X_0 - 100}{15}$$

Note that

$$P(Z > Z_0) = 0.02 \text{ or equivalently } P(Z < Z_0) = 0.98,$$

which gives,  $Z_0 = 2.05$ .

Step 3. By the definition of the Z-score of  $X_0$ , we have

$$\frac{X_0 - 100}{15} = Z_0 = 2.05.$$

Step 4. Therefore,  $X_0 = 100 + (2.05) \times 15 = 130.75$ .

## Problem 15.

A physical fitness association is including the mile run in its secondary-school fitness test. The time for this event for boys in secondary school is known to possess a normal distribution with a mean of 440 seconds and a standard deviation of 60 seconds. Find the probability that a randomly selected boy in secondary school can run the mile in less than 302 seconds.

Answers \*

- ☐ 0.9893
- ☒ 0.0107
- ☐ 0.5107
- ☐ 0.4893
- ☐ cannot be determined

**1. What to Find?**

- ☐ Probability ( $P_0$ )
- ☒ Percentile ( $X_0$ )

---

**2. Which Probability?**

- ☐  $P[V_0 < X < V_1] = ?$
- ☐  $P[X > V_0] = ?$
- ☒  $P[X < V_0] = ?$

**Given Value:  $V_0$**

302

---

**3. Input Information**


**Population Mean:  $\mu$**

440

**Population Standard Deviation:  $\sigma$**


60

---


  
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General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $V_0 = 302$



Standard Normal Distribution



$Z = \frac{X - \mu}{\sigma}$

**Question:**  $P(X < 302) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall the Z-score Transformation

$$Z = \frac{X - 440}{60}$$

Step 2. Z-scores for  $V = 302$  is given by

$$Z_0 = \frac{302 - 440}{60} = -2.3.$$

Step 3. The left-tail probability based on the above z-score is

$$P(Z < -2.3) = 0.0107.$$

Step 4. Note that

$$P(Z < Z_0) = P(Z < -2.3) = 0.0107.$$

Step 5. Therefore,

$$P(X < 302) = 0.0107.$$

### Problem 16

The following are 40 measurements of the iron-solution index of tin-plate specimens, designed to measure the corrosion resistance of tin-plated steel. The original data set has been sorted in ascending order as:

16, 26, 28, 28, 28, 28, 30, 32, 34, 35, 36, 36, 37, 37, 40, 40, 40, 41, 41, 41,  
42, 42, 42, 43, 43, 43, 44, 44, 44, 44, 45, 45, 45, 45, 45, 45, 46, 46, 46, 46,

We want to Construct a frequency table with **five** rows. Which of the following histogram is correct?

Answers \*

☐

[15,31]	7
(31,42]	16
(42,53]	17
(53,64]	0
(64,75]	0

☐

[15,25]	1
(25,35]	9
(35,40]	7
(40,45]	19
(45,50]	4



[15,23]	1
(23,31]	6
(31,39]	7
(39,46]	26
(46,53]	0



[15,22]	1
(22,29]	5
(29,36]	6
(36,43]	14
(43,50]	14

Based on the procedure of constructing histogram of a numerical data set.

**Problem 17.**

The age distribution of students at a community college is given below.

Age (years)	Number of students (f)
Under 21	2189
21-25	2031
26-30	1073
31-35	853
Over 35	221

A student from the community college is selected at random. The event  $E$  is defined as follows.

$E$  = event the student is between 26 and 35 inclusive.

Determine the number of outcomes that comprise the event  $\bar{E}$  (i.e., not in  $E$ ).

Answers \*



4441



5294



4220



1926

$$2189 + 2031 + 221 = 4441.$$

**Problem 18**

Tomkins Associates reports that the mean clear height for a Class A warehouse in the United States is 22 feet. Suppose clear heights are normally distributed and that the standard deviation is 4 feet. A Class A warehouse in the United States is randomly selected,

What is the probability that the clear height is greater than 17 feet?

Answers \*



0.1056



0.8994



1.25



-1.25

#### 1. What to Find?

- ☐ Probability ( $P_0$ )
- ☒ Percentile ( $X_0$ )

#### 2. Which Probability?

- ☒  $P[V_0 < X < V_1] = ?$
- ☐  $P[X > V_0] = ?$
- ☐  $P[X < V_0] = ?$

Given Value:  $V_0$

17

#### 3. Input Information

Population Mean:  $\mu$

22

Population Standard Deviation:  $\sigma$

4

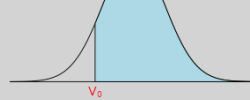


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General Normal Distribution  $N(\mu, \sigma)$

Given Information

$V_0 = 17$



Standard Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$



**Question:**  $P(X > 17) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - \mu}{\sigma}$$

Step 2. Z-scores for  $V = 17$  is given by

$$Z_0 = \frac{17 - 22}{4} = -1.25.$$

Step 3. The left-tail Probability based on the above z-score is

$$P(Z < -1.25) = 0.1056.$$

Step 4. Note that

$$P(Z > Z_1) = 1 - P(Z < -1.25) = 1 - 0.1056 = 0.8944.$$

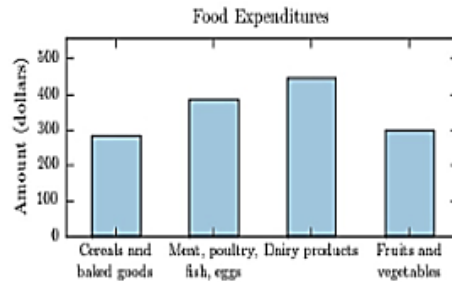
Step 5. Therefore,

$$P(X > 17) = 0.8944.$$

## Problem 19

The following bar graph presents the average amount a certain family spent, in dollars, on various food categories in a recent year.

On which food category was the most money spent?



Answers \*



Dairy products



Fruits and vegetables



Meat poultry, fish, eggs



Cereals and baked goods

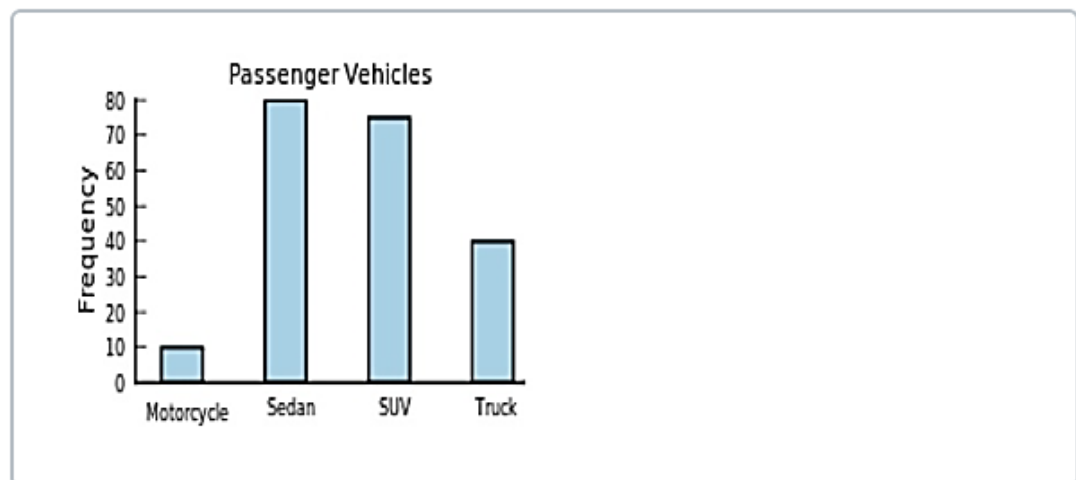
Problem 20

The following frequency distribution presents the frequency of passenger vehicles that pass through a certain intersection from 8:00 AM to 9:00 AM on a particular day.

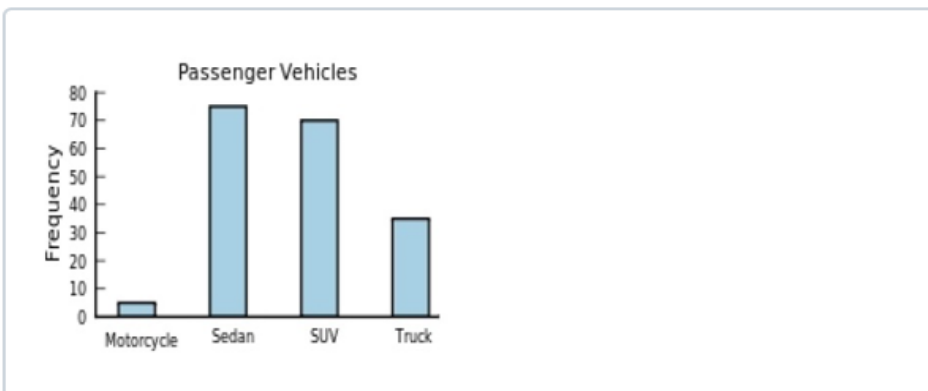
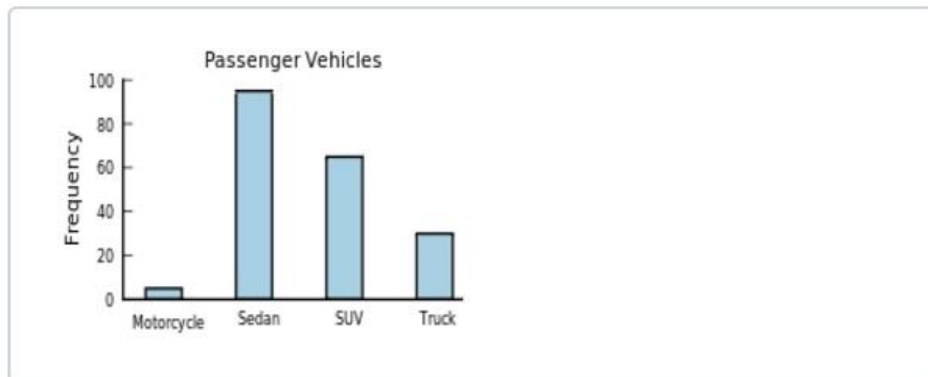
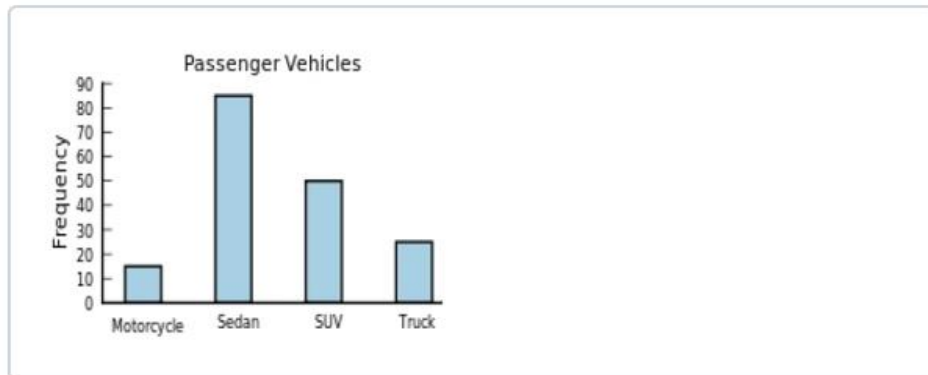
Vehicle Type	Frequency
Motorcycle	5
Sedan	75
SUV	70
Truck	35

Construct a frequency bar graph for the data.

Answers \*





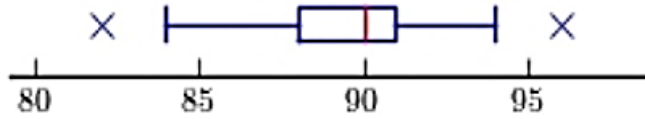
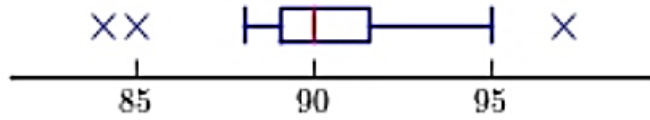
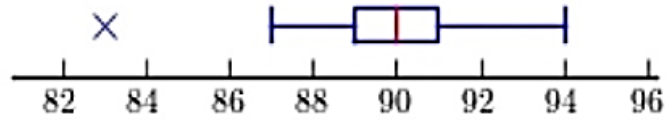


### Problem 21

Construct a boxplot for the data set below.

95	95	95	97	90
89	90	85	91	84
89	91	92	89	89
89	88	89	91	90

Answers \*



## Problem 22

Choose the answer below that best completes the following statement.

A \_\_\_\_\_ is a number that describes a sample.

Answers \*

☐

measurement

☐

population

☒

statistic

☐

parameter

### Problem 23

The sum of the relative frequencies for all classes will always equal

Answers \*

☒

one

☐

the number of classes

☐

the number of items in the study

☐

100

### Problem 24

Let  $Z$  be the standard normal random variable. Given that  $P(Z < Z_0) = 0.758$ , what is  $Z_0$ ?

Answers \*



0.750



0.700



0.242



-0.70

**1. What to Find?**

- Probability ( $P_0$ )
- Percentile ( $X_0$ )

**2.  $X_0$  in Which Probability?**

- $P[X_0 < X < V] = P_0$
- $P[V < X < X_0] = P_0$
- $P[X > X_0] = P_0$
- $P[X < X_0] = P_0$

**Given Probability:  $P_0$**

0.758


**3. Input Information**

**Population Mean:  $\mu$**

0

**Population Standard Deviation:  $\sigma$**

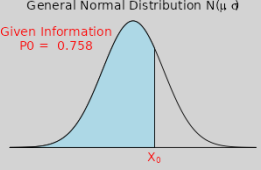
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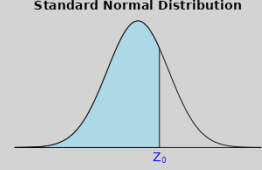
General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $P_0 = 0.758$



$X_0$

Standard Normal Distribution



$Z_0$

$Z = \frac{X - \mu}{\sigma}$

**Question:** Given  $P(X < X_0) = 0.758$ , what is  $X_0$ ?

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - \mu}{\sigma} \text{ and } Z = \frac{X_0 - \mu}{\sigma}$$

Step 2. The given condition  $P(X < X_0) = 0.758$  is equivalent to

$$P(Z < Z_0) = 0.758$$

which gives,  $Z_0 = 0.7$ .

Step 3. Note that,

$$\frac{X_0 - 0}{1} = Z_0 = 0.7.$$

Step 4. Solve for  $X_0$  from the above equation, we have,

$$X_0 = 0 + (0.7) \times 1 = 0.7.$$

### Problem 25

Tomkins Associates reports that the mean clear height for a Class A warehouse in the United States is 22 feet. Suppose clear heights are normally distributed and that the standard deviation is 4 feet.

Find the clear height such that 10% of all clear heights are less than it.

Answers \*

☐

0.9

☐

1.28

☐

-1.28

☒

16.88

**1. What to Find?**

- ☒ Probability ( $P_0$ )
- ☐ Percentile ( $X_0$ )

---

**2.  $X_0$  in Which Probability?**

- ☐  $P[X_0 < X < V] = P_0$
- ☐  $P[V < X < X_0] = P_0$
- ☐  $P[X > X_0] = P_0$
- ☒  $P[X < X_0] = P_0$

**Given Probability:  $P_0$**

0.1

---

**3. Input Information**


**Population Mean:  $\mu$**

22

**Population Standard Deviation:  $\sigma$**

4

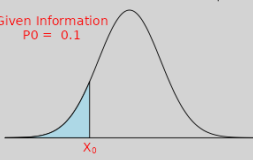
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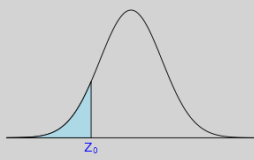
General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $P_0 = 0.1$



$X_0$

Standard Normal Distribution



$Z_0$

$Z = \frac{X - \mu}{\sigma}$

**Question:** Given  $P(X < X_0) = 0.1$ , what is  $X_0$ ?

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - 22}{4} \text{ and } Z = \frac{X_0 - 22}{4}.$$

Step 2. The given condition  $P(X < X_0) = 0.1$  is equivalent to

$$P(Z < Z_0) = 0.1$$

which gives,  $Z_0 = -1.28$ .

Step 3. Note that,

$$\frac{X_0 - 22}{4} = Z_0 = -1.28.$$

Step 4. Solve for  $X_0$  from the above equation, we have,

$$X_0 = 22 + (-1.28) \times 4 = 16.88.$$

### Problem 26.

The GMAT scores of students in a college are normally distributed with a mean of 520 and a standard deviation of 41. What proportion of students have a score higher than 600?

- A) 0.9744
- B) 0.2372
- C) 0.4774
- D) 0.0255

**Answer: D**

**1. What to Find?**

- ☐ Probability ( $P_0$ )
- ☒ Percentile ( $X_0$ )

---

**2. Which Probability?**

- ☒  $P[V_0 < X < V_1] = ?$
- ☐  $P[X > V_0] = ?$
- ☐  $P[X < V_0] = ?$

**Given Value:  $V_0$**

600

---

**3. Input Information**


**Population Mean:  $\mu$**

520

**Population Standard Deviation:  $\sigma$**

41

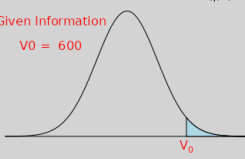
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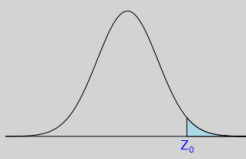
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General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $V_0 = 600$



Standard Normal Distribution



$$Z = \frac{X - \mu}{\sigma}$$

**Question:**  $P(X > 600) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 520}{41}$$

Step 2. Z-scores for  $V = 600$  is given by

$$Z_0 = \frac{600 - 520}{41} = 1.9512.$$

Step 3. The left-tail Probability based on the above z-score is

$$P(Z < 1.9512) = 0.9745.$$

Step 4. Note that

$$P(Z > Z_1) = 1 - P(Z < 1.9512) = 1 - 0.9745 = 0.0255.$$

Step 5. Therefore,

$$P(X > 600) = 0.0255.$$

## Problem 27

A grocery store has a mean accounts receivable of \$264, with a standard deviation of \$55. The accounts receivable are approximately normally distributed. Find the value such that 45% of all the accounts exceed this value. That is, find  $x$  such that:  $P(X > x) = 0.45$ .

- A) \$257.13
- B) \$354.48
- C) \$270.91
- D) \$309.00

**Answer: C**

**1. What to Find?**

- Probability ( $P_0$ )
- Percentile ( $X_0$ )

---

**2.  $X_0$  in Which Probability?**

- $P[X_0 < X < V] = P_0$
- $P[V < X < X_0] = P_0$
- $P[X > X_0] = P_0$
- $P[X < X_0] = P_0$

**Given Probability:  $P_0$**

0.45

---

**3. Input Information**


**Population Mean:  $\mu$**

264

**Population Standard Deviation:  $\sigma$**

55

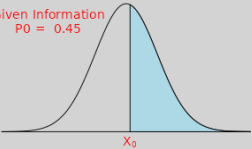
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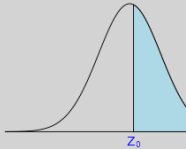
General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $P_0 = 0.45$



$X_0$

Standard Normal Distribution



$Z_0$

$Z = \frac{X - \mu}{\sigma}$

**Question:** Given  $P(X > X_0) = 0.45$ , what is  $X_0$ ?

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 264}{55}.$$

Step 2. The Z-score corresponding to  $X_0$  is given by

$$Z_0 = \frac{X_0 - 264}{55}.$$

Note that

$$P(Z > Z_0) = 0.45 \text{ or equivalently } P(Z < Z_0) = 0.55,$$

which gives,  $Z_0 = 0.13$ .

Step 3. By the definition of the Z-score of  $X_0$ , we have

$$\frac{X_0 - 264}{55} = Z_0 = 0.13.$$

Step 4. Therefore,  $X_0 = 264 + (0.13) \times 55 = 271.15$ .

### Problem 28.

The contents of a particular bottle of shampoo marked as 150 ml are found to be 153 ml at an average, with a standard deviation of 2.5 ml. What proportion of shampoo bottles contain less than the marked quantity? Assume a normal distribution.

- A) 0.2192
- B) 0.1151
- C) 0.4452
- D) 0.0548

**Answer: B.**



### 1. What to Find?

- ☐ Probability ( $P_0$ )
- ☒ Percentile ( $X_0$ )

### 2. Which Probability?

- ☐  $P[V_0 < X < V_1] = ?$
- ☐  $P[X > V_0] = ?$
- ☒  $P[X < V_0] = ?$

Given Value:  $V_0$

### 3. Input Information

Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$

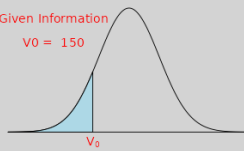


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General Normal Distribution  $N(\mu, \sigma)$

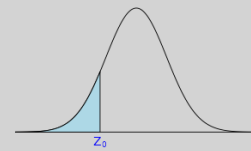
Given Information

$V_0 = 150$



Standard Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$



**Question:**  $P(X < 150) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall the Z-score Transformation

$$Z = \frac{X - \mu}{\sigma}$$

Step 2. Z-scores for  $V = 150$  is given by

$$Z_0 = \frac{150 - 153}{2.5} = -1.2.$$

Step 3. The left-tail probability based on the above z-score is

$$P(Z < -1.2) = 0.1151.$$

Step 4. Note that

$$P(Z < Z_0) = P(Z < -1.2) = 0.1151.$$

Step 5. Therefore,

$$P(X < 150) = 0.1151.$$

## Problem 29

A spark plug manufacturer believes that his plug lasts an average of 30,000 miles, with a standard deviation of 2,500 miles. What is the probability that a given spark plug of this type will last 37,500 miles before replacement?

- A) 0.0228
- B) 0.0114
- C) 0.0013
- D) 0.0714
- E) 0.0833

**Answer: C**

**1. What to Find?**

☐ Probability ( $P_0$ )

☒ Percentile ( $X_0$ )

---

**2. Which Probability?**

☐  $P[V_0 < X < V_1] = ?$

☐  $P[X > V_0] = ?$

☐  $P[X < V_0] = ?$

**Given Value:  $V_0$**

37500

---

**3. Input Information**


**Population Mean:  $\mu$**

30000

**Population Standard Deviation:  $\sigma$**

2500

---

  
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General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $V_0 = 37500$

$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution

**Question:**  $P(X > 37500) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 30000}{2500}.$$

Step 2. Z-scores for  $V = 37500$  is given by

$$Z_0 = \frac{37500 - 30000}{2500} = 3.$$

Step 3. The left-tail Probability based on the above z-score is

$$P(Z < 3) = 0.9987.$$

Step 4. Note that

$$P(Z > Z_1) = 1 - P(Z < 3) = 1 - 0.9987 = 0.0013.$$

Step 5. Therefore,

$$P(X > 37500) = 0.0013.$$

### Problem 30

The IQs of the employees of a company are normally distributed, with a mean of 127 and a standard deviation of 11. What is the probability that the IQ of an employee selected at random will be between 120 and 130?

- A) 0.2389
- B) 0.3453
- C) 0.1064
- D) 0.1325
- E) 0.4638

**Answer: B**

☐ Probability ( $P_0$ )  
☒ Percentile ( $X_0$ )


2. Which Probability?  
☐  $P[V_0 < X < V_1] = ?$   
☒  $P[X > V_0] = ?$   
☐  $P[X < V_0] = ?$

Given Value #1:  $V_0$

Given Value #2:  $V_1$

3. Input Information  
Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$

  
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General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $V_0 = 120$   
 $V_1 = 130$

Standard Normal Distribution

$Z = \frac{X - \mu}{\sigma}$

Question:  $P(120 < X < 130) = ?$

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 127}{11}$$

Step 2. Z-scores for  $V_0 = 120$  and  $V_1 = 130$  are given respectively by

$$Z_0 = \frac{120 - 127}{11} = -0.64 \text{ and } Z_1 = \frac{130 - 127}{11} = 0.27.$$

Step 3. The two left-tail Probabilities are

$$P(Z < 0.27) = 0.6064 \text{ and } P(Z < -0.64) = 0.2611.$$

Step 4. Note that

$$\begin{aligned} P(Z_0 < Z < Z_1) &= P(Z < Z_1) - P(Z < Z_0) \\ &= P(Z < 0.27) - P(Z < -0.64) \\ &= 0.6064 - 0.2611 = 0.3453. \end{aligned}$$

Step 5. Therefore,

$$P(120 < X < 130) = 0.3453.$$

### Problem 31

Which of the following is a correct statement about probability?

- A) Probability values range from 0 to 1, inclusive.
- B) Probabilities may assume negative values.
- C) Probabilities may be greater than 1.
- D) Probabilities are limited to one decimal place.

**Answer: A**

### Problem 32

The lengths of pregnancies of humans are normally distributed with a mean of 268 days and a standard deviation of 15 days. Find the probability of a pregnancy lasting less than 250 days.

- A) 0.1591
- B) 0.0606
- C) 0.0066
- D) 0.115

**Answer: D**

**1. What to Find?**

- ☐ Probability ( $P_0$ )
- ☒ Percentile ( $X_0$ )

---

**2. Which Probability?**

- ☐  $P[V_0 < X < V_1] = ?$
- ☐  $P[X > V_0] = ?$
- ☐  $P[X < V_0] = ?$

**Given Value:  $V_0$**

250

---

**3. Input Information**


**Population Mean:  $\mu$**

268

**Population Standard Deviation:  $\sigma$**

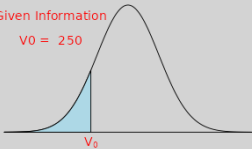
15

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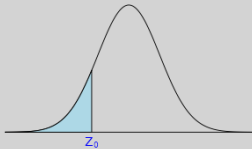
General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $V_0 = 250$



$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



**Question:**  $P(X < 250) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall the Z-score Transformation

$$Z = \frac{X - 268}{15}.$$

Step 2. Z-scores for  $V = 250$  is given by

$$Z_0 = \frac{250 - 268}{15} = -1.2.$$

Step 3. The left-tail probability based on the above z-score is

$$P(Z < -1.2) = 0.1151.$$

Step 4. Note that

$$P(Z < Z_0) = P(Z < -1.2) = 0.1151.$$

Step 5. Therefore,

$$P(X < 250) = 0.1151.$$

### Problem 33

The distribution of cholesterol levels in teenage boys is approximately normal with  $\mu = 170$  and  $\sigma = 30$  (Source: U.S. National Center for Health Statistics). Levels above 200 warrant attention. Find the probability that a teenage boy has a cholesterol level greater than 200.

- A) 0.3419
- B) 0.2138
- C) 0.8413
- D) 0.1587

**Answer: D**

**1. What to Find?**

☐ Probability ( $P_0$ )

☒ Percentile ( $X_0$ )

---

**2. Which Probability?**

☒  $P[V_0 < X < V_1] = ?$

☐  $P[X > V_0] = ?$

☐  $P[X < V_0] = ?$

**Given Value:  $V_0$**

200

---

**3. Input Information**


**Population Mean:  $\mu$**

170

**Population Standard Deviation:  $\sigma$**

30

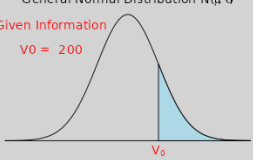
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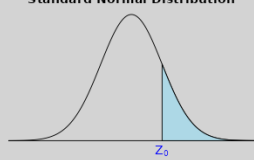
General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $V_0 = 200$



$V_0$

Standard Normal Distribution



$Z_0$

$Z = \frac{X - \mu}{\sigma}$

**Question:**  $P(X > 200) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 170}{30}.$$

Step 2. Z-scores for  $V = 200$  is given by

$$Z_0 = \frac{200 - 170}{30} = 1.$$

Step 3. The left-tail Probability based on the above z-score is

$$P(Z < 1) = 0.8413.$$

Step 4. Note that

$$P(Z > Z_1) = 1 - P(Z < 1) = 1 - 0.8413 = 0.1587.$$

Step 5. Therefore,

$$P(X > 200) = 0.1587.$$

### Problem 34

) IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. What is the IQ scores that correspond to the 85 percentile?

- a. 115.6
- b. 120.3
- c. 126.6
- d. 146.2

**Answer: A**

**1. What to Find?**

- ☒ Probability ( $P_0$ )
- ☐ Percentile ( $X_0$ )

---

**2.  $X_0$  in Which Probability?**

- ☒  $P[X_0 < X < V] = P_0$
- ☐  $P[V < X < X_0] = P_0$
- ☐  $P[X > X_0] = P_0$
- ☐  $P[X < X_0] = P_0$

**Given Probability:  $P_0$**


---

**3. Input Information**

**Population Mean:  $\mu$**

**Population Standard Deviation:  $\sigma$**

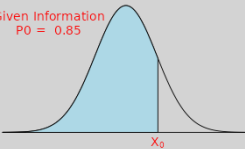
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General Normal Distribution  $N(\mu, \sigma)$

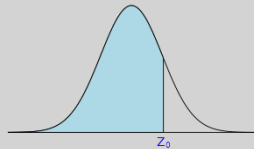
Given Information  
 $P_0 = 0.85$



$X_0$

$$Z = \frac{X - \mu}{\sigma}$$

Standard Normal Distribution



$Z_0$

**Question:** Given  $P(X < X_0) = 0.85$ , what is  $X_0$ ?

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - 100}{15} \text{ and } Z = \frac{X_0 - 100}{15}.$$

Step 2. The given condition  $P(X < X_0) = 0.85$  is equivalent to

$$P(Z < Z_0) = 0.85$$

which gives,  $Z_0 = 1.04$ .

Step 3. Note that,

$$\frac{X_0 - 100}{15} = Z_0 = 1.04.$$

Step 4. Solve for  $X_0$  from the above equation, we have,

$$X_0 = 100 + (1.04) \times 15 = 115.6.$$

### Problem 35

For a population of giraffes, the heights are measured. The sample mean is 16.5 ft; the sample standard deviation is 3.8. What percentage of giraffes are taller than 20 ft.

- a. 8%
- b. 15%
- c. 18%
- d. 21%

**Answer C**

**1. What to Find?**

☐ Probability ( $P_0$ )

☒ Percentile ( $X_0$ )

---

**2. Which Probability?**

☐  $P[V_0 < X < V_1] = ?$

☐  $P[X > V_0] = ?$

☐  $P[X < V_0] = ?$

Given Value:  $V_0$

20

---

**3. Input Information**


Population Mean:  $\mu$

16.5

Population Standard Deviation:  $\sigma$

3.8

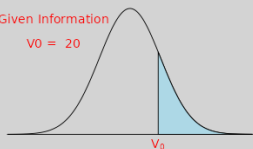
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General Normal Distribution  $N(\mu, \sigma)$

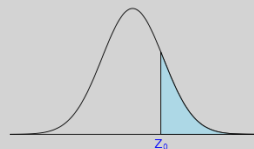
Given Information  
 $V_0 = 20$



$V_0$

$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



$Z_0$

**Question:**  $P(X > 20) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 16.5}{3.8}.$$

Step 2. Z-scores for  $V = 20$  is given by

$$Z_0 = \frac{20 - 16.5}{3.8} = 0.9211.$$

Step 3. The left-tail Probability based on the above z-score is

$$P(Z < 0.9211) = 0.8215.$$

Step 4. Note that

$$P(Z > Z_1) = 1 - P(Z < 0.9211) = 1 - 0.8215 = 0.1785.$$

Step 5. Therefore,

$$P(X > 20) = 0.1785.$$

# Midterm #1 Summary

## 1. Five Number Summary :

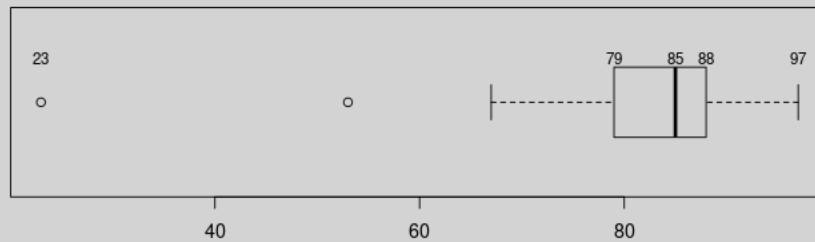
The five-number summary is use used to describe the shape of the distribution of a given numerical data. It consists of five numbers: minimum data value, first quartile, median, the third quartile, and the maximum data value.

The five-number summary of this given data set is:

stats	value
Min.	23.00
1st Qu.	79.00
Median	85.00
3rd Qu.	88.00
Max.	97.00

## 2. Boxplot :

The boxplot is a geometric representation of the five-number summary. The boxplot of the given data set is given below.



The class boundary is: 20, 30,40,50,60,70,80,90,100

cut.data.freq	Freq	midpts	rel.freq	cum.freq	rel.cum.freq
[2e+01,3e+01]	1	25.00	0.03	1	0.03
(3e+01,4e+01]	0	35.00	0.00	1	0.03
(4e+01,5e+01]	0	45.00	0.00	1	0.03
(5e+01,6e+01]	1	55.00	0.03	2	0.06
(6e+01,7e+01]	4	65.00	0.12	6	0.18
(7e+01,8e+01]	4	75.00	0.12	10	0.29
(8e+01,9e+01]	19	85.00	0.56	29	0.85
(9e+01,1e+02]	5	95.00	0.15	34	1.00



