Daily Quiz #10: Applications of Derivatives and Derivative of Implicit Function

Problem 1.

Given $R(x) = 5x - 0.02x^2$, $C(x) = 145 + 1.1x$, find the marginal profit function.

Answers *

- ○ $-0.02 + 145/x^2$
- ✓ $-0.04x + 3.9$
- ○ $-0.02 - 145/x^2$
- ○ cannot be determined

$P(x) = R(x) - C(x)$

$= 5x - 0.02x^2 - [145 + 1.1x]$

$= -0.02x^2 + 3.9x - 145$

$C'(x) = -0.04x + 3.9$

Problem 2.

Given revenue function $R(x) = -0.03x - 3x^3$, find the marginal revenue function.

Answers *

- ○ $-0.03 - 3x^2$
- ○ $-0.03 - 3x^2$
- ✓ $-0.03 - 9x^2$
- ○ $-0.03x + 9x^2$

$R'(x) = [-0.03x - 3x^3]'$

$= -0.03x' - 3(x^3)'$

$= -0.03 - 3 \cdot 3x^2$

$= -0.03 - 9x^2$

Problem 3.

Given cost function C(x) = 175 – 0.8x. What is the marginal cost function?

Answers *

- $175/x - 0.8$
- $175x - 0.8x^2$
- 0.8
- ✓ -0.8

$C'(x) = (175 - 0.8x)'$

$= 175' - 0.8x'$

$= 0 - 0.8 = -0.8$

Problem 4.

The production cost per week for producing x widgets is given by, $C(x) = 500 + 350x - 0.09x^2$ for $0 \le x \le 1000$.

What is the marginal cost at $x = 300$?

Answers *

- ✓ 296
- 97400
- 325
- cannot be determined

$c'(x) = [500 + 350x - 0.09x^2]'$

$= 500' + 350x' - 0.09(x^2)'$

$= 350 - 0.09 \cdot 2x$

$= 350 - 0.18x.$

$c'(300) = 350 - 0.18 \times 300$

$= 350 - 54 = 296$

Problem 5.

Find $y' = dy/dx$ if $x^2 - y^2 = 1$.

$(x^2 - y^2)' = 1'$

Answers *

- [ ] 2x/y
- [x] x/y
- [ ] y/x
- [ ] cannot be determined

$(x^2)' - (y^2)' = 0$

$2x - 2yy' = 0$

$\Rightarrow y' = \frac{x}{y}$

Problem 6.

Find the derivative of y from the implicit function: $3xy + y^2 = 0$.

Answers *

- [x] $-3y/(3x + 2y)$
- [ ] $3x/2y$
- [ ] $-2y/3x$
- [ ] $-3x/2$

$(3xy + y^2)' = 0'$

$3(xy)' + (y^2)' = 0$

$3[x'y + xy'] + 2yy' = 0$

$\Rightarrow 3[y + xy'] + 2yy' = 0$

$3y + 3xy' + 2yy' = 0$

$3y + (3x + 2y)y' = 0$

$y' = -\frac{3y}{3x + 2y}$

Problem 7

Find the derivative of y given that $x^2 + 2xy + y^2 = 1.$

Answers *

- [x] -1
- [ ] -x
- [ ] -1/(x + y)
- [ ] x/y

$$(x^2+2xy+y^2)' = 1'$$

$$(x^2)' + 2(xy)' + (y^2)' = 0$$

$$\Rightarrow 2x + 2[x'y + xy'] + 2yy' = 0$$

$$\Rightarrow \cancel{2}x + \cancel{2}(y + xy') + \cancel{2}yy' = 0$$

$$x + y + xy' + yy' = 0$$

$$\Rightarrow (x+y) + (x+y)y' = 0$$

$$\Rightarrow y' = -1.$$

Problem 8

Suppose that a 2% increase in price results in a 6% decrease in quantity demanded. Own-price elasticity of demand is equal to:

Answers *

- [ ] 1/3
- [ ] 6
- [ ] 2
- [x] 3

$$E = \frac{\text{percent change in demand}}{\text{percent change in price}}$$

$$= -\frac{-0.06}{0.02} = 3.$$

Problem 9

The price decreases from £2,000 to £1,800. Quantity demanded per year increases from 5000 to 6000 units. Which of the following is correct? (use the definition in lecture note or the textbook)

Answers *

- ○ The price elasticity of demand is -1
- ○ The good is inferior
- ✓ Income elasticity is + 0.5
- ○ Income elasticity is + 2

$$E = -\frac{\text{percent change in demand}}{\text{Percent Change in price}}$$

$$= -\frac{\frac{1800-2000}{2000}}{\frac{6000-5000}{5000}} = -\frac{-\frac{200}{2000}}{\frac{1000}{5000}} = \frac{\frac{1}{10}}{\frac{1}{5}} = \frac{1}{2}$$

Problem 10

The price of a commodity rises from 5 to 6 and as a result its demand falls from 100 to 80 units. Find the price elasticity of demand using percentage method

Answers *

- ○ 0.5
- ○ 2
- ✓ 1
- ○ undefined

$$E = -\frac{\text{percent change in demand}}{\text{percent change in price}}$$

$$= -\frac{\frac{6-5}{5}}{\frac{80-100}{100}} = -\frac{\frac{1}{5}}{-\frac{20}{100}} = 1.$$

Problem 11.

When the price decreases from $12 to $6 (50%), the quantity of demand increases from 40 to only 50 (25%). The elasticity coefficient is

Answers *

- 0.6
- .33
- 0.5
- ✓ 2

$$E = -\frac{\text{percent change in demand}}{\text{percent change in price}}$$

$$= -\frac{-0.5}{0.25} = 2$$