Daily Quiz #11: Antiderivatives

Problem 1.

$$\int \sqrt{x}dx = ?$$

$\int \sqrt{x}dx = \int x^{\frac{1}{2}}dx$

$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

Answers *

- $\frac{1}{2\sqrt{x}}$
- $\frac{\sqrt{x}}{2} + C$
- ✓ $\frac{x^{1+1/2}}{1+1/2} + C$ ✓
- $\frac{2x\sqrt{x}}{3}$

Problem 2.

$$\int 2e^x dx =$$

$\int 2e^x dx$

$= 2\int e^x dx$

$= 2 \cdot e^x + C$

Answers *

- $2e^x$
- ✓ $2e^x + C$
- $2xe^x + C$
- $2xe^{x-1}$

Problem 3.

$$\int \frac{3}{x}dx = ?$$

$\int \frac{3}{x}dx$

Answers *

- $3\ln x + C$
- [x] $3\ln|x| + C$
- $-3x^{-2} + C$
- does no exist since the denominator could be 0.

$= 3\int \frac{1}{x}dx$

$= 3\ln|x| + C$

Problem 4.

$$\int \frac{1}{\sqrt{x}}dx =$$

$\int \frac{1}{\sqrt{x}}dx = \int \frac{1}{x^{\frac{1}{2}}}dx$

Answers *

- $\frac{\sqrt{x}}{2} + C$
- $\frac{1}{2\sqrt{x}}$
- [x] $2\sqrt{x} + C$
- $-\frac{1}{2}x^{-3/2} + C$

$= \int x^{-\frac{1}{2}}dx$

$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= 2\sqrt{x} + C$

Problem 5.

$$\int (3-2x)dx =$$

$$\int (3-2x)dx$$

$$= \int 3dx - 2\int x\,dx$$

$$= 3\cdot x - 2\cdot \frac{x^{1+1}}{1+1} + C$$

$$= 3x - x^2 + C$$

Answers *

- $3 - x^2 + C$
- $3x - \frac{x^2}{2} + C$
- $-x^2 + C$
- ✓ $3x - x^2 + C$

Problem 6.

$$\int (x+1)^2 dx =$$

[Hint: expand the integrand before taking integral]

$$\int (x+1)^2 dx$$

$$= \int (x^2 + 2x + 1)\,dx$$

$$= \int x^2 dx + 2\int x\,dx + \int 1\,dx$$

$$= \frac{x^{2+1}}{2+1} + 2\cdot\frac{x^{1+1}}{1+1} + x + C$$

$$= \frac{x^3}{3} + x^2 + x + C$$

Answers *

- ✓ $\frac{x^3}{3} + x^2 + x + C$
- $\frac{(x+1)^3}{3} + C$
- $2(x+1) + C$
- $x^3 + 2x^2 + 1 + C$

Problem 7

$$\int (\sqrt{x} + \frac{1}{\sqrt{x}})^2 dx =$$

[Hint: do expansion first]

Answers *

- $x^2 + 2 + 2\sqrt{x} + C$
- $x^2 + 2 - \frac{1}{2\sqrt{x}} + C$
- ✓ $\frac{x^2}{2} + 2x + \ln|x| + C$
- $\frac{x^2}{2} + 2 + 2\sqrt{x} + C$

$$\int (\sqrt{x} + \frac{1}{\sqrt{x}})^2 dx$$

$$= \int \left[(\sqrt{x})^2 + 2 \cdot \sqrt{x} \cdot \frac{1}{\sqrt{x}} + \left(\frac{1}{\sqrt{x}}\right)^2 \right] dx$$

$$= \int \left(x + 2 + \frac{1}{x} \right) dx$$

$$= \int x\,dx + 2\int 1\,dx + \int \frac{1}{x} dx$$

$$= \frac{x^{1+1}}{1+1} + 2 \cdot x + \ln|x| + C$$

$$= \frac{x^2}{2} + 2x + \ln|x| + C$$

Problem 8

$$\int \frac{2x+1}{2x} dx =$$

Answers *

- ✓ $x + \frac{\ln|x|}{2} + C$
- $1 + \frac{1}{2x} + C$
- $x + 2\ln|x| + C$
- $x + \ln|2x| + C$

$$\int \frac{2x+1}{2x} dx$$

$$= \int \left(\frac{2x}{2x} + \frac{1}{2x} \right) dx$$

$$= \int \left(1 + \frac{1}{2x} \right) dx$$

$$= \int 1\,dx + \frac{1}{2}\int \frac{1}{x} dx$$

$$= x + \frac{1}{2}\ln|x| + C$$

Problem 9

$$\int \sqrt{x}(\sqrt{x}-1)dx =$$

[Hint: rewrite and then integrate]

Answers *

- $\frac{x^2}{2} - x + C$
- $2(x^{3/2} - x) + C$
- $\frac{x^2}{2} - 2\sqrt{x} + C$
- ✓ $\frac{x^2}{2} - \frac{2x^{3/2}}{3} + C$

$$\int \sqrt{x}(\sqrt{x}-1)dx = \int [(\sqrt{x})^2 - \sqrt{x}]dx = \int x\,dx - \int x^{\frac{1}{2}}dx = \frac{x^{1+1}}{1+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{x^2}{2} - \frac{x^{3/2}}{3/2} + C = \frac{x^2}{2} - \frac{2}{3}x^{\frac{3}{2}} + C$$

Problem 10

$$\int x\sqrt{3x}dx =$$

Answers *

- $\frac{5\sqrt{3}}{2}x^{5/2} + C$
- $2\sqrt{3x} + C$
- ✓ $\frac{2\sqrt{3}}{5}x^{5/2} + C$
- $\frac{2\sqrt{3}}{5}x^{3/2} + C$

$$\int x\sqrt{3x}dx = \int \sqrt{3}\cdot x\sqrt{x}\,dx = \sqrt{3}\int x\cdot x^{\frac{1}{2}}dx = \sqrt{3}\int x^{1+\frac{1}{2}}dx = \sqrt{3}\int x^{\frac{3}{2}}dx = \sqrt{3}\cdot\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C = \sqrt{3}\cdot\frac{2}{5}x^{\frac{5}{2}} + C = \frac{2\sqrt{3}}{5}x^{\frac{5}{2}} + C$$