Problem 1.

Find the derivative of the function

$$y = 4x^4 + 7x^3 + 5$$

Answers *

- ○ $16x^3 + 21x^2 - 7$
- ○ $4x^3 + 3x^2 - 7$
- ○ $4x^3 + 3x^2$
- ✓ $16x^3 + 21x^2$

Solution:

$$y' = (4x^4 + 7x^3 + 5)$$
$$= 4(x^4)' + 7(x^3)' + (5)'$$
$$= 16x^3 + 21x^2 + 0$$

Problem 2.

Find the derivative of the function

$$z^{-2} - z^{-1}$$

Answers *

- ○ $-2z^{-3} - \dfrac{1}{z^2}$
- ✓ $-2z^{-3} + \dfrac{1}{z^2}$
- ○ $2z^{-3} - \dfrac{1}{z^2}$
- ○ $z^{-3} + \dfrac{1}{z^2}$

solution:

$$(z^{-2} + z^{-1})'$$
$$= (z^{-2})' + (z^{-1})'$$
$$= -2z^{-2-1} + (-1)z^{-1-1}$$
$$= -2z^{-3} - z^{-2} = -2z^{-3} + \frac{1}{z^2}$$

Problem 3.

Find the derivative of the function

$y = (2x^3 + 4)(5x^7 - 8)$

Answers *

- ✓ $100x^9 + 140x^6 - 48x^2$

- ◯ $8x^9 + 140x^6 - 48x^2$

- ◯ $8x^9 + 140x^6 - 48x$

- ◯ $100x^9 + 140x^6 - 48x$

Solution:

$$y' = \left[(2x^3 + 4)(5x^7 - 8) \right]'$$

$$= (2x^3 + 4)'(5x^7 - 8) + (2x^3 + 4)(5x^7 - 8)'$$

$$= 6x^2(5x^7 - 8) + (2x^3 + 4)(35x^6)$$

$$= 100x^9 + 140x^6 - 48x^2$$

Problem 4.

Find the derivative of the function

$$\frac{9 - x^4}{x^2}$$

Answers *

- ◯ $\dfrac{18}{x^3} + 2x$

- ◯ $-\dfrac{18}{x} + 2x$

- ◯ $-\dfrac{9}{x^3} - 2x$

- ✓ $-\dfrac{18}{x^3} - 2x$

Solution: $\left(\dfrac{9 - x^4}{x^2} \right)'$

$$= \frac{(9 - x^4)' x^2 - (9 - x^4)(x^2)'}{[x^2]^2}$$

$$= \frac{(-4x^3) x^2 - (9 - x^4)(2x)}{x^4}$$

$$= \frac{-4x^5 - 18x + 2x^5}{x^4}$$

$$= -\frac{18}{x^3} - \frac{2x^5}{x^4} = -\frac{18}{x^3} - 2x .$$

Problem 5

Find the derivative of the function

$$y = \frac{x^3}{x - 1}$$

Answers *

- ○ $y' = \dfrac{2x^3 + 3x^2}{(x - 1)^2}$

- ○ $y' = \dfrac{-2x^3 - 3x^2}{(x - 1)^2}$

- ✓ $y' = \dfrac{2x^3 - 3x^2}{(x - 1)^2}$

- ○ $y' = \dfrac{-2x^3 + 3x^2}{(x - 1)^2}$

Solution:

$$y' = \left(\frac{x^3}{x-1}\right)' = \frac{(x^3)'(x-1) - x^3(x-1)'}{(x-1)^2}$$

$$= \frac{3x^2(x-1) - x^3}{(x-1)^2}$$

$$= \frac{3x^3 - 3x^2 - x^3}{(x-1)^2}$$

$$= \frac{2x^3 - 3x^2}{(x-1)^2}.$$

Problem 6

Find the derivative of the function

$$y = \frac{x^2 + 8x + 3}{\sqrt{x}}$$

Answers *

- ○ $y' = \dfrac{2x + 8}{2x^{3/2}}$

- ✓ $y' = \dfrac{3x^2 + 8x - 3}{2x^{3/2}}$

- ○ $y' = \dfrac{2x + 8}{x}$

- ○ $y' = \dfrac{3x^2 + 8x - 3}{x}$

Solution:

$$y' = \left(\frac{x^2 + 8x + 3}{\sqrt{x}}\right)'$$

$$= \frac{(x^2 + 8x + 3)'\sqrt{x} - (x^2 + 8x + 3)(\sqrt{x})'}{(\sqrt{x})^2}$$

$$= \frac{(2x + 8)\sqrt{x} - (x^2 + 8x + 3)\cdot\frac{1}{2}x^{-\frac{1}{2}}}{x}$$

$$= \frac{2(2x + 8)x - (x^2 + 8x + 3)}{2x\sqrt{x}} = \frac{3x^2 + 8x - 3}{2x^{\frac{3}{2}}}$$

Problem 7

Find the derivative of the function

$$s = \frac{t^9 + 8t + 4}{t^2}$$

Answers *

- ✓ $\frac{ds}{dt} = 7t^6 - \frac{8}{t^2} - \frac{8}{t^3}$

- $\frac{ds}{dt} = t^6 - \frac{8}{t^2} - \frac{4}{t^3}$

- $\frac{ds}{dt} = 7t^6 + \frac{8}{t^2} + \frac{8}{t^3}$

- $\frac{ds}{dt} = 8t^{11} + 10t^2 + 8t$

Solution:

$$s' = \left(\frac{t^9 + 8t + 4}{t^2} \right)'$$

$$= \frac{(t^9 + 8t + 4)' \, t^2 - (t^9 + 8t + 4)(t^2)'}{t^4}$$

$$= \frac{(9t^8 + 8)t^2 - (t^9 + 8t + 4) \, 2t}{t^4}$$

$$= \frac{9t^{10} + 8t^2 - 2t^{10} - 16t^2 - 8t}{t^4}$$

$$= \frac{7t^{10} - 8t^2 - 8t}{t^4} = 7t^6 - \frac{8}{t^2} - \frac{8}{t^3}$$

Problem 8

Write the function in the form y = f(u) and u = g(x). Then find dy/dx as function of x.

$$y = (-2x + 10)^3$$

Answers *

- $y = u^3; u = -2x + 10; \frac{dy}{dx} = -2(-2x + 10)^3$

- ✓ $y = u^3; u = -2x + 10; \frac{dy}{dx} = -6(-2x + 10)^2$

- $y = 3u + 10; u = x^3; \frac{dy}{dx} = -6x^2$

- $y = u^3; u = -2x + 10; \frac{dy}{dx} = 3(-2x + 10)^2$

$$f(x) = x^3.$$
$$g(x) = -2x + 10$$

Solution:

$$y' = \left[(-2x + 10)^3 \right]'$$

$$= 3 \left[g(x) \right]^{3-1} \cdot (-2x + 10)'$$

$$= 3(-2x + 10)^2 \cdot (-2)$$

$$= -6(-2x + 10)^2.$$

Problem 9

Find the derivative of the function

$$q = (15r - r^7)^{3/2}$$

Answers *

✓ $\dfrac{3}{2}\sqrt{15r - r^7} \times (15 - 7r^6)$

○ $\dfrac{3}{2}\sqrt{15r - r^7}$

○ $\dfrac{3}{2\sqrt{15 - 7r^6}}$

○ $\dfrac{3}{2\sqrt{15r - r^7}}$

Problem 10

Let

$$f(x) = x^2$$

and

$$g(x) = x + 1$$

. find the derivative of

$$(f \circ g)(x)$$

.

Answers *

✓ $2(x + 1)$

○ $2x$

○ $x^2 + 1$

○ $2x + 1$

Solution:

$$g'\left[\left(15r - r^7\right)^{\frac{3}{2}}\right]'$$

$$= \frac{3}{2}\left(15r - r^7\right)^{\frac{3}{2} - 1} \cdot \left(15r - r^7\right)'$$

$$= \frac{3}{2}\left(15r - r^7\right)^{\frac{1}{2}} \cdot \left(15 - 7r^6\right)$$

$$= \frac{3}{2}\sqrt{15r - r^7}\left(15 - 7r^6\right).$$

Solution:

$$\left[f \circ g(x)\right]'$$

$$= f'(g(x)) \cdot g'(x)$$

$$= 2(g(x))^{2-1} \cdot (x+1)'$$

$$= 2(x+1) \cdot 1 = \boxed{2(x+1)}$$

Problem 11

A total cost function is given by

$$C(x) = 2000(x^2 + 2)^{1/3} + 700$$

Where C(x) is the total cost, in thousands of dollars, of producing x airplanes. Find the rate at which total cost is changing when 20 airplanes have been produced.

[Hint: find the one that is closes to your answer]

Answers *

- ✓ 489
- ○ 1189
- ○ 211
- ○ 700

solution :

$$C'(x) = \left[2000(x^2+2)^{\frac{1}{3}} + 700 \right]'$$

$$= 2000 \left[(x^2+2)^{\frac{1}{3}} \right]' + (700)'$$

$$= 2000 \cdot \left[\frac{1}{3}(x^2+2)^{\frac{1}{3}-1} \cdot (x^2+1)' \right] + 0$$

$$= 2000 \left[\frac{1}{3}(x^2+2)^{-\frac{2}{3}} \cdot (2x) \right] = \frac{4000\,x}{3(x^2+2)^{2/3}}$$

$$C'(20) = \frac{4000 \times 20}{3(20^2+2)^{2/3}} \approx 489 .$$

Problem 12

If $1000 is invested at interest rate r in 3 years will grow an amount A given by

$$1000(1+r)^3$$

Find the instantaneous rate of change of A,

$$\frac{dA}{dr}$$

.

Answers *

- ✓ $3000(1+r)^2$
- ○ $3000r^2$
- ○ $2000(1+r)^2$
- ○ $3000(1+r^2)$

Solution :

$$\frac{dA}{dr} = \left[10000(1+r)^3 \right]'$$

$$= 10000 \left[(1+r)^3 \right]'$$

$$= 10000 \left[3(1+r)^{3-1} \right]$$

$$= 30000 (1+r)^2 .$$