

## **Week 09 Quiz: One-sample Normal Tests**

### **Problem 1**

**The primary goal of a one-sample Z-test is to:**

- a) Estimate the population mean.
- b) Test whether a sample mean is significantly different from a hypothesized population mean.
- c) Compare the means of two independent samples.
- d) Calculate the probability of observing the sample data.

**Answer: B**

### **Problem 2.**

**For a large one-sample Z-test with unknown population variance, the test statistic is calculated as:**

- a)  $(\bar{x} - \mu_0) / (s/\sqrt{n})$
- b)  $(\bar{x} - \mu_0) / (\sigma/\sqrt{n})$
- c)  $(\mu - \mu_0) / (\sigma/\sqrt{n})$
- d)  $(\bar{x} - \mu) / (s/\sqrt{n})$

**Answer: A**

### **Problem 3**

**A sample of size  $n=25$  taken from a normal population has a mean of 110. The hypothesized population mean is 100, and the population standard deviation is known to be 15. What is the Z-test statistic?**

- a) 3.33
- b) 0.67
- c) 10.0
- d) 1.67

**Answer: A**

### **Problem 4**

**A factory process produces steel bars with a mean length of 120 cm and a known  $\sigma=5$  cm. A new process is tested on 40 bars, yielding a sample mean**

of 118.5 cm. What is the calculated z-test statistic for  $H_0: \mu=120$  vs.  $H_1: \mu<120$ ?

- a) 1.5
- b) -1.5
- c) -1.897
- d) -0.237

### Answer: C

**Data Source**

summarized statistics  
 raw data

**sample mean ( $\bar{x}$ )**  
118.5

**sample standard deviation ( $s$ )**  
5

**sample size ( $n$ )**  
40

**Claimed Value ( $\mu_0$ )**  
120

**Claim Type**  
less than

**Significance level  $\alpha$**   
0.05



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**Solution:** This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

**Given sample information:**  $n = 40$ ,  $\bar{x} = 118.5$ ,  $s = 5$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is less than 120.

**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 120$  and  $H_1 : \mu < 120$ .

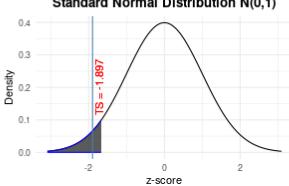
**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{118.5 - 120}{5/\sqrt{40}} = -1.897$

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical value  $z_\alpha$  to be:  $-z_{\alpha} = -z_{0.05} = -1.645$   
The p-value is can be found as  $p\text{-value} \approx 0.029$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 5% significance level, we reject the null hypothesis. ( $p\text{-value} = 0.029$ ).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 5% significance level, we conclude the alternative hypothesis.

**Standard Normal Distribution  $N(0,1)$**



### Problem 5

For a two-tailed one-sample z-test at  $\alpha=0.05$ , what are the critical values that define the rejection region?

- a)  $z = \pm 1.96$
- b)  $z = \pm 1.645$
- c)  $z = +1.96$
- d)  $z = +1.645$

### Answer: A

### Problem 6

**The principal claims the mean IQ of students at a school is 110. A psychologist tests 36 random students, finding a mean of 107. The population standard deviation is known to be 15. If the p-value for the two-sided test is 0.27, what is the correct conclusion at  $\alpha=0.05$ ?**

- a) Reject  $H_0$ ; the mean IQ is significantly different from 110.
- b) Fail to reject  $H_0$ ; there is not enough evidence that the mean IQ is different from 110.
- c) Reject  $H_0$ ; the mean IQ is less than 110.
- d) Accept  $H_0$ ; the mean IQ is exactly 110.

### Answer: D

**Data Source**  
 summarized statistics  
 raw data

**sample mean ( $\bar{x}$ )**  
107

**sample standard deviation ( $s$ )**  
15

**sample size ( $n$ )**  
36

**Claimed Value ( $\mu_0$ )**  
110

**Claim Type**  
equal to

**Significance level  $\alpha$**   
0.01    **0.05**    0.2



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**Solution:** This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

**Given sample information:**  $n = 36$ ,  $\bar{x} = 107$ ,  $s = 15$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is equal to 110.

**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 110$  and  $H_1 : \mu \neq 110$ .

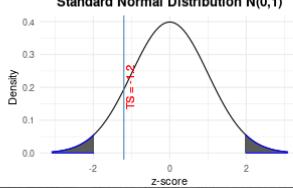
**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{107 - 110}{15/\sqrt{36}} = -1.2$

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical values to be:  $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$   
The p-value is can be found as p-value  $\approx 0.23$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 5% significance level, we do not reject the null hypothesis. (p-value = 0.23).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 5% significance level, we reject the alternative hypothesis .

**Standard Normal Distribution  $N(0,1)$**



### Problem 7

**A pizza restaurant advertises a mean delivery time of 25 minutes. A manager collects a sample of 50 deliveries, finding a mean time of 26.5 minutes with a standard deviation of 4 minutes. The test statistic is 2.65. What is the approximate p-value for this two-tailed test?**

- a) 0.004
- b) 0.008
- c) 0.05
- d) 0.02

## Answer: B

**Data Source**  
 summarized statistics  
 raw data

**sample mean ( $\bar{x}$ )**  
26.5

**sample standard deviation ( $s$ )**  
4

**sample size ( $n$ )**  
50

**Claimed Value ( $\mu_0$ )**  
25

**Claim Type**  
equal to

**Significance level  $\alpha$**   
0.05

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**Solution:** This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

**Given sample information:**  $n = 50$ ,  $\bar{x} = 26.5$ ,  $s = 4$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is equal to 25.

**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 25$  and  $H_1 : \mu \neq 25$ .

**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{26.5 - 25}{4/\sqrt{50}} = 2.652$

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical values to be:  $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$   
The p-value is can be found as p-value  $\approx 0.008000000000000001$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 5% significance level, we reject the null hypothesis. (p-value = 0.008).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 5% significance level, we conclude the alternative hypothesis.

**Standard Normal Distribution  $N(0,1)$**   
Density vs. z-score. The curve is centered at 0. A vertical red line marks the test statistic  $TS = 2.652$  at approximately z=2.65. The area under the curve to the right of this line is shaded blue, representing the p-value.

## Problem 8

**A bottling plant claims its 2-liter bottles contain a mean of 2.01 liters. A regulator measures 36 bottles, finding a mean volume of 2.005 liters with a standard deviation of 0.015 liters. For a two-tailed test at  $\alpha=0.05$ , what is the correct statistical decision?**

- a) Reject  $H_0$  because the sample mean is less than 2.01.
- b) Fail to reject  $H_0$  because the test statistic is within the critical region.
- c) Reject  $H_0$  because the p-value is less than 0.05.
- d) Fail to reject  $H_0$  because the p-value is greater than 0.05.

## Answer: C

**Data Source**

- summarized statistics
- raw data

**sample mean ( $\bar{x}$ )**  
2.005

**sample standard deviation ( $s$ )**  
0.015

**sample size ( $n$ )**  
36

**Claimed Value ( $\mu_0$ )**  
2.01

**Claim Type**  
equal to

**Significance level  $\alpha$**   
0.05



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**Solution:** This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

**Given sample information:**  $n = 36$ ,  $\bar{x} = 2.005$ ,  $s = 0.015$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is equal to 2.01.

**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 2.01$  and  $H_1 : \mu \neq 2.01$ .

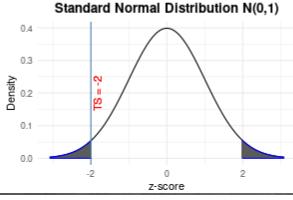
**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.005 - 2.01}{0.015/\sqrt{36}} = -2$

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical values to be:  $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$   
The p-value is can be found as p-value  $\approx 0.046$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 5% significance level, we reject the null hypothesis. (p-value = 0.046).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 5% significance level, we conclude the alternative hypothesis.

**Standard Normal Distribution  $N(0,1)$**



### Problem 9

An ISP guarantees mean network latency is below 50 ms. A user runs 35 speed tests, finding a mean latency of 52 ms with a known population standard deviation of 6 ms. What is the z-test statistic for  $H_1: \mu > 50$ ?

- a) 1.75
- b) 1.97
- c) 2.36
- d) 2.08

**Answer: B**

$$(52-50)/(6/\sqrt{35})=1.972$$

### Problem 10

A car model is rated at 30 mpg highway. A magazine tests 45 cars and finds a mean of 29.2 mpg with a standard deviation of 1.5 mpg. They perform a one-sample Z-test. What is the critical value for a two-tailed test at  $\alpha=0.10$ ?

- a)  $\pm 1.761$

- b)  $\pm 1.645$
- c)  $\pm 1.345$
- d)  $\pm 1.753$

**Answer: B**

**Problem 11**

**A hotel claims its average check-in time is under 2 minutes. A mystery guest records 40 check-ins. The sample mean is 2.1 minutes with a known population standard deviation of 0.4 minutes. For  $H_1: \mu < 2$ , what is the p-value?**

- a) 0.9429
- b) 0.0571
- c) 0.1142
- d) 0.8858

**Answer: A**

**Problem 12**

**A company requires data entry clerks to have a mean speed of 60 words per minute (wpm). A supervisor tests 38 new clerks, finding a mean of 58 wpm and a standard deviation of 5 wpm. For a one-tailed test ( $H_1: \mu < 60$ ) at  $\alpha=0.05$ , what is the decision?**

- a) Reject  $H_0$ ; clerks are too slow.
- b) Fail to reject  $H_0$ ; cannot prove clerks are too slow.
- c) Accept  $H_0$ ; clerks are fast enough.
- d) The result is inconclusive.

**Answer: A**

**Data Source**

summarized statistics  
 raw data

**sample mean ( $\bar{x}$ )**  
58

**sample standard deviation (s)**  
5

**sample size (n)**  
38

**Claimed Value ( $\mu_0$ )**  
60

**Claim Type**  
greater than or equal to

**Significance level  $\alpha$**   
0.05



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**Solution:** This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

**Given sample information:**  $n = 38$ ,  $\bar{x} = 58$ ,  $s = 5$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is greater than or equal to 60.

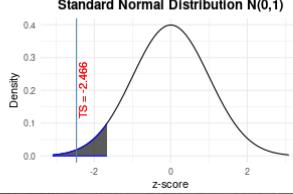
**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 60$  and  $H_1 : \mu < 60$ .

**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{58 - 60}{5/\sqrt{38}} = -2.466$

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical values to be:  $-z_\alpha = -z_{0.05} = -1.645$   
The p-value is can be found as p-value  $\approx 0.007$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 5% significance level, we reject the null hypothesis. (p-value = 0.007).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 5% significance level, we conclude the alternative hypothesis.



### Problem 13

An environmental standard states a river's pH should be 7.0. A researcher takes 36 water samples, finding a mean pH of 6.9 with a standard deviation of 0.15. What is the standard error of the mean?

- a) 0.15
- b) 0.025
- c) 0.06
- d) 0.003

**Answer: B**

### Problem 14

An assembly line is designed to produce one item every 45 seconds. A time-motion study on 50 items finds a mean time of 46 seconds with a known standard deviation of 3 seconds. What is the z-test statistic for  $H_1: \mu \neq 45$ ?

- a) 1.67
- b) 2.36

- c) 3.12
- d) 4.01

## Answer: B

**Data Source**

- summarized statistics
- raw data

**sample mean ( $\bar{x}$ )**  
46

**sample standard deviation ( $s$ )**  
3

**sample size ( $n$ )**  
50

**Claimed Value ( $\mu_0$ )**  
45

**Claim Type**  
equal to

**Significance level  $\alpha$**   
0.05

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**Solution:** This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

**Given sample information:**  $n = 50$ ,  $\bar{x} = 46$ ,  $s = 3$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is equal to 45.

**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 45$  and  $H_1 : \mu \neq 45$ .

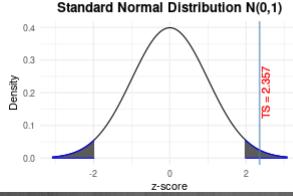
**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{46 - 45}{3/\sqrt{50}} = 2.357$

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical values to be:  $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$ .  
The p-value is can be found as p-value  $\approx 0.018$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 5% significance level, we reject the null hypothesis. (p-value = 0.018).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 5% significance level, we conclude the alternative hypothesis.

**Standard Normal Distribution  $N(0,1)$**



## Problem 15

**A coffee franchise standard requires milk to be steamed for 35 seconds. A regional manager times 36 baristas, finding a mean time of 33 seconds with a standard deviation of 4 seconds. The test is  $H_1: \mu \neq 35$ . What is the correct conclusion at  $\alpha=0.10$ ? (The critical value is 1.96)**

- a) Reject  $H_0$  because  $|t| > 1.96$ .
- b) Fail to reject  $H_0$  because  $|t| < 1.96$ .
- c) Reject  $H_0$  because the sample mean is less than 35.
- d) Accept  $H_0$  because the p-value is large.

## Answer: A

**Data Source**

- summarized statistics
- raw data

**sample mean ( $\bar{x}$ )**  
33

**sample standard deviation ( $s$ )**  
4

**sample size ( $n$ )**  
36

**Claimed Value ( $\mu_0$ )**  
35

**Claim Type**  
equal to

**Significance level  $\alpha$**   
0.05



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**Solution:** This normal test is based on the Central Limit Theorem *CLT*. Since the sample size is larger than 30, the test result is reliable.

**Given sample information:**  $n = 36$ ,  $\bar{x} = 33$ ,  $s = 4$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is equal to 35.

**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 35$  and  $H_1 : \mu \neq 35$ .

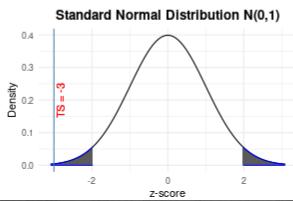
**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{33 - 35}{4/\sqrt{36}} = -3$

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical values to be:  $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$   
The p-value is can be found as p-value  $\approx 0.002$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 5% significance level, we reject the null hypothesis. (p-value = 0.002).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 5% significance level, we conclude the alternative hypothesis.

**Standard Normal Distribution  $N(0,1)$**



## Summary of Weekly Quiz #9

### 1. Five Number Summary :

The five-number summary is used to describe the shape of the distribution of a given numerical data. It consists of five numbers: minimum data value, first quartile, median, the third quartile, and the maximum data value.

The five-number summary of this given data set is:

| stats   | value |
|---------|-------|
| Min.    | 60.00 |
| 1st Qu. | 80.00 |
| Median  | 85.00 |
| 3rd Qu. | 95.00 |
| Max.    | 95.00 |

### 2. Boxplot :

The boxplot is a geometric representation of the five-number summary. The boxplot of the given data set is given below.

