MAT121 Midterm Exam 02

Problem 1.

Assume that the heights of women are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. Which of the following transformation converts this general normal distribution to the standard normal distribution,

A). $Z = (X - 63.6)/2.5$
B). $Z = (X - 2.5)/63.6$
C). $Z = 2.5 X + 63.6$
D). $Z = 63.6X + 2.5$

Answer: A, $Z = (X - 63.6)/2.5$

Problem 2.
Suppose X follows a normal distribution with mean 5 and variance 9. After z-score transformation, Probability $P(X \leq 8)$ is equal to

A $P(Z \leq 1)$

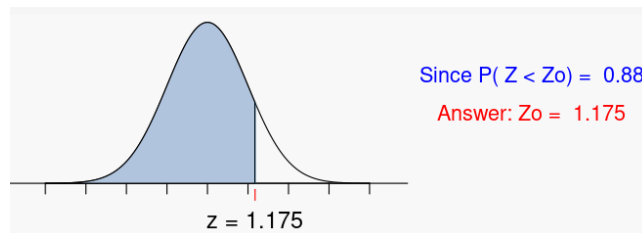B $P(Z \leq -1)$

C $P(Z \leq 0.6)$

D $P(Z \leq -0.6)$

Answer: A. $Z = (8 - 5) / \sqrt{9} = 3/3 = 1.$

Problem 3.

Find the z-score in the standard normal distribution such that the area to the right of z is 0.12.
A. -1.175
B. 0.88
C. -0.88
D. 1.175

Answer: D

Since $P(Z < Z_0) = 0.88$
Answer: $Z_0 = 1.175$

$z = 1.175$

Problem 4

Suppose X is normally distributed with a mean of 5 and a standard deviation of 0.4. Assume that $P(X \leq X_0) = P(Z \leq 1.3)$. Which of the following equations should be solved to get the value of X_0?

    A. $X_0 = 1.3 \times 0.4 + 5$
    B. $X_0 = 1.3 \times 0.4 - 5$
    C. $X_0 = 1.3 \times 5 + 0.4$
    D. $X_0 = 1.3 \times 5 - 0.4$

Answer: A. solve X_0 from $1.3 = (X_0 - 5)/0.4$

Problem 5

The Precision Scientific Instrument Company manufactures thermometers that are supposed to give readings of $0°$ C at the freezing point of water. Tests on a large sample of these instruments reveal that the freezing point of water is around zero (some thermometers give positive degrees, some thermometers give negative degrees), Assume that the mean reading is $0°$ C and the standard deviation of the readings is $1.0°$ C. Assume further that the readings are normally distributed. Find the 3rd percentile of the thermometer readings.

    A. 0.585
    B. 0.675
    C. 0.749
    D. 0.7517

Since $P(Z < Zo) = 0.75$

Answer: $Zo = 0.675$

z = 0.675
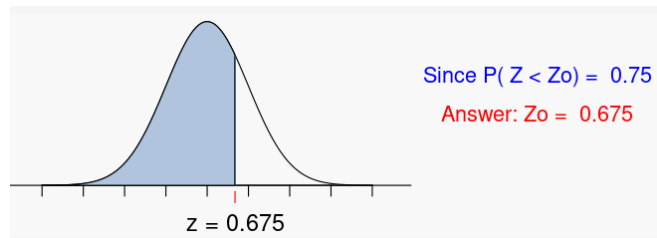
Answer: B

Problem 6.
The height of an adult male is known to be normally distributed with a mean of 175 cm and a standard deviation of 6 cm. We are interested in the 75th percentile in this distribution of heights. Let Q3 be denoted as the 75th percentile. Then Q3 can be solved from which of the following equation?

A.

$$\frac{Q3 - 175}{6} = 0.675$$

B.

$$\frac{Q3 - 175}{36} = 0.675$$

C.

$$\frac{Q3 - 175}{6} = 0.75$$

D.

$$\frac{Q3 - 175}{6} = -0.75$$

Problem 7.

The time that it takes a WCU student to find parking once they have arrived on campus is approximately normally distributed with a mean 25 minutes and a standard deviation 5 minutes. If a student arrives on campus at 8:15 AM, we want to find the probability that the student will find parking before 8:45 AM. Which of the calculation is correct?

A. $P(X < 45\ ) = P(Z < 4) \approx 1$

B. $P(X > 15\ ) = P(Z > -2) \approx 0.9772$

C. $P(X < 30\ ) = P(Z < 1) \approx 0.8413$

D.$P(X > 30\ ) = P(Z > 1) \approx 0.1587$

Problem 8.

A machine that cuts corks for wine bottles operates in such a way that the distribution of the diameter of the corks produced is well approximated by a normal distribution with a mean of 3 cm and a standard deviation of 0.1 cm. The specifications call for corks with diameters between 2.9 and 3.1 cm. A cork **not** meeting specifications is considered defective. We need to use the z-score transformation before using the standard normal distribution table.

Which of the following calculations for the proportion of good corks is correct?

A. $P(2.9 < T < 3.1) = P(-1 < Z < 1) = 0.8413$

B. $P(2.9 < T < 3.1) = P(-1 < Z < 1) = 0.6826$

C. $P(2.9 < T \text{ or } T < 3.1) = P(-1 < Z \text{ or } Z < 1) = .3413$

D. $P(3.1 < T \text{ or } T < 2.9) = P(1 < Z \text{ or } Z < -1) = 0.3174$

Answer: B.

Step 1. Z-score Transformation

$$Z = \frac{\bar{X} - 3}{0.1/\sqrt{1}}$$

Step 2. Z-scores for V_0 and V_1

$$Z_0 = \frac{2.9 - 3}{0.1/\sqrt{1}} = -1$$

$$Z_1 = \frac{3.1 - 3}{0.1/\sqrt{1}} = 1$$

Step 3. Finding the left-tail Probabilities

$$P(Z < 1) = 0.8413$$

$$P(Z < -1) = 0.1587$$

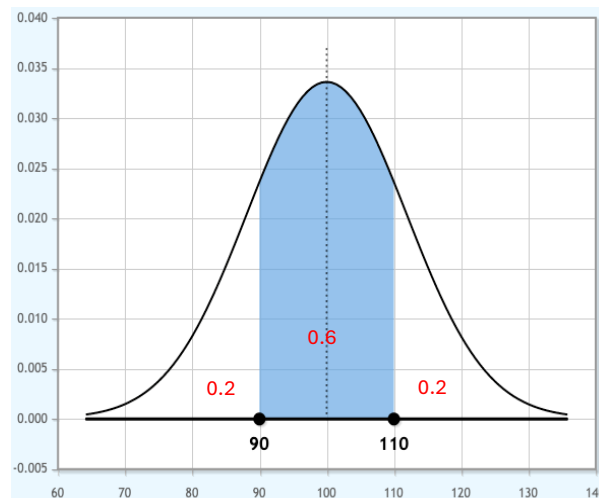$P(2.9 < \bar{X} < 3.1) = P(Z < 1) - P(Z < -1) = 0.8413 - 0.1587 = 0.6826$

Problem 9

Suppose a normally distributed random variable x has a mean of 100 and P(x < 90) = 0.20. What is the probability that x is between 90 and 110? Ie. What is P(90 < x < 110)? [Draw a density curve and label the values, then you can see the region you want to find the area.]

A 0.2
B 0.3
C 0.4
D 0.5
E 0.6

Answer: D
Both tail areas are equal to 0.2.

Problem 10

If you got a 79 on a test in a class with a mean score of 85 and a standard deviation of 4.8, the z-score of your test score would be
A 1.25
B -2.50
C -1.25
D 2.5

Answer: C. Z = (79 – 85)/4.8 = -4/4.8 = - 6/4.8 = - 1.25

Problem 11

For a normal distribution with a mean of 480 and a standard deviation of 32, find the value of Q2 (2nd quartile, or median).
A 458
B 480
C 500
D It depends on the size of the population.

Answer: B. The median (2nd quartile) of the standard normal distribution is Z = 0. That is, the median of the above normal distribution is
$(X_0 – 480)/32 = 0$, implies that $X_0 = 480$.

Problem 12

A normal curve table tells you that the probability lying below z = -1 is .1587. This can be interpreted as:
A 15.87% of the area of the curve lies below z = -1
B 15.87% of the area under the curve lies at or below z = -1
C A random selection from the population has a 15.87% chance of being below z = -1
D All choices are correct.

Answer D.

Problem 13

For a sample taken from a non-normal population, what sample size (n) is considered *insufficiently* large enough to guarantee a good approximation of the sample mean to a normal distribution?

A. n = 50
B. n = 20
C. n = 35
D. n = 41

Answer: B. n = 20 < 30.

Problem 14.

The mean and standard deviation of a sampled population are: μ = 200 and σ = 24. What is the mean and standard deviation of the sample if n = 64?
A. (200, 3)
B. (25, 24)
C. (25, 3)
D. (200, 24)

Answer: A. The sample mean is equal to the population mean 200 and sample standard deviation is equal to 24/sqrt(64) = 24/8 = 3.

Problem 15.

A high school has kept a record of its seniors' SAT scores over the past several years and has found the scores to have a mean of 1256 (out of 1600) and standard deviation of 168. The school randomly sampled 144 students from this population. What is the sample's mean and standard deviation of SAT scores?

A (1256, 1.67)
B. (1256, 14)
C. (89.7, 1.67)
D. (89.7, 12)

Answer: B. Application of the central limit theorem

Problem 16.
Suppose it is known that 23% of Americans will obtain a higher educational degree after completing their undergraduate courses (such as a Master's or Doctoral degree). If a random sample of 25 Americans were surveyed, which of the following statement about the sampling distribution of the sample proportion $\hat{p}$ is correct?

A. The standard deviation of the sample proportion is 23/5.
B. The standard deviation of the sample proportion is 0.23/5.
C. The standard deviation of the sample proportion is 0.23×0.77/5.
D. The standard deviation of the sample proportion is $\sqrt{0.23 \times 0.77/25}$.

Answer: D.

Problem 17.

A sample of 49 workers were asked to rate their job satisfaction on a scale of 1 to 10. The sample mean was 6.8 and the standard deviation is 2.1. To construct a 95% confidence interval for the mean rating scale, what is the margin of error?

A. 1.645×2.1/49
B. 1.645×2.1/$\sqrt{49}$
C. 1.960×2.1/49
D. 1.960×2.1/$\sqrt{49}$

Answer: D. use the sampling distribution of sample mean based on the central limit theorem.

Problem 18

A random sample of 36 public universities were asked to report their semester tuition cost. The mean was found to be $12,000 and standard deviation $1752. The 95% confidence interval of the mean tuition cost is

A. (11427.68, 12572.32)
B. (11519.66, 12480.34)
C. (11919.94, 12080.06)
D. (9117.96, 14882.04)

Answer: A. Following the steps for constructing large sample confidence intervals.

Problem 19.

For sample size 16, the sampling distribution of the mean will be approximately normally distributed

A) if the population is normally distributed.
B) if the sample standard deviation is known.
C) if the shape of the population is symmetrical.
D) regardless of the shape of the population.

Answer: A. This is based on the important fact of normal populations given in the lecture note.

Problem 20.

Which of the following statements about the sampling distribution of the sample mean is **incorrect**?

A) The sampling distribution of the sample mean is approximately normal whenever the sample size is sufficiently large (n > 30).
B) The mean of the sampling distribution of the sample mean is equal to population mean.
C) The standard deviation of the sampling distribution of the sample mean is equal to population standard deviation.
D) The sampling distribution of the sample mean is generated by repeatedly taking samples of size n and computing the sample means.

Answer: C. Based on the CLT.

Problem 21.

Suppose the ages of students in Statistics I follow a right skewed distribution with a mean of 23 years and a standard deviation of 3 years. If we randomly sampled 100 students, which of the following statements about the sampling distribution of the sample mean age is **incorrect**?

A) The standard deviation of the sampling distribution is equal to 3 years.
B) The standard error of the sampling distribution is equal to 0.3 years.
C) The shape of the sampling distribution is approximately normal.
D) The mean of the sampling distribution is equal to 23 years

Answer: A. The standard deviation of the sample mean is 3/sqrt{100}

Problem 22.

Fifty SmartCars were randomly selected and the highway mileage of each was noted. The analysis yielded a mean of 47 miles per gallon and a standard deviation of 5 miles per gallon. Which of the following would represent a 95% confidence interval for the average highway mileage of all SmartCars?

A). $47 \pm 1.345 \times (5/\sqrt{50})$
B). $47 \pm 1.645 \times (5/\sqrt{50})$
C). $47 \pm 1.96 \times (5/\sqrt{50})$
D). $47 \pm 2.575 \times (5/\sqrt{50})$

Answer: C. This is a large sample confidence interval of the population mean.

Problem 23

A sample of 892 people were asked to report their weekly grocery expenditure. The mean was found to be $77.54. Assume σ = $15.43. What is the margin of error for the 95% confidence interval for the mean grocery cost of the population?
A. $1.01
B. $1.49
C. $2.54
D. $0.69

Answer: A. Using the formula of the margin of error.
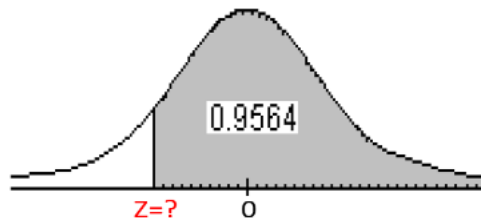
Problem 24.

The owner of a fish market has an assistant who has determined that the weights of catfish are normally distributed, with mean of 3.2 pounds and standard deviation of 0.8 pound. If a sample of 16 fish is taken, what would the standard error of the mean weight equal?

A) 0.800
B) 0.003
C) 0.200
D) 0.050

Answer: C. Using the result of the important fact.

Problem 25.

Find the z-score that corresponds to the given area under the *standard normal* curve.

A 1.71

B -1.71

C 1 – 0.9564

D -1.96

Answer: B

Problem 26.

The role of the sample mean in a confidence interval estimate for the population mean is to:

A determine the amount by which the estimate will miss the true population mean.

B determine the margin of error in the estimate.

C establish the level of confidence for the estimate.

D determine the center of the confidence interval

Answer: D.

Problem 27.

The Central Limit Theorem predicts that

A The sampling distribution of μ will be approximately normal for $n > 30$

B the sampling distribution of sample mean will be approximately normal for $n > 30$

C the sampling distribution of p will be approximately normal for $n > 30$

D the sampling distribution of sample proportion will be approximately normal for $n > 30$

Problem 28.

A study was conducted to determine what proportion of all college students considered themselves as full-time students. A random sample of 300 college students was selected and 210 of the students responded that they considered themselves full-time students. Which of the following would represent the target parameter of interest?

A   mean

B   Proportion

Answer: B.

Problem 29

In a sample of 155 students, it is found that 21 made an A. What is the margin of error of the 95% confidence interval of p constructed based on the sample information?

A
$$E = 1.96 \times \frac{\sqrt{21(155 - 21)}}{155}$$

B
$$E = 1.96 \times \frac{\sqrt{\frac{21}{155}(\frac{155 - 21}{155})}}{155}$$

C
$$E = 1.96 \times \frac{\sqrt{\frac{21}{155}(\frac{155 - 21}{155})}}{\sqrt{155}}$$

D
$$E = 1.96 \times \frac{\sqrt{\frac{21}{155} + \frac{155 - 21}{155}}}{\sqrt{155}}$$

Answer: C.

Problem 30

A randomly selected sample of 400 students at a university with 15-week semesters was asked whether or not they think the semester should be shortened to 14 weeks (with longer classes). Forty-six percent (46%) of the 400 students surveyed answered "yes." Which one of the following statements about the number 46% is correct?

A It is a sample statistic.

B It is a population parameter.

C It is a margin of error.

D It is a standard error.

Answer: A.

Problem 31.

A random sample of 36 customers buying petrol was selected. From this sample, the 95% confidence interval estimate for the mean amount of petrol purchased per customer for the city was calculated to be between 14.5 and 20.5 gallons. Based on this information, the sample mean and the margin of error is given by

A $\bar{X} = (14.5 + 20.5)/2, \quad E = (20.5 - 14.5)/2$.

B $\bar{X} = (14.5 + 20.5)/2, \quad E = (20.5 - 14.5)$.

C $\bar{X} = (14.5 + 20.5)/2, \ E$ cannot be determined

D $\bar{X}$ cannot be determined, $E = (20.5 - 14.5)/2$

Answer: A.

Problem 32.

A 95% confidence interval for the mean reading achievement score for a population of third-grade students is (44.2, 54.2). Suppose you compute a 99% confidence interval using the same information. Which of the following statements is correct?

A The intervals have the same width.

B The 99% interval is shorter.

C The 99% interval is longer.

D The standard deviation of the same mean becomes bigger

Answer: C

Problem 33

A sample of size n is taken from a population that is normally distributed with mean μ and standard deviation σ. Which of the following statements about the sampling distribution of sample mean is true?

A Using the Central Limit Theorem, $\bar{X}$ approximately follows $\bar{X} \sim N(\mu, \ \sigma/\sqrt{n})$

B Using the Central Limit Theorem, $\bar{X}$ exactly follows $\bar{X} \sim N(\mu, \ \sigma/\sqrt{n})$

C Without using the Central Limit Theorem, $\bar{X}$ approximately follows $\bar{X} \sim N(\mu, \ \sigma/\sqrt{n})$

D Using the property of normal distribution, $\bar{X}$ exactly follows $\bar{X} \sim N(\mu, \ \sigma/\sqrt{n})$

Answer: D.

Problem 34.

Explain what the phrase "95% confident" means when we interpret a 95% CI for μ

A   95% of the observations in the population fall within the bounds of the calculated interval.

B In repeated sampling, 95% of constructed intervals would contain the value of μ.

C The probability that the mean falls in the calculated interval is 0.95.

D   95% of similarly constructed intervals would contain the value of the sampled mean.

Answer: B.

Problem 35

An insurance company checks records on 582 accidents selected at random and notes that teenagers were at the wheel in 91 of them. Construct a 95% confidence interval for the proportion of all auto accidents that involve teenage drivers.

A (12.7% , 18.6%)

B (10.3% , 17.2%)

C (10.6% , 18.2%)

D (11.7% , 19.5%)

Answer: A.