

## MAT121 Midterm Exam 04

### Problem 1.

The proportion of defective items is not allowed to be over 15%. A buyer wants to test whether the proportion of defectives exceeds the allowable limit. The buyer takes a random sample of 100 items and finds that 19 are defective. State the null and alternative hypotheses for this test.

- A)  $H_0: p \leq .15$ ,  $H_a: p > .15$
- B)  $H_0: p < .15$ ,  $H_a: p > .15$
- C)  $H_0: p \neq .15$ ,  $H_a: p > .15$
- D)  $H_0: p < .15$ ,  $H_a: p > .15$

**Answer - A:** they want it to be less than 15%, but the buyer thinks that the defective items exceed 15%

### Problem 2.

Conduct a test to determine whether or not the population proportion of voters in favor of proposal A is greater than 50%. In a random sample of 200 voters, 110 said that they were in favor of this proposal. Compute the test statistic. [*Hint: pick the one that is closest to what you obtained.*]

- A)  $TS = 1.41$
- B)  $TS = 40.40$
- C)  $TS = -1.41$
- D)  $TS = 40$

**Answer - A:** you get this answer when you plug it into the calculator and set it to not equal p

## ISLA: ONE SAMPLE NORMAL TEST FOR POPULATION PROPORTION

### Data Source

☒ summarized statistics

### sample proportion ( $\hat{p}$ )

0.55

### sample size ( $n$ )

200

### Claimed Value ( $p_0$ )

0.5

### Claim Type

greater than

### Significance level $\alpha$

0.01 0.05 0.2  
0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



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**Solution:**  $n = 200$ ,  $\hat{p} = 0.55$ .

Since  $n\hat{p} = 110$  and  $n(1 - \hat{p}) = 90$ , the conditions of the central limit are satisfied.

#### Step 1: Identify the claim of the population mean ( $p_0$ ).

The given information indicates that the claim is:  $p_0$  is greater than 0.5.

#### Step 2: Set up the null and alternative hypotheses.

Based on the claim, the null and alternative hypotheses are given by  $H_0 : p = 0.5$  and  $H_1 : p > 0.5$ .

#### Step 3: Evaluate the test statistic.

The test statistic is defined to be:  $TS = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.55 - 0.5}{\sqrt{(0.5(1 - 0.5)/200)}} = 1.414$ .

#### Step 4: Find the critical value and calculate the p-value.

Based on the significance level, we found the critical values to be:  $z_\alpha = z_{0.05} = 1.645$ .

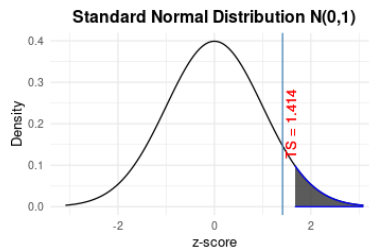
The p-value is can be found as p-value  $\approx 0.079$ .

#### Step 5: Make a statistical decision on $H_0$ .

At the 5% significance level, we do not reject the null hypothesis. ( $p$ -value = 0.079).

#### Step 6: Draw conclusion [justify the claim in step 1].

At the 5% significance level, we reject the alternative hypothesis.



### Problem 3.

The proportion of defective items is not allowed to be over 15%. A buyer wants to test whether the proportion of defectives exceeds the allowable limit. The buyer takes a random sample of 100 items and finds that 16 are defective. Find the p-value. [Hint: pick the one that is closest to what you found from the table]

- A) 0.39
- B) 0.61
- C) 0.42
- D) 0.58

**Answer - A:** Calculate the TS and then use the table to find the right-tail area (this is a right-tailed test).

## ISLA: ONE SAMPLE NORMAL TEST FOR POPULATION PROPORTION

**Data Source**  
☒ summarized statistics


**sample proportion ( $\hat{p}$ )**

**sample size ( $n$ )**

**Claimed Value ( $p_0$ )**

**Claim Type**

**Significance level  $\alpha$**

  
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**Solution:**  $n = 100$ ,  $\hat{p} = 0.16$ .

Since  $n\hat{p} = 16$  and  $n(1 - \hat{p}) = 84$ , the conditions of the central limit are satisfied.

**Step 1: Identify the claim of the population mean ( $p_0$ ).**

The given information indicates that the claim is:  $p_0$  is less than or equal to 0.15.

**Step 2: Set up the null and alternative hypotheses.**

Based on the claim, the null and alternative hypotheses are given by  $H_0 : p = 0.15$  and  $H_1 : p > 0.15$ .

**Step 3: Evaluate the test statistic.**

The test statistic is defined to be:  $TS = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.16 - 0.15}{\sqrt{(0.15(1 - 0.15)/100)}} = 0.28$ .

**Step 4: Find the critical value and calculate the p-value.**

Based on the significance level, we found the critical values to be:  $z_\alpha = z_{0.05} = 1.645$ .

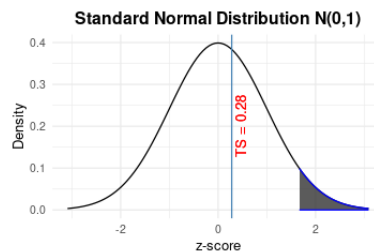
The p-value is can be found as p-value  $\approx 0.39$ .

**Step 5: Make a statistical decision on  $H_0$ .**

At the 5% significance level, we do not reject the null hypothesis. (p-value = 0.39).

**Step 6: Draw conclusion [justify the claim in step 1].**

At the 5% significance level, we reject the alternative hypothesis.



### Problem 4.

A recent newspaper article claimed that the President's approval rating is 46%; however, a sample of 80 U.S. citizens shows that 43 of them approve of the President's performance. At a 0.05 level of significance, you are interested in determining if there is evidence that the President's approval rating is actually higher.

What is the alternative hypothesis for this test?

- A).  $p = .46$
- B).  $p \neq .46$
- C).  $p < .46$
- D).  $p > .46$

**Answer – D.** The keyword “actually higher”. It is a right-tailed test.

### Problem 5.

A recent newspaper article claimed that the President's approval rating is 46%; however, a sample of 80 U.S. citizens shows that 43 of them approve of the President's performance. At a 0.05 level of significance, you are interested in determining if there is evidence that the President's approval rating is actually higher.

What is the test statistic for this hypothesis test? (*Hint: choose the one that is closest to yours*)

- A). TS = -1.39
- B). TS = 1.39
- C). TS = -.538
- D). TS = -.538

**Answer- B.**

## ISLA: ONE SAMPLE NORMAL TEST FOR POPULATION PROPORTION

**Data Source**  
☐ summarized statistics


**sample proportion ( $\hat{p}$ )**

**sample size ( $n$ )**

**Claimed Value ( $p_0$ )**

**Claim Type**

**Significance level  $\alpha$**



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**Solution:**  $n = 80$ ,  $\hat{p} = 0.5375$ .

Since  $n\hat{p} = 43$  and  $n(1 - \hat{p}) = 37$ , the conditions of the central limit are satisfied.

**Step 1: Identify the claim of the population mean ( $p_0$ ).**

The given information indicates that the claim is:  $p_0$  is greater than 0.46.

**Step 2: Set up the null and alternative hypotheses.**

Based on the claim, the null and alternative hypotheses are given by  $H_0 : p = 0.46$  and  $H_1 : p > 0.46$ .

**Step 3: Evaluate the test statistic.**

The test statistic is defined to be:  $TS = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.5375 - 0.46}{\sqrt{(0.46(1 - 0.46))/80}} = 1.391$ .

**Step 4: Find the critical value and calculate the p-value.**

Based on the significance level, we found the critical values to be:  $z_{\alpha} = z_{0.05} = 1.645$ .

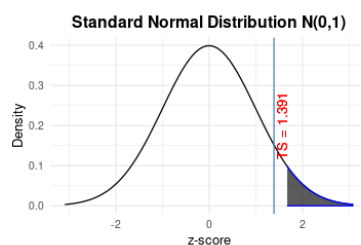
The p-value is can be found as p-value  $\approx 0.082$ .

**Step 5: Make a statistical decision on  $H_0$ .**

At the 5% significance level, we do not reject the null hypothesis. (p-value = 0.082).

**Step 6: Draw conclusion [justify the claim in step 1].**

At the 5% significance level, we reject the alternative hypothesis.



### Problem 6.

A recent newspaper article claimed that the President's approval rating is 46%; however, a sample of 80 U.S. citizens shows that 43 of them approve of the President's performance. At a 0.05 level of significance, you are interested in determining if there is evidence that the President's approval rating is actually higher. What is the p-value for this hypothesis test? (Hint: Draw a normal density curve and label all information on the curve before going to the table. Again, choose the one that is closest to yours.)

- A). 0.082
- B). 0.164
- C). 0.702
- D). 0.298

**Answer-A.** This is a right-tailed test. The p-value is the tail area to the right of the test statistic  $TS = 1.35$

### ISLA: ONE SAMPLE NORMAL TEST FOR POPULATION PROPORTION

**Data Source**  
☐ summarized statistics


**sample proportion ( $\hat{p}$ )**

**sample size ( $n$ )**

**Claimed Value ( $p_0$ )**

**Claim Type**

**Significance level  $\alpha$**

  
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**Solution:**  $n = 80$ ,  $\hat{p} = 0.5375$ .

Since  $n\hat{p} = 43$  and  $n(1 - \hat{p}) = 37$ , the conditions of the central limit are satisfied.

**Step 1: Identify the claim of the population mean ( $p_0$ ).**

The given information indicates that the claim is:  $p_0$  is greater than 0.46.

**Step 2: Set up the null and alternative hypotheses.**

Based on the claim, the null and alternative hypotheses are given by  $H_0 : p = 0.46$  and  $H_1 : p > 0.46$ .

**Step 3: Evaluate the test statistic.**

The test statistic is defined to be:  $TS = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.5375 - 0.46}{\sqrt{(0.46(1 - 0.46))/80}} = 1.391$ .

**Step 4: Find the critical value and calculate the p-value.**

Based on the significance level, we found the critical values to be:  $z_\alpha = z_{0.05} = 1.645$ .

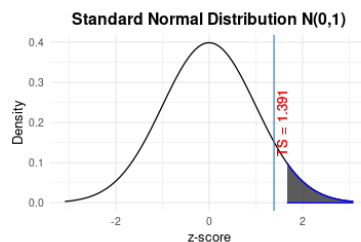
The p-value is can be found as p-value  $\approx 0.082$ .

**Step 5: Make a statistical decision on  $H_0$ .**

At the 5% significance level, we do not reject the null hypothesis. ( $p$ -value = 0.082).

**Step 6: Draw conclusion [justify the claim in step 1].**

At the 5% significance level, we reject the alternative hypothesis .



**Problem 7.**

The heights (in inches) of males in the United States are believed to be approximately normally distributed with a mean  $\mu$ . The average height of a random sample of 25 American adult males is found to be  $\bar{x} = 69.72$  inches, and the standard deviation of the 25 heights is found to be  $s = 4.15$  inches. The standard error (or standard deviation) of  $\bar{x}$  is

- A). 0.17
- B). 0.41
- C). 0.69
- D). 0.83
- E). 2.0

**Answer-D.**  $4.15/\sqrt{25} = 4.15/5 = 0.83$ .

**Problem 8**

An animal rights group has been very supportive of a new silicon product that caps the nails on cats instead of surgically declawing the pets. The company that makes the caps claims they last for an average of 68 days before needing to be replaced. Before publicly advertising their support of the product, the animal rights group plans to run a test to see if the caps last less than 68 days. What would be the appropriate hypotheses for this study?

- A).  $H_0 : \mu \leq 68$  days, vs  $H_a : \mu > 68$  days
- B).  $H_0 : \mu \geq 68$  days, vs  $H_a : \mu < 68$  days
- C).  $H_0 : \mu = 68$  days, vs  $H_a : \mu \neq 68$  days
- D).  $H_0 : \bar{x} \leq 68$  days, vs  $H_a : x < 68$  days
- E).  $H_0 : \bar{x} \geq 68$  days, vs  $H_a : x > 68$  days

**Answer- C.** “... claims they last for an average of 68 days ...” implies  $H_0 : \mu = 68$  days.

**Problem 9**

Researchers have claimed that the average number of headaches per student during a semester of Statistics is 11. Statistics students believe the average is higher. In a sample of  $n = 16$  students, the mean is 12 headaches with a deviation of 2.4. Which of the following represents the null and alternative hypotheses necessary to test the students' beliefs?

- A)  $H_0 : \mu = 11$  vs.  $H_a : \mu \neq 11$
- B)  $H_0 : \mu \geq 11$  vs.  $H_a : \mu < 11$
- C)  $H_0 : \mu < 11$  vs.  $H_a : \mu = 11$
- D)  $H_0 : \mu \leq 11$  vs.  $H_a : \mu > 11$

**Answer- D.** The claim is “the average is higher”. That is  $\mu > 11$ . Therefore,  $H_0 : \mu \leq 11$  vs.  $H_a : \mu > 11$ .

### Problem 10

A consumer product magazine recently ran a story concerning the increasing prices of digital cameras. The story stated that digital camera prices dipped a couple of years ago, but now are beginning to increase in price because of added features. According to the story, the average price of all digital cameras a couple of years ago was \$215.00. A random sample of  $n = 22$  cameras was recently taken and entered into a spreadsheet. It was desired to test to determine if the average price of all digital cameras is now more than \$215.00. Find a rejection region [defined by the critical value(s)] appropriate for this test if we are using  $\alpha = 0.05$ .

- A). Reject  $H_0$  if  $TS > 1.645$
- B). Reject  $H_0$  if  $TS > 2.080$  or  $TS < -2.080$
- C). Reject  $H_0$  if  $TS > 1.72$
- D). Reject  $H_0$  if  $TS > 1.717$

**Answer-C.** “more than \$215” implies that the claim is:  $\mu > 215$ . Therefore, it is a right-tailed test. The only critical value is on the right tail of the t-density curve with  $df = 22 - 1 = 21$ .

### Problem 11

The water diet requires you to drink 2 cups of water every half hour from when you get up until you go to bed but eat anything you want. Four adult volunteers agreed to test this diet. They are weighed prior to beginning the diet and 6 weeks after. Their weights in pounds are

Person	1	2	3	4	mean	s.d.
Weight before	180	125	240	150	173.75	49.56
Weight after	170	130	215	152	166.75	36.09
Difference	10	-5	25	-2	7	13.64

Test the claim that the water diet actually reduces weight at a significance level of 0.05. What is the critical value associated with the test?

- A). -1.645
- B). -2.353
- C). -2.132
- D). -2.575

**Answer-B.** This is a left-tailed t-test with degrees of freedom  $4 - 1 = 3$ .

### Problem 12

A test is conducted for  $H_0: \mu = 34$ , with  $\sigma = 5$ . A sample of size 25 is selected. The standard error of the sampling distribution is

- A). 0.1
- B). 5.0
- C). 1.0
- D). 0.2

**Answer- C.**  $5/\sqrt{25} = 1.0$

### Problem 13

A one-sample  $t$ -test is conducted on  $H_0: \mu = 81.6$  vs  $H_0: \mu \neq 81.6$ . The sample has  $\bar{x} = 84.1$ ,  $s = 3.1$ , and  $n = 25$ . The test statistic is

- A). 4.03
- B). 15.5
- C). 3.95
- D). 3.1

**Answer-A.** simply use the formula to find the test statistic.

## ISLA: ONE SAMPLE T TEST FOR POPULATION MEAN

#### Data Source

- ☐ summarized statistics
- ☒ raw data

#### sample mean ( $\bar{x}$ )

#### sample standard deviation ( $s$ )

#### sample size ( $n$ )

#### Claimed Value ( $\mu_0$ )

#### Claim Type

**Solution:** This  $t$  test is based on the assumption that the population is normal and the population variance is unknown.

**Given sample information:**  $n = 25$ ,  $\bar{x} = 84.1$ ,  $s = 3.1$ .

#### Step 1: Identify the claim of the population mean ( $\mu_0$ ).

The given information indicates that the claim is:  $\mu_0$  is equal to 81.6.

#### Step 2: Set up the null and alternative hypotheses.

Based on the claim, the null and alternative hypotheses are given by  $H_0: \mu = 81.6$  and  $H_1: \mu \neq 81.6$ .

#### Step 3: Evaluate the test statistic.

The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{84.1 - 81.6}{3.1/\sqrt{25}} = 4.032$ .

#### Step 4: Find the critical value and calculate the p-value.

Based on the significance level, we found the critical values to be:  $CV = \pm t_{\alpha/2, df} = \pm t_{0.025, 24} = \pm 2.064$ .

The p-value is can be found as p-value  $\approx 0$ .

#### Step 5: Make a statistical decision on $H_0$ .

At the 5% significance level, we reject the null hypothesis that the true mean is 81.6 ( $p$ -value  $< 0.001$ ).



#### Problem 14

If performing a hypothesis about a population mean of  $\mu = \mu_0$  at significant level 0.05, with  $\sigma$  estimated by  $s$  and  $n = 20$ , the correct critical value for  $t$  is

- A). 1.729
- B). 1.625
- C). 2.186
- D). 2.093

**Answer-D.** This is a two-tailed t-test with a significance level of 0.05.

#### Problem 15

Suppose we were interested in determining if there were differences in the average prices between two local supermarkets. We randomly pick six items to compare at both supermarkets. Which statistical procedure would be best to use for this study?

- A). paired t procedure
- B). One-sample t-test
- C). Two-sample normal test
- D). None of the listed tests

**Answer-A.** The two supermarkets can be considered “before” and “after”, so the same item is considered to be priced “before” and “after”.

#### Problem 16

Two samples of sizes 25 and 35 are independently drawn from two normal populations, where the unknown variances are assumed to be equal. The number of degrees of freedom for the equal-variances t-test statistic is:

- a. 58
- b. 60
- c. 62
- d. 57
- e. 68

**Answer: A,**  $(25+35)-2 = 58$

#### Problem 17

Which of the following statements is correct?

- a. the pooled-variances t-test is used whenever the population standard deviations can be assumed to be equal, regardless of the sample size.

- b. the unequal-variances t-test is used whenever the population standard deviations are unknown and cannot assumed to be equal.
- c. the z-test can be used as a close approximation to the unequal-variances t-test when the population standard deviations are not assumed to be equal but both sample sizes are large (typically greater than 30)
- d. all of the above statements are true.

**Answer: D**

**Problem 18.**

Two independent samples from populations that are normally distributed produced the following statistics: for sample 1 the sample size was 40, the sample mean was 34.2 and the sample standard deviation was 12.6. For sample 2, the sample size was 32, the sample mean was 49.1 and the sample standard deviation was 19.4. Assume that population variances are equal. Given a significance level of 5%, at what approximate value of t should you reject the null hypothesis that states that the two population means are equal, in favor of the two-sided alternative?

- a.  $\pm 2.32$
- b.  $\pm 1.285$
- c.  $\pm 1.68$
- d.  $\pm 1.96$
- e.  $\pm 2.13$

**Answer D.**

**Problem 19.**

The owner of Bun & Run Hamburgers wishes to compare the sales per day at different locations. The mean number of hamburgers sold for 10 randomly selected days at Northside was 83.55 with a sample standard deviation of 10.50. For a randomly selected 12 days at Southside, the mean number of hamburgers sold was 69.54 with a sample standard deviation of 14.25. We wish to test whether there is a difference in the mean number of hamburgers sold at the two locations using a 5% significance level. What is the value of the test statistic in this case?

- a. 1.84
- b. 0.24
- c. 2.57
- d. 1.71
- e. 2.20

**Answer C:**

## ISLA: Two-Sample Test for the Difference of Two Population Means

### Sample #1

sample mean ( $\bar{x}_1$ )

83.55

sample variance ( $s_1^2$ )

110.25

sample size ( $n_1$ )

10

### Sample #2

sample mean ( $\bar{x}_2$ )

69.54

sample variance ( $s_2^2$ )

203.0625

sample size ( $n_2$ )

12

Claimed Value ( $\mu_1 - \mu_2$ )

**Solution:** Since one of the sample sizes is less than 31. The following normal test assumes both populations are normal and the two unknown population variances are equal.

**Given sample information:**  $n_1 = 10$ ,  $\bar{x}_1 = 83.55$ ,  $s_1^2 = 110.25$ .  
 $n_2 = 12$ ,  $\bar{x}_2 = 69.54$ ,  $s_2^2 = 203.0625$ .

**Step 1: Identify the claim of the population mean ( $\mu_1 - \mu_2$ ).**

The given information indicates that the claim is:  $\mu_1 - \mu_2$  is equal to 0.

**Step 2: Set up the null and alternative hypotheses.**

Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu_1 - \mu_2 = 0$  and  $H_1 : \mu_1 - \mu_2 \neq 0$ .

**Step 3: Evaluate the test statistic.**

We first find the pooled sample variance in the following

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(10 - 1)110.25 + (12 - 1)203.0625}{10 + 12 - 2} = 161.297.$$

The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_{pool}^2/n_1 + s_{pool}^2/n_2}} = \frac{(83.55 - 69.54) - 0}{\sqrt{161.297/10 + 161.297/12}} = 2.576.$$

**Step 4: Find the critical value and calculate the p-value.**

Based on the significance level, we found the critical values to be:  $\pm t_{\alpha/2, df} = \pm t_{0.025, 20} = \pm 2.086$ .

**Step 5: Make a statistical decision on  $H_0$ .**

At the 5% significance level, we reject the null hypothesis that the true mean is 0 ( $p$ -value = 0.018).

**Step 6: Draw conclusion [Justify the claim in step 1].**

At the 5% significance level, we conclude the alternative hypothesis. The claim is addressed using relationship between the alternative hypothesis and the claim.

### Problem 20

A researcher randomly sampled 30 graduates of an MBA program and recorded data concerning their starting salaries. The sample comprised 18 women whose average starting salary is R48000, and 12 men whose average starting salary is R55000. It is known that the sample standard deviations of starting salaries for women and men are R11500 and R13000 respectively. The researcher was attempting to show that female MBA graduates have significantly lower average starting salaries than male MBA graduates. What is the value of the test statistic in this case?

- 1.55
- 2.16
- 0.86
- 1.40
- 1.68

### Answer A.

**Step 3: Evaluate the test statistic.**

We first find the pooled sample variance in the following

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(18 - 1)132250000 + (12 - 1)169000000}{18 + 12 - 2} = 146687500$$

The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(48000 - 55000) - 0}{\sqrt{146687500/18 + 146687500/12}} = -1.551$$

**Problem 21**

It is known that the sample variances of final exam marks for first-year statistics students at a particular South African university are 45.3 for female students and 52.1 for male students. Samples of 27 female and 31 male first-year statistics students from the university are selected and the sample exam marks are calculated. For females, the sample mean mark is 52.3 % and for males, the sample mean mark is 55.4 %. If we wish to test whether females have, on average, higher exam marks than males, what would the test statistic value of the hypothesis test in this case be?

- a. 1.04
- b. 1.58
- c. 0.49
- d. -1.15
- e. -1.68

**Answer E.**

**Step 3: Evaluate the test statistic.**

We first find the pooled sample variance in the following

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(27 - 1)45.3 + (31 - 1)52.1}{27 + 31 - 2} = 48.943$$

The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(52.3 - 55.4) - 0}{\sqrt{48.943/27 + 48.943/31}} = -1.683$$

**Problem 22**

A new Grade 9 math curriculum is to be tested. Several students in a Grade 9 math course at a particular high school were randomly divided into two groups. The experimental group received teaching according to the new curriculum while the control group did not. All the students were given a test of computational skill (out of a total of 5 points) after the course and the results were as follows: for the experimental group, the sample size was 28, the sample mean was 1.99 and the sample variance was 85. For the control group, the sample size was 31, the sample mean was 3.54, and the sample variance was 80. We wish to test at the 5 % level of significance whether there is any difference in the mean test scores. We assume that the underlying population variances are equal. What is the value of the test statistic for this hypothesis test?

- a. -1.03
- b. 0.44
- c. -0.80
- d. -0.66
- e. 0.13

**Answer: D.**

**Step 3: Evaluate the test statistic.**

We first find the pooled sample variance in the following

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(28 - 1)85 + (31 - 1)80}{28 + 31 - 2} = 82.368$$

The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(1.99 - 3.54) - 0}{\sqrt{82.368/28 + 82.368/31}} = -0.655$$

**Problem 23.**

A comparison of the price-earnings (P/E) ratio for the top and bottom 100 companies on the JSE is being prepared. A financial advisor randomly sampled each group to determine whether there was any difference in the P/E ratios of the two groups of companies. Assume unequal population variances but that the populations are normally distributed. For 6 top 100 companies, the average P/E ratio was 21.03 with a sample variance of 128.17. For 8 of the bottom 100 companies, the average P/E ratio was 10.67 with a sample variance of 125.15. The test is conducted at the 5 % level of significance. What would be the value of the test statistic for the hypothesis test in this case?

- a. 1.34
- b. 0.74
- c. 1.67
- d. 1.72
- e. 2.25

**Answer: D.**

**Step 3: Evaluate the test statistic.**

We first find the pooled sample variance in the following

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(6 - 1)128.17 + (8 - 1)125.15}{6 + 8 - 2} = 126.408$$

The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(21.03 - 10.6) - 0}{\sqrt{126.408/6 + 126.408/8}} = 1.718$$

**Problem 24.**

A sociologist wants to test if there is any difference between the mean income of farm workers in two different areas. In area A, a random sample of 41 workers yielded a mean income of R21000 with a standard deviation of R9010. In area B a random sample of 9 workers yielded a mean income of R15078 and a standard deviation of R5624. If we assume that the population variances are not equal, what would be the value of the test statistic for the hypothesis test in this case?

- a. 0.21
- b. 1.25
- c. 0.67
- d. 1.88
- e. 2.56

**Answer: D.**

**Step 3: Evaluate the test statistic.**

We first find the pooled sample variance in the following

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(41 - 1)81180100 + (9 - 1)31629376}{41 + 9 - 2} = 72921646$$

The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(21000 - 15078) - 0}{\sqrt{72921646/41 + 72921646/9}} = 1.884$$

**Problem 25**

The national electricity supplier claims that switching off the hot water cylinder at night does not result in saving electricity. In order to test this claim, a newspaper reporter obtained the cooperation of 16 house owners with similar houses and salaries. Eight of the selected owners switch their cylinders off at night. The consumption of electricity in each house over a period of 30 days is measured; the units are kWh (kilowatt-hours). For households that switched off their hot water cylinders, average consumption over the 30 days was 680kWh with a variance of 425kWh<sup>2</sup>. For those that did not switch off their hot water cylinders, average electricity consumption was 700kWh with a variance of 300kWh<sup>2</sup>. If we wish to test the assumption of equal population variances at the 5 % level of significance, what is the value of the test statistic?

- a. -1.50
- b. -2.10
- c. -1.29
- e. -1.95

**Answer: B.**

**Step 3: Evaluate the test statistic.**

We first find the pooled sample variance in the following

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(8 - 1)425 + (8 - 1)300}{8 + 8 - 2} = 362.5$$

The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(680 - 700) - 0}{\sqrt{362.5/8 + 362.5/8}} = -2.101$$

**Problem 26**

A teacher is interested in performing a hypothesis test to compare the mean math score of the girls and the mean math score of the boys. She randomly selects 10 girls from the class and then randomly selects 10 boys. She arranges the girls' names alphabetically and uses this list to assign each girl a number between 1 and 10. She does the same thing for the boys.

- A) Paired t-test. Since the boys and girls are in the same class and are hence dependent samples, they can be linked.
- B) 1-sample t-test. The teacher should compare the sample mean for the girls against the population mean for the boys.
- C) Two-sample t-test. There is no natural pairing between the two populations.
- D) Paired t-test. Since there are 10 boys and 10 girls, we can link the two samples.

**Answer: C**

**Problem 27**

The first 10 students who arrived for the Friday lecture filled out a questionnaire on their attitudes toward the instructor. The first 10 who were late for the lecture were spotted and afterward filled out the same questionnaire. The appropriate design for testing the significance of the difference between the means is related to

- A) paired-sample t-test.
- B) independent 2-sample t-test.
- C) one-sample t-test.
- D) one-sample z-test.

**Answer: B.**

**Problem 28**

The owner of Bun & Run Hamburgers wishes to compare the sales per day at two different locations. The mean number of hamburgers sold for 10 randomly selected days at Northside was 83.55 with a population standard deviation of 10.50. For a randomly selected 12 days at Southside, the mean number of hamburgers sold was 69.54 with a population standard deviation of 14.25. We wish to test whether there is a difference in the mean number of hamburgers sold at the two locations using a 5% significance level. Assume that distributions of the sales at two locations are normally distributed and with equal variances. What is the value of the test statistic in this case?

- A 1.84
- B 0.24
- C 2.56
- D 1.71

**Answer: C.**

**Step 3: Evaluate the test statistic.**

We first find the pooled sample variance in the following

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(10 - 1)110.25 + (12 - 1)203.06}{10 + 12 - 2} = 161.296.$$

The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_{pool}^2/n_1 + s_{pool}^2/n_2}} = \frac{(83.5 - 69.54) - 0}{\sqrt{161.296/10 + 161.296/12}} = 2.567.$$

**Problem 29**

The health of the bear population in Yellowstone National Park is monitored by periodic measurements taken from anesthetized bears. In a sample of 100 bears, the mean weight was found to be 185 lbs. Assume that  $\sigma$  (population standard deviation) is known to be 125 lbs., use a 0.03 significance level to test the claim that the population mean weight of bears is equal to 210 lbs. What is the value of the test statistic?

- A 2.00
- B 0.0228
- C -2.00
- D 0.0456

**Answer: C.**

**Step 3: Evaluate the test statistic.**

The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{185 - 210}{125/\sqrt{100}} = -2$

**Step 4: Find the critical value and calculate the p-value.**

Based on the significance level, we found the critical values to be :  $\pm z_{\alpha/2} = \pm z_{0.015} = \pm 2.17$

The p-value is can be found as p-value  $\approx 0.046$ .

### Problem 30

In a survey, 1,865 out of 2,246 randomly selected adults in the United States said that texting while driving should be illegal. Using these results, conduct a hypothesis test at the 5% significance level to test the claim that more than 80% of adults believe that texting while driving should be illegal. What is the P-value?

- A  $> 0.999$
- B  $< 0.001$
- C  $> 3.55$
- D = 1.86

**Answer: B**

**Step 3: Evaluate the test statistic.**

The test statistic is defined to be:

$$TS = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.83 - 0.8}{\sqrt{(0.8(1 - 0.8))/2246}} = 3.554.$$

**Step 4: Find the critical value and calculate the p-value.**

Based on the significance level, we found the critical values to be :  $z_{\alpha} = z_{0.05} = 1.645$ .

The p-value is can be found as p-value  $\approx 0$ .

### Problem 31



According to a certain TV broadcast station, the average number of violent incidents shown per episode of a TV series is 7. A researcher believes that this has increased in the last few years. A random sample of 16 recent episodes was selected which produced a sample mean of 6.9 violent incidents and a standard deviation of 1.2. *Assume that the number of violent incidents follows a normal distribution.* If we were to perform a hypothesis test at the level of 0.05 in order to test whether the researcher's belief is accurate or not, what would be the critical value?

- A 2.31
- B 1.746
- C 1.645
- D 2.131

**Answer: D.**

**Step 3: Evaluate the test statistic.**

The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{6.9 - 7}{1.2/\sqrt{16}} = -0.333.$

**Step 4: Find the critical value and calculate the p-value.**

Based on the significance level, we found the critical values to be :

$$CV = \pm t_{\alpha/2, df} = \pm t_{0.025, 15} = \pm 2.131.$$

The p-value is can be found as p-value  $\approx 0.744.$

### Problem 32

A hypothesis test is conducted to test whether the mean age of clients at a certain health spa is equal to 25 or not. 36 clients were randomly selected, and their ages were recorded, with the sample mean age being 27.8 and a standard deviation of 10. Assume that the population distribution of ages is skewed to the right. What is the p-value? *[Hint: if your answer is different from any of the provided ones, choose the closest one as your answer]*

- A 1.68
- B 0.9535
- C 0.0465
- D 0.092

**Answer: D**

**Step 2: Set up the null and alternative hypotheses.**

Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 25$  and  $H_1 : \mu \neq 25$ .

**Step 3: Evaluate the test statistic.**

The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{27.8 - 25}{10/\sqrt{36}} = 1.68$

**Step 4: Find the critical value and calculate the p-value.**

Based on the significance level, we found the critical values to be:  $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$

The p-value is can be found as p-value  $\approx 0.0920000000000001$ .

**Step 5: Make a statistical decision on  $H_0$ .**

At the 5% significance level, we do not reject the null hypothesis. ( $p$ -value = 0.092).

**Problem 33**

The mean life of a battery used in a digital clock is 305 days. The lives of the batteries follow a normal distribution. The battery was recently modified to last longer. A sample of 20 of the modified batteries had a mean life of 307 days with a standard deviation of 12 days. A hypothesis test is undertaken to determine whether the modification *increased* the battery life. The null and alternative hypotheses are

- A  $H_0: \mu \leq 305$  vs  $H_a: \mu > 305$
- B  $H_0: \mu = 305$  vs  $H_a: \mu \neq 305$
- C  $H_0: \mu < 305$  vs  $H_a: \mu \geq 305$
- D  $H_0: \mu > 305$  vs  $H_a: \mu \leq 305$

**Answer: A.**

**Problem 34**

A hypothesis test is to be conducted to test whether a certain population mean is equal to or greater than 24.4. A sample of size 64 is selected from the population and the sample mean is calculated as being 26.52 and standard deviation 7.6. What is the value of the test statistic for this test?

- A 1.14
- B 0.12
- C 2.23
- D -0.90

**Answer C**

**Step 3: Evaluate the test statistic.**

The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{26.52 - 24.4}{7.6/\sqrt{64}} = 2.232$