

## Week 05 Quiz: Sampling Distribution

### Problem 1

The daily revenue at a university snack bar has been recorded for the past five years. Records indicate that the mean daily revenue is \$1500, and the standard deviation is \$500. The distribution is skewed to the right due to several high-volume days (football game days). Suppose that 100 days are randomly selected and the average daily revenue computed. Which of the following describes the sampling distribution of the sample mean?

- A). normally distributed with a mean of \$1500 and a standard deviation of \$500
- B). normally distributed with a mean of \$1500 and a standard deviation of \$50
- C). normally distributed with a mean of \$150 and a standard deviation of \$50
- D). skewed to the right with a mean of \$1500 and a standard deviation of \$500

**Answer:** B) standard error  $s_{\bar{X}} = 500/\sqrt{100}=50$ ,  $\bar{X} \sim N(1500, 50)$

### Problem 2.

Suppose students' ages follow a skewed right distribution with a mean of 23 years old and a standard deviation of 4 years. If we randomly sample 200 students, which of the following statements about the sampling distribution of the sample mean age is incorrect?

- A) The mean of sampling distribution is approximately 23 years old.
- B) The standard deviation of the sampling distribution is equal to 4 years.
- C) The shape of the sampling distribution is approximately normal

**Answer:** C). This is based on the central limit theorem.  $s_{\bar{X}} = 4/\sqrt{200}=0.283$ ,  $\bar{X} \sim N(23, 0.283)$

### Problem 3

A random sample of  $n = 600$  measurements is drawn from a binomial population with probability of success .08. What is the sampling distribution of the sample proportion,  $\hat{p}$ .

- A)  $N(.08; .011)$
- B)  $N(.92; .003)$
- C)  $N(.08; .003)$
- D)  $N(.92; .011)$

**Answer:** A)  $s_{\hat{p}} = \sqrt{0.08(1 - 0.08)/600} = 0.0111$

#### **Problem 4**

You form a distribution of the means of all samples of size 36 drawn from an infinite population that is skewed to the left. The population from which the samples are drawn has a mean of 50 and a standard deviation of 12 . Which one of the following statements is true of this distribution?

- (a)  $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 12$ , the sampling distribution is skewed somewhat to the left.
- (b)  $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 2$ , the sampling distribution is skewed somewhat to the left.
- (c)  $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 12$ , the sampling distribution is approximately normal.
- (d)  $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 2$ , the sampling distribution is approximately normal.

**Answer: D)**

#### **Problem 5**

An auto insurance company has 32,000 clients, and 5% of their clients submitted a claim in the past year. We will take a sample of 3,200 clients and determine how many of them have submitted a claim in the past year. What is the sampling distribution of  $\hat{p}$  ?

- a)  $\hat{p} \sim N(3200, 0.2)$
- b)  $\hat{p} \sim N(160, 152)$
- c)  $\hat{p} \sim N(0.05, 0.003852)$
- d) Can not be determined

$$\text{Answer: C)} \quad \sigma_{\hat{p}} = \sqrt{\frac{0.05(1-0.05)}{3200}} = 0.00385$$

#### **Problem 6**

A survey indicates that 60% of adults in a certain city support a new public transportation project. If a random sample of 200 adults is taken, what is the probability that the sample proportion supporting the project is greater than 65%?

- a) Approximately 0.0745
- b) Approximately 0.1151
- c) Approximately 0.2611
- d) Approximately 0.3821

$$\text{Answer: A)} \quad \sigma_{\hat{p}} = \sqrt{\frac{0.6(1-0.6)}{200}} = 0.0346, \text{ by CLT } \hat{p} \sim N(0.6, 0.0346). \text{ Next, we find z-score of 0.65: } z_0 = (0.65-0.6)/0.0346 = 1.445. P(Z > 1.445) = 0.0742$$

The primary interest of applying the CLT to sample proportion is to find the probability of an event defined by the sampling distribution of sample proportions.

**1. Which Probability to Find?**

- $P[p_0 < \hat{p} < p_1] = ?$
- $P[\hat{p} > p_0] = ?$
- $P[\hat{p} < p_0] = ?$

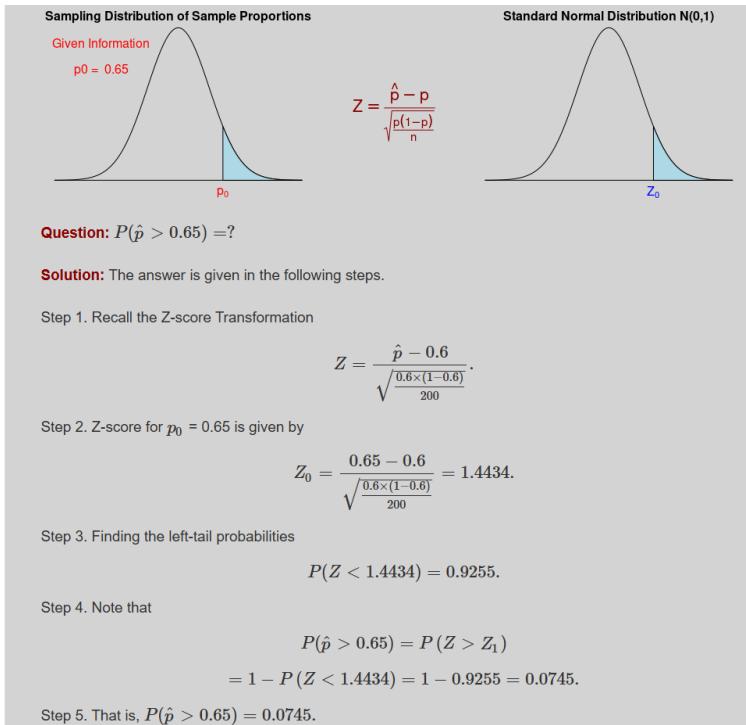
**Given Value:  $p_0$**

**2. Input Information**

**Population Proportion:  $p$**

**Sample Size:  $n$**

  
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## Problem 7

In a large population, 40% of individuals have a certain genetic trait. A random sample of 400 individuals is selected. Which of the following statements about the sampling distribution of the sample proportion is **incorrect**?

- a) The mean of the sampling distribution is 0.40.
- b) The standard deviation of the sampling distribution is approximately 0.0245.
- c) The shape of the sampling distribution is approximately normal.
- d) The standard deviation of the sampling distribution is larger than the standard deviation of the population.

**Answer: D)** Population standard deviation  $\sqrt{0.4(1 - 0.4)} = \sqrt{0.24}$ , while the standard deviation of the sampling distribution is  $\sqrt{0.4(1 - 0.4)/400} = \sqrt{0.24}/20$ . Therefore, the standard deviation of the sampling distribution is **smaller** than the standard deviation of the population.

## Problem 8

A population has a mean of 50 and a standard deviation of 8. A random sample of size 64 is taken. Which of the following statements about the sampling distribution of the sample mean is **incorrect**?

- a) The mean of the sampling distribution is 50.
- b) The standard deviation of the sampling distribution is 1.

- c) The sampling distribution is approximately normal.  
d) The standard deviation of the sampling distribution is 8.

**Answer: D)** The standard deviation of the sampling distribution is  $8/\sqrt{50} \approx 1.131$

### Problem 9

A sample of size 49 is taken from a population with a mean of 200 and a standard deviation of 14. What is the probability that the sample mean will be less than 196?

- a) 0.0013
- b) 0.0228
- c) 0.0475
- d) 0.1587

**Answer: B)** Using the CLT,  $\bar{X} \sim N\left(200, \frac{14}{\sqrt{49}}\right) = N(200, 2)$ .  $Z_0 = (196-200)/2 = -2 \rightarrow P(Z < -2) = 0.0228$

**1. What to Find?**

- Probability ( $P_0$ )
- Percentile ( $X_0$ )

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**2. Which Probability?**

- $P[V_0 < \bar{X} < V_1] = ?$
- $P[\bar{X} > V_0] = ?$
- $P[\bar{X} < V_0] = ?$

**Given Value:  $V_0$**



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**3. Input Information**

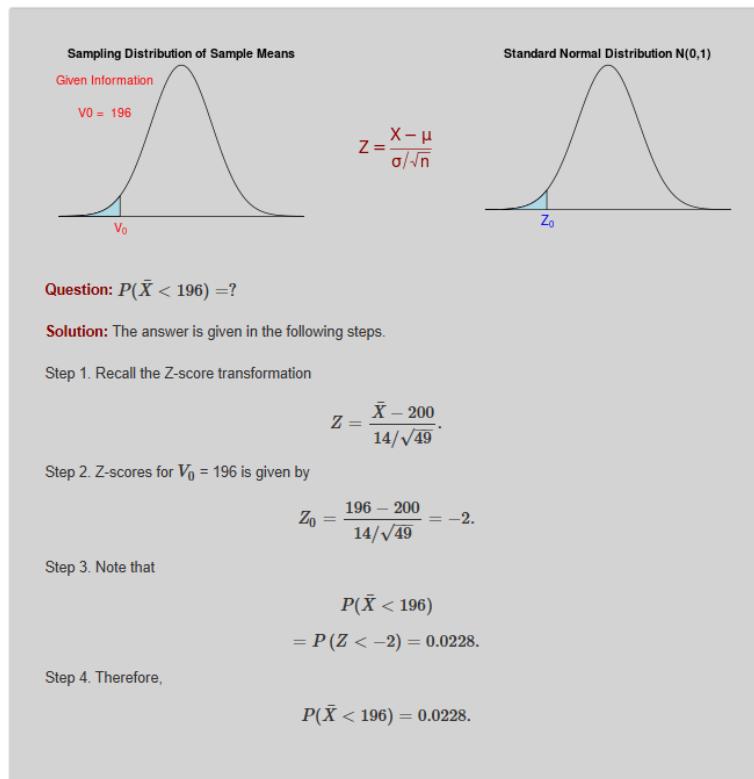
Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$

Sample Size:  $n$



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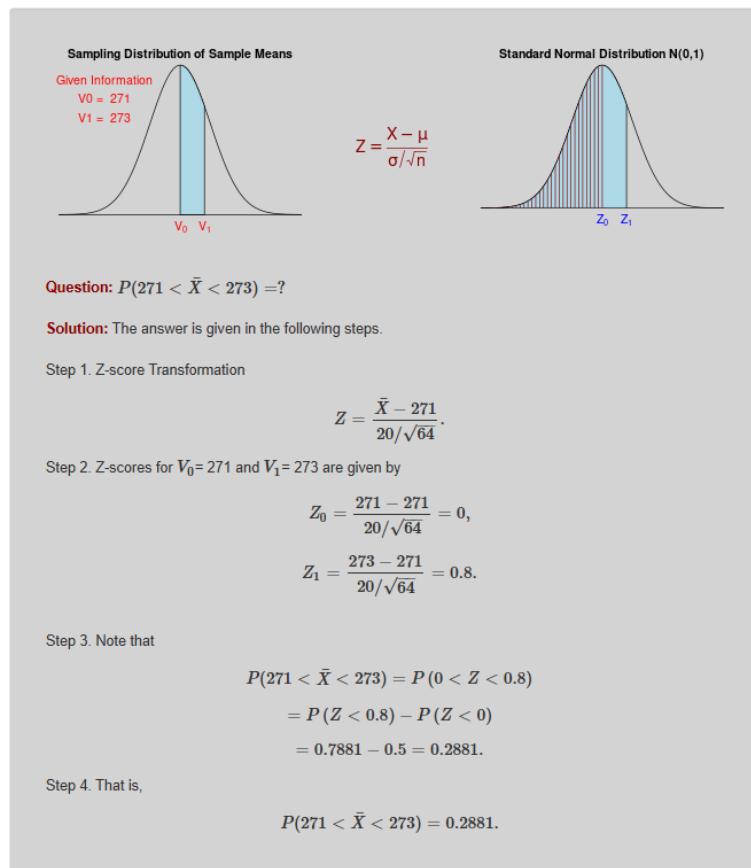
## Problem 10

The lengths of pregnancies are normally distributed with a mean of 271 days and a standard deviation of 20 days. If 64 women are randomly selected, find the probability that they have a mean pregnancy between 271 days and 273 days.

- A). 0.2119
- B). 0.5517
- C). 0.2881
- D). 0.7881

**Answer: C).** First  $\bar{X} \sim N\left(271, \frac{20}{\sqrt{64}}\right) = N(271, 2.5)$ , Two corresponding z-scores are:  $z_1 = (271-271)/2.5 = 0$ ,  $z_2 = (273-271)/2.5 = 0.8$ .  $\rightarrow P(Z < 0.8) - P(Z < 0) = 0.7881 - 0.5 = 0.2881$

<b>1. What to Find?</b>	
<input checked="" type="radio"/> Probability ( $P_0$ ) <input type="radio"/> Percentile ( $X_0$ )	
<hr/>	
<b>2. Which Probability?</b>	
<input checked="" type="radio"/> $P[V_0 < \bar{X} < V_1] = ?$ <input type="radio"/> $P[\bar{X} > V_0] = ?$ <input type="radio"/> $P[\bar{X} < V_0] = ?$	
<hr/>	
<b>Given Value #1: <math>V_0</math></b>	
271	
<b>Given Value #2: <math>V_1</math></b>	
273	
<hr/>	
<b>3. Input Information</b>	
Population Mean: $\mu$	
271	
Population Standard Deviation: $\sigma$	
20	
Sample Size: $n$	
64	
<hr/>	
	
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### Problem 11

Assume that the heights of women are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. If 100 women are randomly selected, find the probability that they have a mean height greater than 63.0 inches.

- A). 0.2881
- B). 0.0082
- C). 0.9918
- D). 0.8989

**Answer: C.** First  $\bar{X} \sim N\left(63.6, \frac{2.5}{\sqrt{100}}\right) = N(63.6, 0.25)$ , the z-scores is:  $z_1 = (63.0 - 63.6)/0.25 = -2.4$ ,  $\rightarrow P(Z < -2.4) = 0.0082 \rightarrow P(\bar{X} > 63.0) = 1 - 0.0082 = 0.9918$ .

1. What to Find?

- Probability ( $P_0$ )
- Percentile ( $X_0$ )

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2. Which Probability?

- $P[V_0 < \bar{X} < V_1] = ?$
- $P[\bar{X} > V_0] = ?$
- $P[\bar{X} < V_0] = ?$

Given Value:  $V_0$

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3. Input Information

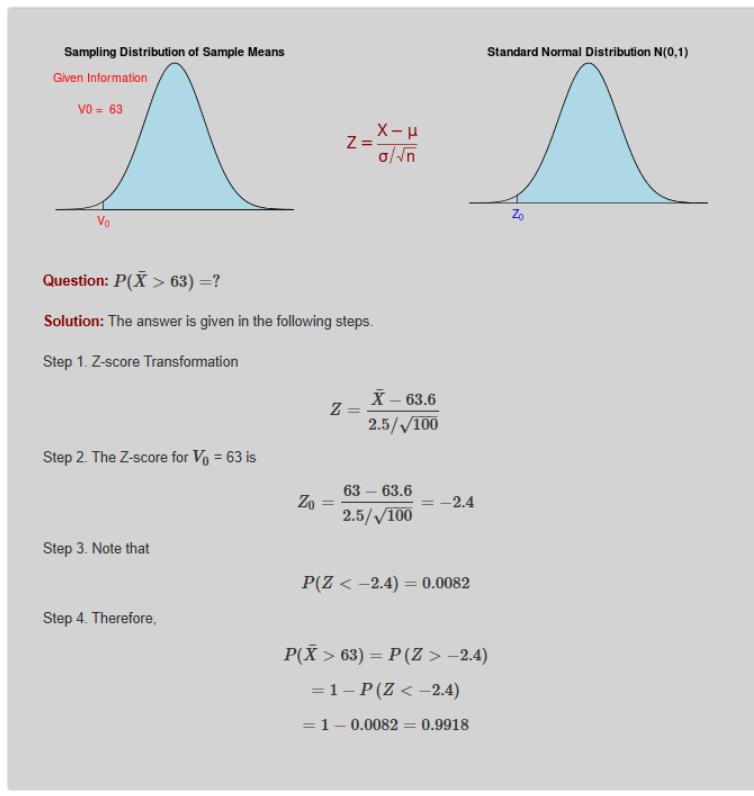
Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$

Sample Size:  $n$

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### Problem 12

The body temperatures of adults are normally distributed with a mean of  $98.6^\circ F$  and a standard deviation of  $0.60^\circ F$ . If 36 adults are randomly selected, find the probability that their mean body temperature is greater than  $98.4^\circ F$ .

- A). 0.9360
- B). 0.0228
- C). 0.8188
- D). 0.9772

**Answer D).** First  $\bar{X} \sim N\left(98.6, \frac{0.6}{\sqrt{36}}\right) = N(98.6, 0.1)$ , the z-scores is:  $z_1 = (98.4 - 98.6)/0.1 = -2$ ,  $\rightarrow P(Z > -2) = 1 - P(Z < 2) = 1 - 0.02275 = 0.9772$ .

**1. What to Find?**

- Probability ( $P_0$ )
- Percentile ( $X_0$ )

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**2. Which Probability?**

- $P[V_0 < \bar{X} < V_1] = ?$
- $P[\bar{X} > V_0] = ?$
- $P[\bar{X} < V_0] = ?$

**Given Value:  $V_0$**

**3. Input Information**

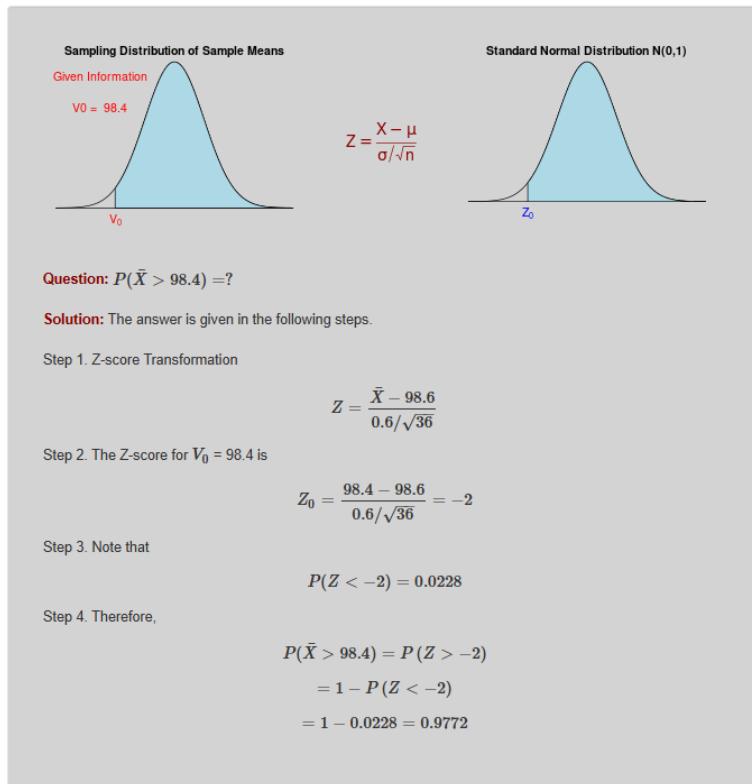
Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$

Sample Size:  $n$



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### Problem 13

A random sample of  $n = 300$  measurements is drawn from a binomial population with probability of success .43. Give the mean and the standard deviation of the sampling distribution of the sample proportion,  $\hat{p}$ .

- A).  $N(.57; .029)$
- B).  $N(.43; .014)$
- C).  $N(.57; .014)$
- D).  $N(.43; .029)$

**Answer D).**  $\sigma_{\hat{p}} = \sqrt{0.43(1 - 0.43)/300} \approx 0.0285$ .

### Problem 14.

The heights of adult women in a city are normally distributed with a mean of 165 cm and a standard deviation of 5 cm. What is the height that 95% of women are shorter than?

- a) 173.25 cm
- b) 171.45 cm
- c) 175.00 cm
- d) 174.60 cm

**Answer: A.**

1. What to Find?

- Probability ( $P_0$ )
- Percentile ( $X_0$ )

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2.  $X_0$  in Which Probability?

- $P[X_0 < X < V] = P_0$
- $P[V < X < X_0] = P_0$
- $P[X > X_0] = P_0$
- $P[X < X_0] = P_0$

Given Probability:  $P_0$

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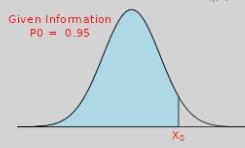
3. Input Information

Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$

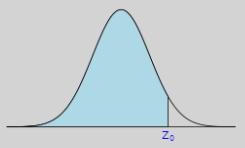
General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $P_0 = 0.95$



$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



Question: Given  $P(X < X_0) = 0.95$ , what is  $X_0$ ?

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - 165}{5} \text{ and } Z = \frac{X_0 - 165}{5}.$$

Step 2. The given condition  $P(X < X_0) = 0.95$  is equivalent to

$$P(Z < Z_0) = 0.95$$

which gives,  $Z_0 = 1.64$ .

Step 3. Note that,

$$\frac{X_0 - 165}{5} = Z_0 = 1.64.$$

Step 4. Solve for  $X_0$  from the above equation, we have,

$$X_0 = 165 + (1.64) \times 5 = 173.2.$$

### Problem 15.

The test scores in a large class are normally distributed with a mean of 75 and a standard deviation of 10. What is the probability that a randomly selected student scored between 70 and 85?

- a) 0.5328
- b) 0.6826
- c) 0.7745
- d) 0.3413

**Answer: A**

**1. What to Find?**

- Probability ( $P_0$ )
- Percentile ( $X_0$ )

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**2. Which Probability?**

- $P[V_0 < X < V_1] = ?$
- $P[X > V_0] = ?$
- $P[X < V_0] = ?$

Given Value #1:  $V_0$

Given Value #2:  $V_1$



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**3. Input Information**

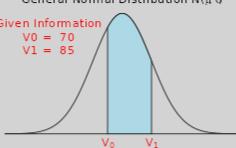
Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$



General Normal Distribution  $N(\mu, \sigma^2)$

Given Information  
 $V_0 = 70$   
 $V_1 = 85$



Question:  $P(70 < X < 85) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - \mu}{\sigma}$$

Step 2. Z-scores for  $V_0 = 70$  and  $V_1 = 85$  are given respectively by

$$Z_0 = \frac{70 - 75}{10} = -0.5 \text{ and } Z_1 = \frac{85 - 75}{10} = 1.$$

Step 3. The two left-tail Probabilities are

$$P(Z < 1) = 0.8413 \text{ and } P(Z < -0.5) = 0.3085.$$

Step 4. Note that

$$\begin{aligned} P(Z_0 < Z < Z_1) &= P(Z < Z_1) - P(Z < Z_0) \\ &= P(Z < 1) - P(Z < -0.5) \\ &= 0.8413 - 0.3085 = 0.5328. \end{aligned}$$

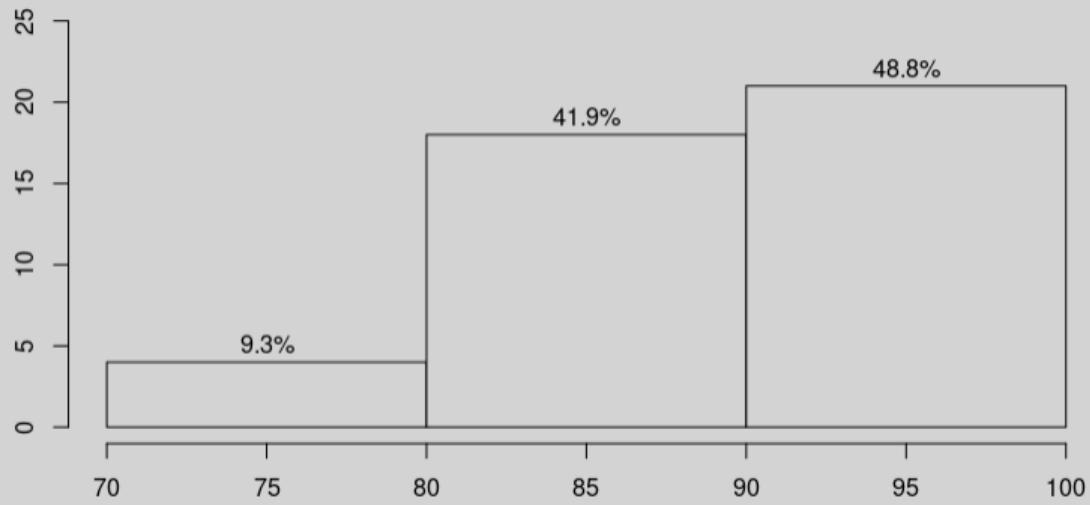
Step 5. Therefore

## Summary of Weekly Assignment #5

The class boundary is: 70,80,90,100

cut.data.freq	Freq	midpts	rel.freq	cum.freq	rel.cum.freq
[7e+01,8e+01]	4	75.00	0.09	4	0.09
(8e+01,9e+01]	18	85.00	0.42	22	0.51
(9e+01,1e+02]	21	95.00	0.49	43	1.00

### Probability Distribution Histogram



#### 1. Five Number Summary :

The five-number summary is used to describe the shape of the distribution of a given numerical data. It consists of five numbers: minimum data value, first quartile, median, the third quartile, and the maximum data value.

The five-number summary of this given data set is:

stats	value
Min.	70.00
1st Qu.	90.00
Median	90.00
3rd Qu.	95.00
Max.	95.00

#### 2. Boxplot :

The boxplot is a geometric representation of the five-number summary. The boxplot of the given data set is given below.

