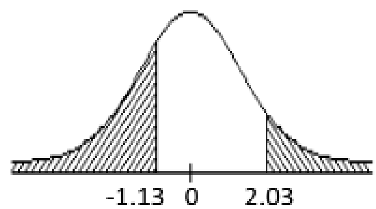
MAT 121 Statistics I
Midterm Exam #2

Time: 120 minutes

Problem 1

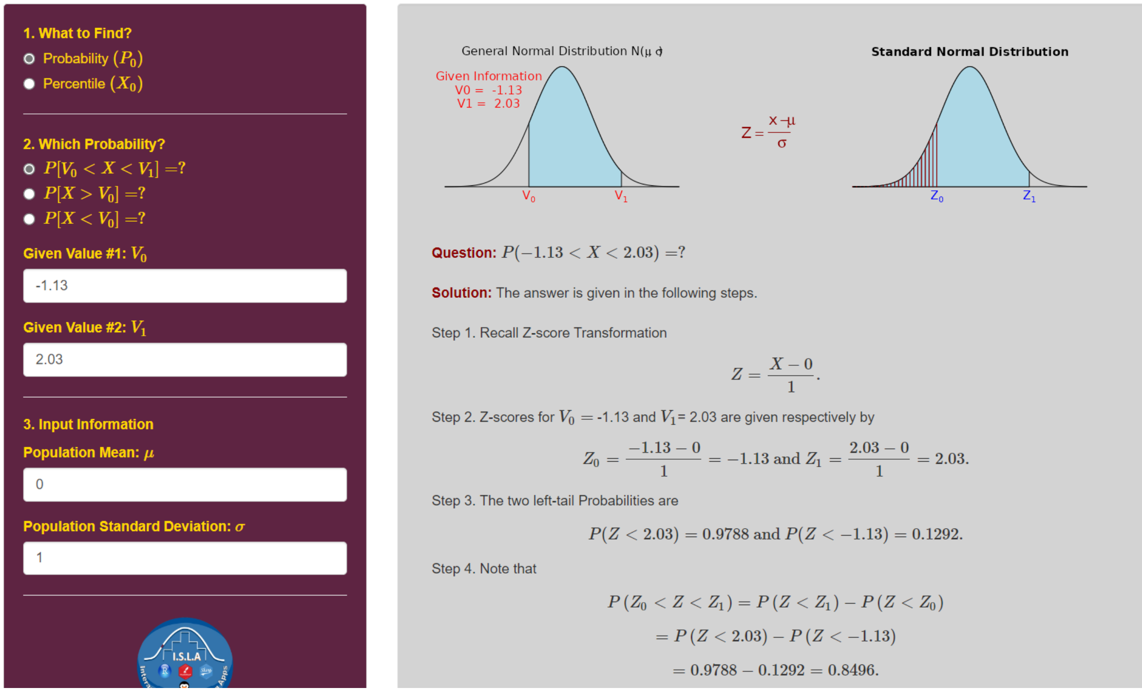
Find the area of the indicated region under the standard normal curve



Answers *

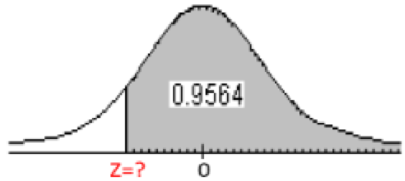
- 0.8489
- 0.1292
- ✓ 0.1504
- 0.0212
- 0.9788

Answer: C 1 – 0.8496 = 0.1504



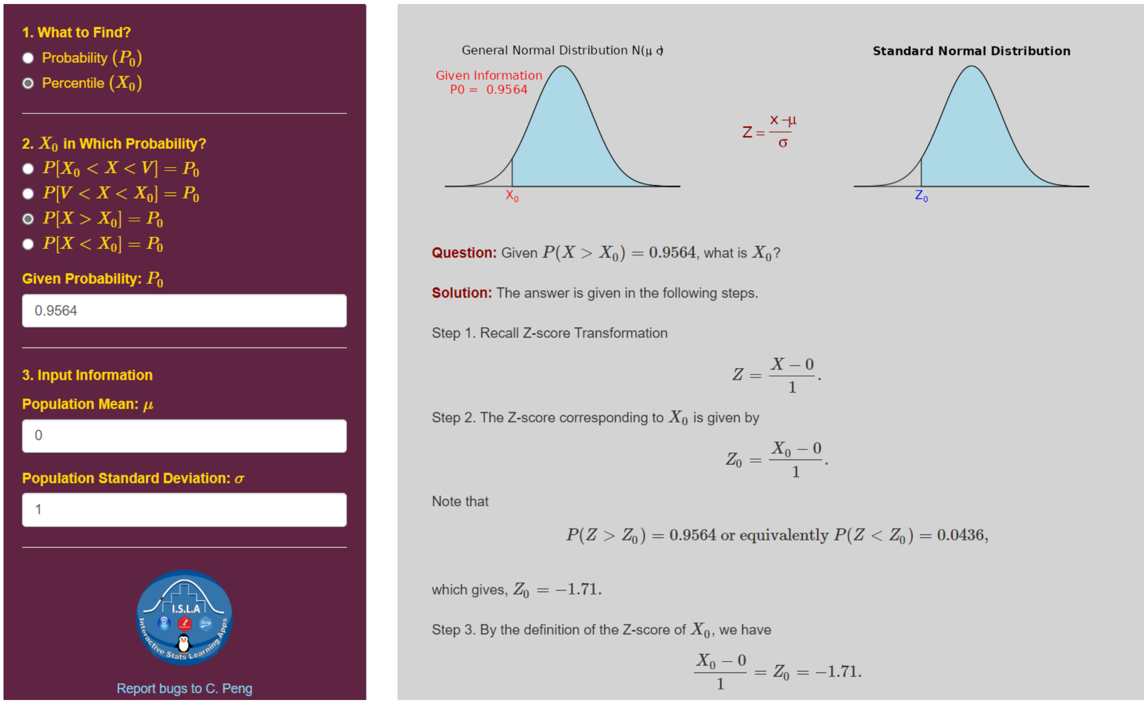
Problem 2

Find the z-score that corresponds to the given area under the *standard normal* curve.



Answers *

- 1.71
- ✓ -1.71
- 1-0.9564
- -1.96
- -0.0436


1. What to Find?
Probability (P_0)
Percentile (X_0)
2. X_0 in Which Probability?
$P[X_0 < X < V] = P_0$
$P[V < X < X_0] = P_0$
$P[X > X_0] = P_0$
$P[X < X_0] = P_0$
Given Probability: P_0
0.9564
3. Input Information
Population Mean: μ
0
Population Standard Deviation: σ
1
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General Normal Distribution N(μ σ)
Given Information
P0 = 0.9564
X_0
$Z = \frac{X-\mu}{\sigma}$
Standard Normal Distribution
Z_0
Question: Given $P(X > X_0) = 0.9564$, what is X_0?
Solution: The answer is given in the following steps.
Step 1. Recall Z-score Transformation
$Z = \frac{X - 0}{1}.$
Step 2. The Z-score corresponding to X_0 is given by
$Z_0 = \frac{X_0 - 0}{1}.$
Note that
$P(Z > Z_0) = 0.9564 \text{ or equivalently } P(Z < Z_0) = 0.0436,$
which gives, $Z_0 = -1.71.$
Step 3. By the definition of the Z-score of X_0, we have
$\frac{X_0 - 0}{1} = Z_0 = -1.71.$

Problem 3

The role of the sample mean in a confidence interval estimate for the population mean is to:

Answers *

- [ ] determine the amount by which the estimate will miss the true population mean.
- [ ] determine the margin of error in the estimate.
- [ ] establish the level of confidence for the estimate.
- [x] determine the center of the confidence interval
- [ ] None of the listed answers is true

Problem 4

The Central Limit Theorem predicts that

Answers *

- [ ] The sampling distribution of μ will be approximately normal for n >30
- [x] the sampling distribution of $\bar{x}$ will be approximately normal for $n > 30$
- [ ] the sampling distribution of p will be approximately normal for $n > 30$
- [ ] the sampling distribution of π will be approximately normal for $n > 30$

Problem 5

A study was conducted to determine what proportion of all college students considered themselves as full-time students. A random sample of 300 college students was selected and 210 of the students responded that they considered themselves full-time students. Which of the following would represent the target parameter of interest?

Answers *

- mean
- ✓ Proportion

Problem 6

In a sample of 155 students, it is found that 21 made an A. What is the margin of error of the 95% confidence interval of p constructed based on the sample information?

Answers *

- ( ) $E = 1.96 \times \dfrac{\sqrt{21(155-21)}}{155}$
- ( ) $E = 1.96 \times \dfrac{\sqrt{\frac{21}{155}(\frac{155-21}{155})}}{155}$
- (✓) $E = 1.96 \times \dfrac{\sqrt{\frac{21}{155}(\frac{155-21}{155})}}{\sqrt{155}}$
- ( ) $E = 1.96 \times \dfrac{\sqrt{\frac{21}{155}+\frac{155-21}{155}}}{\sqrt{155}}$
- ( ) $E = 0.95 \times \dfrac{\sqrt{\frac{21}{155}(\frac{155-21}{155})}}{\sqrt{155}}$

Problem 7

The heights (in inches) of adult males in the United States are believed to be Normally distributed with mean μ. The average height of a random sample of 25 American adult males is found to be $\bar{x}$ = 69.72 inches, and the standard deviation of the 25 heights is found to be s = 4.15. A 90% confidence interval for μ is

Answers *

- $69.72 \pm (1.708) \times 4.15/\sqrt{25}$
- ✓ $69.72 \pm (1.711) \times 4.15/\sqrt{25}$
- $69.72 \pm (1.708) \times 4.15/\sqrt{24}$
- $69.72 \pm (1.316) \times 4.15/\sqrt{25}$

Problem 8

It is IMPOSSIBLE to construct a confidence interval for the population mean under which of the following circumstance.

Answers *

- a non-normal population with a large sample size and unknown population variance.
- a normal population with a large sample size and known population variance.
- ✓ a non-normal population with a small sample size and unknown population variance.
- a normal population with a small sample size and unknown population variance.
- a non-normal population with a large sample size and known population variance.

Problem 9

Which of the following **is** a property of the sampling distribution of sample proportion?

Answers *

- An increase in the sample size n will result in an increase in the standard deviation of $\hat{p}$.
- The mean of $\hat{p}$ is different from the population p.
- ✓ The sampling distribution will be approximately normally distributed when np and n(1-p) are both bigger than 5.

Problem 10

Which of the statement about the sampling distribution of a sample mean is INCORRECT?

Answers *

- A large sample ($n > 30$) from a normal population with known variance, then $\bar{x}$ is normally distributed.
- A large sample ($n > 30$) from a normal population with unknown variance, then $\bar{x}$ is approximately normally distributed.
- A small sample ($n < 30$) from a normal population with known variance, then $\bar{x}$ is a normal distribution.
- ✓ A small sample ($n < 30$) from a normal population with unknown variance, then $\bar{x}$ is a t distribution with n-1 degrees of freedom.

Problem 11

In developing a confidence interval for the population mean, the t-distribution is used to obtain the critical value when

Answers *

- the sample contains some extreme values that skew the results.
- the population standard deviation is unknown.
- the sample is not a random sample.
- the confidence level is low
- ✓ the population is normal, the sample size is small, and the standard deviation is unknown.

Problem 12

A sample of size n is taken from a population that is normally distributed with mean μ and standard deviation σ. Which of the following statements about the sampling distribution of sample mean is true?

Answers *

- Using the Central Limit Theorem, $\bar{X}$ approximately follows $\bar{X} \sim N(\mu,\ \sigma/\sqrt{n})$
- Using the Central Limit Theorem, $\bar{X}$ exactly follows $\bar{X} \sim N(\mu,\ \sigma/\sqrt{n})$
- Without using the Central Limit Theorem, $\bar{X}$ approximately follows $\bar{X} \sim N(\mu,\ \sigma/\sqrt{n})$
- ✓ Using the property of normal distribution, $\bar{X}$ exactly follows $\bar{X} \sim N(\mu,\ \sigma/\sqrt{n})$
- Only if the sample size is large, $\bar{X}$ exactly follows $\bar{X} \sim N(\mu,\ \sigma/\sqrt{n})$

Problem 13

Explain what the phrase "95% confident" means when we interpret a 95% CI for μ

Answers *

- [ ] 95% of the observations in the population fall within the bounds of the calculated interval.
- [x] In repeated sampling, 95% of constructed intervals would contain the value of μ.
- [ ] The probability that the mean falls in the calculated interval is 0.95.
- [ ] 95% of similarly constructed intervals would contain the value of the sampled mean.
- [ ] The confidence interval has a 95% chance to be correct.

Problem 14

A simple random sample of 20 US adults is obtained and each person's red blood cell count (in cells per microliter) is measured. A sample mean is 4.63. Let be a mean blood cell count for all US adults.

Suppose the population standard deviation is unknown, but a sample standard deviation is given. If you wanted to compute a 90% confidence interval for and use a t-interval procedure, what is the appropriate t-value you need to use in computing of a margin of error in your confidence interval, give appropriate degrees of freedom?

Answers *

- [x] df=19, t=1.729
- [ ] df=20, t=1.725
- [ ] df=19, t=1.328
- [ ] df=20, t=1.325

Problem 15

A random sample of 36 customers buying petrol was selected. From this sample, the 95% confidence interval estimate for the mean amount of petrol purchased per customer for the city was calculated to be between 14.5 and 20.5 gallons. Based on this information, the sample mean and the margin of error is given by

Answers *

- [x] $\bar{X} = (14.5 + 20.5)/2, \quad E = (20.5 - 14.5)/2$
- [ ] $\bar{X} = (14.5 + 20.5)/2, \quad E = (20.5 - 14.5)$
- [ ] $\bar{X} = (14.5 + 20.5)/2,$ E cannot be determined
- [ ] $\bar{X}$ cannot be determined, $E = (20.5 - 14.5)/2$
- [ ] Both $\bar{X}$ and E cannot be determined.

Problem 16

The effect of acid rain upon the yield of crops is of concern in many places. To determine baseline yields, a sample of 13 fields was selected, and the yields of barley (g/400 m^2) were obtained and the sample mean and standard deviation are given, respectively, by 220 and 60. Assume that yields of barley are normally distributed. A 95% confidence interval for the mean yield is

Answers *

- $220 \pm 1.96(60/\sqrt{13})$
- $220 \pm 1.96(60/\sqrt{12})$
- ✓ $220 \pm 2.18(60/\sqrt{13})$
- $220 \pm 2.16(60/\sqrt{13})$
- $220 \pm 2.18(60/\sqrt{12})$

Problem 17

Which of the following statements about the relationship between confidence level and critical value is correct? [*Hint: you can see the formula on the last page of the note used in week 07*].

Answers *

- The lower the confidence level, the larger the critical value (implying a wider confidence interval).
- The lower the confidence level, the smaller the critical value (implying a narrower confidence interval).
- ✓ The higher the confidence level, the larger the critical value (implying a wider confidence interval).
- The higher the confidence level, the smaller the critical value (implying a narrower confidence interval).
- The confidence level is independent of the critical value.

Problem 18

A 95% confidence interval for the mean reading achievement score for a population of third-grade students is (44.2, 54.2). Suppose you compute a 99% confidence interval using the same information. Which of the following statements is correct?

Answers *

- [ ] The intervals have the same width.
- [ ] The 99% interval is shorter.
- [x] The 99% interval is longer.
- [ ] The standard deviation of the same mean becomes bigger
- [ ] The standard deviation of the same mean becomes smaller.

Problem 19

A publishing company is studying the sales of various franchises in their chain of stores. They draw a random sample of 75 stores in the chain, and measure the average daily sales. They find that $\bar{x} = \$5{,}670$ and $s = \$1{,}750$. Calculate a 90% confidence interval for μ, the mean daily sales of their stores.

Answers *

- ($2791.3 , $8548.8)
- ✓ ($5337.6 , $6002.4)
- ($5149.5 , $6190.5)
- ($5410.9 , $5929.1)
- ($3920.0 , $7420.0)

Problem 20

A simple random sample of 30 US adults is obtained and each person's red blood cell count (in cells per microliter) is measured. A sample mean is 4.63. Let mu be a mean blood cell count for all US adults.

Suppose a 95 % confidence interval is: (4.13, 5.13). Based on that interval do you think it is reasonable to assume that mu is equal to 4 cells per microliter? Select appropriate answer from the following [*Hint: think about the interpretation of CI*]:

Answers *

- ✓ No, because the lower endpoint of CI is above 4 cells per microliter
- Yes, because CI is above 4 cells per microliter
- Yes, because the sample mean was only slightly above 4 cells per microliter
- No, because the sample mean was above 4 cells per microliter
- Not enough information to answer the question.

Problem 21

The rates of return on 7 natural resources mutual funds are given below:

14.75 15.01 16.95 18.07 14.81 15.59 17.86

The mean and standard deviation of the above sample are $\bar{x} = 16.15, s = 1.45$. Calculate a 99% confidence interval for μ, the mean rate of return of natural resources mutual funds.

Answers *

- ( ) (14.74 , 17.56)
- ( ) (14.23 , 18.07)
- ( ) (14.31 , 17.99)
- (✓) (14.12 , 18.18)
- ( ) (15.36 , 16.94)

1. Sample Statistics

Sample Mean: $\bar{X}$

16.15

Standard Deviation: s **or** σ_0

1.45

2. Sample Size and Confidence Level

Sample Size: n

7

Confidence Level: $1-\alpha$

80% 99%

80 82 84 86 88 90 92 94 96 98 99

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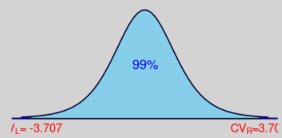
Solution: The desired confidence interval is constructed in the following steps.

Step 1. The given confidence level.
conf.level $= 1-\alpha$ = 1 - 0.01 = 99% .

Step 2. CV on the t density curve.
$CV = t(\alpha/2, df) = t($ 0.01 , 6 $)$ = 3.707 .



Step 3: Margin of Error

$$E = CV \times \frac{s}{\sqrt{n}} = 3.707 \times \frac{1.45}{\sqrt{7}} = 2.032.$$

Step 4: Expression of Confidence Interval

$$(\bar{X}-E, \bar{X}+E) = (16.15-2.032, 16.15+2.032) = (14.118, 18.182).$$

Step 5: Interpretation of Confidence Interval

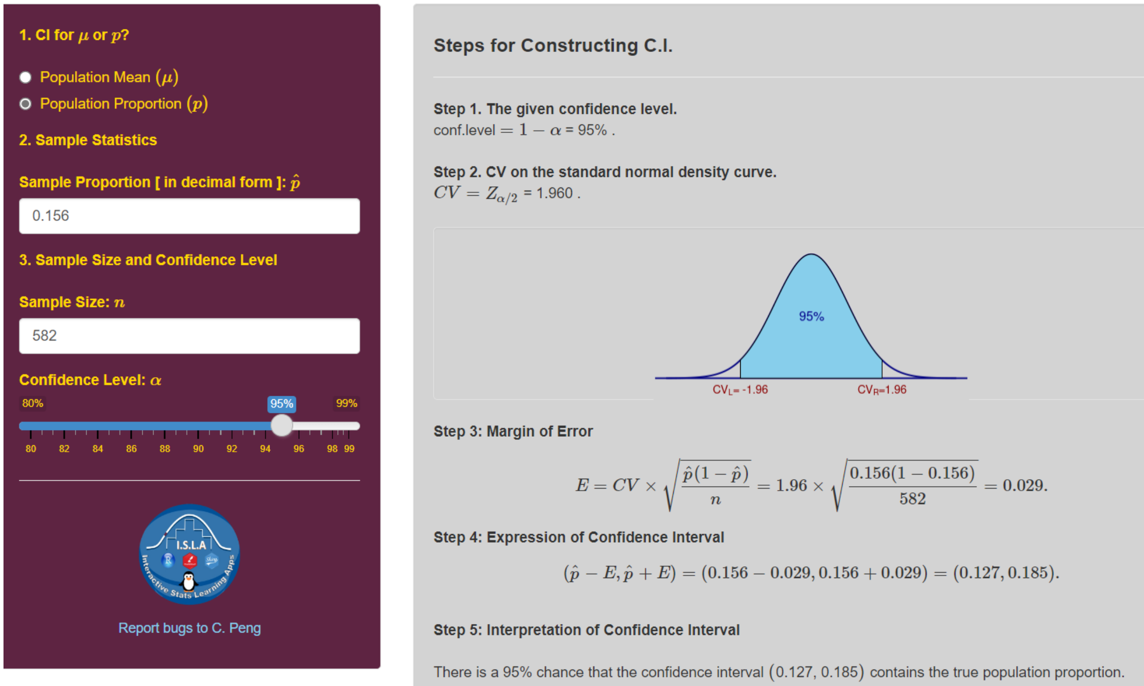
There is a 99% chance that the confidence interval (14.118, 18.182) contains the true population mean.

Problem 22

An insurance company checks records on 582 accidents selected at random and notes that teenagers were at the wheel in 91 of them. Construct a 95% confidence interval for the proportion of all auto accidents that involve teenage drivers.

Answers *

- [x] (12.7% , 18.6%)
- [ ] (10.3% , 17.2%)
- [ ] (10.6% , 18.2%)
- [ ] (11.7% , 19.5%)


1. CI for μ or p?
Population Mean (μ)
Population Proportion (p)
2. Sample Statistics
Sample Proportion [in decimal form]: $\hat{p}$
0.156
3. Sample Size and Confidence Level
Sample Size: n
582
Confidence Level: α
80%
95%
99%
80 82 84 86 88 90 92 94 96 98 99
I.S.L.A
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Steps for Constructing C.I.
Step 1. The given confidence level.
conf.level $= 1 - \alpha$ = 95% .
Step 2. CV on the standard normal density curve.
$CV = Z_{\alpha/2}$ = 1.960 .
95%
CV_L= -1.96
CV_R=1.96
Step 3: Margin of Error
$E = CV \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96 \times \sqrt{\frac{0.156(1-0.156)}{582}} = 0.029.$
Step 4: Expression of Confidence Interval
$(\hat{p} - E, \hat{p} + E) = (0.156 - 0.029, 0.156 + 0.029) = (0.127, 0.185).$
Step 5: Interpretation of Confidence Interval
There is a 95% chance that the confidence interval $(0.127, 0.185)$ contains the true population proportion.

Problem 23

A random sample of n = 300 measurements is drawn from a binomial population with probability of success $\hat{p}$ =.43. Give the mean and the standard deviation of the sampling distribution of the sample proportion.

Answers *

- .57; .029
- .43; .014
- 57; .014
- ✓ .43; .029

Standard error: 0.02857143

Problem 24

A randomly selected sample of 400 students at a university with 15-week semesters was asked whether or not they think the semester should be shortened to 14 weeks (with longer classes). Forty-six percent (46%) of the 400 students surveyed answered "yes." Which one of the following statements about the number 46% is correct?

Answers *

- [x] It is a sample statistic.
- [ ] It is a population parameter.
- [ ] It is a margin of error.
- [ ] It is a standard error.

Problem 25

In a random sample of 28 families, the average weekly food expense was \$95.60 with a standard deviation of \$22.50. Determine whether a normal distribution or a t -distribution should be used or whether neither of these can be used to construct a confidence interval. Assume the distribution of weekly food expenses is normally shaped.

Answers *

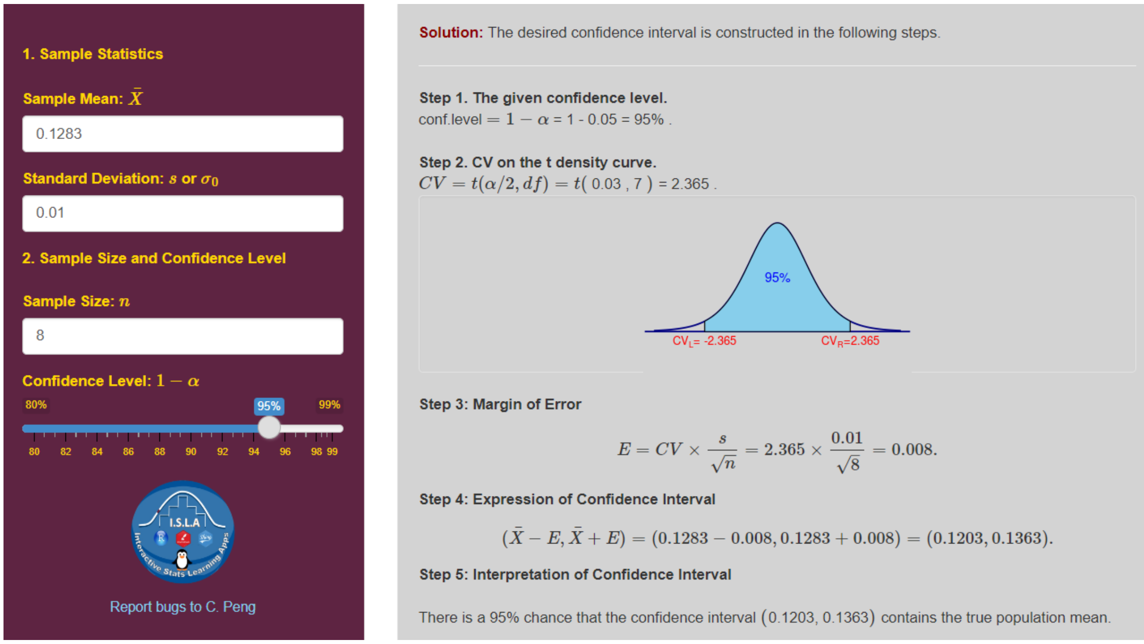
- [ ] Cannot use normal distribution or t-distribution.
- [ ] Use normal distribution.
- [x] Use the t-distribution.

Problem 26

Pennies are made primarily out of Copper, with trace amounts of Zinc. A sample of 8 pennies were tested for total zinc contents, in grams. The weights were found to be: .125, .135, .123, .111, .124, .143, .136, and .129. Assuming this sample is representative of all lincoln pennies, and the weight is normally distributed. Construct a 95% confidence interval estimate for the mean mass of zinc in pennies.

A) (.118,.134)
B) (.115,.141)
C) (.125,.131)
D) (.120,.136)

Answer:D.

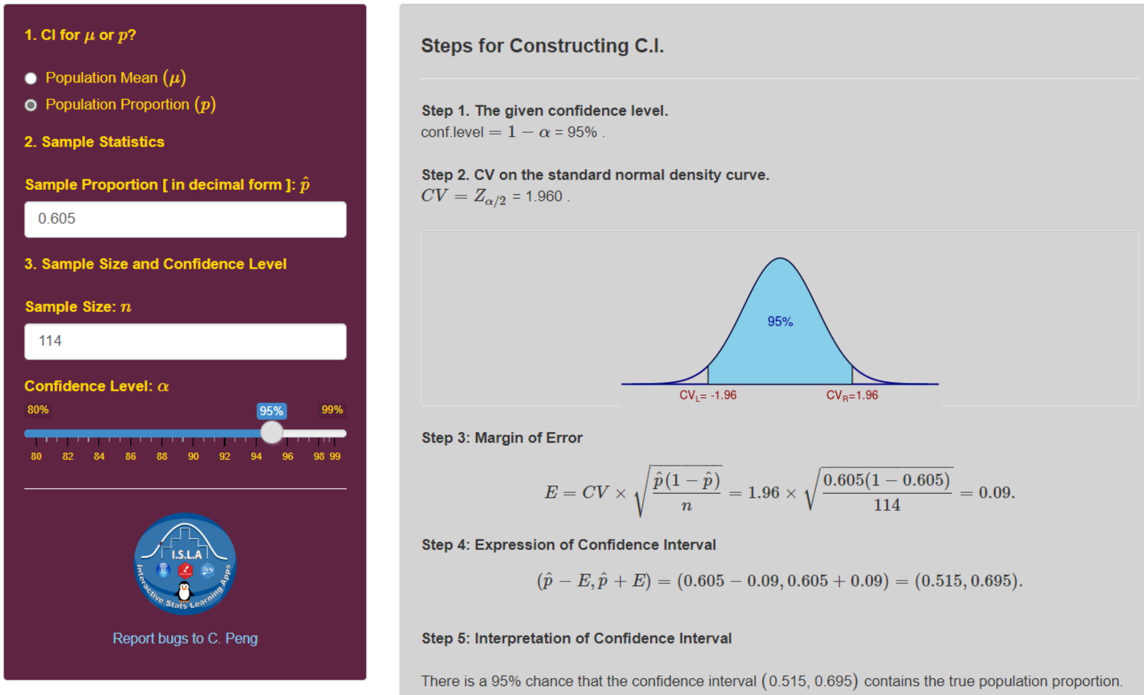


Problem 27.

Sally and Timmy are running for Class President of Student Council at their local highschool. An exit poll of 114 students who voted showed 69 Students voted for Timmy. Create a 95% confidence interval to show the true proportion of students who voted for Timmy.

A) (.555,.645)
B) (.538,.671)
C) (.515,.695)
D) (.497,.713)

Answer C.

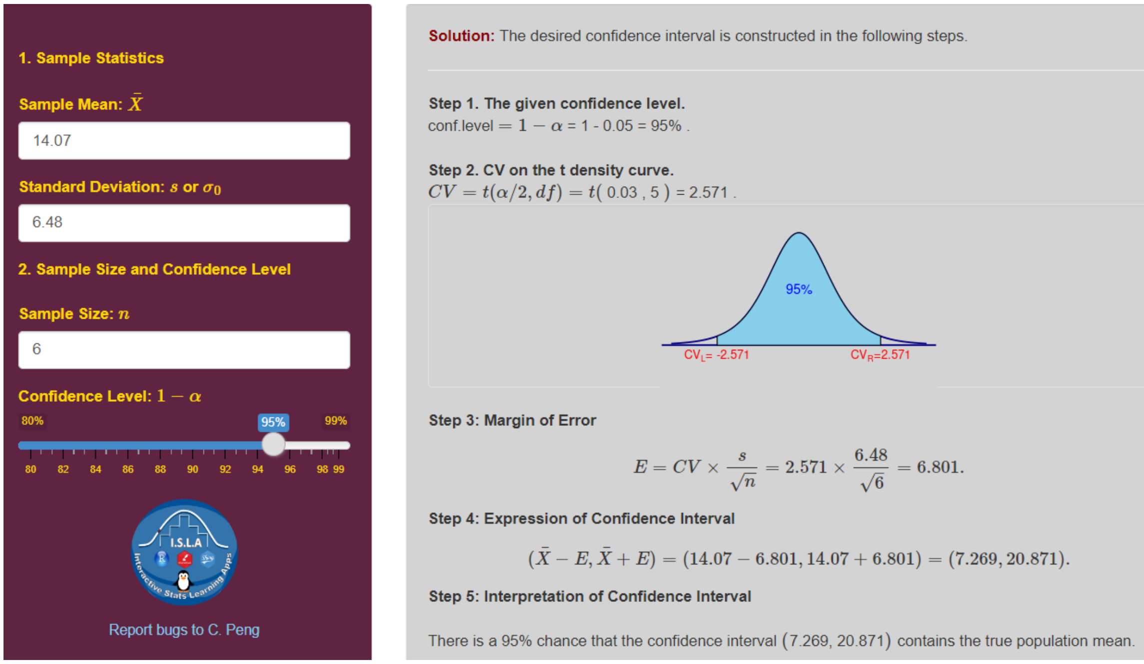


Problem 28.

An Apiarist collected the honeycomb from a sample of honeybee hives at the local farm, wants to predict the true mean of honeycomb, in grams, a hive on the farm will give. A sample of 6 collected honeycombs showed weights of 11.32, 14.53, 17.45, 15.67, 19.48, and 15.99 grams. The sample mean and srandard deviation are given by 14.07 and 6.48 respectively. Assuming normality and a random sample, construct a 95% confidence interval for the true mean weight of harvested honeyccomb per hive.

A. (7.26, 20.87)
B. (8.50, 19.74)
C. (7.37, 20.77)
D. (6.03, 22.12)

Answer: A

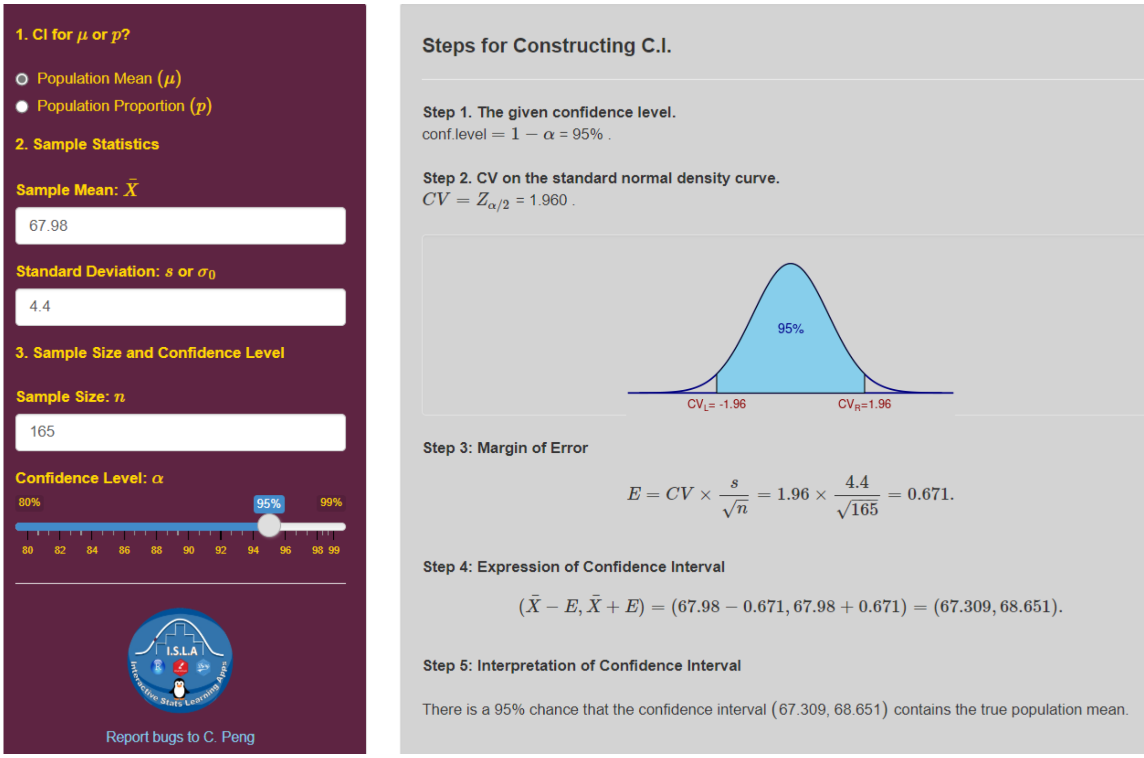


Problem 29

Suppose you wanted to estimate the average height of every student at West Chester University. A sample of 165 students is taken. The average length, in inches, was recorded to be 67.98, with a standard deviation of 4.4 inches. Create a 95% confidence interval for the true population mean of the height of student at West Chester University.

A. (67.309,68.651)
B. (67.331,68.629)
C. (67.002,68.958)
D. (67.293,68.667)

Answer: A

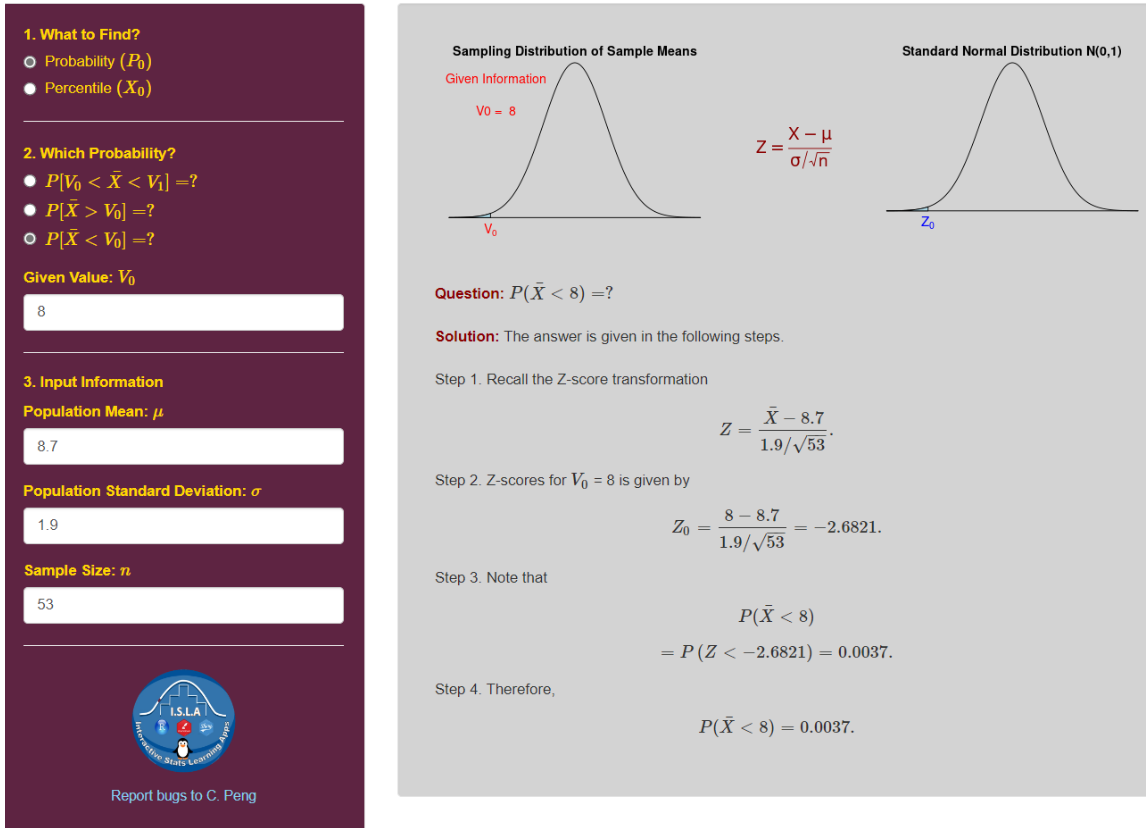


Problem 30

Suppose a certain type of caterpiller is expected to spend 8.7 days on average in its chrysalis before morphing into a butterfly, with a standard deviation of 1.9 days. A sample batch of 52 of these caterpillers was collected. What is the probability of observing an average below 8 days in this sample?

A) .090
B) .004
C) .205
D) .491

Answer: B.

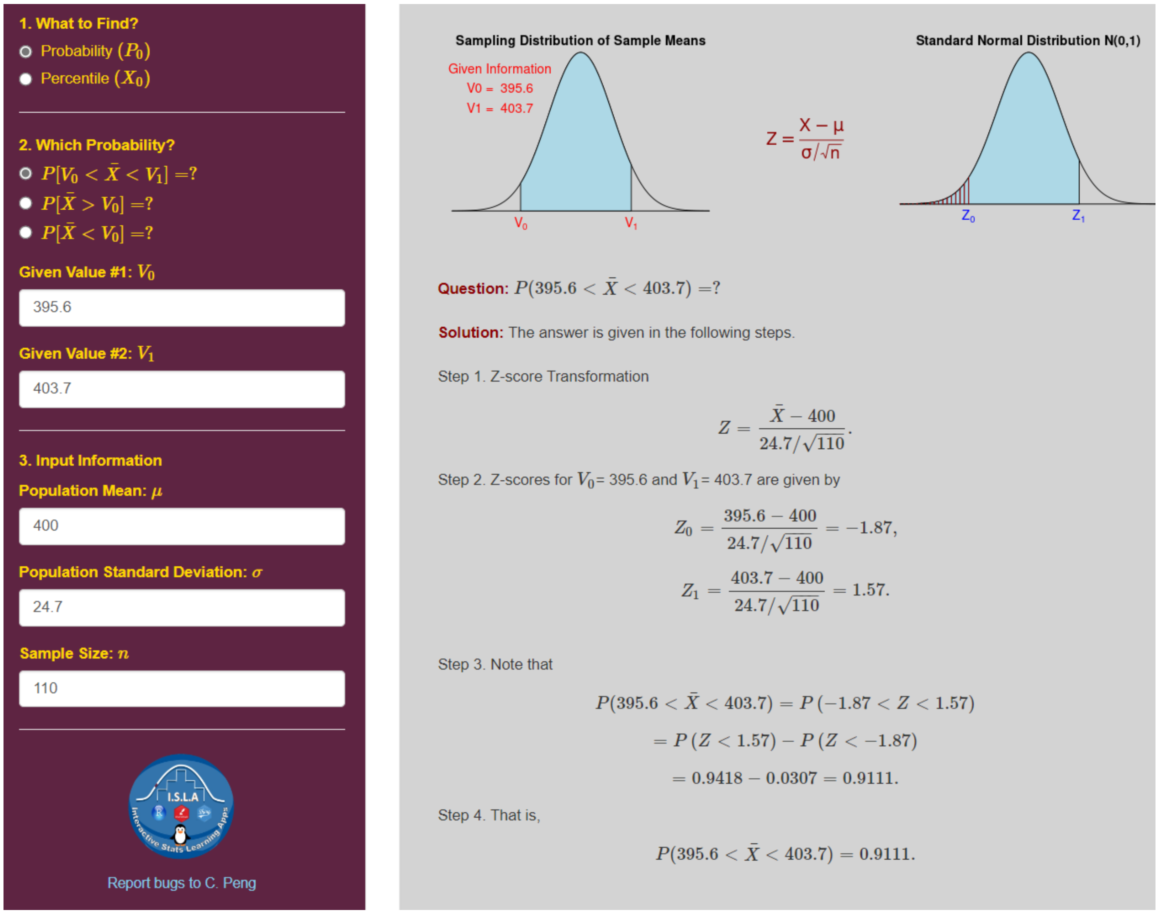


Problem 31

Suppose the average Bite Force of all Gray Wolves is 400 PSI, with a standard deviation of 24.7 PSI. A sample of 110 graywolves is taken. What is the probability that their average Bite Force is between 395.6 and 403.7

A) .776
B) .864
C) .892
D) .911

Answer: D

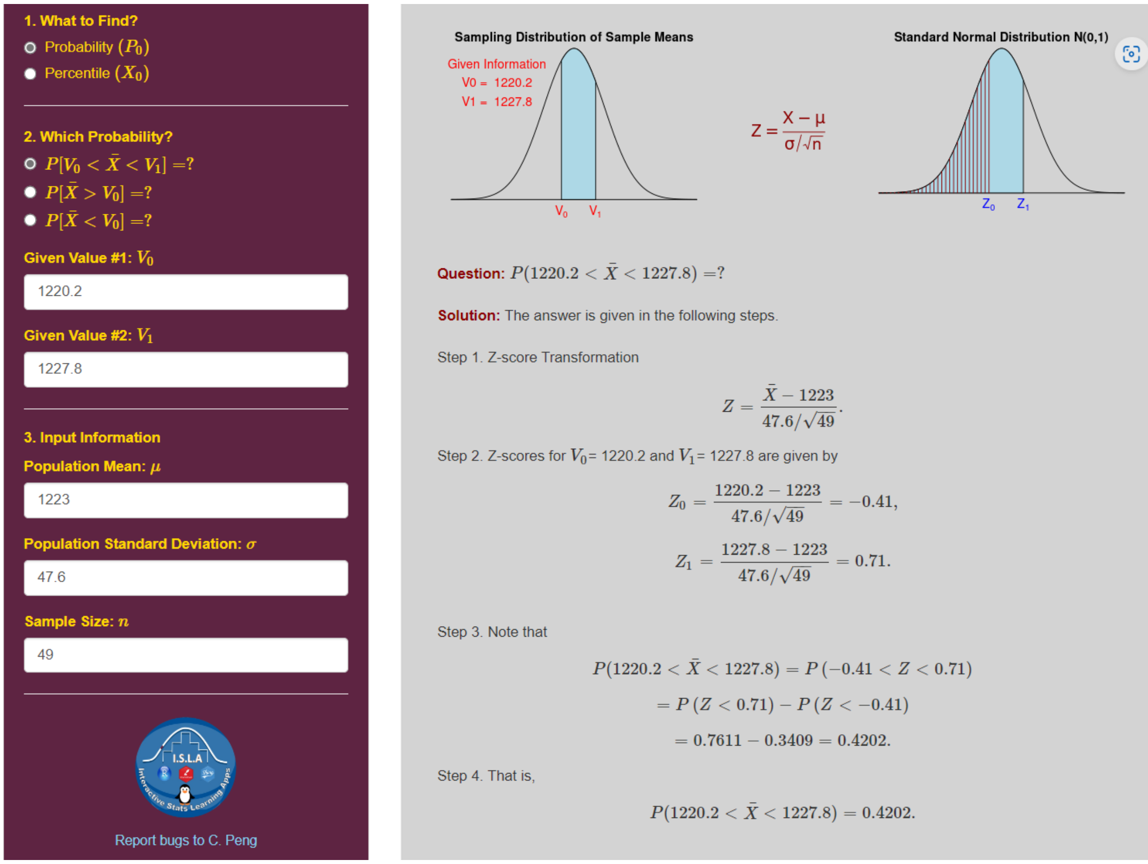


Problem 32

The average weight of a certain species of giraffe is found to be 1,223 pounds, with a standard deviation of 47.6 pounds. A sample of 49 of this species of Giraffe is taken. What is the probability the average weight is between 1,220.2 and 1227.8?

A) .4202
B) .4906
C) .3920
D) .3873

Answer: A.

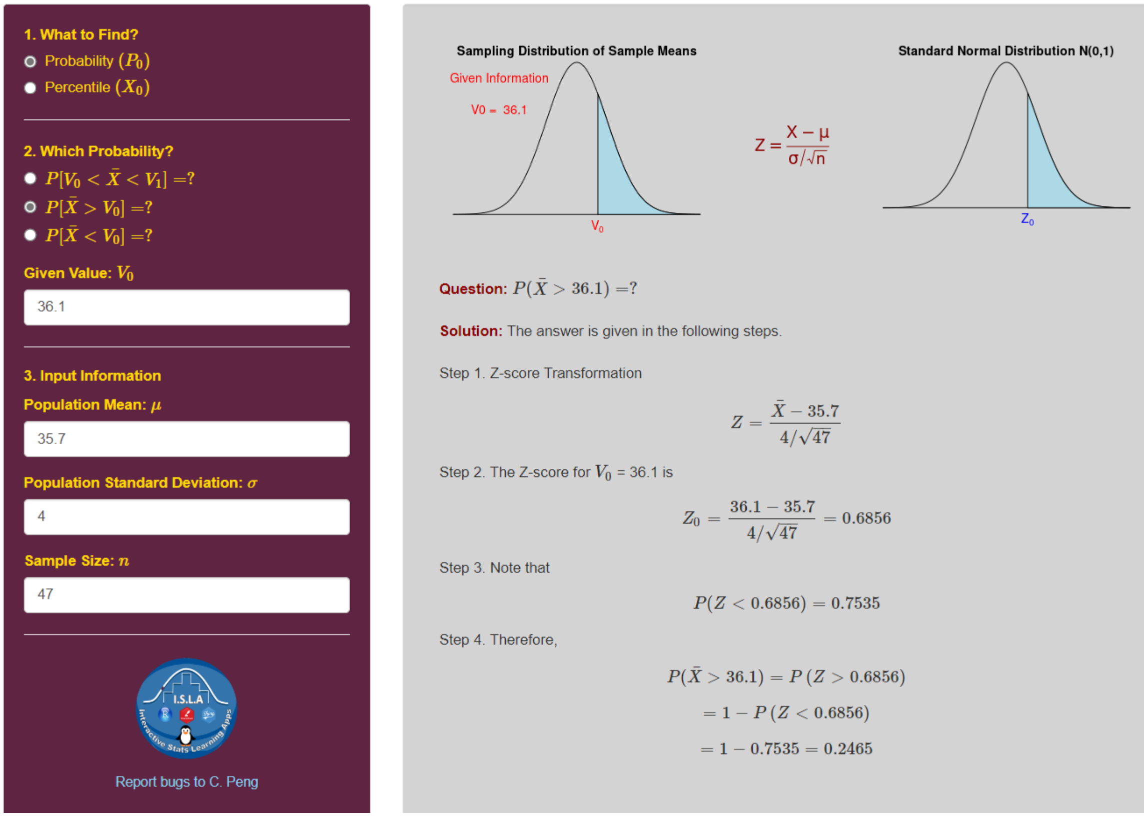


Problem 33

A certain species of tree has an average height of 35.7 feet, with a standard deviation of 4 feet. A cluster of 47 of these trees was found. What is the probability that their average height is greater than 36.1 feet?

A) .141
B) .247
C) .356
D) .222

Answer: B



Problem 34

Different Species of Penguins have different Flipper Lengths. Suppose Gentoo Penguins have a mean flipper length of 22 millimeters and a standard deviation of 6.0 millimeters. If we take a sample of 37 Penguins, what is the probability that their average flipper length is between 21 and 23?

A) .912
B) .688
C) 1
D) .999

Answer: B

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. Which Probability?

- $P[V_0 < \bar{X} < V_1] = ?$
- $P[\bar{X} > V_0] = ?$
- $P[\bar{X} < V_0] = ?$

Given Value #1: V_0

21

Given Value #2: V_1

23

3. Input Information

Population Mean: μ

22

Population Standard Deviation: σ

6

Sample Size: n

37

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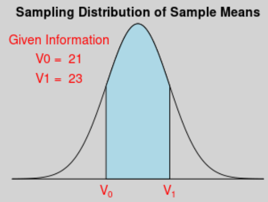
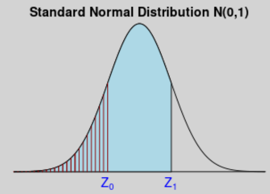


$$Z = \frac{X - \mu}{\sigma/\sqrt{n}}$$



Question: $P(21 < \bar{X} < 23) = ?$

Solution: The answer is given in the following steps.

Step 1. Z-score Transformation

$$Z = \frac{\bar{X} - 22}{6/\sqrt{37}}.$$

Step 2. Z-scores for $V_0 = 21$ and $V_1 = 23$ are given by

$$Z_0 = \frac{21 - 22}{6/\sqrt{37}} = -1.01,$$

$$Z_1 = \frac{23 - 22}{6/\sqrt{37}} = 1.01.$$

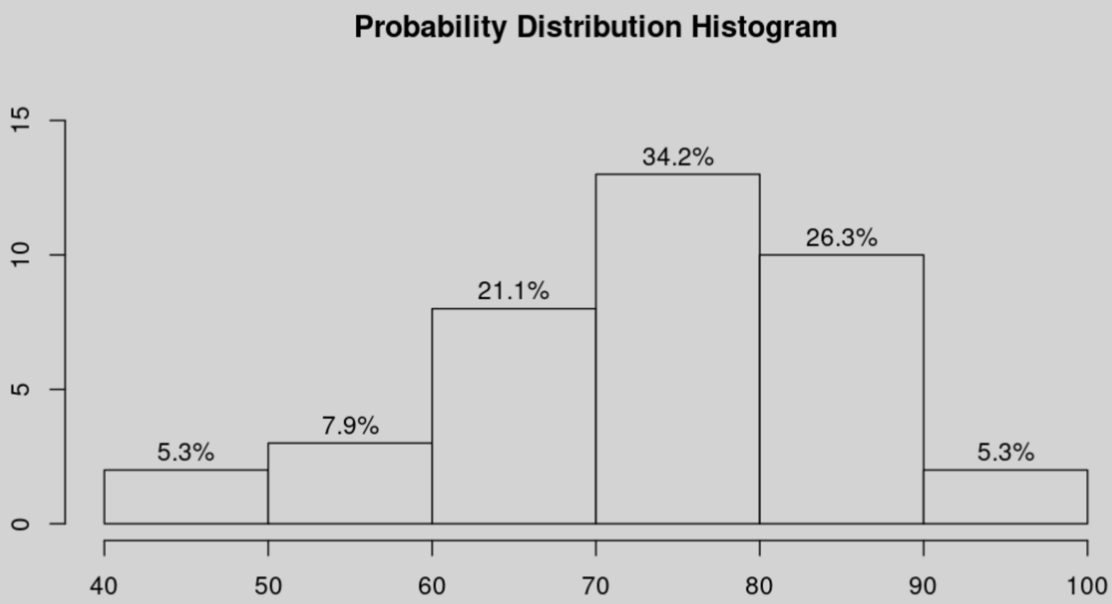
Step 3. Note that

$$\begin{aligned} P(21 < \bar{X} < 23) &= P(-1.01 < Z < 1.01) \\ &= P(Z < 1.01) - P(Z < -1.01) \\ &= 0.8438 - 0.1562 = 0.6876. \end{aligned}$$

Step 4. That is,

$$P(21 < \bar{X} < 23) = 0.6876.$$

Midterm Exam #2 Summary



The five-number summary of this given data set is:

stats	value
Min.	40.00
1st Qu.	70.00
Median	74.50
3rd Qu.	82.00
Max.	91.00

2. Boxplot :

The boxplot is a geometric representation of the five-number summary. The boxplot of the given data set is given below.