

## Week 10 Quiz: One-sample Tests

### Problem 1

A researcher wants to use a one-sample t-test to see if the average height of a certain species of plant differs from the known national average of 15 cm. The most important assumption for this test is:

- a) The sample of plants was randomly selected.
- b) The heights of the plants are normally distributed in the sample.
- c) The population standard deviation is known.
- d) The sample size is greater than 30.

**Answer: B**

### Problem 2.

A quality control manager is testing if the average weight of cereal boxes is 500 grams. He weighs 25 boxes. For the one-sample t-test to be valid, what assumption is made about the 25 weight measurements?

- a) They are a simple random sample from the population of cereal boxes.
- b) They are independent of each other.
- c) They come from a population that is approximately normally distributed.
- d) All of the above.

**Answer: D**

### Problem 3

The degrees of freedom for a one-sample t-test are calculated as:

- a)  $n$
- b)  $n - 1$
- c)  $\sigma$
- d)  $\sqrt{n}$

**Answer: B**

#### Problem 4

The general formula for the test statistic in a one-sample t-test is:

- a)  $t = \frac{\bar{x} - \mu}{s}$
- b)  $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
- c)  $t = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
- d)  $t = \frac{\bar{x} - s}{\mu/\sqrt{n}}$

**Answer: C**

-

#### Problem 5

A researcher calculates a sample mean ( $\bar{x}$ ) of 110, a sample standard deviation ( $s$ ) of 15, from a sample size ( $n$ ) of 16. If the null hypothesis states the population mean ( $\mu_0$ ) is 100, what is the calculated t-statistic?

- a) 0.67
- b) 2.67
- c) 10.0
- d) 1.67

**Answer: B**

#### Problem 6

If a sample mean is 50, the hypothesized population mean is 52, and the standard error is 0.8, what is the value of the t-test statistic?

- a) -2.5
- b) 2.5
- c) -0.25
- d) 1.6

**Answer: A**

### Problem 7

The margin of error for a one-sample t confidence interval is calculated as:

- a)  $t_{\alpha/2} \cdot s$
- b)  $t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$
- c)  $z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$
- d)  $\bar{x} \pm t_{\alpha/2}$

**Answer: B**

### Problem 8

Which factor does NOT affect the width of the margin of error in a one-sample t-test confidence interval?

- a) The sample mean ( $\bar{x}$ )
- b) The sample size (n)
- c) The sample standard deviation (s)
- d) The confidence level

**Answer: A**

### Problem 9

A seed packet claims that the average height of a certain sunflower variety is 72 inches. A gardener plants 14 seeds, and the average height at maturity is 70 inches with a standard deviation of 4 inches. She tests if the mean is different from the claim using  $\alpha=0.10$ . What is the decision?

- a) Fail to reject  $H_0$ . The average height is not significantly different from 72 inches.
- b) Reject  $H_0$ . The average height is significantly different from 72 inches.
- c) Reject  $H_0$ . The average height is exactly 72 inches.
- d) Fail to reject  $H_0$ . The seeds grew taller than claimed.

**Answer: B**

**Data Source**

- ☒ summarized statistics
- ☐ raw data

**sample mean ( $\bar{x}$ )**

**sample standard deviation ( $s$ )**

**sample size ( $n$ )**

**Claimed Value ( $\mu_0$ )**

**Claim Type**

not equal to

**Significance level  $\alpha$**

0.01 0.1 0.2

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2

Report bugs to C. Peng

Solution: This t test is based on the assumption that the population is normal and the population variance is unknown.

Given sample information:  $n = 14$ ,  $\bar{x} = 70$ ,  $s = 4$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is not equal to 72.

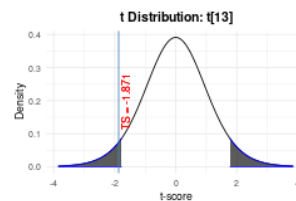
**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 72$  and  $H_1 : \mu \neq 72$ .

**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{70 - 72}{4/\sqrt{14}} = -1.871$ .

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical values to be:  $CV = \pm t_{\alpha/2, df} = \pm t_{0.05, 13} = \pm 1.771$ .  
The p-value is can be found as p-value  $\approx 0.084$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 10% significance level, we reject the null hypothesis that the true mean is 72 (p-value = 0.084).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 10% significance level, we conclude the alternative hypothesis.



## Problem 10

For a left-tailed test ( $H_a: \mu < \mu_0$ ) with  $\alpha=0.01$  and  $df=22$ , the critical value is -2.508. If the calculated t-statistic is -1.85, the decision is:

- a) Reject  $H_0$ , as  $-1.85 < -2.508$ .
- b) Fail to reject  $H_0$ , as  $-1.85 > -2.508$ .
- c) Reject  $H_0$ , as -1.85 is negative.
- d) Fail to reject  $H_0$ , as -1.85 is not less than -2.508.

**Answer: B**

## Problem 11

A car company claims its new model gets 40 MPG. A consumer group tests 9 cars and finds a mean of 38 MPG with a standard deviation of 3 MPG. They want to test if the true average is less than 40 MPG ( $\alpha=0.05$ , critical value = -1.860). The calculated t-statistic is -2.0. What should they conclude?

- a) Reject  $H_0$ . There is sufficient evidence that the average MPG is less than 40.

- b) Fail to reject  $H_0$ . There is not sufficient evidence that the average MPG is less than 40.
- c) Reject  $H_0$ . There is sufficient evidence that the average MPG is 40.
- d) Fail to reject  $H_0$ . There is sufficient evidence that the average MPG is 40.

### Answer: A

**Data Source**

☒ summarized statistics

☐ raw data

**sample mean ( $\bar{x}$ )**

**sample standard deviation ( $s$ )**

**sample size ( $n$ )**

**Claimed Value ( $\mu_0$ )**


**Claim Type**

less than ▼

**Significance level  $\alpha$**

0.01  0.2

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



Report bugs to C. Peng

**Solution:** This t test is based on the assumption that the population is normal and the population variance is unknown.

Given sample information:  $n = 9$ ,  $\bar{x} = 38$ ,  $s = 3$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is less than 40.

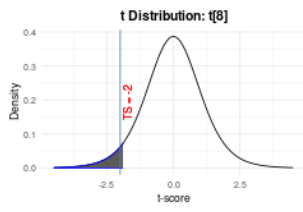
**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 40$  and  $H_1 : \mu < 40$ .

**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{38 - 40}{3/\sqrt{9}} = -2$ .

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical values to be  $-t_{\alpha, df} = -t_{0.05, 8} = -1.86$ .  
The p-value is can be found as  $p\text{-value} \approx 0.04$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 5% significance level, we reject the null hypothesis that the true mean is 40 ( $p\text{-value} = 0.04$ ).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 5% significance level, we conclude the alternative hypothesis.



### Problem 12

A smartphone manufacturer claims its battery lasts 15 hours on a full charge. A tech reviewer tests 22 phones and finds a mean battery life of 14.5 hours with a standard deviation of 1.2 hours. He conducts a left-tailed test at  $\alpha=0.05$ . The critical t-value is -1.721. What is the calculated t-statistic?

- a) -0.42
- b) -1.96
- c) -2.05
- d) -1.72

### Answer: B

$$\frac{14.5-15}{1.2/\sqrt{22}} = \frac{-0.5}{0.2558} \approx -1.955.$$

**Problem 13**

A piano tuner knows that middle C should have a frequency of 261.6 Hz. He checks the tuning of 8 pianos in a concert hall and finds the average frequency for middle C is 262.1 Hz with a standard deviation of 0.5 Hz. He performs a two-tailed test to see if the pianos are out of tune ( $\alpha=0.05$ , critical values =  $\pm 2.365$ ). What is the value of the test statistic?

- a) 0.63
- b) 1.00
- c) 2.53
- d) 2.83

**Answer: D**

$$\frac{262.1 - 261.6}{0.5/\sqrt{8}} = \frac{0.5}{0.1768} \approx 2.83$$

**Problem 14**

A coffee shop's menu states that its large coffee contains 16 oz. An employee thinks the machine is dispensing more and secretly measures 12 cups, finding a mean of 16.4 oz with a standard deviation of 0.6 oz. She runs a right-tailed test at  $\alpha=0.01$ . What is the conclusion?

- a) Reject  $H_0$ . There is significant evidence that the mean volume is greater than 16 oz.
- b) Fail to reject  $H_0$ . There is not significant evidence that the mean volume is greater than 16 oz.
- c) Reject  $H_0$ . The machine is dispensing exactly 16 oz.
- d) Fail to reject  $H_0$ . The machine is under-filling the cups.

**Answer: B**

**Data Source**

☒ summarized statistics

☐ raw data

**sample mean ( $\bar{x}$ )**

16.4

**sample standard deviation ( $s$ )**

0.6

**sample size ( $n$ )**

12

**Claimed Value ( $\mu_0$ )**

16


**Claim Type**

greater than

**Significance level  $\alpha$**

0.01

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



Report bugs to C. Pang

**Solution:** This t test is based on the assumption that the population is normal and the population variance is unknown.

Given sample information:  $n = 12$ ,  $\bar{x} = 16.4$ ,  $s = 0.6$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**

The given information indicates that the claim is:  $\mu_0$  is greater than 16.

**Step 2: Set up the null and alternative hypotheses.**

Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 16$  and  $H_1 : \mu > 16$ .

**Step 3: Evaluate the test statistic.**

The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{16.4 - 16}{0.6/\sqrt{12}} = 2.309$ .

**Step 4: Find the critical value and calculate the p-value.**

Based on the significance level, we found the critical values to be:  $t_{\alpha, df} = t_{0.01, 11} = 2.718$ .

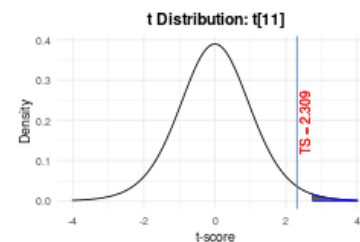
The p-value is can be found as p-value  $\approx 0.021$ .

**Step 5: Make a statistical decision on  $H_0$ .**

At the 1% significance level, we do not reject the null hypothesis that the true mean is 16 ( $p\text{-value} = 0.021$ ).

**Step 6: Draw conclusion [justify the claim in step 1].**

At the 1% significance level, we reject the alternative hypothesis .



## Problem 15

A mint produces gold coins that should weigh 1 ounce. A collector weighs 9 coins and finds a mean weight of 1.02 ounces with a standard deviation of 0.04 ounces. To test if the coins are overweight (a right-tailed test) at  $\alpha=0.01$ . What is the correct decision?

- Reject  $H_0$ . There is evidence the coins are overweight.
- Fail to reject  $H_0$ . There is no evidence the coins are overweight.
- Reject  $H_0$ . The coins are underweight.
- Fail to reject  $H_0$ . The coins weigh exactly 1 ounce.

**Answer: B**

**Data Source**

☒ summarized statistics

☐ raw data

**sample mean ( $\bar{x}$ )**

**sample standard deviation ( $s$ )**

**sample size ( $n$ )**

**Claimed Value ( $\mu_0$ )**


**Claim Type**

greater than

**Significance level  $\alpha$**

0.2

0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.2



Report bugs to C. Peng

**Solution:** This t test is based on the assumption that the population is normal and the population variance is unknown.

**Given sample information:**  $n = 9$ ,  $\bar{x} = 1.02$ ,  $s = 0.04$ .

**Step 1: Identify the claim of the population mean ( $\mu_0$ ).**  
The given information indicates that the claim is:  $\mu_0$  is greater than 1.

**Step 2: Set up the null and alternative hypotheses.**  
Based on the claim, the null and alternative hypotheses are given by  $H_0 : \mu = 1$  and  $H_1 : \mu > 1$ .

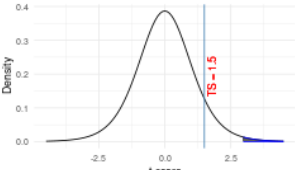
**Step 3: Evaluate the test statistic.**  
The test statistic is defined to be:  $TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1.02 - 1}{0.04/\sqrt{9}} = 1.5$ .

**Step 4: Find the critical value and calculate the p-value.**  
Based on the significance level, we found the critical values to be:  $t_{\alpha, df} = t_{0.01, 8} = 2.896$ .  
The p-value is can be found as p-value  $\approx 0.086$ .

**Step 5: Make a statistical decision on  $H_0$ .**  
At the 1% significance level, we do not reject the null hypothesis that the true mean is 1 (p-value = 0.086).

**Step 6: Draw conclusion [justify the claim in step 1].**  
At the 1% significance level, we reject the alternative hypothesis .

**t Distribution: t[8]**



## Summary of the Weekly Assignment

The boxplot is a geometric representation of the five-number summary. The boxplot of the given data set is given below.

