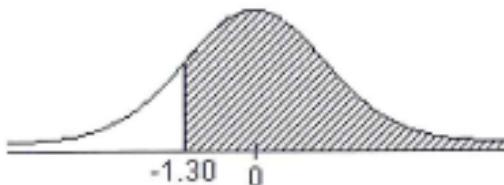


Week 04 Quiz

Problem 1

Find the area of the indicated region under the standard normal curve?

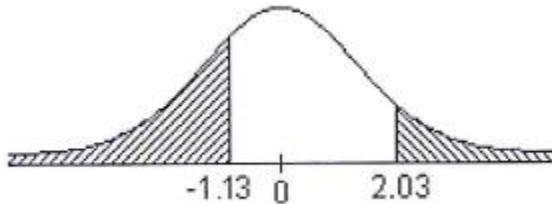


- A) 0.9177
- B) 0.0968
- C) 0.0823
- D) 0.9032

Answer: D) $1 - \text{value-from-normal-table}(z = -1.3)$

Problem 2.

Find the area of the indicated region under the standard normal curve?



- A) 0.0212
- B) 0.8489
- C) 0.1504
- D) 0.1292

Answer: C). $0.1292381 + (1 - 0.9788217) = 0.1504$

Problem 3

The lengths of pregnancies of humans are normally distributed with a mean of 268 days and a standard deviation of 15 days. Find the probability of a pregnancy lasting more than 300 days. [Hint: draw related density curves and label the given information on the curves]

- A) 0.3189
- B) 0.9834
- C) 0.0166
- D) 0.2375

Answer: C) $Z = (300 - 268)/15 = 2.1333$, right tail area: $1 - P(Z < 2.1333) = 0.0166$

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. Which Probability?

- $P[V_0 < X < V_1] = ?$
- $P[X > V_0] = ?$
- $P[X < V_0] = ?$

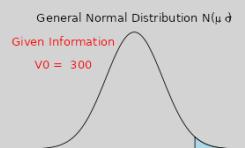
Given Value: V_0

3. Input Information

Population Mean: μ

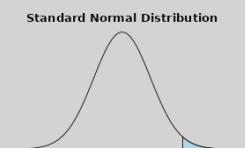
Population Standard Deviation: σ

General Normal Distribution $N(\mu, \sigma)$
Given Information
 $V_0 = 300$



$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



Question: $P(X > 300) = ?$

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 268}{15}$$

Step 2. Z-scores for $V = 300$ is given by

$$Z_0 = \frac{300 - 268}{15} = 2.1333.$$

Step 3. The left-tail Probability based on the above z-score is

$$P(Z < 2.1333) = 0.9835.$$

Step 4. Note that

$$P(Z > Z_1) = 1 - P(Z < Z_1) = 1 - 0.9835 = 0.0165.$$

Step 5. Therefore,

$$P(X > 300) = 0.0165.$$

Problem 4

The distribution of cholesterol levels in teenage boys is approximately normal with $\mu = 170$ and $\sigma = 30$ (Source: U.S. National Center for Health Statistics). Levels above 200 warrant attention. What percent of teenage boys have between 170 and 225? [Hint: draw related density curves and label the given information on the curves]

- A). 46.66%
- B). 3.36%
- C). 6.06%
- D). 56.13%

Answer: A) $Z_1 = (225-170)/30 = 1.833$, $z_2 = (170-170)/30 = 0$

$$P(Z < 1.833) - P(Z < 0) = 0.966621 - 0.5 = 0.4664$$

1. What to Find?

Probability (P_0)
 Percentile (X_0)

2. Which Probability?

$P[V_0 < X < V_1] = ?$
 $P[X > V_0] = ?$
 $P[X < V_0] = ?$

Given Value #1: V_0
170

Given Value #2: V_1
225

3. Input Information

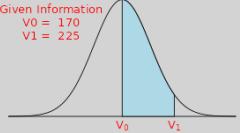
Population Mean: μ
170

Population Standard Deviation: σ
30



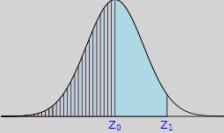
General Normal Distribution $N(\mu, \sigma)$

Given Information
 $V_0 = 170$
 $V_1 = 225$



$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



Question: $P(170 < X < 225) = ?$

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - \mu}{\sigma}$$

Step 2. Z-scores for $V_0 = 170$ and $V_1 = 225$ are given respectively by

$$Z_0 = \frac{170 - 170}{30} = 0 \text{ and } Z_1 = \frac{225 - 170}{30} = 1.83.$$

Step 3. The two left-tail Probabilities are

$$P(Z < 1.83) = 0.9664 \text{ and } P(Z < 0) = 0.5.$$

Step 4. Note that

$$\begin{aligned} P(Z_0 < Z < Z_1) &= P(Z < Z_1) - P(Z < Z_0) \\ &= P(Z < 1.83) - P(Z < 0) \\ &= 0.9664 - 0.5 = 0.4664. \end{aligned}$$

Problem 5

Assume that blood pressure readings are normally distributed with $\mu=120$ and $\sigma = 8$. A blood pressure reading of 145 or more may require medical attention. What percent of people have blood pressure reading greater than 145?

- A). 6.06%
- B). 0.09%
- C). 99.91%
- D). 11.09%

Answer: B) $Z = (145-120)/8 = 3.125$. $1 - P(Z < 3.125) = 1 - 0.999111 = 0.000889$

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. Which Probability?

- $P[V_0 < X < V_1] = ?$
- $P[X > V_0] = ?$
- $P[X < V_0] = ?$

Given Value: V_0

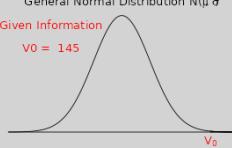
3. Input Information

Population Mean: μ

Population Standard Deviation: σ

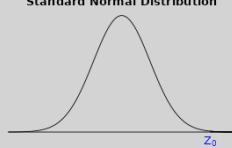

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General Normal Distribution $N(\mu, \sigma)$
Given Information
 $V_0 = 145$



$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



Question: $P(X > 145) = ?$

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 120}{8}.$$

Step 2. Z-scores for $V = 145$ is given by

$$Z_0 = \frac{145 - 120}{8} = 3.125.$$

Step 3. The left-tail Probability based on the above z-score is

$$P(Z < 3.125) = 0.9991.$$

Step 4. Note that

$$P(Z > Z_1) = 1 - P(Z < Z_1) = 1 - 0.9991 = 9e - 04.$$

Step 5. Therefore,

$$P(X > 145) = 9e - 04.$$

Problem 6

Find the 30th percentile of the standard normal distribution. (*Hint: choose the closest to your answer*)

- A). -0.52
- B). -0.98
- C). -0.47
- D). -0.81

Answer: A) $P(Z < Z_0) = 0.70 \rightarrow Z_0 = 0.5244$ (from normal table)

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. X_0 in Which Probability?

- $P[X_0 < X < V] = P_0$
- $P[V < X < X_0] = P_0$
- $P[X > X_0] = P_0$
- $P[X < X_0] = P_0$

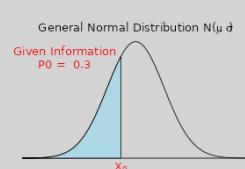
Given Probability: P_0

3. Input Information

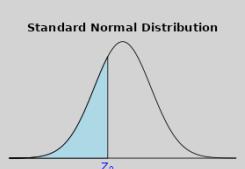
Population Mean: μ

Population Standard Deviation: σ

General Normal Distribution $N(\mu, \sigma^2)$
Given Information $P_0 = 0.3$


$$Z = \frac{X - \mu}{\sigma}$$

Standard Normal Distribution



Question: Given $P(X < X_0) = 0.3$, what is X_0 ?

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - 0}{1} \text{ and } Z = \frac{X_0 - 0}{1}.$$

Step 2. The given condition $P(X < X_0) = 0.3$ is equivalent to

$$P(Z < Z_0) = 0.3$$

which gives, $Z_0 = -0.52$.

Step 3. Note that,

$$\frac{X_0 - 0}{1} = Z_0 = -0.52.$$

Step 4. Solve for X_0 from the above equation, we have,

$$X_0 = 0 + (-0.52) \times 1 = -0.52.$$

Problem 7

For the standard normal curve, find the z-score that corresponds to the 90th percentile.

- A). 0.28
- B). 2.81
- C). 1.28
- D). 1.52

Answer: C) $P(Z < Z_0) = 0.9 \rightarrow Z_0 = 1.28$

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. X_0 in Which Probability?

- $P[X_0 < X < V] = P_0$
- $P[V < X < X_0] = P_0$
- $P[X > X_0] = P_0$
- $P[X < X_0] = P_0$

Given Probability: P_0

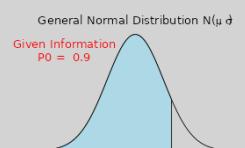
3. Input Information

Population Mean: μ

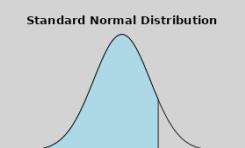
Population Standard Deviation: σ

General Normal Distribution $N(\mu, \sigma)$

Given Information
 $P_0 = 0.9$


$$Z = \frac{X - \mu}{\sigma}$$

Standard Normal Distribution



Question: Given $P(X < X_0) = 0.9$, what is X_0 ?

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - 0}{1} \text{ and } Z = \frac{X_0 - 0}{1}.$$

Step 2. The given condition $P(X < X_0) = 0.9$ is equivalent to

$$P(Z < Z_0) = 0.9$$

which gives, $Z_0 = 1.28$.

Step 3. Note that,

$$\frac{X_0 - 0}{1} = Z_0 = 1.28.$$

Step 4. Solve for X_0 from the above equation, we have,

$$X_0 = 0 + (1.28) \times 1 = 1.28.$$

Problem 8

IQ test scores are normally distributed with a mean of 100 and a standard deviation of 15. Find the x-score that corresponds to a z-score of 2.33.

- A). 142.35
- B). 134.95
- C). 139.55
- D). 125.95

Answer: B) $(X - 100)/15 = 2.33 \rightarrow X = 134.95$

Problem 9

Assume that the salaries of elementary school teachers in the United States are normally distributed with a mean of \$39,000 and a standard deviation of \$3000. What is the cutoff salary for teachers in the bottom 10%?

- A). \$35,160
- B). \$43,935
- C). \$34,065
- D). \$42,400

Answer: A) $P(Z < Z_0) = 0.1 \rightarrow Z_0 = -1.28$. Plug in the percentile -1.28 to the z-score transformation, $(X-3900)/3000 = -1.28 \rightarrow X = 3900 - 1.28 \times 3000 = 35160$

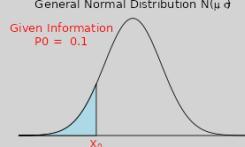
1. What to Find?
 Probability (P_0)
 Percentile (X_0)

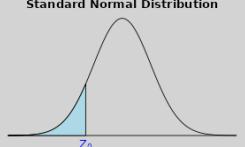
2. X_0 in Which Probability?
 $P[X_0 < X < V] = P_0$
 $P[V < X < X_0] = P_0$
 $P[X > X_0] = P_0$
 $P[X < X_0] = P_0$

Given Probability: P_0
0.10

3. Input Information
Population Mean: μ
39000
Population Standard Deviation: σ
3000


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General Normal Distribution $N(\mu, \sigma)$
Given Information $P_0 = 0.1$


Standard Normal Distribution
 $Z = \frac{X - \mu}{\sigma}$


Question: Given $P(X < X_0) = 0.1$, what is X_0 ?

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - 39000}{3000} \text{ and } Z = \frac{X_0 - 39000}{3000}.$$

Step 2. The given condition $P(X < X_0) = 0.1$ is equivalent to

$$P(Z < Z_0) = 0.1$$

which gives, $Z_0 = -1.28$.

Step 3. Note that,

$$\frac{X_0 - 39000}{3000} = Z_0 = -1.28.$$

Step 4. Solve for X_0 from the above equation, we have,

$$X_0 = 39000 + (-1.28) \times 3000 = 35160.$$

Problem 10

The lengths of pregnancies are normally distributed with a mean of 271 days and a standard deviation of 20 days. If a woman is randomly selected, find the probability that she has a pregnancy between 271 days and 273 days.

- A). 0.02119
- B). 0.57936
- C). 0.52881
- D). 0.03980

Answer: D)

$$Z_1 = (271 - 271)/20 = 0, Z_2 = (273 - 271)/20 = 0.1,$$
$$P(Z_1 < Z < Z_2) = P(Z < Z_2) - P(Z < Z_1) = 0.5398 - 0.5 = 0.0398$$

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. Which Probability?

- $P[V_0 < X < V_1] = ?$
- $P[X > V_0] = ?$
- $P[X < V_0] = ?$

Given Value #1: V_0

Given Value #2: V_1

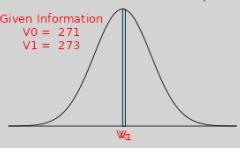
3. Input Information

Population Mean: μ

Population Standard Deviation: σ

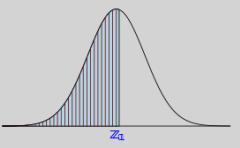
General Normal Distribution $N(\mu, \sigma)$

Given Information
 $V_0 = 271$
 $V_1 = 273$



$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



Z_0

Question: $P(271 < X < 273) = ?$

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 271}{20}.$$

Step 2. Z-scores for $V_0 = 271$ and $V_1 = 273$ are given respectively by

$$Z_0 = \frac{271 - 271}{20} = 0 \text{ and } Z_1 = \frac{273 - 271}{20} = 0.1.$$

Step 3. The two left-tail Probabilities are

$$P(Z < 0.1) = 0.5398 \text{ and } P(Z < 0) = 0.5.$$

Step 4. Note that

$$\begin{aligned} P(Z_0 < Z < Z_1) &= P(Z < Z_1) - P(Z < Z_0) \\ &= P(Z < 0.1) - P(Z < 0) \\ &= 0.5398 - 0.5 = 0.0398. \end{aligned}$$

Problem 11

The body temperatures of adults are normally distributed with a mean of $98.6^\circ F$ and a standard deviation of $0.60^\circ F$. If one adult is randomly selected, find the probability that his body temperature is greater than $98.4^\circ F$.

- A). 0.63056

- B). 0.0228
 C). 0.1368
 D). 0.3694

Answer: A) $Z = (98.4 - 98.6)/0.6 = -0.333$, $1 - P(Z < -0.333) = 1 - 0.3694 = 0.63056$

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. Which Probability?

- $P[V_0 < X < V_1] = ?$
- $P[X > V_0] = ?$
- $P[X < V_0] = ?$

Given Value: V_0

3. Input Information

Population Mean: μ

Population Standard Deviation: σ

General Normal Distribution $N(\mu, \sigma)$
Given Information
 $V_0 = 98.4$

Standard Normal Distribution

Question: $P(X > 98.4) = ?$

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - \mu}{\sigma}$$

Step 2. Z-scores for $V = 98.4$ is given by

$$Z_0 = \frac{98.4 - 98.6}{0.6} = -0.3333.$$

Step 3. The left-tail Probability based on the above z-score is

$$P(Z < -0.3333) = 0.3695.$$

Step 4. Note that

$$P(Z > Z_1) = 1 - P(Z < -0.3333) = 1 - 0.3695 = 0.6305.$$

Step 5. Therefore,

$$P(X > 98.4) = 0.6305.$$

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Problem 12

A physical fitness association is including the mile run in its secondary-school fitness test. The time for this event for boys in secondary school is known to possess a normal distribution with a mean of 440 seconds and a standard deviation of 60 seconds. Find the probability that a randomly selected boy in secondary school can run the mile in less than 302 seconds.

- A) .9893
- B) .0107
- C) .5107
- D) .4893

Answer B. $Z_0 = (302 - 440)/60 = -2.3$, $P(Z < -2.3) = 0.0107$

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. Which Probability?

- $P[V_0 < X < V_1] = ?$
- $P[X > V_0] = ?$
- $P[X < V_0] = ?$

Given Value: V_0

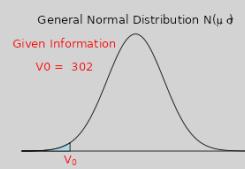
3. Input Information

Population Mean: μ

Population Standard Deviation: σ

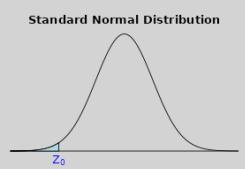
General Normal Distribution $N(\mu, \sigma)$

Given Information
 $V_0 = 302$



$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



Question: $P(X < 302) = ?$

Solution: The answer is given in the following steps.

Step 1. Recall the Z-score Transformation

$$Z = \frac{X - 440}{60}.$$

Step 2. Z-scores for $V = 302$ is given by

$$Z_0 = \frac{302 - 440}{60} = -2.3.$$

Step 3. The left-tail probability based on the above z-score is

$$P(Z < -2.3) = 0.0107.$$

Step 4. Note that

$$P(Z < Z_0) = P(Z < -2.3) = 0.0107.$$

Step 5. Therefore,

$$P(X < 302) = 0.0107.$$

Problem 13

The weights of certain machine components are normally distributed with a mean of 8.98 g and a standard deviation of 0.05 g. Find the 97th percentile.

- A). 9.00 g
- B). 9.07 g
- C). 9.12 g
- D). 8.99 g

Answer B. $P(Z < Z_0) = 0.97 \rightarrow Z_0 = 1.88$. Therefore, by z-score transformation, $(x - 8.98)/0.05 = 1.88 \rightarrow X = 8.98 + 1.88 \times 0.05 = 9.074$

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. X_0 in Which Probability?

- $P[X_0 < X < V] = P_0$
- $P[V < X < X_0] = P_0$
- $P[X > X_0] = P_0$
- $P[X < X_0] = P_0$

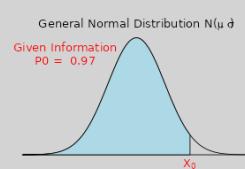
Given Probability: P_0

3. Input Information

Population Mean: μ

Population Standard Deviation: σ

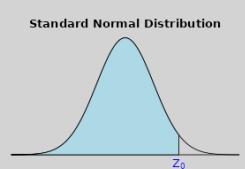
General Normal Distribution $N(\mu, \sigma)$



Given Information
 $P_0 = 0.97$

$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



Z_0

Question: Given $P(X < X_0) = 0.97$, what is X_0 ?

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - 8.98}{0.05} \quad \text{and} \quad Z = \frac{X_0 - 8.98}{0.05}.$$

Step 2. The given condition $P(X < X_0) = 0.97$ is equivalent to

$$P(Z < Z_0) = 0.97$$

which gives, $Z_0 = 1.88$.

Step 3. Note that,

$$\frac{X_0 - 8.98}{0.05} = Z_0 = 1.88.$$

Step 4. Solve for X_0 from the above equation, we have,

$$X_0 = 8.98 + (1.88) \times 0.05 = 9.07.$$

Problem 14

The tread life of a particular brand of tire is a random variable best described by a normal distribution with a mean of 60,000 miles and a standard deviation of 1300 miles. What warranty should the company use if they want 96% of the tires to outlast the warranty?

- A). 61,300 miles
- B). 62,275 miles
- C). 57,725 miles
- D). 58,700 miles

Answer B. $P(Z < Z_0) = 0.96 \rightarrow Z_0 = 1.75$, therefore, $(X - 60000)/1300 = 1.75 \rightarrow Z = 62275$.

1. What to Find?

- Probability (P_0)
- Percentile (X_0)

2. X_0 in Which Probability?

- $P[X_0 < X < V] = P_0$
- $P[V < X < X_0] = P_0$
- $P[X > X_0] = P_0$
- $P[X < X_0] = P_0$

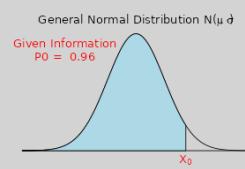
Given Probability: P_0

3. Input Information

Population Mean: μ

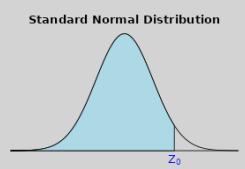
Population Standard Deviation: σ

General Normal Distribution $N(\mu, \sigma)$
Given Information $P_0 = 0.96$



$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



Question: Given $P(X < X_0) = 0.96$, what is X_0 ?

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - 60000}{1300} \quad \text{and} \quad Z = \frac{X_0 - 60000}{1300}.$$

Step 2. The given condition $P(X < X_0) = 0.96$ is equivalent to

$$P(Z < Z_0) = 0.96$$

which gives, $Z_0 = 1.75$.

Step 3. Note that,

$$\frac{X_0 - 60000}{1300} = Z_0 = 1.75.$$

Step 4. Solve for X_0 from the above equation, we have,

$$X_0 = 60000 + (1.75) \times 1300 = 62275.$$

Problem 15

Assume that the heights of women are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. If a woman is randomly selected, find the probability that her height is 63.0 inches.

- A). 0.28810
- B). 0.0000
- C). 0.9918
- D). 0.8989

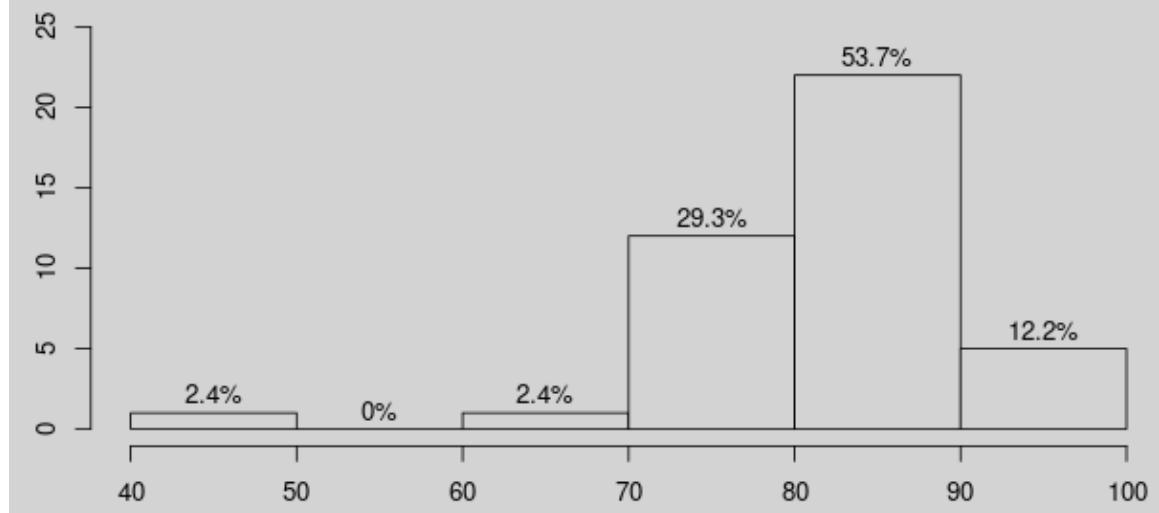
Answer B. The probability of continuous distribution at any specific value is always 0.

Week #4 Assignment Summary

The class boundary is: 40,50,60,70,80,90,100

cut.data.freq	Freq	midpts	rel.freq	cum.freq	rel.cum.freq
[4e+01,5e+01]	1	45.00	0.02	1	0.02
(5e+01,6e+01]	0	55.00	0.00	1	0.02
(6e+01,7e+01]	1	65.00	0.02	2	0.05
(7e+01,8e+01]	12	75.00	0.29	14	0.34
(8e+01,9e+01]	22	85.00	0.54	36	0.88
(9e+01,1e+02]	5	95.00	0.12	41	1.00

Probability Distribution Histogram



The five-number summary of this given data set is:

stats	value
Min.	40.00
1st Qu.	80.00
Median	85.00
3rd Qu.	90.00
Max.	100.00

