Problem 1.

Find the average rate of change of the following function over the given interval.

$y = x^2 + x, [1, 9]$

Answers *

- 10
- 88/9
- 45/4
- ✓ 11

Solution :

$$\frac{f(9)-f(1)}{9-1} = \frac{(9^2+9)-(1^2+1)}{8}$$

$$= \frac{81+9-2}{8} = \frac{88}{8} = 11$$

Problem2.

Find the average rate of change of the function over the given interval.

$f(x) = 3/(x-2), [4, 7]$

Answers *

- 2
- 7
- 1/3
- ✓ 3/10

$$\frac{f(7)-f(4)}{7-4} = \frac{\left(\frac{3}{7-2}\right)-\left(\frac{3}{4-2}\right)}{3}$$

$$= \frac{\frac{3}{5}-\frac{3}{2}}{3} = \frac{\frac{3\times 2 - 3\times 5}{10}}{3}$$

$$= \frac{\frac{6-15}{10}}{3} = \frac{-9}{30} = -\frac{3}{10}$$

Problem 3.

Find the slope of the tangent line of the curve at given point P.

$y = x^2 + 5x$, P(4, 36)

Answers *

- 1/20
- ✓ 13
- -4/25
- -39

The slope of a tangent line is the instantaneous rate of change at $x = 4$.

$$\lim_{h\to 0} \frac{(4+h)^2 + 5(4+h) - [4^2 + 5\times 4]}{h}$$

$$= \lim_{h\to 0} \frac{16 + 8h + h^2 + 20 + 5h - 36}{h}$$

$$= \lim_{h\to 0} \frac{13h + h^2}{h} = \lim_{h\to 0} (13 + h) = 13$$

Problem 4.

Evaluate

$$\lim_{h\to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

for the given x_0 and function f(x):

$f(x) = 5x^2$ for $x_0 = 3$.

$$\lim_{h\to 0} \frac{5(x_0+h)^2 - 5x_0^2}{h} = \lim_{h\to 0} \frac{5(x_0^2 + 2x_0h + h^2) - 5x_0^2}{h}$$

Answers *

- ✓ 30
- 15
- does not exist
- 45

$$= \lim_{h\to 0} \frac{5x_0^2 + 10x_0h + 5h^2 - 5x_0^2}{h}$$

$$= \lim_{h\to 0} (10x_0 + 5h) = 10x_0$$

$$= 10 \times 3 = 30$$

(since $x_0 = 3$).

Problem 5

Evaluate

$$\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$$

for the given x_0 and function:

f(x) = 4/x for x_0 = -6.

Answers *

- does not exist
- 24
- ✓ -1/9
- 2/3

Solution:

$$\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{4}{x_0+h}-\frac{4}{x_0}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{4x_0-4(x_0+h)}{x_0(x_0+h)}}{h}$$

$$=\lim_{h\to 0}\frac{-4h}{x_0(x_0+h)\,h}$$

$$=\lim_{h\to 0}-\frac{4}{x_0(x_0+h)}=-\frac{4}{x_0^2}$$

$$=-\frac{4}{6^2}=\boxed{-\frac{1}{9}}$$

Problem 6

Find limit

$$\lim_{x\to 1^-}\sqrt{\frac{x+1}{x+5}}$$

Answers *

- 0
- does not exist
- 1/3
- ✓ $1/\sqrt{3}$

solution: using substitution, we have

$$\lim_{h\to 1}\sqrt{\frac{x+1}{x+5}}=\sqrt{\frac{1+1}{1+5}}=\sqrt{\frac{2}{6}}=\sqrt{\frac{1}{3}}$$

$$=\frac{1}{\sqrt{3}}$$

Problem 7

Find the intervals on which the function is continuous.

$$f(x) = \frac{2}{x+5} - 3x$$

Answers *

- [ ] discontinuous only when x =5
- [ ] discontinuous everywhere
- [x] discontinuous only when x = -5
- [ ] discontinuous only when x = -8

Solution:

Since $\frac{2}{x+5}$ is undefined if $x+5=0$

Therefore, $f(x)$ is discontinous at $x=-5$.

Problem 8

Find the value of x on which the function is discontinuous.

$$f(x) = \frac{2}{x^2-9}$$

Answers *

- [ ] discontinuous only when x = 9
- [ ] discontinuous only when x = -3
- [ ] discontinuous only when x = -9 or x = 9
- [x] discontinuous when x = 3 or x = -3

Solution:

$f(x)$ is undefined when $x^2 - 9 = 0$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$.

Problem 9

Find limit

$$\lim_{x\to\infty} \frac{6}{7-(5/x^2)}$$

Answers *

- [ ] 3
- [ ] 6
- [x] 6/7
- [ ] negative infinity

Solution: by substitution method

$$\lim_{x\to\infty} \frac{6}{7-\frac{5}{x^2}} = \frac{6}{7-\frac{5}{\infty^2}} = \frac{6}{7-0} = \frac{6}{7}.$$

Problem 10

Divide numerator and denominator by the highest power of x in the denominator to find the limit.

$$\lim_{x\to\infty} \sqrt{\frac{25x^2}{3+9x^2}}$$

Answers *

- [ ] 25/3
- [x] 5/3
- [ ] 25/9
- [ ] does not exist

Solution: direct substitution will end up with an indeterminate form $\frac{\infty}{\infty}$

we divide both numerator and the denominator by x^2 (in the square root sign).

$$\lim_{x\to\infty} \sqrt{\frac{25x^2}{3+9x^2}} = \lim_{x\to\infty} \sqrt{\frac{\frac{25x^2}{x^2}}{\frac{3+9x^2}{x^2}}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{25}{\frac{3}{x^2}+9}} = \sqrt{\frac{25}{\frac{3}{\infty}+9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}.$$

Problem 11

Divide numerator and denominator by the highest power of x in the denominator to find the limit.

$$\lim_{x\to\infty}\sqrt{\frac{25x^2}{3+9x^2}}$$

Answers *

- [ ] 25/3
- [x] 5/3
- [ ] 25/9
- [ ] does not exist

Same as problem 11.

Problem 12

Find the limit

$$\lim_{x\to\sqrt[3]{3}}\left(\frac{x^2}{3}-\frac{1}{x}\right)$$

Solution:

We do direct substitution

$$\lim_{x\to\sqrt[3]{3}}\left(\frac{x^2}{3}-\frac{1}{x}\right)=\frac{(\sqrt[3]{3})^2}{3}-\frac{1}{\sqrt[3]{3}}$$

$$=3^{\frac{2}{3}-1}-3^{-\frac{1}{3}}=0.$$

Answers *

- [ ] negative infinity
- [x] 0
- [ ] 2/3
- [ ] positive infinity

Problem 13

Find the derivative of function

$$f(x) = x^{1/2}$$

Answers *

- $-\frac{1}{2\sqrt{x}}$
- $\frac{1}{\sqrt{x}}$
- ✓ $\frac{1}{2\sqrt{x}}$
- $\sqrt{x}$

Solution:

$f'(x) = (x^{\frac{1}{2}})'$

$= \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$

Problem 14

Find the derivative of function

$$f(x) = 2023$$

Answers *

- 2022
- 2023
- ✓ 0
- ∞

Solution:

$f'(x) = (2023)' = 0$

Problem 15

Find the derivative of function

$$f(x) = x^2 + 6x$$

Answers *

- [x] $2x + 6$
- [ ] $x^2 + 6$
- [ ] $2x$
- [ ] $2x^2 + 6$

Solution:

$$f'(x) = (x^2 + 6x)'$$

$$= (x^2)' + (6x)'$$

$$= 2x + 6$$