Practice the following problems related to exponential growth models.

Section 2.4.

10. Compound interest. If an amount P_0 is deposited in a savings account and interest is compounded continuously at 4.3% per year, the balance P grows at the rate given by

$$\frac{dP}{dt} = 0.043P.$$

a) Find the function that satisfies the equation. Write it in terms of P_0 and 0.043.
b) Suppose \$20,000 is deposited. What is the balance after 1 yr? After 2 yr?
c) What is the rate of change of the balance after 1 yr? After 2 yr?

10. (a) $P(t) = P_0e^{0.043t}$; **(b)** \$20,878.76, \$21,796.13; **(c)** \$897.79/yr, \$937.23/yr

16. Consumer price index. The *consumer price index* compares the costs, c, of goods and services over various years, where 1983 is used as a base ($t = 0$). The same goods and services that cost \$100 in 1983 cost \$243 in 2017. (*Source:* Bureau of Labor Statistics.)

a) Model c as an exponential function, rounding the growth rate k to six decimal places. Let t be the number of years after 1983.
b) Estimate what the goods and services costing \$100 in 1983 will cost in 2023.
c) Estimate the rate of change in 2023 of the cost of goods and services that cost \$100 in 1983.

16. (a) $c(t) = 100e^{0.026114t}$; **(b)** \$284.21; **(c)** \$7.42/yr

22. Average salary of Major League baseball players. In 1970, the average salary of Major League baseball players was \$29,303. In 2016, the average salary was \$4,400,000. (*Source:* mlb.com.) Assuming exponential growth occurred, what was the growth rate to the nearest hundredth of a percent? What will the average salary be in 2025? Round your answer to the nearest thousand dollars.

22. 10.89%; $11,698,000

The following is the detailed solution to Problem 22. [You can modify this to get detailed solutions for problems 2 and 16]

The current salary can be expressed as $C(t) = P_0 e^{rt}$. We use two points (0, 29303) based on 1970 and (46, 4400000) to find Po and r using the above exponential model.

$$29303 = P_0 e^{r\times 0}$$

This gives $P_0 = 29303$. From 2016 data, we have

$$4400000 = 29303 e^{r\times 46}$$

Re-express the above equation, we have

$$\frac{4400000}{29303} = e^{r\times 46}$$

Taking natural log on both sides of the above equation, we have

$$\ln\left(\frac{4400000}{29303}\right) = r \times 46.$$

Therefore,

$$r = \frac{1}{46}\ln\left(\frac{4400000}{29303}\right) = 0.1089$$

The exponential growth model is

$$C(t) = 29303 e^{0.1089t}.$$

The projected salary of 2025 (55 years from 1970) is

$$C(55) = 29303 e^{0.1089\times 55} = 11698000\,.$$