MAT143 Brief Calculus

Midterm Exam #1

Monday, 6/5/2023

Time: 60 minutes

Name: ____________________________________
(Please print)

Instructions: This is a closed-book exam. No notes, books, or a computer should be used for the exam. However, you can use a calculator (TI or scientific calculator) for the exam. The first part consists of 6 multiple-choice problems and 2nd part consists of 3 show-your-work problems.

Part I: Multiple-choice problems. (50 points)

1. The average rate of change of f(x) from A and B (see Figure 1) is given by

   A). $(y_1 - y_2) / (x_2 - x_1)$

   B). $(x_1 - x_2) / (y_1 - y_2)$

   C). $(y_1 - y_2) / (x_2 - x_1)$

   D). $(y_2 - y_1) / (x_2 - x_1)$

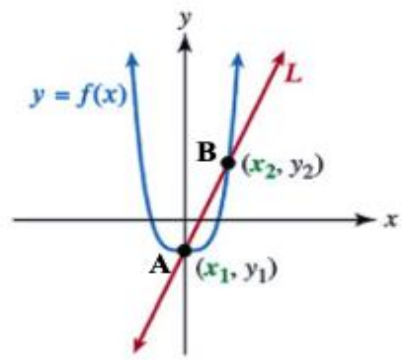

Figure 1

2. The difference quotient is defined by

$$\frac{f(x+h)-f(x)}{h}$$

   which is

   A). the intercept of tangent line PQ

   B). the slope of tangent line PQ

   C). the slope of the secant line PQ

   D). the derivative of *f(x)*

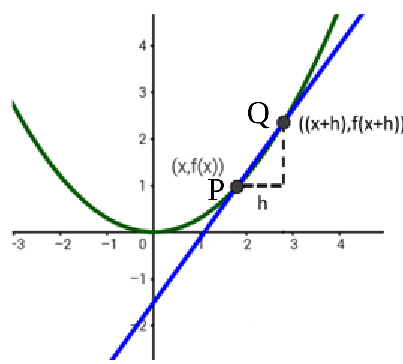

Figure 2

because the line passes through 2 points

3. Let f(x) = 2023. Then the derivative of f(x) is

A). 25
B). 1
C). 0
D). 25x

4. Let f(x) = x^{2023}. What rule of the derivative must be used to find the derivative of f(x)?

A). additive rule
B). multiplicative rule
C). power rule
D). chain rule

5. Let f(x) = $(\sqrt{x} + 1)^{1/3}$, what rule of the derivative must NOT be used to find the derivative of f(x)?

A). additive rule
B). multiplicative rule
C). power rule
D). chain rule

6. Refer to the following figure 3, what is the left limit as x $\rightarrow 6^-$

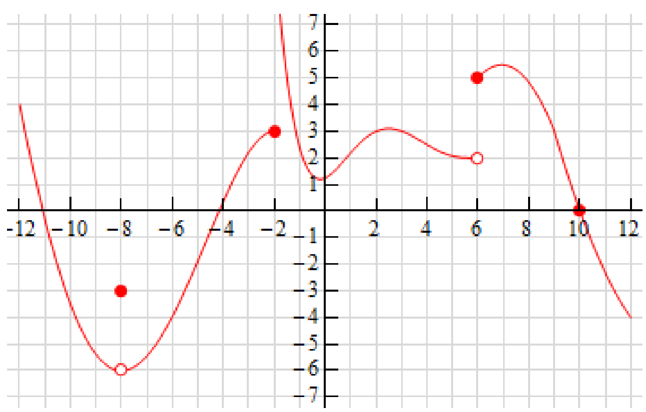

Figure 3

A). 6
B). 0
C). 2 ← trace the curve from the left - to right.
D). 5

7. $\lim_{x \to 0} \frac{x^3 - 8}{x^2 - 4} = ?$

A) 4
B) 0
C) 1
D) 2

direct substitution

8. $\lim_{x \to 0} \frac{-8}{x^2} = ?$

A) 1
B) −1
C) 0
D) does not exist

9. If $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$. $f(g(x)) = ?$

A) $\sqrt{x^2 + 1}$
B) $x + 1$
C) $\sqrt{x} + 1$
D) $\sqrt{x + 1}$

10. Find the limit

$$\lim_{x \to \infty} \frac{-8}{7 - (1/x^2)}$$

A) $-8/7$
B) ∞
C) 0
D) Does not exist

Part I: Show your work to receive credit. (50 points)

Problem 1. (**10 points**) For the function $f(x)$ whose graph is given in Figure 4, answer the following questions.

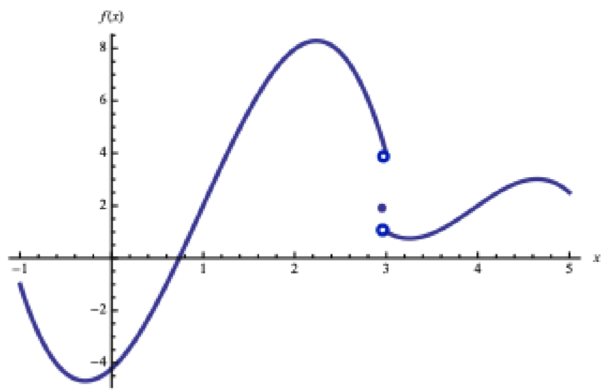

Figure 4

(1). Does $\lim_{x\to 3} f(x)$ exist? Why?

No. Because the left limit is not equal to the right limit.

(2). Is $f(x)$ continuous at $x = 3$? Why?

No. Because the actual value of y at $x=3$ is not equal to the limit (actually the limit of $f(x)$ does not exist at $x = 3$).

Problem 2. Finding limits (20 points)

1. $\lim_{x\to\infty} \frac{2x^2}{1+x^2}$.

$$= \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}}{\frac{1+x^2}{x^2}} = \lim_{x\to\infty} \frac{2}{\frac{1}{x^2}+1}$$

$$\underset{\text{substitution}}{=} \frac{2}{\frac{1}{\infty^2}+1} = 2.$$

2. $\lim\limits_{x \to 0} \frac{\sqrt{2+x^2}}{1+x^2}$.

$\frac{\text{direct}}{\text{substitution}}$ $\frac{\sqrt{2+0^2}}{1+0^2} = \frac{\sqrt{2}}{1} = \sqrt{2}$

Problem 3. Finding Derivatives (20 points)

1. Find the derivative of $f(x) = (x^3 + 1)^3$.

Using chain rule:

$f'(x) = \left([\underbrace{x^3+1}_{u}]^3\right)' = 3(x^3+1)^{3-1} \cdot (x^3+1)'$

$= 3(x^3+1)^2 \cdot (3x^{3-1}+0) = 9x^2(x^3+1)^2.$

2. Let $f(x) = x + \frac{1}{x} + \sqrt{x^3}$, find the value of $f'(x)$.

$f'(x) = (x + x^{-1} + x^{\frac{3}{2}})' = (x)' + (x^{-1})' + (x^{3/2})'$

$= 1 - 1x^{-1-1} + \frac{3}{2}x^{\frac{3}{2}-1} = x - x^{-2} + \frac{3}{2}x^{\frac{1}{2}}$

$= x - \frac{1}{x^2} + \frac{3\sqrt{x}}{2}.$

3. Let $f(x) = x(x^2 + 1)$, find the derivative of $f(x)$.

$f'(x) = (x)'(x^2+1) + x(x^2+1)' = (x^2+1) + x(2x+0)$

$= x^2+1+2x^2 = 3x^2+1$

OR: $f'(x) = [x(x^2+1)]' = (x^3+x)' = (x^3)' + (x)'$

$= 3x^2+1.$

4. Find the derivative of $f(x) = \frac{x+1}{x^2}$.

$f'(x) = \left[\frac{x+1}{x^2}\right]' = \frac{(x+1)' \cdot x^2 - (x+1)\cdot(x^2)'}{(x^2)^2} = \frac{(1+0)x^2 - (x+1)\cdot 2x}{x^4}$

$= \frac{x^2 - 2x^2 - 2x}{x^4} = \frac{-x^2 - 2x}{x^4} = -\frac{x+2}{x^3}.$

OR: $f' = \left[\frac{x+1}{x^2}\right]' = \left[\frac{x}{x^2} + \frac{1}{x^2}\right]' = (x^{-1} + x^{-2})'$

$= (x^{-1})' + (x^{-2})' = -1x^{-1-1} - 2x^{-2-1} = -x^{-2} - 2x^{-3}$