

MAT 121 Statistics I

Midterm Exam #1

10/25/2022

Time: 75 minutes

Name _____
(Please print)

Instruction: This is a closed-book exam. You can prepare a formula sheet for the exam. Examples and definitions are NOT allowed to be put on the formula sheet. You need to turn in your formula sheet with your exam.

Part I: Multiple choice questions (12 questions, each is worth 5 points: total 60 pints)

Problem 1.

The National Weather Service keeps records of snowfall in mountain ranges. Records indicate that in a certain range, the annual snowfall has a mean of 110 inches and a standard deviation of 14 inches. Suppose the snowfalls are sampled during randomly picked years. For samples of size 49, determine the mean and standard deviation of \bar{x} .

- A) $\mu_{\bar{x}} = 2$; $\sigma_{\bar{x}} = 110$
- B) $\mu_{\bar{x}} = 110$; $\sigma_{\bar{x}} = 14$
- C) $\mu_{\bar{x}} = 14$; $\sigma_{\bar{x}} = 110$
- D) $\mu_{\bar{x}} = 110$; $\sigma_{\bar{x}} = 2$

Answer: D. This is based on the conclusion of the CLT.

Problem 2.

The heights of people in a certain population are normally distributed with a mean of 68 inches and a standard deviation of 3.1 inches. Determine the sampling distribution of the mean for samples of size 44.

- A) Normal, mean = 68 inches, standard deviation = 3.1 inches
- B) Normal, mean = 68 inches, standard deviation = 0.07 inches
- C) Approximately normal, mean = 68 inches, standard deviation = 0.47 inches
- D) Approximately normal, mean = 68 inches, standard deviation = 0.07 inches

Answer: C. This is based on the result of the CLT.

Problem 3

The monthly expenditures on food by single adults in one city are normally distributed with a mean of \$410 and a standard deviation of \$70. What is the probability that the mean monthly

expenditure of all single adults in that city is between \$400 and \$420 by the mean of a random sample of 49 such adults? Hint: choose the one that is closest to what you found.

- A) 0.9990
- B) 0.8413
- C) 0.6826
- D) 0.1587

Answer. C. Application of the CLT.

Problem 4.

The mean annual income for adult women in one city is \$28520 and the standard deviation of the incomes is \$5190. The distribution of incomes is skewed to the right. For samples of size 50, which of the following statements best describes the sampling distribution of the mean \bar{x} ?

- A) \bar{x} is approximately normally distributed.
- B) Nothing can be said about the distribution of \bar{x} .
- C) The distribution of \bar{x} is skewed to the right.
- D) \bar{x} is normally distributed.

Answer A. This is based on the CLT. Choice D is correct if the population is normal (based on the important fact).

Problem 5.

Based on a sample of 40 randomly selected years, a 90% confidence interval for the mean annual precipitation in one city is from 42.7 inches to 45.3 inches. Find the margin of error.

- A) 2.6 inches
- B) 0.34
- C) There is not enough information to find the margin of error
- D) 1.3 inches

Answer: D. $(45.3 - 42.7)/2 = 2.6/2 = 1.3$.

Problem 6.

Suppose it is known that 23% of Americans will obtain a higher educational degree after completing their undergraduate courses (such as a Master's or Doctoral degree). If a random sample of 25 Americans were surveyed, which of the following statement about the sampling distribution of the sample proportion \hat{p} is correct?

- A). The standard deviation of the sample proportion is $23/5$.
- B). The standard deviation of the sample proportion is $0.23/5$.
- C). The standard deviation of the sample proportion is $0.23 \times 0.77/5$.
- D). The standard deviation of the sample proportion is $\sqrt{0.23 \times 0.77/25}$.

Answer. D. This is the result of the sampling distribution of sample proportion since $0.23 \cdot 25 = 5.75 > 5$ and $(1-0.23) \cdot 25 = 19.25 > 5$.

Problem 7

Which of the following statements about the sampling distribution of the sample mean is **incorrect**?

- A). The sampling distribution of the sample mean is approximately normal whenever the sample size is sufficiently large ($n > 30$).
- B). The mean of the sampling distribution of the sample mean is equal to the population mean.
- C). The standard deviation of the sampling distribution of the sample mean is equal to the population standard deviation.
- D). The sampling distribution of the sample mean is generated by repeatedly taking samples of size n and computing the sample means.

Answer. C. This is again the result of the CLT.

Problem 8

The owner of a fish market has an assistant who has determined that the weights of catfish are normally distributed, with a mean of 3.2 pounds and a standard deviation of 0.8 pound. If a sample of 16 fish is taken, what would the standard error of the mean weight equal?

- A). 0.8
- B). 0.003
- C). 0.2
- D). 0.05

Answer. C. The sampling distribution of sample means.

Problem 9

For a sample taken from a non-normal population, what sample size (n) is considered *insufficiently* large enough to guarantee a good approximation of the sample mean to a normal distribution?

- A). $n = 50$.
- B). $n = 20$.
- C). $n = 35$.
- D). $n = 41$.

Answer. B. Large sample condition: $n > 30$.

Problem 10.

What is the critical t-value for finding a 90% confidence interval estimate from a sample of 15 observations?

- A). 1.341
- B). 1.761
- C). 1.350
- D). 1.753

Answer. B. Use the t-table.

Problem 11

Suppose (25, 30) is a 90% confidence interval estimate for a population mean μ . Which of the following is the margin of error?

- A). 5
- B). 2.5
- C). 27.5
- D). 1.645

Answer. B. The margin of error is equal to half of the confidence interval.

Problem 12

A confidence interval estimate is determined from the GPAs of a simple random sample of n students. All other things being equal, which of the following will result in a smaller margin of error?

- A). A larger sample standard deviation
- B). A smaller sample size
- C). A lower confidence level
- D). A smaller sample mean

Answer: C. A lower confidence level yields a smaller critical value, hence, a smaller margin of error.

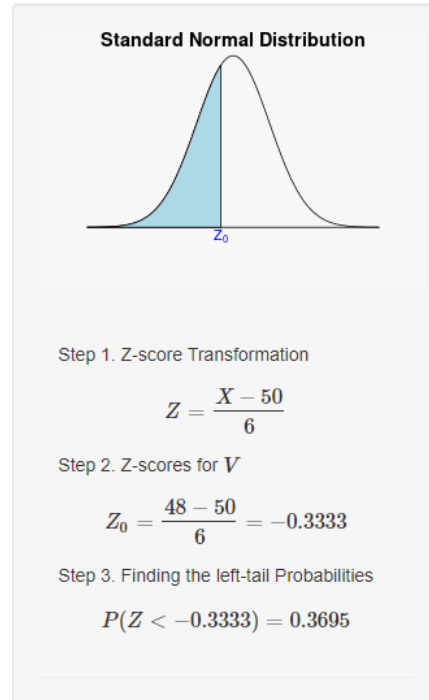
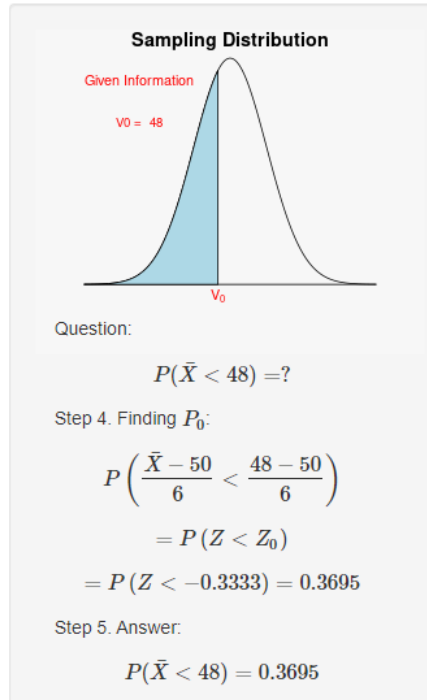
Part II: Show your work problems

Please show your work to earn credit. A Correct answer with no supporting details will earn about 25% of the credit. However, with correct details, even a wrong answer could earn up to 75% of the credit. Details are VERY important!

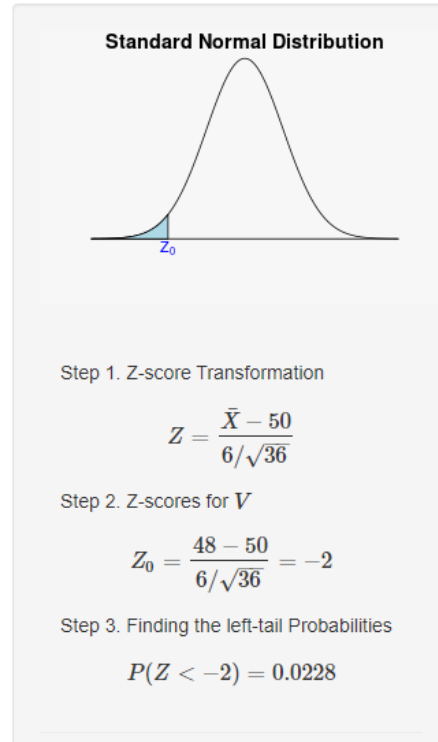
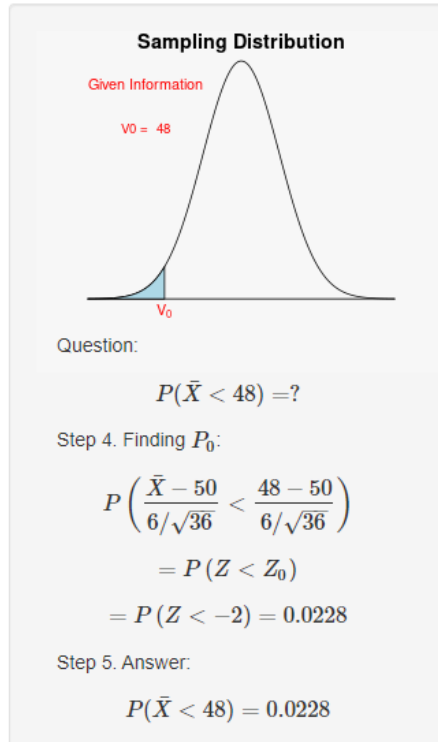
Problem 1 (10 points)

An automobile battery manufacturer claims that its midgrade battery has a mean life of 50 months with a standard deviation of 6 months. Suppose the distribution of battery lives of this particular brand is approximately normal.

1. On the assumption that the manufacturer's claims are true, find the probability that one randomly selected battery of this type will last less than 48 months.



2. Find the probability that the mean of a random sample of 36 such batteries will be less than 48 months.



Problem 2. (15 pints, each step is worth 3 points)

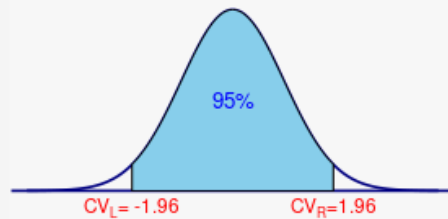
At the beginning of the Fall 2022 semester, a representative sample of 625 students at a university was surveyed and asked if they owned a dog. The sample proportion was 0.56. Construct a 95% confidence interval for the proportion of all students who own a dog.

Step 1. The given confidence level.

$$\text{conf.level} = 1 - \alpha = 95\%$$

Step 2. CV on the standard normal density curve.

$$CV = Z_{\alpha/2} = 1.960$$



Step 3: Margin of Error

$$E = CV \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 1.96 \times \sqrt{\frac{0.56(1 - 0.56)}{625}} = 0.039$$

Step 4: Expression of Confidence Interval

$$(\hat{p} - E, \hat{p} + E) = (0.56 - 0.039, 0.56 + 0.039) = (0.521, 0.599)$$

Step 5: Interpretation of Confidence Interval

There is a 95 % chance that the confidence interval (0.521 , 0.599) contains the true population proportion.

Problem 3. (15 points, each step is worth 5 points)

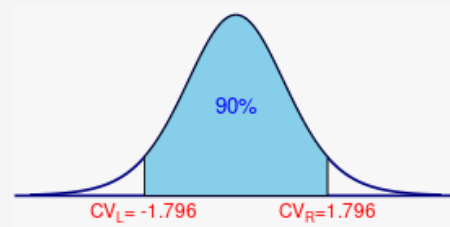
A random sample of 12 students from a large university yields a mean GPA of 2.71 with a sample standard deviation of 0.51. Construct a 90% confidence interval for the mean GPA of all students at the university. Assume that the population GPAs have a normal distribution.

Step 1. The given confidence level.

$$\text{conf.level} = 1 - \alpha = 90\%$$

Step 2. CV on the t density curve.

$$CV = t_{\alpha/2, df} = 1.796$$



Step 3: Margin of Error

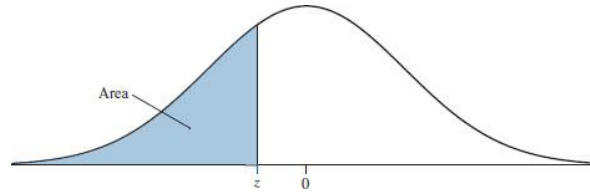
$$E = CV \times \frac{s}{\sqrt{n}} = 1.796 \times \frac{0.51}{\sqrt{12}} = 0.264$$

Step 4: Expression of Confidence Interval

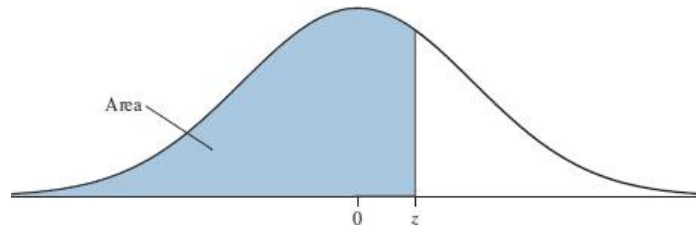
$$(\bar{X} - E, \bar{X} + E) = (2.71 - 0.264, 2.71 + 0.264) = (2.446, 2.974)$$

Step 5: Interpretation of Confidence Interval

There is a 90 % chance that the confidence interval (2.446 , 2.974) contains the true population mean.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999