

MAT121 Confidence Intervals

This quiz focuses on the normal confidence intervals for the population means and proportions and the t-confidence interval for the population mean. Please follow the steps that I used in the lecture notes and the videos to write the detailed steps before you choose the correct answer. **If the confidence level is not given in the story problem, use our default 95% level.**

Problem 1.

What is the primary purpose of a confidence interval for a mean?

- A. to estimate a sample mean.
- B. to find the precision of a sample mean.
- C. to estimate a population mean.
- D. to provide an interval that covers 95% of the individual values in the population.
- E. to find the accuracy of a sample mean.

Correct Answer: C.

Problem 2

Which of the following statements about constructing the confidence interval of a population mean is correct from a large random sample?

- A. The critical value is equal to the population standard deviation divided by the sample size.
- B. The critical value increases as the sample size increases.
- C. The confidence level increases as the sample size increases.
- D. The critical value is dependent on the confidence level.
- E. The critical value is always bigger than the sample mean.

Correct Answer: D.

The critical value is the percentile of the standard normal distribution. The higher the confidence level, the bigger the critical value.

Problem 3

Which of the following statement is correct about confidence intervals?

- A. The margin of error does not change as the sample size increases.
- B. The margin of error increases as the sample size increases.
- C. The margin of error decreases as the sample size increases.
- D. The margin of error decreases as the confidence level increases.
- E. The margin of error does not change as the confidence level changes.

Correct Answer: C

Problem 4

Suppose that I want to calculate the 80% confidence interval for the population mean value of a normally distributed population with a known standard deviation of 40. Which of the following critical values should I use to perform this calculation?

- A. 1.645
- B. 1.28
- C. 1.96
- D. 80% critical value of the t with 39 degrees of freedom
- E. 80% critical value of the t with 40 degrees of freedom

Correct Answer: B.

The sampling distribution of the sample mean is normally distributed. The normal critical value should be used.

Problem 5

Under which of the following circumstances is it impossible to construct a confidence interval for the population mean?

- A. A non-normal population with a large sample and an unknown population variance
- B. A normal population with a large sample and a known population variance
- C. Non-normal population with a small sample and an unknown population variance
- D. A normal population with a small sample and an unknown population variance
- E. A normal population with a large sample and an unknown population variance

Correct Answer: C

Problem 6

A random sample of 144 full-grown lobsters had a mean weight of 16 ounces and a standard deviation of 3.0 ounces. To construct a 95 percent confidence interval for the population mean μ , what is the margin of error?

- A. ≈ 0.4112
- B. ≈ 0.4900
- C. ≈ 0.3204
- D. ≈ 0.3342
- E. ≈ 0.3352

Correct Answer: 0.49

$$E = 1.96 \times \frac{3}{\sqrt{144}} = 0.49$$

Problem 7

A 95% confidence interval for the mean reading achievement score for a population of third-grade students is (44.2, 54.2). Which of the following statements is correct?

- A. The margin of error is 10
- B. The margin of error is 5
- C. The margin of error is 1.96×5
- D. The margin of error is 1.96×10
- E. Cannot be determined.

Correct Answer: B.

The margin of error is the half-width of the confidence interval. i.e., $(54.2 - 44.2)/2 = 5$.

Problem 8

Market researchers use the number of sentences per advertisement as a measure of readability for magazine advertisements. A random sample of the number of sentences found in 25 advertisements was obtained. The mean and standard deviation of this sample are 12.4 and 5 respectively. What is the 95% confidence interval for the average number of sentences in magazine advertisements?

- A. (10.44, 14.36)
- B. (10.76, 14.045)
- C. (10.34, 14.46)
- D. (10.69, 14.11)
- E. Cannot be determined. Need more information on the population distribution.

Correct Answer: E.

Need to assume the normal population to use a t distribution to construct the confidence interval.

Problem 9

A college admissions director wishes to estimate the mean age of all students currently enrolled. In a random sample of 36 students, the mean age is found to be 22.9 years. From past studies, the standard deviation is known to be 1.8 years. The historical data showed that the ages of enrolled students are distributed asymmetrically. The margin of error E in the used for the 90% confidence interval of the population mean age is given by

- A. $Z_{0.05} \times \frac{1.8}{\sqrt{36}}$
- B. $\frac{1.8}{\sqrt{36}}$
- C. $t_{35,0.05} \times \frac{1.8}{\sqrt{36}}$
- D. $Z_{0.05} \times \frac{1.8}{36}$
- E. $t_{35,0.05} \times \frac{1.8}{36}$

Correct Answer: A.

Sampling distribution of the mean based on the central limit theorem.

Problem 10

A team of efficiency experts intends to use the mean of a random sample of size $n = 100$ to estimate the average mechanical aptitude of assembly-line workers in a large industry (as measured by a certain standardized test) and found that the sample mean is 19.9 minutes and the sample standard deviation is 5.73 minutes. The objective is to construct a 95% confidence interval for the average mechanical aptitude of assembly line workers in the given industry. The form of the confidence interval is given by

- A. $19.9 \pm 1.96 \times \frac{5.73}{\sqrt{100-1}}$
- B. $19.9 \pm 0.95 \times \frac{5.73}{\sqrt{100}}$
- C. $19.9 \pm 0.975 \times \frac{5.73}{\sqrt{100-1}}$
- D. $19.9 \pm 0.025 \times \frac{5.73}{\sqrt{100}}$
- E. $19.9 \pm 1.96 \times \frac{5.73}{\sqrt{100}}$

Correct Answer: E

Because the confidence level and the critical value are different.

Problem 11

We want to construct a confidence interval for a population with mean and standard deviation denoted by μ and σ , respectively, using a small random sample taking from the population with $n < 30$. Let (\bar{X}, s) be the sample mean and standard deviation. Then

- A. \bar{X} is normally distributed with mean μ and standard deviation σ/\sqrt{n}

- B. \bar{X} is normally distributed with mean μ and standard deviation s/\sqrt{n}
- C. \bar{X} is a t distribution with mean μ and standard deviation σ/\sqrt{n}
- D. \bar{X} is t distribution with $n - 1$ degrees of freedom.
- E. The distribution of \bar{X} cannot be determined with the given information.

Correct Answer: E.

This is a small sample problem with no information about population distribution. The sampling distribution of sample means cannot be determined.

Problem 12

The college newspaper of a large university periodically conducts a survey of students on campus to determine the attitude on campus concerning issues of interest. Pictures of the students interviewed along with quotes of their responses are printed in the paper. Students are interviewed by a reporter "roaming" the campus selecting students to interview "haphazardly." On a particular day, the reporter interviews five students and asks them if they feel there is adequate student parking on campus. Four of the students say, "no."

Which of the following conditions for inference about a proportion using a confidence interval are violated in this example?

- I The data represents the population of interest.
- II The population is at least ten times as large as the sample.
- III $n\hat{p} \geq 5$ and $n(1 - \hat{p}) \geq 5$.

- A. I only
- B. I and II only
- C. III only
- D. II and III only
- E. I, II, and III.

Correct Answer: E.

Problem 13

An agricultural researcher plants 25 plots with a new variety of corn. The average yield for these plots is $\bar{X} = 150$ bushels per acre. Assume that the yield per acre for the new variety of corn follows a normal distribution with unknown mean μ and standard deviation $s = 10$ bushels. A 90% confidence interval form is

- A. 150 ± 2.00 .
- B. 150 ± 3.29 .
- C. 150 ± 3.42 .
- D. 150 ± 16.45 .
- E. 150 ± 32 .

Correct Answer: B

This is a large sample of the normal confidence interval. The critical value is 1.645. The margin of error $E = 1.645 \times \frac{10}{\sqrt{25}} = 3.29$.

Problem 14

Other things being equal, the margin of error of a confidence interval increases as

- A. the sample size increases.
- B. the sample mean increases.
- C. the population standard deviation increases.
- D. the confidence level decreases.
- E. none of the above.

Correct Answer: C.

This is obvious if you look at the form of the margin of error: $E = CV \times \frac{\sigma}{\sqrt{n}}$

Problem 15

A professor sampled 46 students from a large university to obtain a 95% confidence interval for the proportion of students in favor of raising ASB fees. The interval was (.356, .397). If the professor had used a 90% confidence interval instead, the confidence interval would have been

- A. Wider and would have a smaller chance of missing the true proportion.
- B. Narrower and would have a larger chance of missing the true proportion.
- C. Narrower and would have a smaller chance of missing the true proportion.
- D. Wider and would have a larger chance of missing the true proportion.
- E. Wider, but the chance of missing the true proportion cannot be determined

Correct Answer B.

A lower confidence level leads to a small critical value, consequently leading to a narrower confidence interval. A narrower confidence interval will have more chances to miss the true value of the parameter.

Problem 16

The rates of return on 7 natural resources mutual funds are given below:

14.75, 15.01, 16.95, 18.07, 14.81, 15.59, 17.86

The sample mean and standard deviation are given by $\bar{x} = 16.15$ and $s = 1.45$, respectively.

Calculate a 99% confidence interval for μ , the mean rate of return of natural resources mutual funds.

- a. (14.74 , 17.56)
- b. (14.23 , 18.07)
- c. (14.31 , 17.99)
- d. (14.12 , 18.18)
- e. (15.36 , 16.94)

Answer: D.

Problem 17

A random sample of 100 preschool children in Camperdown revealed that only 60 had been vaccinated. Provide an approximate 95% confidence interval for the proportion vaccinated in that suburb. Exactly one option must be correct)

- A. $0.6 \pm 1.96 \times \sqrt{\frac{0.6 \times 0.4}{100}}$
- B. $0.6 \pm 1.645 \times \sqrt{\frac{0.6 \times 0.4}{100}}$
- C. $0.6 \pm 0.95 \times \sqrt{\frac{0.6 \times 0.4}{100}}$
- D. $0.6 \pm 0.975 \times \sqrt{\frac{0.6 \times 0.4}{100}}$
- E. $0.6 \pm 1.96 \times \sqrt{\frac{0.6 + 0.4}{100}}$

Correct Answer: A

Simply using the sampling distribution of sample proportion.

Problem 18

The heights (in inches) of adult males in the United States are believed to be Normally distributed with mean μ . The average height of a random sample of 25 American adult males is found to be $\bar{x} = 69.72$ inches, and the standard deviation of the 25 heights is found to be $s = 4.15$. A 90% confidence interval for μ is

- A. 69.72 ± 1.09
- B. 69.72 ± 1.37
- C. 69.72 ± 1.42
- D. 69.72 ± 1.25

Answer: C. The t critical value for 90% CI for $df = 24$ is 1.711. So margin of error $E = (1.711) \cdot 4.15 / \sqrt{25} = (1.711) \cdot (0.83) = 1.42$

Problem 19.

In a study of sleep patterns of eleven 6-month-old infants, a researcher reported the following results from 11 randomly selected infants: (The data is the number of hours out of every 24 hours that each infant slept).

13.3, 12.8, 12.6, 14.3, 13.3, 12.6, 12.8, 12.2, 12.7, 12.7, 12.2

The sample mean and standard deviation are given by $\bar{x} = 12.86$ and $s = 0.59$, respectively. We assume that the data is normally distributed. What is the margin of error (E) for a 99% confidence interval estimate of the true mean number of hours in a 24-hour period that 6-month-old infants sleep?

- a. 0.564
- b. 0.606
- c. 0.442
- d. 0.396

Answer. A.

Problem 20

In a recent Zogby International survey, 11% of 10,000 Americans under 50 said they would be willing to implant a device in their brain to be connected to the Internet if it could be done safely. What is the margin of error at the 99% confidence level?

A. $\sqrt{\frac{1100 \times 8900}{10000}}$

B. $1.96 \times \sqrt{\frac{50 \times 50}{10000}}$

C. $2.575 \times \sqrt{\frac{0.11 \times 0.89}{10000}}$

D. $1.96 \times \sqrt{\frac{0.11 \times 0.89}{10000}}$

Answer: C