MAT121 Quiz 04

This quiz focuses on the application of normal distribution. Except for a few questions related to the concepts, all other questions are categorized into one of the two basic types of questions: finding probability and finding percentile. You are strongly encouraged to draw at least one density curve for each of the questions and then use the standard normal distribution table in the textbook.

Problem 1.

Assume that the heights of women are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. Which of the following transformations converts this general normal distribution to the standard normal distribution,

A). $Z = (X - 63.6)/2.5$
B). $Z = (X - 2.5)/63.6$
C). $Z = 2.5X + 63.6$
D). $Z = 63.6X + 2.5$

Answer: A, $Z = (X - 63.6)/2.5$

Problem 2.

Suppose X follows a normal distribution with a mean of 5 and a variance of 9. After z-score transformation, Probability $P(X \le 8)$ is equal to

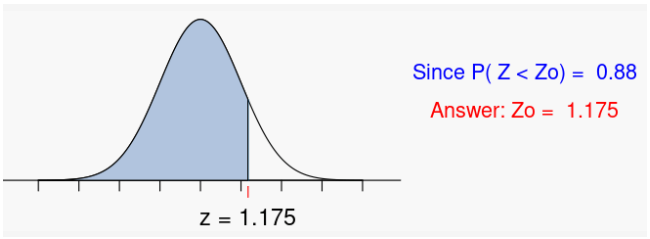
A $P(Z \le 1)$
B $P(Z \le -1)$
C $P(Z \le 0.6)$
D $P(Z \le -0.6)$

Answer: A. $Z = (8 - 5) / \sqrt{9} = 3/3 = 1.$

Problem 3.

Find the z-score in the standard normal distribution such that the area to the right of z is 0.12.
A. – 1.175
B. 0.88
C. – 0.88
D. 1.175

Answer: D



Problem 4

Suppose X is normally distributed with a mean of 5 and a standard deviation of 0.4. Assume that $P(X \leq X_0) = P(Z \leq 1.3)$. Which of the following equations should be solved to get the value of X_0?

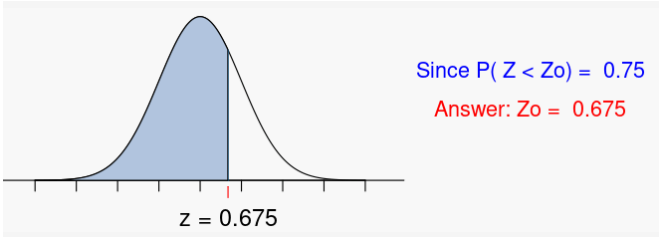
A. $X_0 = 1.3 \times 0.4 + 5$
B. $X_0 = 1.3 \times 0.4 - 5$
C. $X_0 = 1.3 \times 5 + 0.4$
D. $X_0 = 1.3 \times 5 - 0.4$

Answer: A. solve X_0 from $1.3 = (X_0 - 5)/0.4$

Problem 5

The Precision Scientific Instrument Company manufactures thermometers that are supposed to give readings of 0^0 C at the freezing point of water. Tests on a large sample of these instruments reveal that the freezing point of water is around zero (some thermometers give positive degrees, some thermometers give negative degrees), Assume that the mean reading is 0^0 C and the standard deviation of the readings is 1.0^0 C. Assume further that the readings are normally distributed. Find the 3rd percentile of the thermometer readings.

A. 0.585
B. 0.675
C. 0.749
D. 0.7517



Answer: B

Problem 6.
The height of an adult male is known to be normally distributed with a mean of 175 cm and a standard deviation of 6 cm. We are interested in the 75th percentile in this distribution of heights. Let Q3 be denoted as the 75th percentile. Then Q3 can be solved from which of the following equations?

A.

$$\frac{Q3 - 175}{6} = 0.675$$

B.

$$\frac{Q3 - 175}{36} = 0.675$$

C.

$$\frac{Q3 - 175}{6} = 0.75$$

D.

$$\frac{Q3 - 175}{6} = -0.75$$

Answer: A

Problem 7.

The time that it takes a WCU student to find parking once they have arrived on campus is approximately normally distributed with a mean of 25 minutes and a standard deviation of 5 minutes. If a student arrives on campus at 8:15 AM, we want to find the probability that the student will find parking before 8:45 AM. Which of the calculations is correct?

A.

$P(X < 45\ ) = P(Z < 4) \approx 1$

B.

$P(X > 15\ ) = P(Z > -2) \approx 0.9772$

C.

$P(X < 30\ ) = P(Z < 1) \approx 0.8413$

D.

$P(X > 30\ ) = P(Z > 1) \approx 0.1587$

Correct Answer: C.

Problem 8.

A machine that cuts corks for wine bottles operates in such a way that the distribution of the diameter of the corks produced is well approximated by a normal distribution with a mean of 3 cm and a standard deviation of 0.1 cm. The specifications call for corks with diameters between 2.9 and 3.1 cm. A cork **not** meeting specifications is considered defective. We need to use the z-score transformation before using the standard normal distribution table.

Which of the following calculations for the proportion of good corks is correct?

A.

$P(2.9 < T < 3.1) = P(-1 < Z < 1) = 0.8413$

B.
$P(2.9 < T < 3.1) = P(-1 < Z < 1) = 0.6826$

C.
$P(2.9 < T \ or \ T < 3.1) = P(-1 < Z \ or \ Z < 1) = .3413$

D.
$P(3.1 < T \ or \ T < 2.9) = P(1 < Z \ or \ Z < -1) = 0.3174$

Answer: B.

Step 1. Z-score Transformation

$$Z = \frac{\bar{X} - 3}{0.1/\sqrt{1}}$$

Step 2. Z-scores for V_0 and V_1

$$Z_0 = \frac{2.9 - 3}{0.1/\sqrt{1}} = -1$$

$$Z_1 = \frac{3.1 - 3}{0.1/\sqrt{1}} = 1$$

Step 3. Finding the left-tail Probabilities

$$P(Z < 1) = 0.8413$$

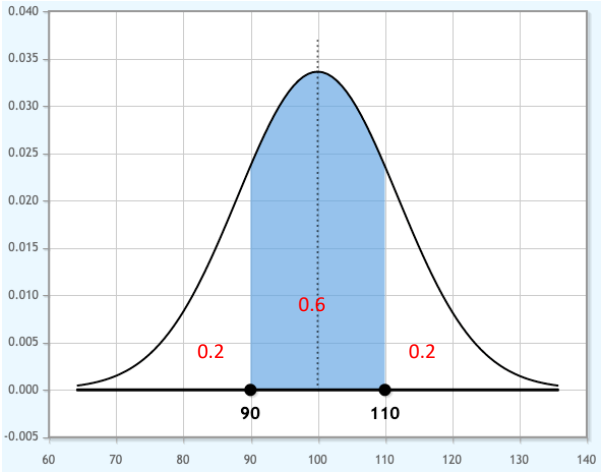
$$P(Z < -1) = 0.1587$$

$$P(2.9 < \bar{X} < 3.1) = P(Z < 1) - P(Z < -1) = 0.8413 - 0.1587 = 0.6826$$

Problem 9

Suppose a normally distributed random variable x has a mean of 100 and P(x < 90) = 0.20. What is the probability that x is between 90 and 110? Ie. What is P(90 < x < 110)? [Draw a density curve and label the values, then you can see the region you want to find the area.]

A 0.2
B 0.3
C 0.4
D 0.5
E 0.6



Answer: D
Both tail areas are equal to 0.2.
The middle area is 1- 0.4 = 0.6.
The correct answer is E.

Problem 10

If you got a 79 on a test in a class with a mean score of 85 and a standard deviation of 4.8, the z-score of your test score would be
A 1.25
B -2.50
C -1.25
D 2.5

Answer: C. Z = (79 – 85)/4.8 = -4/4.8 = - 6/4.8 = - 1.25

Problem 11

For a normal distribution with a mean of 480 and a standard deviation of 32, find the value of Q2 (2^{nd} quartile, or median).
A 458
B 480
C 500
D It depends on the size of the population.

Answer: B. The median (2^{nd} quartile) of the standard normal distribution is Z = 0. That is, the median of the above normal distribution is
$(X_0 – 480)/32 = 0$, implies that $X_0 = 480$.

Problem 12

A normal curve table tells you that the probability lying below z = -1 is .1587. This can be interpreted as:
A 15.87% of the area of the curve lies below z = -1
B 15.87% of the area under the curve lies at or below z = -1
C A random selection from the population has a 15.87% chance of being below z = -1
D All choices are correct.

Answer D.

Summary of Quiz #4

The class boundary is: 10,20,30,40,50,60,70,80,90,100

cut.data.freq	Freq	midpts	rel.freq	cum.freq	rel.cum.freq
[1e+01,2e+01]	1	15.00	0.01	1	0.01
(2e+01,3e+01]	0	25.00	0.00	1	0.01
(3e+01,4e+01]	1	35.00	0.01	2	0.02
(4e+01,5e+01]	0	45.00	0.00	2	0.02
(5e+01,6e+01]	1	55.00	0.01	3	0.04
(6e+01,7e+01]	2	65.00	0.02	5	0.06
(7e+01,8e+01]	7	75.00	0.09	12	0.15
(8e+01,9e+01]	32	85.00	0.39	44	0.54
(9e+01,1e+02]	38	95.00	0.46	82	1.00