MAT143 Final Exam Review Topics

1. **Limits and Derivatives of Functions**
   - Limits of a function
   - Continuity of a function
   - Rate of change of a function and the slope of the secant line.
   - Instantaneous rate of change and the slope of the tangent line.
   - Derivative of a function
   - Rules of derivatives
     1) Constant rule
     2) Power rule
     3) Additive and difference rule
     4) Multiplicative rule
     5) Quotient rule
     6) Chain rule (for composite functions)
   - High-order derivatives

2. **Exponential and Logarithmic Functions**
   - The natural base exponential function
   - Properties of natural base exponential functions
     1) $e^0 = 1$
     2) $e^{x+y} = e^x \times e^y$
     3) $e^{x-y} = e^x / e^y$
     4) $e^{kx} = [e^x]^k$
   - Natural-base logarithmic function
   - Properties of natural log
     1) $\ln(1) = 0$
     2) $\ln(xy) = \ln(x) + \ln(y)$
     3) $\ln(x/y) = \ln(x) - \ln(y)$
     4) $\ln(x^k) = k \ln(x)$
   - derivative of natural log and exponential functions
     1) $[e^x]' = e^x$
     2) $[\ln x]' = 1/x$
   - Exponential and logarithmic functions with arbitrary bases
   - Derivatives of general exponential and logarithmic functions
     1) $[a^x]' = a^x \ln a$
     2) $[\log_a x]' = 1/(x \ln a)$

3. **Applications Derivatives - Optimization**
   - Optimizations problems and concepts
     1) Finding critical values
     2) Concave-up and concave-down
     3) Relative maxima and minima

4) Absolute maximum and minimum
- Business Applications – exponential models
    1) Compound interests
    2) Depreciation model.
- Cost, Revenue, and Profit
    1) Relationship between the three functions
    2) Maximization/minimization of these functions
    3) Marginal analysis of these functions
    4) Average cost, revenue, and profit functions

4. Integration and Applications
- Antiderivative /Indefinite integral
- Four basic rules of integral
    1) Constant rule: $\int k \, dx = kx + C$
    2) Power rule: $\int x^k \, dx = \frac{1}{1+k} x^{k+1} + C$, for $k \neq -1$
    3) Natural log: $\int \frac{1}{x} dx = \ln |x| + C$
    4) Natural exponential: $\int e^x \, dx = e^x + C$
- Two basic properties
    1) $\int kf(x) dx = k \int f(x) dx$
    2) $\int [af(x) \pm bg(x)] dx = a \int f(x) dx + b \int g(x) dx$
- Integration by substitution: the reverse of the chain rule
    1) Identify the candidate u-function (substitution)
    2) Convert the problem to one of the four rules
    3) Substitute back to the original variable.
- Applications
    1) For a given marginal cost (revenue, profit) function to find the actual cost function.
    2) Initial value problem to find the integral constant C.

5. Multi-variable Functions and Partial Derivatives

- Concepts of a function with more than variable
- Partial derivatives
    1) Definition and geometry
    2) Calculation
- Applications – production function as an example
    1) Finding marginal productivity of a specific variable
    2) Evaluating marginal productivity