MAT143 Brief Calculus
Midterm Exam #2

Friday, 3/10/2023

Time: 50 minutes

Name: ___________________________________ WCU-ID ___________________
(Please print)

Instructions: This is a closed-book exam. No notes, books or a computer should be used for the exam. However, you can use a calculator (TI or scientific calculator) for the exam. The first part consists of 6 multiple-choice problems and 2nd part consists of 3 show-your-work problems.

Part I: Multiple-choice problems. (50 points)

1. **Marginal profit**. The profit, in thousands of dollars, from the sale of x thousand candles can be estimated by

$$P(x) = 2x - 0.3x\ln x$$

Find the marginal profit function.

A. $P'(x) = 1.7 - 0.3\ln x$

B. $P'(x) = 1.7$

C. $P'(x) = 2 - 0.3\ln x$

D. $P'(x) = 2x - 0.3\ln x$

$P'(x) = (2x - 0.3x\ln x)'$
$= (2x)' - 0.3[x\ln x]'$
$= 2 - 0.3[(x)'\ln x + x(\ln x)']$
$= 2 - 0.3[\ln x + x \cdot \frac{1}{x}]$
$= 2 - 0.3\ln x - 0.3 = 1.7 - 0.3\ln x$

2. Find the derivative of

$$y = \ln(x^2)$$

A $\frac{2}{x}$

B $\frac{1}{2x}$

C $\frac{2}{x^2}$

D. $2 + \frac{1}{x}$

$y = 2\ln x$

$y' = [2\ln x]' = 2[\ln x]' = \frac{2}{x}$

3. Find the derivative of

$$y = x^e + e^x$$

e is the natural base.

power exp

$$y' = (x^e + e^x)' = (x^e)' + (e^x)'$$

$$= e x^{e-1} + e^x$$

A $x^e + e^x$

B $x^e + xe^{x-1}$

C $ex^{e-1} + e^x$

D $ex^{e-1} + xe^{x-1}$

4. The total cost, in millions of dollars, for Greenleaf Construction is given by

$$C(x) = 100 - 50e^{-x}$$

where x is the number of houses built. Find the **marginal cost function** (i.e., the derivative of C(x).

A $-50e^{-x}$

B $50e^{-x}$

C $100 + 50e^{-x}$

D $100 + 50xe^{-x}$

$$C'(x) = (100 - 50e^{-x})'$$

$$= (100)' - 50(e^{-x})'$$

$$= 0 - 50 \cdot e^{-x}(-x)'$$

$$= -50 \cdot e^{-x} \times (-1) = 50e^{-x}$$

5. Find the derivative of

$$y = 10^x.$$

$$y' = [10^x]' = 10^x \ln 10$$

A $x10^{x-1}$

B $10^{x-1} \log_{10} e$

C $10^x \ln 10$

D $x10^{x-1} \ln 10$

6. Find the derivative of

$$y = 9e^{x^2}$$

$$y' = (9e^{x^2})' = 9[e^{x^2}]'$$

$$= 9 \cdot e^{x^2} \cdot (x^2)' = 9 \cdot e^{x^2} \cdot 2x$$

$$= 18xe^{x^2}$$

A $9 + e^{x^2}$

B $2x^2e^{x^2}$

C $2xe^{x^2}$

D $18xe^{x^2}$

7. What is the derivative of

$$f(x) = \ln(x^2 + 1)$$

$f'(x) = [\ln(x^2+1)]' = \frac{(x^2+1)'}{x^2+1} = \frac{2x}{x^2+1}$

A $\frac{1}{x^2+1}$

B $\frac{2}{x^2+1}$

C $\frac{2x}{x^2+1}$

D $\frac{1}{x^2}$

8. Find the derivative of

$$f(x) = \log_2 x^2$$

$f'(x) = [\log_2 x^2]' = [2\log_2 x]' = 2[\log_2 x]'$

$= \frac{2}{x\ln 2}$

A $\frac{2}{x^2}$

B $\frac{2\ln 2}{x}$

C $\frac{2}{x\ln 2}$

D $\frac{2}{x^2(\log_x 2)}$

9. Find the derivative of

$$y = 3^{2x} + 1$$

$y' = [3^{2x}+1]' = (3^{2x})' + 1' = 3^{2x}\ln 3 \cdot (2x)'$

$= (2\ln 3)\cdot 3^{2x}$

A $2x3^{2x-1}$

B $2x3^{2x}\ln 3$

C $(2\ln 3)\,3^{2x}$

D $(2\ln 3)\,3^{2x} + 1$

10. Find the derivative of $y = \ln 3^{2x} + 1$

$y = \ln 3^{2x} + 1 = 2x\ln 3 + 1$

$y' = [(2\ln 3)x]' + 1' = 2\ln 3\,(x)' + 0 = 2\ln 3$

A $\frac{1}{\ln 3^{2x}}$

B $\frac{2}{\ln 3^{2x}}$

C $2\ln 3$

D $2x\ln 3$

Part I: Show your work to receive credit*. (50 points).*

Problem 1. (**8 points**) Find the derivative of the function.

$$y = \ln \frac{1 - x}{(x+5)^3}$$

$$y = \ln(1-x) - \ln(x+5)^3 = \ln(1-x) - 3\ln(x+5)$$

$$y' = [\ln(1-x)]' - 3[\ln(x+5)]'$$

$$= \frac{(1-x)'}{1-x} - 3\cdot\frac{(x+5)'}{x+5} = \frac{-1}{1-x} - 3\cdot\frac{1}{x+5}$$

$$= -\frac{3}{x+5} - \frac{1}{1-x}.$$

Problem 2. (10 points) Find the derivative of the following function

$$f(x) = \frac{x^2+1}{e^x}$$

$$f'(x) = \left[\frac{x^2+1}{e^x}\right]' = \frac{(x^2+1)'\,e^x - (x^2+1)\cdot[e^x]'}{(e^x)^2} = \frac{2x\cancel{e^x} - (x^2+1)\cancel{e^x}}{e^{\cancel{2}x}}$$

$$= -\frac{x^2-2x+1}{e^x} = \frac{-(x-1)^2}{e^x}$$

Problem 3. (10 points) Find the derivative of the following function

$$f(x) = \ln[(x^3+4)/x]$$

$$f(x) = \ln\left(\frac{x^3+4}{x}\right) = \ln(x^3+4) - \ln x$$

$$f'(x) = [\ln(x^3+4) - \ln x]' = [\ln(x^3+4)]' - [\ln x]'$$

$$= \frac{(x^3+4)'}{x^3+4} - \frac{1}{x} = \frac{3x^2}{x^3+4} - \frac{1}{x}.$$

Problem 4.(10 points) Interest Compounded Continuously. Suppose P_0, in dollars, is invested in the Von Neumann Hi-Yield Fund, with interest compounded continuously at 7% per year. That is, at any point in time after t years, the balance P, in dollars per year, is given by

$$P(t) = P_0 e^{0.07t}$$

Find the growing rate of the balance of $P(t)$ at t = 5.

$$P'(t) = (P_0 e^{0.07t})' = P_0 (e^{0.07t})'$$

$$= P_0 e^{0.07t} \cdot (0.07t)' = 0.07 P_0 e^{0.07t}.$$

$$P(5) = 0.07 P_0 e^{0.35}.$$

Problem 5 (10 points) An office machine is purchased for \$5200. Assume that its salvage value, V, in dollars, depreciates, according to a method called double declining balance, by 20% each year and is given by

$$V(t) = 5200(0.80)^t$$

where t is the time, in years, after purchase. Find $V'(5)$, and explain its meaning.

$$V'(t) = [5200 \times 0.8^t]' = 5200 (0.8^t)'$$

$$= (5200 \times \ln 0.8) \cdot 0.8^t$$

$$= -1160.35 \times 0.8^t.$$

Therefore,

$$V'(5) = -1160.35 \times e^5 = -380.22$$