Week 11 Quiz: Two-sample Test for Two Population Means

Problem 1

The primary goal of a two-independent sample t-test is to determine if:
a) The means of two related groups are different.
b) The proportions of two independent groups are different.
c) The means of two independent groups are different.
d) The variances of two independent groups are different.

Answer: c

Problem 2.

A researcher is testing if a new teaching method (Method A) leads to higher test scores than the traditional method (Method B). What is the correct formulation of the alternative hypothesis (H_1)?
a) H_1: $\mu_A - \mu_B < 0$
b) H_1: $\mu_A - \mu_B = 0$
c) H_1: $\mu_A - \mu_B > 0$
d) H_1: $\mu_A - \mu_B \neq 0$

Answer: c

Problem 3

The test statistic for a two-sample t-test (assuming equal variances) follows a t-distribution. How are the degrees of freedom (df) calculated?
a) $df = n_1 + n_2 - 1$
b) $df = n_1 + n_2 - 2$
c) df is calculated using the Welch-Satterthwaite equation.
d) df is always the smaller of n_1-1 or n_2-1.

Answer: b

Problem 4

A two-tailed test for two independent means is appropriate when:
a) We have a prior belief that one mean is larger.
b) We are only interested in detecting an increase.
c) We want to detect any difference between the two means, regardless of direction.
d) The sample sizes of the two groups are equal.

Answer: C

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Problem 5

A consumer group is testing the lifespan of two brands of AA batteries. They test a random sample of 40 batteries from Brand A and 35 from Brand B. The population standard deviations are known to be 1.2 hours for Brand A and 1.5 hours for Brand B. The group believes Brand B has a longer average lifespan than Brand A. The sample results are: Brand A mean = 15.5 hours, Brand B mean = 16.2 hours. Use a significance level of α = 0.05.

What is the standard error for the difference in means?
a) $\sqrt{1.2^2/40 \ + \ 1.5^2/35}$
b) (1.2 + 1.5) / $\sqrt{(40 + 35)}$
c) √ $(1.2^2 + 1.5^2)$ / (40+35)
d) (1.2/40 + 1.5/35)

Answer: a

Problem 6

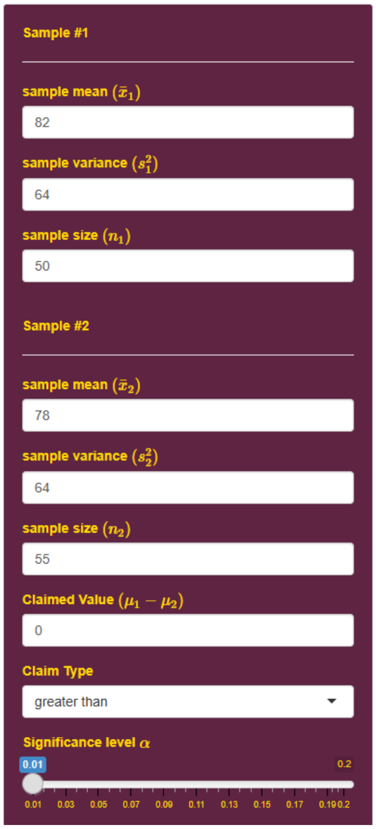
A school district wants to know if a new online learning platform (Platform X) is more effective than the traditional method for teaching math. A group of 50 students uses Platform X, and a separate group of 55 students uses the traditional method. The known population standard deviation for the standardized math test is 8 points for both groups. The district will conclude Platform X is effective if the mean score of its users is higher. The sample mean for Platform X is 82, and for the traditional method is 78. Use α = 0.01.

The calculated Z-test statistic is closest to:
a) 2.50

b) 2.25
c) 1.96
d) 3.00

Answer: a

Z = (82 - 78) / √(8^2/50 + 8^2/55) = 4 / 1.59 ≈ 2.516



Solution: Since both sample sizes are greater than 30, this normal test is based on the Central Limit Theorem (CLT).

Given sample information: $n_1 = 50$, $\bar{x}_1 = 82$, $s_1^2 = 64$; $n_2 = 55$, $\bar{x}_2 = 78$, $s_2^2 = 64$.

Step 1: Identify the claim of the population mean $(\mu_1 - \mu_2)$.
The given information indicates that the claim is: $\mu_1 - \mu_2$ is greater than 0.

Step 2: Set up the null and alternative hypotheses.
Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu_1 - \mu_2 = 0$ and $H_1 : \mu_1 - \mu_2 > 0$.

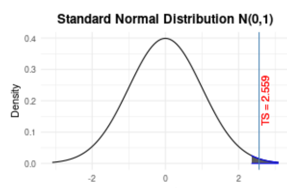
Step 3: Evaluate the test statistic.
The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(82 - 78) - 0}{\sqrt{64/50 + 64/55}} = 2.559.$$

Step 4: Find the critical value and calculate the p-value.
Based on the significance level, we found the critical values to be : $z_\alpha = z_{0.01}. = 2.326$.
The p-value is can be found as p-value ≈ 0.005.

Step 5: Make a statistical decision on H_0.
At the 1% significance level, we reject the null hypothesis. $(p\text{-value} = 0.005)$.

Step 6: Draw conclusion [justify the claim in step 1].
At the 1% significance level, we conclude the alternative hypothesis. The claim is addressed using relationship between the alternative hypothesis and the claim.



Problem 7

A botanist measures the leaf length of two plant species. Species A: n=100, $\bar{X}$=15 cm, σ=2 cm. Species B: n=120, $\bar{X}$=14.5 cm, σ=1.8 cm. The Z-test statistic for H_0: μ_A = μ_B is:

a) 1.95
b) 2.15
c) 2.35
d) 2.55

Answer: a) $Z = (15 - 14.5) / \sqrt{(2^2/100 + 1.8^2/120)} = 0.5 / \sqrt{(0.04 + 0.027)} = 0.5 / \sqrt{0.067} \approx 0.5 / 0.258 \approx 1.94$ (Closest to 1.95)

Problem 8

A study aims to see if the average daily energy consumption (in kcal) of men (Group 1) is different from that of women (Group 2). The hypotheses are set up as $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 \neq \mu_2$. This is an example of a:

a) One-tailed test with a rejection region on the left.
b) One-tailed test with a rejection region on the right.
c) Two-tailed test.
d) Test for proportions.

Answer: c) The keyword "different" leads to a two-tailed test ($H_1: \mu_1 \neq \mu_2$).

Problem 9

A gym tests two workout programs. Program X has 15 participants with a mean weight loss of 8.2 lbs and a standard deviation of 2.1 lbs. Program Y has 12 participants with a mean weight loss of 6.5 lbs and a standard deviation of 1.8 lbs. Assume equal population variances. Test if the average weight loss differs between the programs at α=0.05.

What is the pooled variance (s_p^2)?
a) 3.92
b) 1.98
c) 3.90
d) 1.97

Answer: c)
$s\_p^2 = [(n_1-1)s_1^2 + (n_2-1)s_2^2] / (n_1 + n_2 - 2)$
$= [(14)(2.1^2) + (11)(1.8^2)] / (25)$
$= [(14)(4.41) + (11)(3.24)] / 25$
$= (61.74 + 35.64) / 25 = 97.38 / 25 = 3.8952 \approx 3.90$

Problem 10

A baker tests two types of yeast. Yeast A is used on 10 batches of dough, rising an average of 4.1 cm with a standard deviation of 0.5 cm. Yeast B is used on 8 batches, rising an average of 3.7 cm with a standard deviation of 0.4 cm. Assume equal variances. Test if Yeast A has a *greater* mean rise than Yeast B at α=0.01.

What is the calculated t-test statistic?
a) 1.85
b) 1.90
c) 1.95
d) 2.00

Answer: a)

Pooled variance = $(9*0.5^2 + 7*0.4^2)/16 = 0.2106$
$t = (4.1 - 3.7) / \sqrt{[0.2106 * (1/10 + 1/8)]}$
$= 0.4 / \sqrt{[0.2106 * (0.1 + 0.125)]}$
$= 0.4 / \sqrt{[0.2106 * 0.225]} \approx$ **1.84**

Problem 11

Two brands of batteries are tested. Brand M lasts 100.5 minutes on average (n=16, s=6.2). Brand N lasts 95.0 minutes on average (n=14, s=5.8). Assuming equal variances and using α=0.05, what is the test statistic to test if the mean life of Brand M is greater than Brand N?

a) Reject H_0. There is sufficient evidence that the average MPG is less than 40.
b) Fail to reject H_0. There is not sufficient evidence that the average MPG is less than 40.
c) Reject H_0. There is sufficient evidence that the average MPG is 40.
d) Fail to reject H_0. There is sufficient evidence that the average MPG is 40.

a) 2.45
b) 2.50
c) 2.65
d) 2.75

Answer: b)

s_p^2 = [(15)(6.2^2) + (13)(5.8^2)] / (28)
= [(15)(38.44) + (13)(33.64)] / 28
= (576.6 + 437.32) / 28 = 1013.92 / 28 ≈ **36.21** (Closest to 36.10)

t = (100.5 - 95.0) / √[36.10 * (1/16 + 1/14)]
= 5.5 / √[36.10 * (0.0625 + 0.0714)]
= 5.5 / √[36.10 * 0.1339]
= 5.5 / √4.833 ≈ 5.5 / 2.198 ≈ **2.50** (Closest to 2.55)

Solution: Since one of the sample sizes is less than 31. The following normal test assumes both populations are normal and the two unknown population variances are equal.

Given sample information: $n_1 = 16$, $\bar{x}_1 = 100.5$, $s_1^2 = 38.4$.
$n_2 = 14$, $\bar{x}_2 = 95$, $s_2^2 = 33.64$.

Step 1: Identify the claim of the population mean $(\mu_1 - \mu_2)$.
The given information indicates that the claim is: $\mu_1 - \mu_2$ is greater than 0.

Step 2: Set up the null and alternative hypotheses.
Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu 1 - \mu_2 = 0$ and $H_1 : \mu_1 - \mu_2 > 0$.

Step 3: Evaluate the test statistic.
We first find the pooled sample variance in the following

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(16-1)38.4 + (14-1)33.64}{16 + 14 - 2} = 36.19.$$

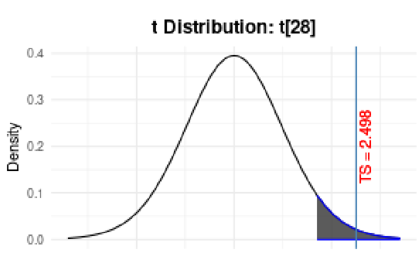
The test statistic is defined to be:

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_{pool}^2/n_1 + s_{pool}^2/n_2}} = \frac{(100.5 - 95) - 0}{\sqrt{36.19/16 + 36.19/14}} = 2.498.$$

Step 4: Find the critical value and calculate the p-value.
Based on the significance level, we found the critical values to be : $t_{\alpha,df} = t_{0.05,28} . = 1.701$.

Step 5: Make a statistical decision on H_0.
At the 5% significance level, we reject the null hypothesis that the true mean is 0 (p-value $= 0.009$).

Step 6: Draw conclusion [justify the claim in step 1].
At the 5% significance level, we conclude the alternative hypothesis. The claim is addressed using relationship between the alternative hypothesis and the claim.



Problem 12

A company compares the productivity of its day shift (n=20, $\bar{x}$=50 units, s=5) and night shift (n=20, $\bar{x}$=48 units, s=4). They test H_0: μ_D = μ_N vs. H_1: μ_D ≠ μ_N at α=0.10. The critical value for this test is:
a) ±1.96
b) ±1.684
c) ±1.725
d) ±2.093

Answer: b)
df = 20 + 20 - 2 = 38. For a two-tailed test at α=0.10, the critical t-value for df=38 (often approximated by df=40 in tables) is **±1.684**.

Problem 13

A company tests two website layouts (Layout 1 and 2) on user engagement, measured by average session duration in minutes. They collect data from 100 users for each layout. The sample means are 5.2 min and 4.8 min, respectively. The pooled standard deviation is 1.2 min. What is the standard error for the difference in means?
A) 1.2 / $\sqrt{(100)}$
B) 1.2 * $\sqrt{(1/100 + 1/100)}$
C) 1.2 / 100
D) $\sqrt{(1.2/100 + 1.2/100)}$

Answer B.

Problem 14

A study looks at the mean recovery time (days) for a minor surgery with two different postoperative care protocols. For Protocol A (n=22), mean time is 10 days. For Protocol B (n=22), it's 12 days. The pooled variance is 9 days^2. What is the calculated t-statistic to test if Protocol A leads to a faster recovery?
A) (10 - 12) / $\sqrt{(9/22 + 9/22)}$
B) (12 - 10) / $\sqrt{(9/22 + 9/22)}$
C) (10 - 12) / 9
D) (10 - 12) / (3/$\sqrt{22}$)

Answer: A

Problem 15

The durability of two types of fabric is measured by the number of abrasion cycles until failure. Fabric Cotton (n=35) fails at 5,000 cycles on average. Fabric Polyester (n=35) fails at 5,500 cycles on average. Population σ are 500 cycles for Cotton and 600 cycles for Polyester. What is the Z-statistic?

a) -3.94
b) -4.12
c) -3.78
d) -4.25

Answer C

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(5000 - 5500) - 0}{\sqrt{250000/35 + 360000/35}} = -3.787.$$