

MAT143 Brief Calculus

Midterm Exam #3

Friday, 4/14/2023

Time: 50 minutes

Name: _____ WCU-ID _____
(Please print)

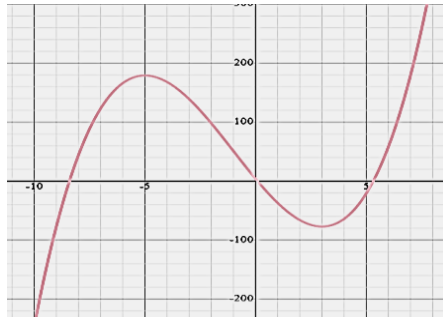
Instructions: This is a closed-book exam. No notes, books or a computer should be used for the exam. However, you can use a calculator (TI or scientific calculator) for the exam. The first part consists of 6 multiple-choice problems and the 2nd part consists of 3 show-your-work problems.

Part I: Multiple-choice problems. (50 points)

1. Which of the following functions is increasing $(0, \infty)$?

A) $f(x) = 13 - x^3$ $f'(x) = -3x^2 < 0$
B) $f(x) = -4x + 1$ $f'(x) = -4 < 0$
C) $f(x) = -4x^2$ $f'(x) = -8x < 0$
D) $f(x) = x^2$ $f'(x) = 2x > 0$

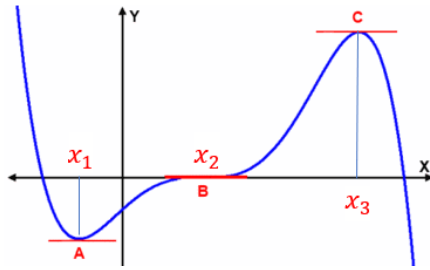
2. Let $f(x) = x^3 + 3x^2 - 45x + 4$.



Then the local extrema of $f(x)$ are

- A). A local minimum of -179 at $x = 5$ and a local maximum of 77 at $x = -3$.
B). A local minimum of -77 at $x = 3$ and a local maximum of 179 at $x = -5$.
C). A local minimum of -179 at $x = -5$ and a local maximum of -77 at $x = 3$.
D). A local minimum of -77 at $x = 3$ and a local maximum of 77 at $x = 5$.

3. Which of the following statements is true based on the given figure?



- A) A, B, and C are critical points.
 B) C and A are inflection points.
 C) A, B, and C are relative extrema.
 D) Only A and C are critical points.
4. Find the relative extrema for the following function by (1) determining the critical value(s) and (2) determining whether at these critical values, the function is a relative maximum or minimum (or possible inflection point).

$$f(x) = (x - 1)^3$$

- A) $x = 1$, an inflection point
 B) $x = 1$, relative minimum
 C) $x = 1$, relative maximum
 D) $x = 1$ is not a critical value

5. For what value of x does the function $y = x^3 - 6x$ have a local minimum?

- A) 0
 B) $\sqrt{2}$
 C) $-\sqrt{2}$
 D) 6

$$y' = 3x^2 - 6, \quad y'' = 6x$$

$$\Rightarrow x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$y''(\sqrt{2}) > 0 \rightarrow \text{min}$$

$$y''(-\sqrt{2}) < 0 \rightarrow \text{max}$$

6. The function $f(x) = x^2 + 2x^3$ has

- A) no relative extrema
 B) two relative extrema and one inflection point
 C) one relative extremum and two inflection points
 D) one relative extremum and one inflection point
 E) three relative extrema and two inflection point



$$f'(x) = 2x + 6x^2 = 2x(1 + 3x) \Rightarrow x = 0 \text{ or } x = -\frac{1}{3}$$

$$f''(x) = 2 + 12x = 0, \quad x = -\frac{1}{6}$$

$$f''(0) = 2 > 0 \rightarrow \text{local min}$$

$$f''(-\frac{1}{3}) = 2 + 12(-\frac{1}{3}) = -2 < 0 \rightarrow \text{max}$$

7. Find the x-coordinate(s) of the inflection point(s) of the curve of the following function

$$y = \frac{x^3}{3} - x^2$$

$$y' = x^2 - 2x$$

$$y'' = 2x - 2 = 2(x - 1)$$

$$\text{set } 2(x - 1) = 0$$

$$\Rightarrow x = 1$$

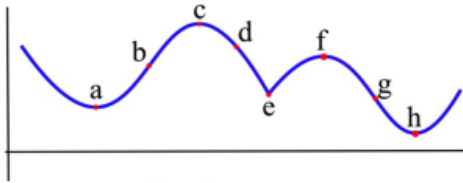
- A) 1
- B) 0, 2
- C) 0, 1
- D) 1, 2

8. Given revenue function $R(x) = -0.03x - 3x^3$, find the marginal revenue function.

- A). $-0.03 - 3x^2$
- B). $-0.03 - 3x^2$
- C). $-0.03 - 9x^2$
- D). $-0.03x + 9x^2$

$$R'(x) = -0.03 - 9x^2$$

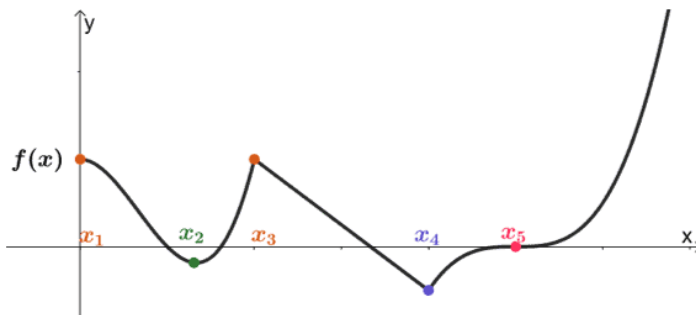
9. Assume that the graph of function $f(x)$ is given below.



Which of the following statement is true about the sign of $f''(x)$ at $x = a$?

- A) Positive because it is a relative minimum.
- B). Negative because it is a relative minimum.
- C). Positive because it is an absolute minimum.
- D). Negative because it is an inflection point.

10. Which of the following x values can be found from $f'(x) = 0$?



- A) x_2, x_3, x_4, x_5
- B) x_3, x_4
- C) x_2, x_5
- D) x_2

Part I: Show your work to receive credit. (50 points).

Problem 1. (20 points)

Using the following steps to find the absolute extreme values of the function $g(x) = x^3 - 3$ on the interval $[-2, 2]$.

1. Find all critical value(s) of $g(x)$.

$$g'(x) = (x^3 - 3)' = 3x^2$$

$$\text{Set } g'(x) = 0, \quad 3x^2 = 0 \Rightarrow x = 0$$

\Rightarrow There is only one critical value.

2. Identify relative maxima and minima.

$$g''(x) = 6x$$

$g''(0) = 6(0) = 0$, $x = 0$ is neither local maximum nor minimum.

3. Find the inflection point.

$$\text{Set } g''(x) = 0 \rightarrow 6x = 0$$

$$\Rightarrow x = 0, \quad g(0) = 0^3 - 3 \cdot 0 = 0$$

\Rightarrow The inflection point is $(0, -3)$.

4. Find the absolute maximum and minimum of $g(x)$ on $[-2, 2]$

$$g(-2) = (-2)^3 - 3 = -11$$

$$g(2) = (2)^3 - 3 = 5$$

The absolute maximum is 5 when $x = 2$.

The absolute minimum is -11 when $x = -2$.

Problem 2. (15 points) A company manufactures and sells ribbon winders. The ribbon winders cost \$30 apiece to manufacture, plus there are fixed costs of \$9000 per year.

1. What is the cost function in terms of manufacturing x ribbon winders?

$$C(x) = 9000 + 30x$$

2. What is the marginal cost function?

$$C'(x) = 30.$$

3. Assuming that the demand of ribbon winders is dependent on the unit sale price p is given by $D(p) = 300 - 3p$. Find the elastic coefficient using the definition of elastic demand

$$E = -\frac{pD'(p)}{D(p)}.$$

$$D'(p) = (300 - 3p)' = -3$$

$$E = -\frac{p \times (-3)}{300 - 3p} = \frac{3p}{300 - 3p} = \frac{p}{100 - p}.$$

Problem 3. (5 points). The price decreases from £2,000 to £1,800. Quantity demanded per year increases from 5000 to 6000 units. Use the definition in the lecture note or the textbook to find the elastic demand coefficient.

$$E = -\frac{(6000 - 5000)/5000}{(1800 - 2000)/2000} = -\frac{1000/5000}{-200/2000} = \frac{0.2}{0.1} = 2.$$