Daily Quiz #8

Problem 1.

Which of the following functions is increasing $(0, \infty)$?

Answers *

- ( ) $f(x) = 13 - x^3$ — $f'(x) = 0 - 3x^2 < 0$, always decreasing
- ( ) $f(x) = -4x + 1$ — $f'(x) = -4 < 0$, always decreasing
- ( ) $f(x) = -4x^2$ — $f'(x) = -8x$, > 0 If $x < 0 \rightarrow$ increasing, < 0, If $x > 0$, $\rightarrow$ decreasing
- (✓) x^2 ✓ — $f'(x) = 2x > 0$ on $(0, \infty)$.

Problem 2.

Which of the following functions, when defined on the set of non-negative real numbers, is decreasing? $\Rightarrow x \geq 0$. $f'(x) < 0$

Answers *

- ( ) $f(x) = 2x$ — $f'(x) = 2 > 0$, ↑
- ( ) $f(x) = x^{1/2}$ — $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} > 0$ when $x \geq 0$ ↑
- ( ) $f(x) = x^3 + 4x^2 + x + 1$ — $f'(x) = 3x^2 + 8x + 1 > 0$ when $x \geq 0$ ↑
- (✓) $f(x) = -2(x^2 + 9)$ ✓ — $f'(x) = -2(x^2+9)' = -2(2x+0) = -4x < 0$ when $x > 0$ ↓

Problem 3.

Let $f(x) = x^3 + 3x^2 - 45x + 4$. Then the local extrema of $f(x)$ are

① $f'(x) = 3x^2 + 6x - 45$ ② set $f'(x) = 0$

$3x^2 + 6x - 45 = 0$

$\Rightarrow x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$\Rightarrow x = -5$

or $x = 3$

Answers *

- A local minimum of -179 at $x = 5$ and a local maximum of 77 at $x = -3$.
- ✓ A local minimum of -77 at $x = 3$ and a local maximum of 179 at $x = -5$.
- A local minimum of -179 at $x = -5$ and a local maximum of -77 at $x = 3$.
- A local minimum of -77 at $x = 3$ and a local maximum of 77 at $x = 5$.

$f''(x) = (3x^2 + 6x - 45)' = 6x + 6 = 6(x+1)$

$f''(-5) = 6(-5+1) < 0$ $f''(3) = 6(3+1) > 0$

Problem 4

For what values of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum?

Answers *

- ✓ 10
- 4
- 3
- -4

$f'(x) = [x^3 - 9x^2 - 120x + 6]'$

$= 3x^2 - 18x - 120$

set $f'(x) = 0$, $3x^2 - 18x - 120 = 0$

$3(x^2 - 6x - 40) = 0$

$3(x-10)(x+4) = 0 \to$ $x = 10$ or $x = -4$

$f''(x) = [3x^2 - 18x - 120]' = 6x - 18 = 6(x-3)$

$f''(10) = 6(10-3) = 42 > 0$ $\to$ min

$f''(-4) = 6(-4-3) = -42 < 0$ $\to$ max

Problem 5.

The graph of $y = x^3 - 5x^2 + 4x + 2$ has a local minimum at

$y' = 3x^2 - 10x + 4$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{10 \pm \sqrt{10^2 - 4 \times 3 \times 4}}{2 \times 3} = \frac{10 \pm \sqrt{48}}{6}$

Answers *

- (0.46, 2.87)
- (0.46, 0)
- ✓ (2.94, -4.05) 2.87
- (4.06, 2.87)

$x = \frac{10 + \sqrt{48}}{6} \approx 2.87$

or $x = \frac{10 - \sqrt{48}}{6} \approx 0.464$

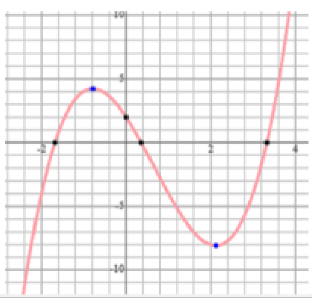
$y'' = (3x^2 - 10x + 4)' = 6x - 10 = 2(3x - 5)$

$y''(2.87) = 2(3 \times 2.87 - 5) > 0$

$y''(0.464) = 2(3 \times 0.464 - 5) < 0$

Problem 6.

The graph of $y = x^3 - 2x^2 - 5x + 2$ has a local maximum at



4×19

$y' = 3x^2 - 4x - 5 = 0$

$x = \frac{4 \pm \sqrt{(4)^2 - 4 \times 3 \times (-5)}}{2 \times 3} = \frac{4 \pm \sqrt{16 + 60}}{6}$

$= \frac{4 + \sqrt{76}}{6} = \frac{4 \pm \sqrt{4 \times 19}}{6} = \frac{4 \pm 2\sqrt{19}}{6}$

$= \frac{2 \pm \sqrt{19}}{3} = 2.120$ or -0.786

Answers *

- (2.12, 0)
- (2.12, -8.061)
- (-0.786, 0)
- ✓ (-0.786, 4.209)

$y'' = (3x^2 - 4x - 5)' = 6x - 4 = 2(3x - 2)$

$y''(2.12) = 2(3 \times 2.12 - 2) > 0$

$y''(-0.786) = 2(3 \times (-0.7867) - 2) < 0$

Problem 7.

Find the relative extrema for the following functions by (1) determining the critical value(s) and (2) determining whether at these critical values the function is a relative maximum or minimum (or possible inflection point).

$$f(x) = -8x^2 + 12x + 3$$

$$f'(x) = [-8x^2 + 12x + 3]' = -16x + 12$$

$$f'(x) = 0 \rightarrow -16x + 12 = 0 \Rightarrow x = 3/4$$

$$f''(x) = (-16x+12)' = -16 < 0$$

Answers *

- ✓ x = 3 /4 , relative maximum ✓
- x = 3/ 4 , relative minimum
- x = – 3 /4 , relative minimum
- x = – 3 /4 , relative maximum

Problem 8.

Find the relative extrema for the following functions by (1) determining the critical value(s) and (2) determining whether at these critical values the function is a relative maximum or minimum (or possible inflection point).

$$f(x) = (x - 1)^3$$

$$f'(x) = [(x-1)^3]' = 3(x-1)^2 \cdot (x-1)' = 3(x-1)^2$$

$$f'(x) = 0 \Rightarrow x = 1.$$

$$f''(x) = 3[(x-1)^2]' = 3[2 \cdot (x-1)] = 6(x-1)$$

$$f''(1) = 6(1-1) = 0$$

$\Rightarrow$ $x = 1$ is an inflection point.

Answers *

- ✓ x = 1, inflection point ✓
- x = 1, relative minimum
- x = 1, relative maximum
- x = 1 is not a critical value

Problem 9.

Optimize the following function by (1) finding the critical value(s) at which the function is optimized and (2) testing the second-order condition to distinguish between a relative maximum or minimum and (3) the values of the relative extrema for the function. For the following given function

$$f(x) = x^2 + 6x + 9$$

Which if the following is correct?

$f'(x) = [x^2 + 6x + 9]'$
$= 2x + 6$
$f'(x) = 0 \Rightarrow 2x + 6 = 0$
$\Rightarrow x = -3.$

Answers *

- x = −3, relative maximum, f(−3) = 0
- x = 3, relative minimum, f(−3) = 0
- ✓ x = −3, relative minimum, f(−3) = 0
- x = −3, relative minimum, f(−3) = −1

$f''(x) = [2x+6]' = 2 > 0$

$f(-3) = (-3)^2 + 6(-3) + 9$
$= 9 - 18 + 9 = 0$

Problem 10.

Optimize the following function by (1) finding the critical value(s) at which the function is optimized and (2) testing the second-order condition to distinguish between a relative maximum or minimum and (3) the value(s) of the relative extrema for the function. For the following giving function

$$f(x) = x^3 + 6x^2 - 96x + 23$$

Which of the following choices is correct?

$f'(x) = [x^3 + 6x^2 - 96x + 23]'$
$= 3x^2 + 12x - 96$

$f'(x) = 0 \Rightarrow 3x^2 + 12x - 96 = 0$

Answers *

- ✓ x = 4, relative minimum; x = −8, relative maximum; f(4) = −201 and f(−8) = 663
- x = −4, relative minimum; x = 8, relative maximum; f(−4) = 200 and f(8) = −665
- x = 4, relative minimum; x = 8, relative maximum; f(4) = −201 and f(8) = 663
- x = 4, relative minimum; x = −8, relative maximum; f(4) = −154 and f(−8) = 653

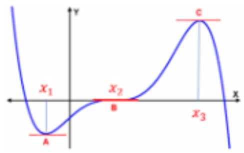
$3(x^2 + 4x - 32) = 0$
$3(x+8)(x-4) = 0$
$x = -8$ or $x = 4.$

$f''(x) = [3x^2 + 12x - 96]'$
$= 6x + 12$
$= 6(x+2).$

$f''(-8) = 6(-8+2) < 0$
$f''(4) = 6(4+2) > 0$
$f(4) = 4^3 + 6 \times 4^2 - 96 \times 4 + 23 = -201,$

Problem 11

Which of the following statements is true based on the given figure.

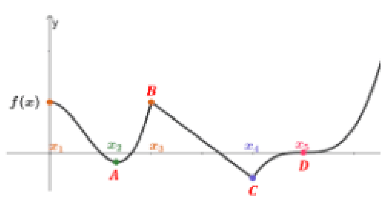


Answers *

- [x] A, B, and C are critical points.
- [ ] C and A are inflection points.
- [ ] A, B, and C are relative extrema.
- [ ] Only A and C are critical points

Problem 12.

Find the critical points in the following figure at which the derivative does **not** exist.

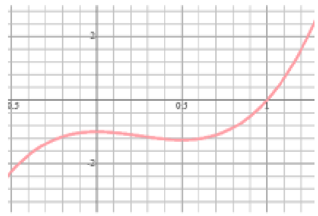


Answers *

- [x] Points B, C, and D
- [ ] Points B and C
- [ ] Point D
- [ ] A and D

Problem 13.

Find the intervals of concavity of the function $f(x) = 4x^3 - 3x^2 - 1$



by solving equation $f''(x) = 0$ (see the definition of concavity).

$f'(x) = [4x^3 - 3x^2 - 1]'$

$= 12x^2 - 6x$

for finding concavity, we need

$f''(x) = [12x^2 - 6x]'$

$= 24x - 6$

set $f''(x) = 0 \Rightarrow 24x - 6 = 0$

$\Rightarrow x = \frac{6}{24} = \frac{1}{4}$.

$f''(x) = 24(x - \frac{1}{4})$

If $x < 1/4$, $f''(x) < 0$ (down)

If $x > \frac{1}{4}$, $f''(x) > 0$, (up)

Answers *

- Concave up on $(\frac{1}{4}, \infty)$ and concave down on $(-\infty, \frac{1}{4})$
- Concave up on $(\frac{\sqrt{2}}{2}, \infty)$ and concave down on $(-\infty, \frac{\sqrt{2}}{2})$
- Concave up on $(0.5, \infty)$ and concave down on $(-\infty, 0.5)$
- Concave up on $(-\frac{\sqrt{2}}{2}, \infty)$ and concave down on $(-\infty, -\frac{\sqrt{2}}{2})$

Problem 14.

The local extrema of $f(x) = -x^3 + 6x^2 + 6$ occur at which of the following x-values?

$f'(x) = [-x^3 + 6x^2 + 6]' = -3x^2 + 12x$

Set $f'(x) = 0 \Rightarrow -3x(x-4) = 0 \Rightarrow x = 0$ or 4

$f''(x) = [-3x^2 + 12x]' = -6x + 12$

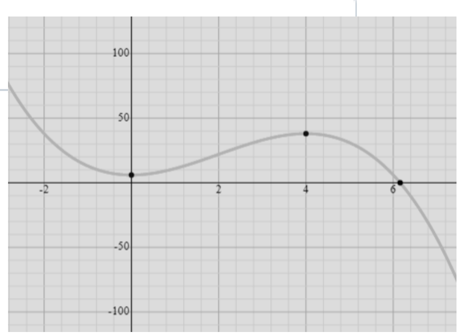
$= -6(x-2)$.

$f''(0) = -6(0-2) > 0$

$f''(4) = -6(4-2) = -12 < 0$

Answers *

- local maximum at $x = 0$, local minimum at $x = 4$
- local maximum at $x = 4$, local minimum at $x = 0$
- local minimum at $x = 0, 4$
- local maximum at $x = 2$



Problem 15.

What is he x-coordinate of the inflection point on the graph of

$$f(x) = -x^3/3 + 5x^2 + 24$$

$$f'(x) = \left[-\frac{x^3}{3} + 5x^2 + 24\right]'$$

$$= -x^2 + 10x = -x(x-10).$$

$$f''(x) = -2x + 10$$

set $f''(x) = 0$, $-2x + 10 = 0$

$$\Rightarrow x = 5$$

Answers *

- ✓ 5 ✓
- 0
- -5
- -10