Topic #1 Homework

1. If $f(x) = x^2 - 2x - 1, f(-2) = ?$

A. 7
B. -1
C. 1
D. -3

$f(-2) = (-2)^2 - 2\times(-2) - 1$
$= 4 + 4 - 1 = 7.$

Answer: A

2. The domain of $f(x) = \frac{x-1}{x^2+1}$ is

A. $x \neq -1$
B. $x \neq 1$
C. all real numbers
D. $x \neq \pm 1$

no matter what x value one chooses $f(x)$ is always well-defined.

Answer: C

3. If $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$. $f(g(x)) = ?$

A. $\sqrt{x^2+1}$
B. $x + 1$
C. $\sqrt{x} + 1$
D. $\sqrt{x+1}$

$f(g(x)) = \sqrt{g(x)} = \sqrt{x^2+1}$

Answer: A

4. $f(x) = x^3 + 2$, the inverse function is $f^{-1}(x) = ?$

A. $\frac{1}{x^3+2}$
B. $\frac{1}{x^3+2}$
C. $\sqrt[3]{x-2}$
D. $-(x^3+2)$

① $y = x^3 + 2$

② $x = y^3 + 2$

$y^3 = x - 2$

$y = \sqrt[3]{x-2}$

Answer: C

5. Let $f(x) = x^2 + 1$, evaluate $f(x + h) =?$

A. $x^2 + h^2 + 1$
B. $x^2 + h^2$
C. $x^2 + 2xh^2 + 1$
D. $x^2 + 2hx + h^2 + 1$

$f(x+h) = (x+h)^2 + 1 = x^2 + 2xh + h^2 + 1$

Answer: D

6. Simplify expression $\frac{x^2-1}{x+1}$.

A. $x + 1$
B. $x - 1$
C. $\frac{1}{x+1}$.
D. $\frac{-1}{x+1}$.

$\frac{(x^2-1)}{x+1} = \frac{(x-1)(x+1)}{x+1} = x - 1 \quad (x \neq -1)$.

Answer: B

7. Let $f(x) = \sqrt{x+2}$, then $\lim_{x\to 0} f(x) =?$

A. 2
B. $\sqrt{2}$
C. $\sqrt{-2}$
D. does not exist.

By direct substitution

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \sqrt{x+2} = \sqrt{0+2} = \sqrt{2}$

Answer: B

8. $\lim_{x\to -1} \frac{x^2-1}{x+1} =?$

A. ∞
B. does not exist
C. 2
D. -2

Answer: D

Direct substitution gives $\frac{0}{0}$. we need to do some algebra:

$\frac{x^2-1}{x+1} = \frac{(x-1)(x+1)}{x+1} = x - 1$

$\lim_{x\to 0} \frac{x^2-1}{x+1} = \lim_{x\to 0} (x-1) = 0 - 1 = -1$

↑ direct substitution!

9. Let $f(x) = x^2$ find $f(x+h) - f(x)$ and simplify it.

A. h^2
B. $h^2 + 2xh$
C. $h^2 + 2h$
D. $2x^2 + h^2 + 2h$

$f(x+h) - f(x) = (x+h)^2 - x^2$

$= x^2 + 2xh + h^2 - x^2 = 2xh + h^2$

$=$

Answer: B

10. $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = ?$

A. 0
B. ∞
C. 1/2
D. does not exist.

The initial direct substitution gives $\frac{0}{0}$!

We need some algebra:

$$\frac{\sqrt{x}-1}{x-1} = \frac{\sqrt{x}-1}{(\sqrt{x})^2-1} = \frac{\sqrt{x}-1}{(\sqrt{x}+1)(\sqrt{x}-1)}$$

$$= \frac{1}{\sqrt{x}+1}$$

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}.$$

Answer: C