MAT143 Brief Calculus
Midterm Exam #2

Friday, 3/10/2023

Time: 50 minutes

Name: _____ WCU-ID _____
(Please print)

Instructions: This is a closed-book exam. No notes, books or a computer should be used for the exam. However, you can use a calculator (TI or scientific calculator) for the exam. The first part consists of 6 multiple-choice problems and 2nd part consists of 3 show-your-work problems.

Part I: Multiple-choice problems. (50 points)

1. **Marginal profit**. The profit, in thousands of dollars, from the sale of x thousand candles can be estimated by

$$P(x) = 2x - 0.3x \ln x$$

Find the marginal profit function.

A. $P'(x) = 1.7 - 0.3 \ln x$

B. $P'(x) = 1.7$

C. $P'(x) = 2 - 0.3 \ln x$

D. $P'(x) = 2x - 0.3 \ln x$

$P'(x) = (2x - 0.3x\ln x)'$
$= (2x)' - 0.3[x \ln x]'$
$= 2 - 0.3[(x)'\ln x + x(\ln x)']$
$= 2 - 0.3[\ln x + x \cdot \frac{1}{x}]$
$= 2 - 0.3\ln x - 0.3 = 1.7 - 0.3\ln x$

2. Find the derivative of

$$y = \ln(x^2)$$

A $\frac{2}{x}$

B $\frac{1}{2x}$

C $\frac{2}{x^2}$

D. $2 + \frac{1}{x}$

$y = 2\ln x$
$y' = [2\ln x]' = 2[\ln x]' = \frac{2}{x}$

3. Find the derivative of

$$y = x^e + e^x$$

e is the natural base.

A $x^e + e^x$

B $x^e + xe^{x-1}$

C $ex^{e-1} + e^x$

D $ex^{e-1} + xe^{x-1}$

power exp

$y' = (x^e + e^x)' = (x^e)' + (e^x)'$

$= e x^{e-1} + e^x$

4. The total cost, in millions of dollars, for Greenleaf Construction is given by

$$C(x) = 100 - 50e^{-x}$$

where x is the number of houses built. Find the **marginal cost function** (i.e., the derivative of C(x).

A $-50e^{-x}$

B $50e^{-x}$

C $100 + 50e^{-x}$

D $100 + 50xe^{-x}$

$C'(x) = (100 - 50e^{-x})'$

$= (100)' - 50(e^{-x})'$

$= 0 - 50 \cdot e^{-x}(-x)'$

$= -50 \cdot e^{-x} \times (-1) = 50e^{-x}$

5. Find the derivative of

$$y = 10^x.$$

A $x10^{x-1}$

B $10^{x-1} \log_{10} e$

C $10^x \ln 10$

D $x10^{x-1} \ln 10$

$y' = [10^x]' = 10^x \ln 10$

6. Find the derivative of

$$y = 9e^{x^2}$$

A $9 + e^{x^2}$

B $2x^2e^{x^2}$

C $2xe^{x^2}$

D $18xe^{x^2}$

$y' = (9e^{x^2})' = 9[e^{x^2}]'$

$= 9 \cdot e^{x^2} \cdot (x^2)' = 9 \cdot e^{x^2} \cdot 2x$

$= 18x e^{x^2}$

7. What is the derivative of

$$f(x) = \ln(x^2 + 1)$$

A $\frac{1}{x^2+1}$

B $\frac{2}{x^2+1}$

C $\frac{2x}{x^2+1}$

D $\frac{1}{x^2}$

$f'(x) = [\ln(x^2+1)]' = \frac{(x^2+1)'}{x^2+1} = \frac{2x}{x^2+1}$

8. Find the derivative of

$$f(x) = \log_2 x^2$$

A $\frac{2}{x^2}$

B $\frac{2\ln 2}{x}$

C $\frac{2}{x\ln 2}$

D $\frac{2}{x^2(\log_x 2)}$

$f'(x) = [\log_2 x^2]' = [2\log_2 x]' = 2[\log_2 x]' = \frac{2}{x\ln 2}$

9. Find the derivative of

$$y = 3^{2x} + 1$$

A $2x3^{2x-1}$

B $2x3^{2x}\ln 3$

C $(2\ln 3)\,3^{2x}$

D $(2\ln 3)\,3^{2x} + 1$

$y' = [3^{2x}+1]' = (3^{2x})' + 1' = 3^{2x}\ln 3 \cdot (2x)' = (2\ln 3)\cdot 3^{2x}$

10. Find the derivative of $y = \ln 3^{2x} + 1$

A $\frac{1}{\ln 3^{2x}}$

B $\frac{2}{\ln 3^{2x}}$

C $2\ln 3$

D $2x\ln 3$

$y = \ln 3^{2x} + 1 = 2x\ln 3 + 1$

$y' = [(2\ln 3)x]' + 1' = 2\ln 3\,(x)' + 0 = 2\ln 3$

***Part I: Show your work to receive credit**. (50 points).*

Problem 1. (10 point each)

1. Find the derivative of the following function

$$f(x) = \frac{x+1}{e^x}$$

$$f'(x) = \left[\frac{x+1}{e^x}\right]' = \frac{(x+1)'e^x - (x+1)\cdot[e^x]'}{(e^x)^2} = \frac{e^x - (x+1)e^x}{e^{2x}}$$

$$= \frac{-x}{e^x}$$

2. Find the derivative of the following function

$$f(x) = \ln[(x+4)/x]$$

$$f(x) = \ln(x+4) - \ln x$$

$$f'(x) = [\ln(x+4)]' - [\ln x]' = \frac{1}{x+4}(x+4)' - \frac{1}{x}$$

$$= \frac{1}{x+4} - \frac{1}{x}$$

Problem 2. (10 points each)

1. Find the derivative of the following function

$$f(x) = log_{10}(x+4)$$

$$f'(x) = \frac{(x+4)'}{(x+4)\ln 10} = \frac{1}{(x+4)\ln 10}.$$

2. Find the derivative of the following function

$$f(x) = x2^x$$

$f'(x) = (x2^x)' = (x)' \cdot 2^x + x \cdot [2^x]'$

$= 2^x + x \cdot 2^x \cdot \ln 2 \quad = 2^x(1 + x\ln 2)$

Problem 3 (5 points each)

Interest Compounded Continuously. Suppose $P_0 = 10000$, in dollars, is invested in the Von Neumann Hi-Yield Fund, with interest compounded continuously at 7% per year. That is, at any point in time after t years, the balance P, in dollars per year, is given by

$$P(t) = 10000e^{0.07t}$$

(1). What is the account balance at the end of fifth year, i.e., P(5)?

$P(5) = 10000\, e^{0.07 \times 5} = 10000\, e^{0.35}$

≈ 14190.68

(2). Find the growing rate of the balance, i.e., $P'(5)$. [*Hint: find the derivative of P(t) first, then evaluate P'(5)*].

$P'(t) = [10000e^{0.07t}]' = 10000\,(e^{0.07t})'$

$= 10000\, e^{0.07t} \cdot (0.07t)'$

$= 10000 \times 0.07\, e^{0.07t} = 700\, e^{0.07t}$

$P'(5) = 700e^{0.07 \times 5} = 700\, e^{0.35} \approx 993.34$