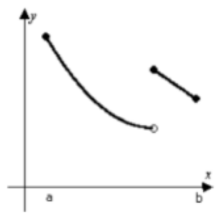
Daily Quiz #9

Problem 1.

Determine from the graph whether the function has any absolute extreme values on the interval [a, b].

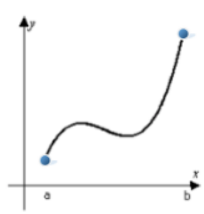


Answers *

- Absolute minimum only.
- Absolute minimum and absolute maximum
- No absolute extrema
- ✓ Absolute maximum only ← by the definition !

Problem 2.

Determine from the graph whether the function has any absolute extreme values on the interval [a, b].



Answers *

- No absolute extrema.
- ✓ Absolute minimum and absolute maximum. ✓
- Absolute maximum only.
- Absolute minimum only.

$f'(x) = (-x^2+11x-30)'$

$= -2x+11$

Problem 3.

set $f'(x)=0 \Rightarrow -2x+11=0$

$\Rightarrow x = 11/2 = 5.5$

$f''(x) = -2 < 0$

Find the absolute extreme values of the function on the interval.

$g(x) = -x^2 + 11x - 30, \quad 5 \le x \le 6$

$f(\frac{11}{2}) = -(\frac{11}{2})^2 + 11 \times \frac{11}{2} - 30 = -\frac{11^2}{4} + \frac{11^2}{2} - 30 = \frac{1}{4}$

$f(5) = -5^2 + 11 \times 5 - 30 = 0$

$f(6) = -6^2 + 11 \times 6 - 30 = 0$

Answers *

- absolute maximum is 5/4 at x = 13/2 ; absolute minimum is 0 at 6 and 0 at x = 5
- absolute maximum is 241/4 at x = 11/2 ; absolute minimum is 0 at 6 and 0 at x = 5
- ✓ absolute maximum is 1/4 at x = 11/2 ; absolute minimum is 0 at 6 and 0 at x = 5 ✓
- absolute maximum is 1/4 at x = 13/2 ; absolute minimum is 0 at 6 and 0 at x = 5

Problem 4

Find the absolute extreme values of the function on the interval.

$F(x) = -\frac{1}{x^2}, \quad 0.5 \le x \le 5$

$f(x) = -x^{-2}$,

$f'(x) = -1 \times (-2)\, x^{-2-1} = \frac{1}{x^3}$

set $f'(x)=0 \Rightarrow \frac{1}{x^3} = 0$ No root, i.e. No critical value!

Answers *

- absolute maximum is – 1/25 at x = 5; absolute minimum is -4 at x = - 1/2

$f(5) = -\frac{1}{5^2} = -\frac{1}{25}$, $f(0.5) = -\frac{1}{0.5^2} = -\frac{1}{0.25} = -4$

- absolute maximum is – 1/25 at x = 1/2 ; absolute minimum is -4 at x = -5
- ✓ absolute maximum is – 1/25 at x = 5; absolute minimum is -4 at x = 1/2
- absolute maximum is 1/25 at x = 1/2 ; absolute minimum is -4 at x =5

Problem 5.

$g'(x) = (10-6x^2)' = 0-12x$, setting $g'(x)=0 \Rightarrow -12x=0$
$\Rightarrow x=0$

$g(0)=10$, $g(-2)=10-6\times(-2)^2 = 10-24 = -12$

Find the absolute extreme values of the function on the interval.

$g(x) = 10 - 6x^2, \quad -2 \le x \le 5$

$g(5) = 10 - 6\times(5)^2 = 10 - 6\times 25 = 10-150 = -140.$

Answers *

- absolute maximum is 20 at x = 0; absolute minimum is -14 at x = 5
- absolute maximum is 10 at x = 0; absolute minimum is -140 at x = 5 ✓
- absolute maximum is 6 at x = 0; absolute minimum is -160 at x = 5
- absolute maximum is 60 at x = 0; absolute minimum is -14 at x = -2

Problem 6.

Find the extreme values of the function and where they occur.

$$f(x) = x^2 + 2x - 3$$

Answers *

$f'(x) = [x^2+2x-3]' = 2x+2$
Setting $f'(x)=0 \Rightarrow 2x+2=0$
$\Rightarrow x=-1.$
$f''(x) = 2 > 0$ ∪ → minimum
$f(-1) = (-1)^2 + 2(-1) - 3 = -4.$

- Absolute minimum is -1 at x = 4.
- Absolute minimum is 1 at x = 4.
- Absolute minimum is 1 at x = -4.
- Absolute minimum is -4 at x = -1 ✓

Problem 7.

Find the extreme values of the function and where they occur.

$f(x) = (x - 4)^{2/3}$

$f'(x) = \left[(x-4)^{\frac{2}{3}}\right]' = \frac{2}{3} \cdot (x-4)^{\frac{2}{3}-1} \cdot (x-4)'$

$= \frac{2}{3}(x-4)^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x-4}}$.

$f'(x)$ does not exist if $x = 4$

$\Rightarrow x = 4$ is a critical value.

Note that $f(x) = (x-4)^{\frac{2}{3}} \geq 0$.

$f(4) = (4-4)^{\frac{2}{3}} = 0$

Therefore $f(x)$ has an absolute minimum.

Answers *

- Absolute minimum value is 0 at x = -4.
- There are no definable extrema.
- Absolute maximum value is 0 at x = -4.
- Absolute minimum value is 0 at x = 4 ✓

Problem 8.

Identify the critical values of function $y = 2x^3 - 3x^2$.

$y' = (2x^3 - 3x^2)' = 6x^2 - 6x = 6x(x-1)$

$y' = 0 \Rightarrow 6x(x-1) = 0$

$\Rightarrow x = 0$ or $x = 1$.

Answers *

- -1, 1
- 0, 0
- 0, -1
- 0, 1 ✓

Problem 9

For what value of x does the function $y = x^3 - 6x$ have a local minimum?

$y' = [x^3 - 6x]' = 3x^2 - 6$

$y' = 0 \Rightarrow 3x^2 - 6 = 0 \quad 3(x^2 - 2) = 0$

$\Rightarrow x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2}.$

Answers *

- ( ) 0

$f''(x) = 6x$

- (✓) $\sqrt{2}$ ✓

$f''(\sqrt{2}) = 6\sqrt{2} > 0 \quad \cup \Rightarrow$ minimum

$f''(-\sqrt{2}) = -6\sqrt{2} < 0 \quad \cap \Rightarrow$ maximum

- ( ) $-\sqrt{2}$
- ( ) 6

Problem 10.

Find the x-coordinate(s) of the inflection point(s) of the curve of the following function

$y = \frac{x^3}{3} - x^2$

$y' = \left[\frac{x^3}{3} - x^2\right]' = x^2 - 2x$

$y'' = [y']' = [x^2 - 2x]' = 2x - 2$

Answers *

$y'' = 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1.$

- (✓) 1 ✓
- ( ) 0, 2
- ( ) 0, 1
- ( ) 1, 2

Problem 11.

The function $f(x)=x^2+2x^3$ has

$y' = (x^2+2x^3)' = 2x+6x^2$

$y'=0 \Rightarrow 2x+6x^2=0$

$2x(1+3x)=0$

$\Rightarrow x=0$ or $x=-\frac{1}{3}$

Answers *

- no relative extrema
- ✓ two relative extrema and one inflection point
- one relative extrema and two inflection points
- one relative extrema and one inflection point
- three relative extrema and two inflection point

$y'' = [y']' = [2x+6x^2]' = 2+12x$

$y''(0) = 2+12\times 0 = 2 > 0$ $\cup$

$y''(-\frac{1}{3}) = 2+12\times(-\frac{1}{3}) = -2 < 0$ $\cap$

Note also that $f''(x)=0$

gives $12x+1=0$

$\Rightarrow x=-\frac{1}{12}$