Problem 1.

Find the derivative of the function

$y = 4x^4 + 7x^3 + 5$

solution:

$$y' = (4x^4 + 7x^3 + 5)'$$
$$= 4(x^4)' + 7(x^3)' + (5)'$$
$$= 16x^3 + 21x^2 + 0$$

Answers *

- ○ $16x^3 + 21x^2 - 7$
- ○ $4x^3 + 3x^2 - 7$
- ○ $4x^3 + 3x^2$
- ✓ $16x^3 + 21x^2$

Problem 2.

Find the derivative of the function

$$z^{-2} - z^{-1}$$

Answers *

solution:

$$(z^{-2} + z^{-1})'$$
$$= (z^{-2})' + (z^{-1})'$$
$$= -2z^{-2-1} + (-1)z^{-1-1}$$
$$= -2z^{-3} - z^{-2} = -2z^{-3} + \frac{1}{z^2}$$

- ○ $-2z^{-3} - \frac{1}{z^2}$
- ✓ $-2z^{-3} + \frac{1}{z^2}$
- ○ $2z^{-3} - \frac{1}{z^2}$
- ○ $z^{-3} + \frac{1}{z^2}$

Problem 3.

Find the derivative of the function

$y = (2x^3 + 4)(5x^7 - 8)$

Answers *

- [x] $100x^9 + 140x^6 - 48x^2$
- [ ] $8x^9 + 140x^6 - 48x^2$
- [ ] $8x^9 + 140x^6 - 48x$
- [ ] $100x^9 + 140x^6 - 48x$

Solution:

$$y' = \left[(2x^3+4)(5x^7-8)\right]'$$

$$= (2x^3+4)'(5x^7-8) + (2x^3+4)(5x^7-8)'$$

$$= 6x^2(5x^7-8) + (2x^3+4)(35x^6)$$

$$= 100x^9 + 140x^6 - 48x^2$$

Problem 4.

Find the derivative of the function

$$\frac{9 - x^4}{x^2}$$

Answers *

- [ ] $\frac{18}{x^3} + 2x$
- [ ] $-\frac{18}{x} + 2x$
- [ ] $-\frac{9}{x^3} - 2x$
- [x] $-\frac{18}{x^3} - 2x$

Solution: $\left(\frac{9-x^4}{x^2}\right)'$

$$= \frac{(9-x^4)'x^2 - (9-x^4)(x^2)'}{[x^2]^2}$$

$$= \frac{(-4x^3)x^2 - (9-x^4)(2x)}{x^4}$$

$$= \frac{-4x^5 - 18x + 2x^5}{x^4}$$

$$= -\frac{18}{x^3} - \frac{2x^5}{x^4} = -\frac{18}{x^3} - 2x.$$

Problem 5

Find the derivative of the function

$$y = \frac{x^3}{x - 1}$$

Answers *

- $y' = \dfrac{2x^3 + 3x^2}{(x-1)^2}$
- $y' = \dfrac{-2x^3 - 3x^2}{(x-1)^2}$
- ✓ $y' = \dfrac{2x^3 - 3x^2}{(x-1)^2}$
- $y' = \dfrac{-2x^3 + 3x^2}{(x-1)^2}$

Solution:

$$y' = \left(\frac{x^3}{x-1}\right)' = \frac{(x^3)'(x-1) - x^3(x-1)'}{(x-1)^2}$$

$$= \frac{3x^2(x-1) - x^3}{(x-1)^2}$$

$$= \frac{3x^3 - 3x^2 - x^3}{(x-1)^2}$$

$$= \frac{2x^3 - 3x^2}{(x-1)^2}$$

Problem 6

Find the derivative of the function

$$y = \frac{x^2 + 8x + 3}{\sqrt{x}}$$

Answers *

- $y' = \dfrac{2x + 8}{2x^{3/2}}$
- ✓ $y' = \dfrac{3x^2 + 8x - 3}{2x^{3/2}}$
- $y' = \dfrac{2x + 8}{x}$
- $y' = \dfrac{3x^2 + 8x - 3}{x}$

solution:

$$y' = \left(\frac{x^2 + 8x + 3}{\sqrt{x}}\right)'$$

$$= \frac{(x^2+8x+3)'\sqrt{x} - (x^2+8x+3)(\sqrt{x})'}{(\sqrt{x})^2}$$

$$= \frac{(2x+8)\sqrt{x} - (x^2+8x+3)\cdot\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)}{x}$$

$$= \frac{2(2x+8)x - (x^2+8x+3)}{2x\sqrt{x}} = \frac{3x^2+8x-3}{2x^{3/2}}$$

Problem 7

Find the derivative of the function

$s = \frac{t^9 + 8t + 4}{t^2}$

Answers *

- [x] $\frac{ds}{dt} = 7t^6 - \frac{8}{t^2} - \frac{8}{t^3}$
- [ ] $\frac{ds}{dt} = t^6 - \frac{8}{t^2} - \frac{4}{t^3}$
- [ ] $\frac{ds}{dt} = 7t^6 + \frac{8}{t^2} + \frac{8}{t^3}$
- [ ] $\frac{ds}{dt} = 8t^{11} + 10t^2 + 8t$

Solution:

$$s' = \left(\frac{t^9+8t+4}{t^2}\right)'$$

$$= \frac{(t^9+8t+4)'t^2 - (t^9+8t+4)(t^2)'}{t^4}$$

$$= \frac{(9t^8+8)t^2 - (t^9+8t+4)2t}{t^4}$$

$$= \frac{9t^{10}+8t^2 - 2t^{10} - 16t^2 - 8t}{t^4}$$

$$= \frac{7t^{10} - 8t^2 - 8t}{t^4} = 7t^6 - \frac{8}{t^2} - \frac{8}{t^3}.$$

Problem 8

Find an equation for the tangent line of the curve given below at the point (1,3).

$y = \frac{6x}{x^2 + 1}$

[*Hint*: *Find the slope of the tangent at (1,3) first, then use the point and slope formula to find the equation of the tangent line.*]

Answers *

- [x] y = 3
- [ ] y = 3x
- [ ] y = 0
- [ ] y = x + 3

Solution:

$$y' = \left(\frac{6x}{x^2+1}\right)' = \frac{(6x)'(x^2+1) - 6x(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{6(x^2+1) - 6x(2x)}{(x^2+1)^2}$$

$$= \frac{6x^2+6 - 12x^2}{(x^2+1)^2} = \frac{6-6x^2}{(x^2+1)^2}$$

$$\text{slope} = y'(1) = \frac{6-6\times 1^2}{(1^2+1)^2} = 0.$$

The tangent line: $y - 3 = 0(x-1) = 0$

$$\Rightarrow \boxed{y = 3}$$

Problem 9

Write the function in the form y = f(u) and u = g(x). Then find dy/dx as function of x.

$y = (-2x + 10)^3$

Answers *

- ○ $y = u^3;\ u = -2x + 10;\ \frac{dy}{dx} = -2(-2x + 10)^3$
- ✓ $y = u^3;\ u = -2x + 10;\ \frac{dy}{dx} = -6(-2x + 10)^2$
- ○ $y = 3u + 10;\ u = x^3;\ \frac{dy}{dx} = -6x^2$
- ○ $y = u^3;\ u = -2x + 10;\ \frac{dy}{dx} = 3(-2x + 10)^2$

$f(x) = x^3.$

$g(x) = -2x + 10$

Solution:

$y' = \left[(-2x+10)^3\right]'$

$= 3\left[g(x)\right]^{3-1} \cdot (-2x+10)'$

$= 3(-2x+10)^2 \cdot (-2)$

$= -6(-2x+10)^2.$

Problem 10

Find the derivative of the function

$$q = (15r - r^7)^{3/2}$$

Answers *

- ✓ $\frac{3}{2}\sqrt{15r - r^7} \times (15 - 7r^6)$
- ○ $\frac{3}{2}\sqrt{15r - r^7}$
- ○ $\frac{3}{2\sqrt{15 - 7r^6}}$
- ○ $\frac{3}{2\sqrt{15r - r^7}}$

Solution:

$q' \left[(15r - r^7)^{\frac{3}{2}}\right]'$

$= \frac{3}{2}(15r - r^7)^{\frac{3}{2}-1} \cdot (15r - r^7)'$

$= \frac{3}{2}(15r - r^7)^{\frac{1}{2}} \cdot (15 - 7r^6)$

$= \frac{3}{2}\sqrt{15r - r^7}\,(15 - 7r^6).$

Problem 11

Let

$$f(x) = x^2$$

and

$$g(x) = x + 1$$

. find the derivative of

$$(f \circ g)(x)$$

.

Answers *

- ✓ $2(x+1)$
- $2x$
- $x^2 + 1$
- $2x + 1$

Solution:

$$[f \circ g(x)]'$$

$$= f'(g(x)) \cdot g'(x)$$

$$= 2(g(x))^{2-1} \cdot (x+1)'$$

$$= 2(x+1) \cdot 1 = \boxed{2(x+1)}$$

Problem 12

A total cost function is given by

$$C(x) = 2000(x^2+2)^{1/3} + 700$$

Where C(x) is the total cost, in thousands of dollars, of producing x airplanes. Find the rate at which total cost is changing when 20 airplanes have been produced.

[Hint: find the one that is closes to your answer]

Answers *

- ✓ 489
- 1189
- 211
- 700

solution:

$$C'(x) = \left[2000(x^2+2)^{\frac{1}{3}} + 700\right]'$$

$$= 2000\left[(x^2+2)^{\frac{1}{3}}\right]' + (700)'$$

$$= 2000 \cdot \left[\frac{1}{3}(x^2+2)^{\frac{1}{3}-1} \cdot (x^2+1)'\right] + 0$$

$$= 2000\left[\frac{1}{3}(x^2+2)^{-\frac{2}{3}} \cdot (2x)\right] = \frac{4000x}{3(x^2+2)^{2/3}}$$

$$C'(20) = \frac{4000 \times 20}{3(20^2+2)^{2/3}} \approx 489.$$

Problem 13

If \$1000 is invested at interest rate r in 3 years will grow an amount A given by

$$1000(1+r)^3$$

Find the instantaneous rate of change of A,

$$\frac{dA}{dr}$$

.

Answers *

- ✓ $3000(1+r)^2$
- $3000r^2$
- $2000(1+r)^2$
- $3000(1+r^2)$

Solution :

$$\frac{dA}{dr} = \left[10000\,(1+r)^3\right]'$$

$$= 10000\left[(1+r)^3\right]'$$

$$= 10000\left[3\,(1+r)^{3-1}\right]$$

$$= 30000\,(1+r)^2.$$