Week 09 Quiz: One-sample Normal Tests

Problem 1

The primary goal of a one-sample Z-test is to:
a) Estimate the population mean.
b) Test whether a sample mean is significantly different from a hypothesized population mean.
c) Compare the means of two independent samples.
d) Calculate the probability of observing the sample data.

Answer: B

Problem 2.

For a large one-sample Z-test with unknown population variance, the test statistic is calculated as:
a) $(\bar{x} - \mu_0) / (s/\sqrt{n})$
b) $(\bar{x} - \mu_0) / (\sigma/\sqrt{n})$
c) $(\mu - \mu_0) / (\sigma/\sqrt{n})$
d) $(\bar{x} - \mu) / (s/\sqrt{n})$

Answer: A

Problem 3

A sample of size n=25 taken from a normal population has a mean of 110. The hypothesized population mean is 100, and the population standard deviation is known to be 15. What is the Z-test statistic?
a) 3.33
b) 0.67
c) 10.0
d) 1.67

Answer: A

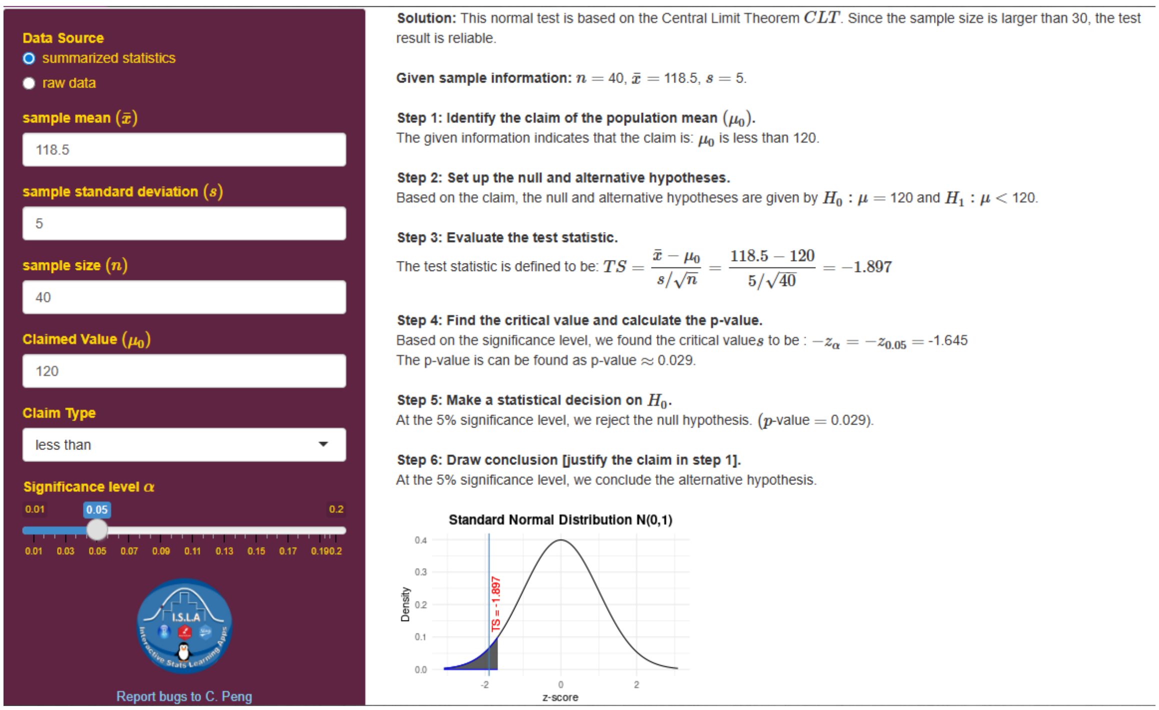
Problem 4

A factory process produces steel bars with a mean length of 120 cm and a known σ=5 cm. A new process is tested on 40 bars, yielding a sample mean

of 118.5 cm. What is the calculated z-test statistic for H_0: μ=120 vs. H_1: μ<120?

a) 1.5
b) -1.5
c) -1.897
d) -0.237

Answer: C



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Problem 5

For a two-tailed one-sample z-test at α=0.05, what are the critical values that define the rejection region?

a) z = ±1.96
b) z = ±1.645
c) z = +1.96
d) z = +1.645

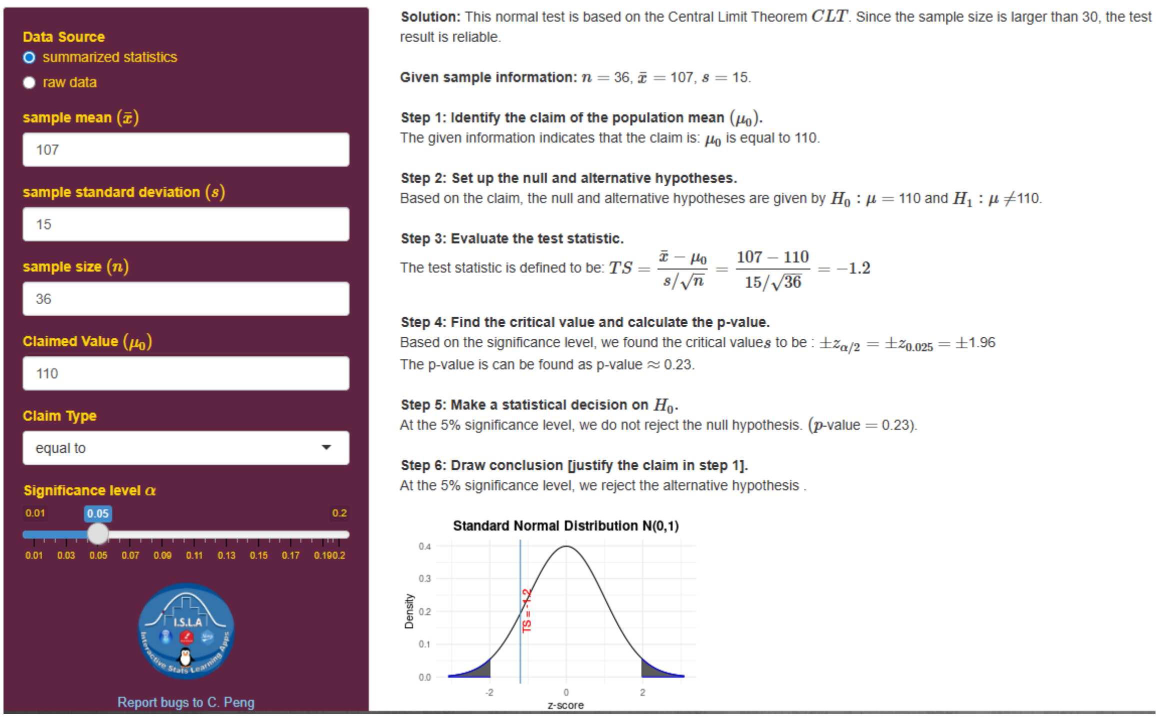
Answer: A

Problem 6

The principal claims the mean IQ of students at a school is 110. A psychologist tests 36 random students, finding a mean of 107. The population standard deviation is known to be 15. If the p-value for the two-sided test is 0.27, what is the correct conclusion at α=0.05?

a) Reject H_0; the mean IQ is significantly different from 110.
b) Fail to reject H_0; there is not enough evidence that the mean IQ is different from 110.
c) Reject H_0; the mean IQ is less than 110.
d) Accept H_0; the mean IQ is exactly 110.

Answer: D

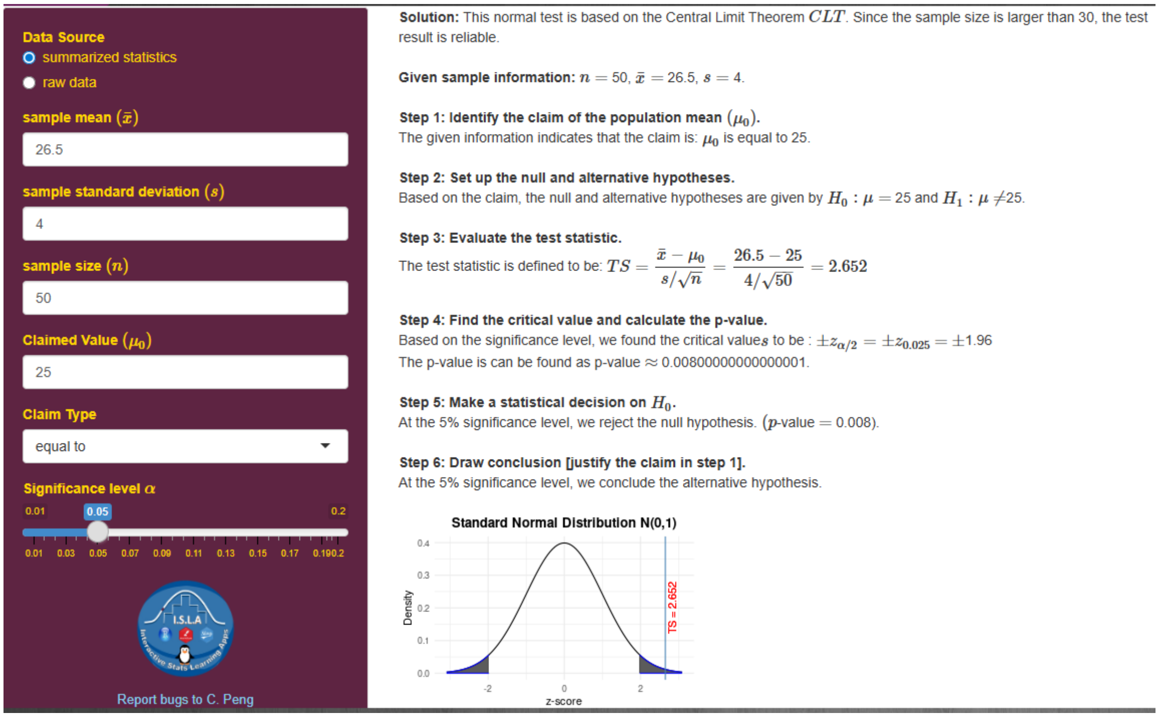


Problem 7

A pizza restaurant advertises a mean delivery time of 25 minutes. A manager collects a sample of 50 deliveries, finding a mean time of 26.5 minutes with a standard deviation of 4 minutes. The test statistic is 2.65. What is the approximate p-value for this two-tailed test?

a) 0.004
b) 0.008
c) 0.05
d) 0.02

Answer: B

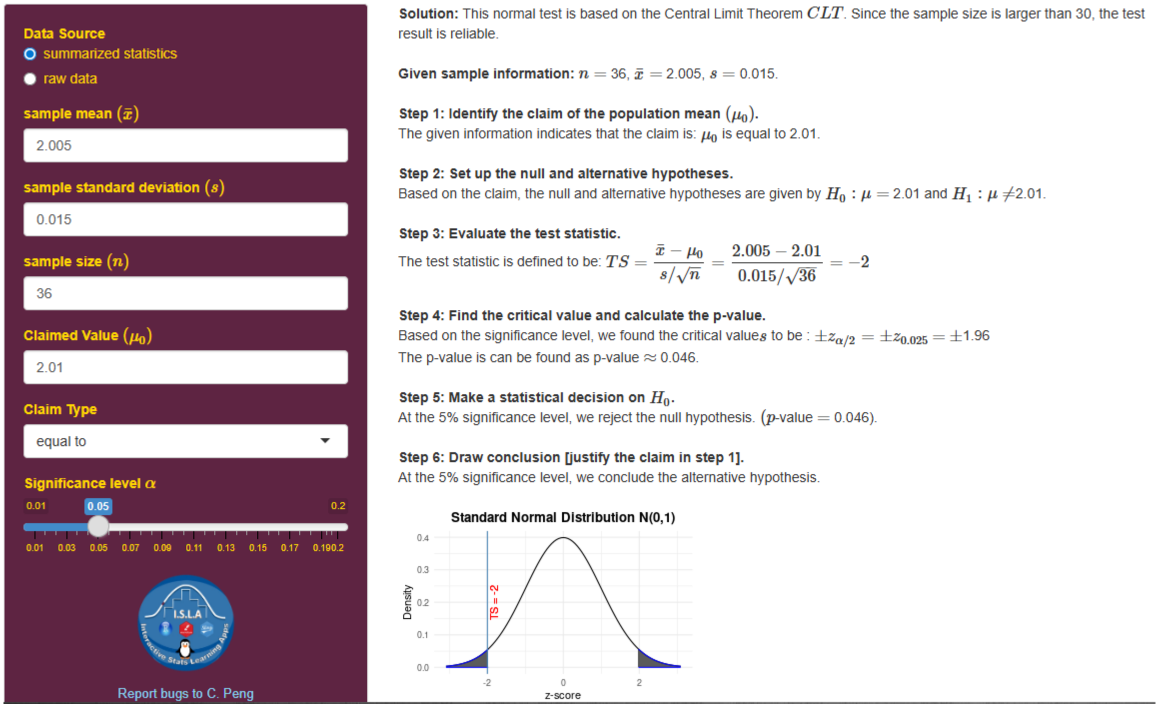


Problem 8

A bottling plant claims its 2-liter bottles contain a mean of 2.01 liters. A regulator measures 36 bottles, finding a mean volume of 2.005 liters with a standard deviation of 0.015 liters. For a two-tailed test at α=0.05, what is the correct statistical decision?

a) Reject H_0 because the sample mean is less than 2.01.
b) Fail to reject H_0 because the test statistic is within the critical region.
c) Reject H_0 because the p-value is less than 0.05.
d) Fail to reject H_0 because the p-value is greater than 0.05.

Answer: C



Problem 9

An ISP guarantees mean network latency is below 50 ms. A user runs 35 speed tests, finding a mean latency of 52 ms with a known population standard deviation of 6 ms. What is the z-test statistic for H_1: μ > 50?

a) 1.75
b) 1.97
c) 2.36
d) 2.08

Answer: B
(52-50/(6/sqrt(35))=1.972

Problem 10

A car model is rated at 30 mpg highway. A magazine tests 45 cars and finds a mean of 29.2 mpg with a standard deviation of 1.5 mpg. They perform a one-sample Z-test. What is the critical value for a two-tailed test at α=0.10?

a) ±1.761

b) ± 1.645
c) ± 1.345
d) ± 1.753

Answer: B

Problem 11

A hotel claims its average check-in time is under 2 minutes. A mystery guest records 40 check-ins. The sample mean is 2.1 minutes with a known population standard deviation of 0.4 minutes. For H_1: $\mu < 2$, what is the p-value?

a) 0.9429
b) 0.0571
c) 0.1142
d) 0.8858

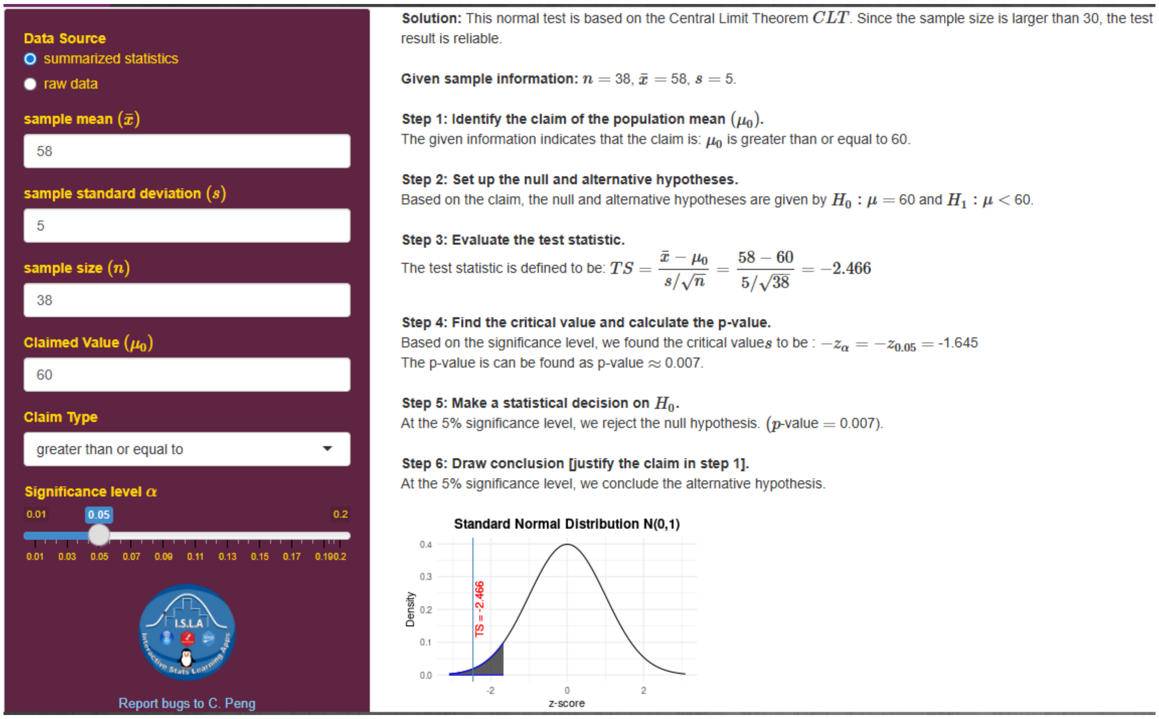
Answer: A

Problem 12

A company requires data entry clerks to have a mean speed of 60 words per minute (wpm). A supervisor tests 38 new clerks, finding a mean of 58 wpm and a standard deviation of 5 wpm. For a one-tailed test (H_1: $\mu < 60$) at α=0.05, what is the decision?

a) Reject H_0; clerks are too slow.
b) Fail to reject H_0; cannot prove clerks are too slow.
c) Accept H_0; clerks are fast enough.
d) The result is inconclusive.

Answer: A



Problem 13

An environmental standard states a river's pH should be 7.0. A researcher takes 36 water samples, finding a mean pH of 6.9 with a standard deviation of 0.15. What is the standard error of the mean?

a) 0.15
b) 0.025
c) 0.06
d) 0.003

Answer: B

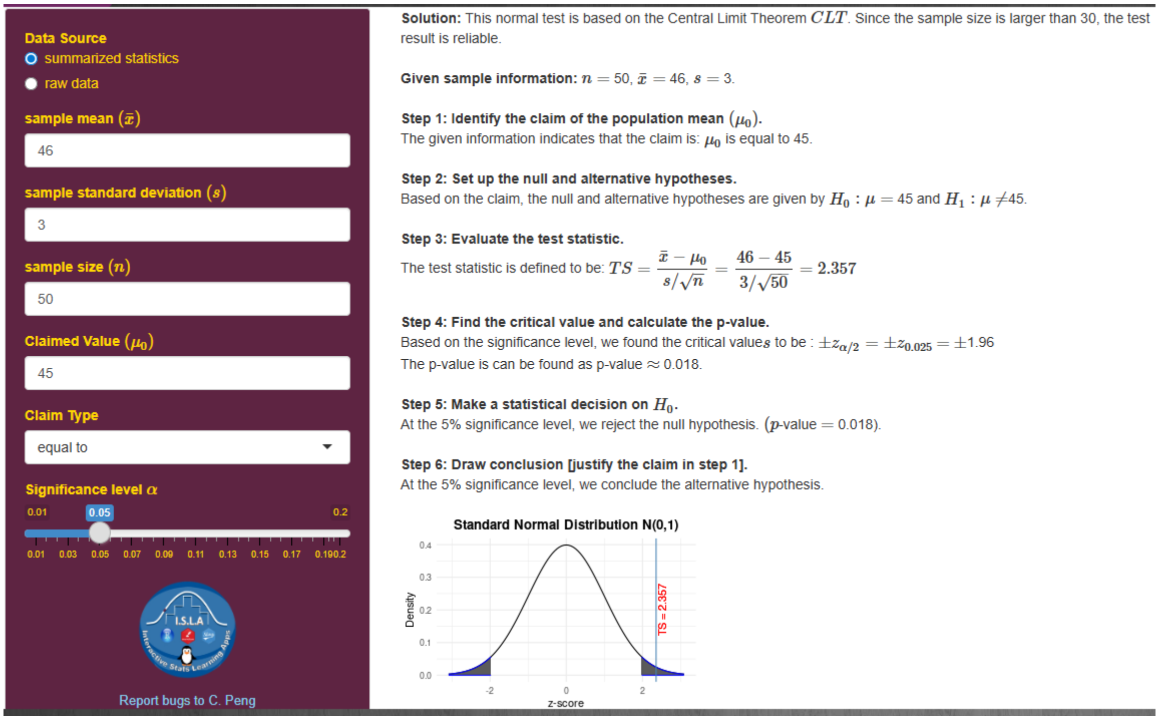
Problem 14

An assembly line is designed to produce one item every 45 seconds. A time-motion study on 50 items finds a mean time of 46 seconds with a known standard deviation of 3 seconds. What is the z-test statistic for H_1: $\mu \neq 45$?

a) 1.67
b) 2.36

c) 3.12
d) 4.01

Answer: B



Problem 15

A coffee franchise standard requires milk to be steamed for 35 seconds. A regional manager times 36 baristas, finding a mean time of 33 seconds with a standard deviation of 4 seconds. The test is H_1: $\mu \neq 35$. What is the correct conclusion at α=0.10? (The critical value is 1.96)

a) Reject H_0 because |t| > 196.
b) Fail to reject H_0 because |t| < 1.96.
c) Reject H_0 because the sample mean is less than 35.
d) Accept H_0 because the p-value is large.

Answer: A

Data Source

- summarized statistics
- raw data

sample mean $(\bar{x})$

33

sample standard deviation (s)

4

sample size (n)

36

Claimed Value (μ_0)

35

Claim Type

equal to

Significance level α

0.01 0.05 0.2

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2

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Solution: This normal test is based on the Central Limit Theorem CLT. Since the sample size is larger than 30, the test result is reliable.

Given sample information: $n = 36$, $\bar{x} = 33$, $s = 4$.

Step 1: Identify the claim of the population mean (μ_0).
The given information indicates that the claim is: μ_0 is equal to 35.

Step 2: Set up the null and alternative hypotheses.
Based on the claim, the null and alternative hypotheses are given by $H_0 : \mu = 35$ and $H_1 : \mu \neq 35$.

Step 3: Evaluate the test statistic.
The test statistic is defined to be: $TS = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{33 - 35}{4/\sqrt{36}} = -3$

Step 4: Find the critical value and calculate the p-value.
Based on the significance level, we found the critical value$s$ to be : $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$
The p-value is can be found as p-value ≈ 0.002.

Step 5: Make a statistical decision on H_0.
At the 5% significance level, we reject the null hypothesis. (p-value $= 0.002$).

Step 6: Draw conclusion [justify the claim in step 1].
At the 5% significance level, we conclude the alternative hypothesis.