MAT143 Brief Calculus
Midterm Exam #1

Friday, 2/17/2023

Time: 50 minutes

Name: _____
(Please print)

Instructions: This is a closed-book exam. No notes, books, or a computer should be used for the exam. However, you can use a calculator (TI or scientific calculator) for the exam. The first part consists of 6 multiple-choice problems and 2nd part consists of 3 show-your-work problems.

Part I: Multiple-choice problems. (42 points)

1. The average rate of change of f(x) from A and B (see Figure 1) is given by

   A). $(y_1 - y_2) / (x_2 - x_1)$

   B). $(x_1 - x_2) / (y_1 - y_2)$

   C). $(y_1 - y_2) / (x_2 - x_1)$
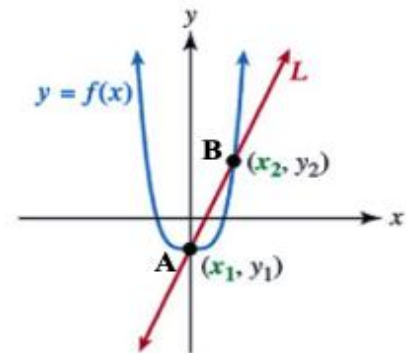
   D). $(y_2 - y_1) / (x_2 - x_1)$

Figure 1

2. The difference quotient is defined by
$$\frac{f(x+h) - f(x)}{h}$$

   A). the intercept of tangent line PQ
   B). the slope of tangent line PQ
   C). the slope of the secant line PQ
   D). the derivative of $f(x)$
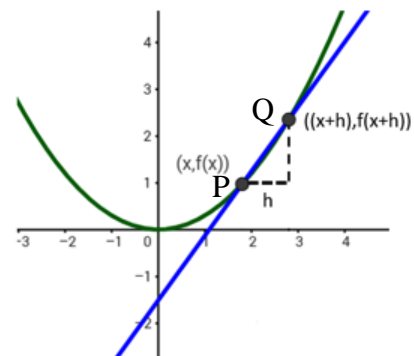

Figure 2

3. Let f(x) = 25 (a constant function). Then the derivative of f(x) is

   A). 25
   B). 1
   C). 0
   D). 25x

4. Let f(x) = x^{2023}. What rule of the derivative must be used to find the derivative of f(x)?

   A). additive rule
   B). multiplicative rule
   C). power rule
   D). chain rule

5. Let f(x) = $(x^2 + 1)^{1/3}$, what rule of the derivative must be used to find the derivative of f(x)?

   A). additive rule
   B). multiplicative rule
   C). power rule
   D). chain rule

6. Refer to Figure 3. What is the left limit as x → 1⁻
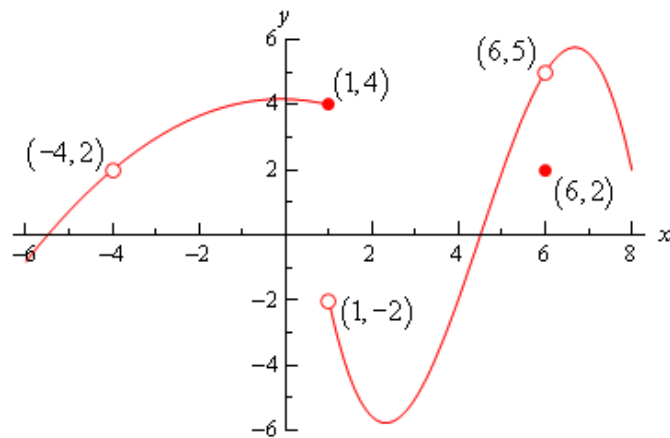
   A). 1
   B). 0
   C). -2
   D). 4

Figure 3

Part I: Show your work to receive credit. *(58 points)*

Problem 1. (**8 points**) For the function $f(x)$ whose graph is given in Figure 4, answer the following questions.
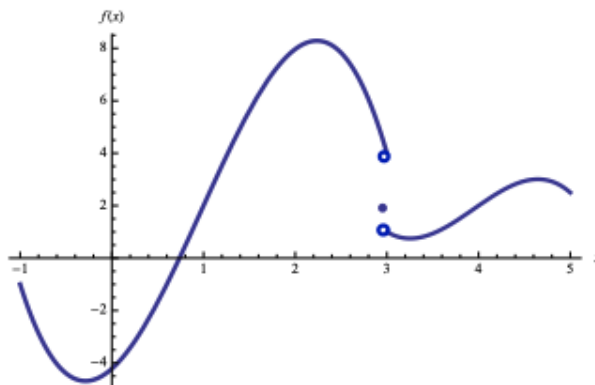

Figure 4

(1). Does $\lim\limits_{x\to 3} f(x)$ exist? Why?

No. Because the left limit is NOT equal to the right limit.

(2). Is $f(x)$ continuous at $x = 3$? Why?

No. the left and the right limits are NOT equal.

Problem 2. Finding limits (20 points)

1. $\lim\limits_{x\to\infty} \dfrac{2x^2}{1+x^2}$.

The direct substitution will result in $\frac{\infty}{\infty}$, we need to do algebraic manupition before doing substitution.

$\lim\limits_{x\to\infty} \dfrac{2x^2}{1+x^2} = \lim\limits_{x\to\infty} \dfrac{2}{\frac{1}{x^2}+1} = \dfrac{2}{\frac{1}{\infty^2}+1} = 2.$

2. $\lim\limits_{x\to 0} \dfrac{\sqrt{2+x^2}}{1+x^2}$.

Direct substitution gives

$$\lim\limits_{x\to 0} \frac{\sqrt{2+x^2}}{1+x^2} = \frac{\sqrt{2+0^2}}{1+0^2} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

Problem 3. Finding Derivatives (30 points)

1. For the given function

$$f(x) = (2x^3 + 2)^3.$$

Find the derivative of $f(x)$.

$$f'(x) = [(2x^3 + 2)^3]' = 3(2x^3 + 2)^{3-1} \cdot (2x^3 + 2)'$$

$$= 3(2x^3 + 2)^2 \cdot (2 \times 3x^{3-1} + 0)$$

$$= 18x^2(2x^3 + 2)$$

2. If $f(x) = \sqrt{x^3}$, find the value of $f'(4)$.

$$f'(x) = (\sqrt{x^3})' = (x^{\frac{3}{2}})' = \frac{3}{2} x^{\frac{3}{2}-1} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$

$$f'(4) = \frac{3}{2} \times \sqrt{4} = \frac{3}{2} \times 2 = 3$$

3. Let If $f(x) = (x + 1)(x^2 + 1)$, find the derivative of $f(x)$.

$$f'(x) = [(x+1)(x^2+1)]' = (x+1)'(x^2+1) + (x+1)(x^2+1)'$$

$$= 1 \times (x^2+1) + (x+1)(2x+0)$$

$$= x^2+1 + 2x(x+1)$$

$$= x^2+1 + 2x^2 + 2x = 3x^2 + 2x + 1.$$