

Week 10 Quiz: One-sample t-Tests

Problem 1

A researcher wants to use a one-sample t-test to see if the average height of a certain species of plant differs from the known national average of 15 cm. The most important assumption for this test is:

- a) The sample of plants was randomly selected.
- b) The heights of the plants are normally distributed in the sample.
- c) The population standard deviation is known.
- d) The sample size is greater than 30.

Answer: B

Problem 2.

A quality control manager is testing if the average weight of cereal boxes is 500 grams. He weighs 25 boxes. For the one-sample t-test to be valid, what assumption is made about the 25 weight measurements?

- a) They are a simple random sample from the population of cereal boxes.
- b) They are independent of each other.
- c) They come from a population that is approximately normally distributed.
- d) All of the above.

Answer: D

Problem 3

The degrees of freedom for a one-sample t-test are calculated as:

- a) n
- b) n - 1
- c) σ
- d) \sqrt{n}

Answer: B

Problem 4

The general formula for the test statistic in a one-sample t-test is:

- a) $t = \frac{\bar{x}-\mu}{s}$
- b) $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$
- c) $t = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$
- d) $t = \frac{\bar{x}-s}{\mu/\sqrt{n}}$

Answer: C

Problem 5

A researcher calculates a sample mean (\bar{x}) of 110, a sample standard deviation (s) of 15, from a sample size (n) of 16. If the null hypothesis states the population mean (μ_0) is 100, what is the calculated t-statistic?

- a) 0.67
- b) 2.67
- c) 10.0
- d) 1.67

Answer: B

Problem 6

If a sample mean is 50, the hypothesized population mean is 52, and the standard error is 0.8, what is the value of the t-test statistic?

- a) -2.5
- b) 2.5
- c) -0.25
- d) 1.6

Answer: A

Problem 7

The margin of error for a one-sample t confidence interval is calculated as:

- a) $t_{\alpha/2} \cdot s$
- b) $t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$
- c) $z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$
- d) $\bar{x} \pm t_{\alpha/2}$

Answer: B

Problem 8

Which factor does NOT affect the width of the margin of error in a one-sample t-test confidence interval?

- a) The sample mean (\bar{x})
- b) The sample size (n)
- c) The sample standard deviation (s)
- d) The confidence level

Answer: A

Problem 9

A seed packet claims that the average height of a certain sunflower variety is 72 inches. A gardener plants 14 seeds, and the average height at maturity is 70 inches with a standard deviation of 4 inches. She tests if the mean is different from the claim using $\alpha=0.10$. What is the decision?

- a) Fail to reject H_0 . The average height is not significantly different from 72 inches.
- b) Reject H_0 . The average height is significantly different from 72 inches.
- c) Reject H_0 . The average height is exactly 72 inches.
- d) Fail to reject H_0 . The seeds grew taller than claimed.

Answer: B

Data Source

- summarized statistics
- raw data

sample mean (\bar{x})
70

sample standard deviation (s)
4

sample size (n)
14

Claimed Value (μ_0)
72

Claim Type
not equal to

Significance level α
0.01 0.1 0.2
0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



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Solution: This t test is based on the assumption that the population is normal and the population variance is unknown.

Given sample information: $n = 14$, $\bar{x} = 70$, $s = 4$.

Step 1: Identify the claim of the population mean (μ_0).
The given information indicates that the claim is: μ_0 is not equal to 72.

Step 2: Set up the null and alternative hypotheses.
Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 72$ and $H_1 : \mu \neq 72$.

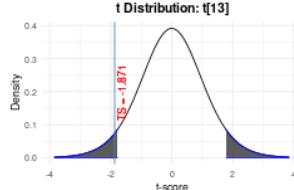
Step 3: Evaluate the test statistic.
The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{70 - 72}{4/\sqrt{14}} = -1.871$.

Step 4: Find the critical value and calculate the p-value.
Based on the significance level, we found the critical values to be: $CV = \pm t_{\alpha/2, df} = \pm t_{0.05, 13} = \pm 1.771$.
The p-value is can be found as p-value ≈ 0.084 .

Step 5: Make a statistical decision on H_0 .
At the 10% significance level, we reject the null hypothesis that the true mean is 72 (p-value = 0.084).

Step 6: Draw conclusion [justify the claim in step 1].
At the 10% significance level, we conclude the alternative hypothesis.

t Distribution: t[13]



Problem 10

For a left-tailed test ($H_a: \mu < \mu_0$) with $\alpha=0.01$ and $df=22$, the critical value is -2.508. If the calculated t-statistic is -1.85, the decision is:

- a) Reject H_0 , as $-1.85 < -2.508$.
- b) Fail to reject H_0 , as $-1.85 > -2.508$.
- c) Reject H_0 , as -1.85 is negative.
- d) Fail to reject H_0 , as -1.85 is not less than -2.508.

Answer: B

Problem 11

A car company claims its new model gets 40 MPG. A consumer group tests 9 cars and finds a mean of 38 MPG with a standard deviation of 3 MPG. They want to test if the true average is less than 40 MPG ($\alpha=0.05$, critical value = -1.860). The calculated t-statistic is -2.0. What should they conclude?

- a) Reject H_0 . There is sufficient evidence that the average MPG is less than 40.

- b) Fail to reject H_0 . There is not sufficient evidence that the average MPG is less than 40.
- c) Reject H_0 . There is sufficient evidence that the average MPG is 40.
- d) Fail to reject H_0 . There is sufficient evidence that the average MPG is 40.

Answer: A

Data Source
 summarized statistics
 raw data

sample mean (\bar{x})

sample standard deviation (s)

sample size (n)

Claimed Value (μ_0)

Claim Type
 less than

Significance level α
 0.05
 0.01
 0.025
 0.05
 0.075
 0.1
 0.15
 0.2

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Solution: This t test is based on the assumption that the population is normal and the population variance is unknown.

Given sample information: $n = 9$, $\bar{x} = 38$, $s = 3$.

Step 1: Identify the claim of the population mean (μ_0).
The given information indicates that the claim is: μ_0 is less than 40.

Step 2: Set up the null and alternative hypotheses.
Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 40$ and $H_1 : \mu < 40$.

Step 3: Evaluate the test statistic.
The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{38 - 40}{3/\sqrt{9}} = -2$.

Step 4: Find the critical value and calculate the p-value.
Based on the significance level, we find the critical values to be: $-t_{\alpha, df} = -t_{0.05, 8} = -1.86$.
The p-value is can be found as p-value ≈ 0.04 .

Step 5: Make a statistical decision on H_0 .
At the 5% significance level, we reject the null hypothesis that the true mean is 40 (p-value = 0.04).

Step 6: Draw conclusion [justify the claim in step 1].
At the 5% significance level, we conclude the alternative hypothesis.

Problem 12

A smartphone manufacturer claims its battery lasts 15 hours on a full charge. A tech reviewer tests 22 phones and finds a mean battery life of 14.5 hours with a standard deviation of 1.2 hours. He conducts a left-tailed test at $\alpha=0.05$. The critical t-value is -1.721. What is the calculated t-statistic?

- a) -0.42
b) -1.96
c) -2.05
d) -1.72

Answer: B

$$\frac{14.5-15}{1.2/\sqrt{22}} = \frac{-0.5}{0.2558} \approx -1.955.$$

Problem 13

A piano tuner knows that middle C should have a frequency of 261.6 Hz. He checks the tuning of 8 pianos in a concert hall and finds the average frequency for middle C is 262.1 Hz with a standard deviation of 0.5 Hz. He performs a two-tailed test to see if the pianos are out of tune ($\alpha=0.05$, critical values = ± 2.365). What is the value of the test statistic?

- a) 0.63
- b) 1.00
- c) 2.53
- d) 2.83

Answer: D

$$\frac{262.1 - 261.6}{0.5/\sqrt{8}} = \frac{0.5}{0.1768} \approx 2.83$$

Problem 14

A coffee shop's menu states that its large coffee contains 16 oz. An employee thinks the machine is dispensing more and secretly measures 12 cups, finding a mean of 16.4 oz with a standard deviation of 0.6 oz. She runs a right-tailed test at $\alpha=0.01$. What is the conclusion?

- a) Reject H_0 . There is significant evidence that the mean volume is greater than 16 oz.
- b) Fail to reject H_0 . There is not significant evidence that the mean volume is greater than 16 oz.
- c) Reject H_0 . The machine is dispensing exactly 16 oz.
- d) Fail to reject H_0 . The machine is under-filling the cups.

Answer: B

Data Source

- summarized statistics
- raw data

sample mean (\bar{x})

sample standard deviation (s)

sample size (n)

Claimed Value (μ_0)

Claim Type

greater than

Significance level α

0.01



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Solution: This t test is based on the assumption that the population is normal and the population variance is unknown.

Given sample information: $n = 12$, $\bar{x} = 16.4$, $s = 0.6$.

Step 1: Identify the claim of the population mean (μ_0).

The given information indicates that the claim is: μ_0 is greater than 16.

Step 2: Set up the null and alternative hypotheses.

Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 16$ and $H_1 : \mu > 16$.

Step 3: Evaluate the test statistic.

The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{16.4 - 16}{0.6/\sqrt{12}} = 2.309$.

Step 4: Find the critical value and calculate the p-value.

Based on the significance level, we found the critical values to be: $t_{\alpha, df} = t_{0.01, 11} = 2.718$.

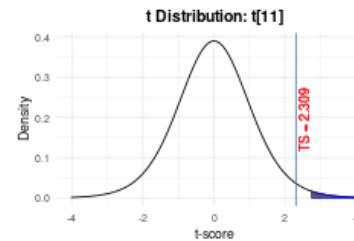
The p-value is can be found as p-value ≈ 0.021 .

Step 5: Make a statistical decision on H_0 .

At the 1% significance level, we do not reject the null hypothesis that the true mean is 16 (p-value = 0.021).

Step 6: Draw conclusion [justify the claim in step 1].

At the 1% significance level, we reject the alternative hypothesis .



Problem 15

A mint produces gold coins that should weigh 1 ounce. A collector weighs 9 coins and finds a mean weight of 1.02 ounces with a standard deviation of 0.04 ounces. To test if the coins are overweight (a right-tailed test) at $\alpha=0.01$. What is the correct decision?

- Reject H_0 . There is evidence the coins are overweight.
- Fail to reject H_0 . There is no evidence the coins are overweight.
- Reject H_0 . The coins are underweight.
- Fail to reject H_0 . The coins weigh exactly 1 ounce.

Answer: B

Data Source

- summarized statistics
- raw data

sample mean (\bar{x})
1.02

sample standard deviation (s)
0.04

sample size (n)
9

Claimed Value (μ_0)
1

Claim Type
greater than

Significance level α
0.01



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Solution: This t test is based on the assumption that the population is normal and the population variance is unknown.

Given sample information: $n = 9$, $\bar{x} = 1.02$, $s = 0.04$.

Step 1: Identify the claim of the population mean (μ_0).
The given information indicates that the claim is: μ_0 is greater than 1.

Step 2: Set up the null and alternative hypotheses.
Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 1$ and $H_1 : \mu > 1$.

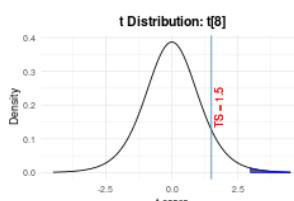
Step 3: Evaluate the test statistic.
The test statistic is defined to be: $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1.02 - 1}{0.04/\sqrt{9}} = 1.5$.

Step 4: Find the critical value and calculate the p-value.
Based on the significance level, we found the critical values to be: $t_{\alpha, df} = t_{0.01, 8} = 2.896$.
The p-value is can be found as p-value ≈ 0.086 .

Step 5: Make a statistical decision on H_0 .
At the 1% significance level, we do not reject the null hypothesis that the true mean is 1 (p-value = 0.086).

Step 6: Draw conclusion [justify the claim in step 1].
At the 1% significance level, we reject the alternative hypothesis .

t Distribution: t(8)



Summary of the Weekly Assignment

1. Five Number Summary :

The five-number summary is used to describe the shape of the distribution of a given numerical data. It consists of five numbers: minimum data value, first quartile, median, the third quartile, and the maximum data value.

The five-number summary of this given data set is:

stats	value
Min.	60.00
1st Qu.	90.00
Median	90.00
3rd Qu.	95.00
Max.	100.00

2. Boxplot :

The boxplot is a geometric representation of the five-number summary. The boxplot of the given data set is given below.

