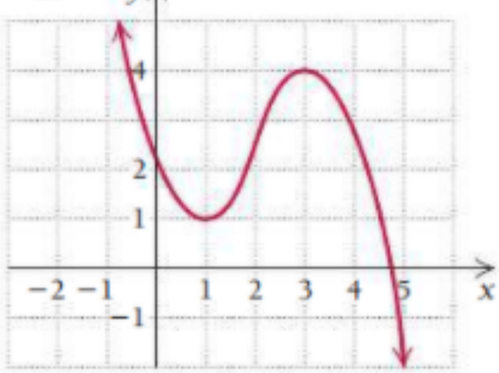
Practice the following problems related to critical values

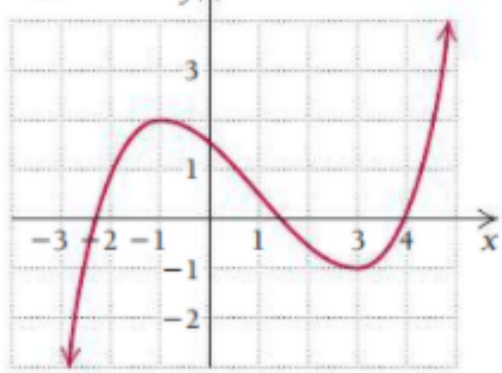
Section 3.1

For each graph in Exercises 1–4, identify all (a) critical values, (b) relative minima, (c) relative maxima, (d) relative minimum points, and (e) relative maximum points.

1.

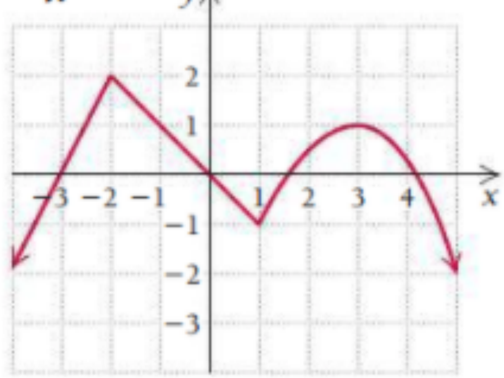


2.



3.

4.



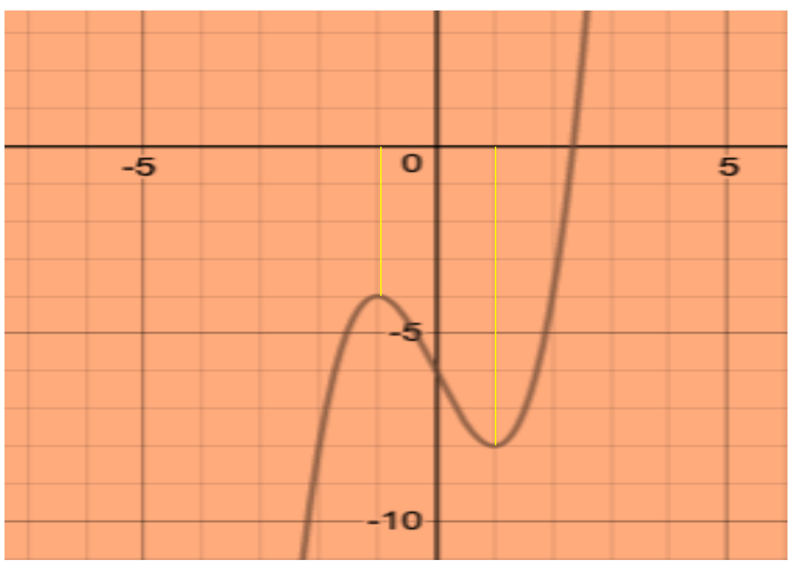
1. (a) $x = 1, x = 3$; **(b)** relative minimum 1; **(c)** relative maximum 4; **(d)** relative minimum point $(1, 1)$; **(e)** relative maximum point $(3, 4)$ **2. (a)** $x = -1, x = 3$; **(b)** relative minimum -1; **(c)** relative maximum 2; **(d)** relative minimum point $(3, -1)$; **(e)** relative maximum point $(-1, 2)$

3. **(a)** $x = -2, x = 2$; **(b)** relative minimum -3; **(c)** relative maximum -1; **(d)** relative minimum point $(2, -3)$; **(e)** relative maximum point $(-2, -1)$ **4.** **(a)** $x = -2, x = 1, x = 3$; **(b)** relative minimum -1; **(c)** relative maxima 2, 1; **(d)** relative minimum point $(1, -1)$; **(e)** relative maximum points $(-2, 2)$ and $(3, 1)$

For each function given in Exercises 17–32, find (a) any critical values and (b) any relative extrema.

20. $g(x) = x^3 - 3x - 6$

20. **(a)** $x = -1, x = 1$;
(b) relative maximum $(-1, -4)$, relative minimum $(1, -8)$



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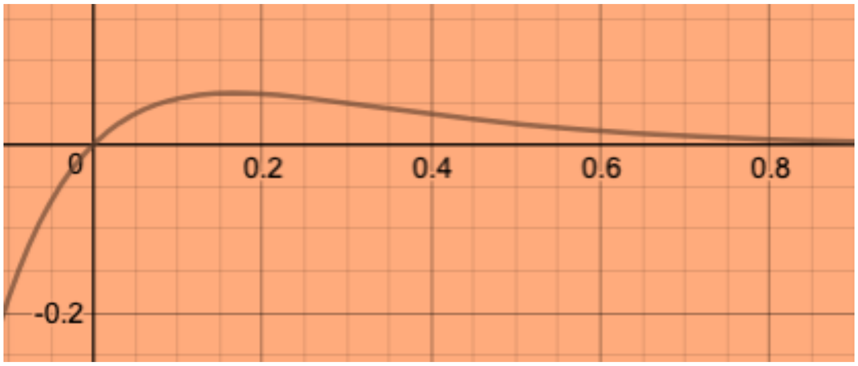
30. $f(x) = xe^{-6x}$



Let. $f'(x) = 0$, we have $e^{-6x} + x\,e^{-6x}(-6) = 0$ that is equivalent to

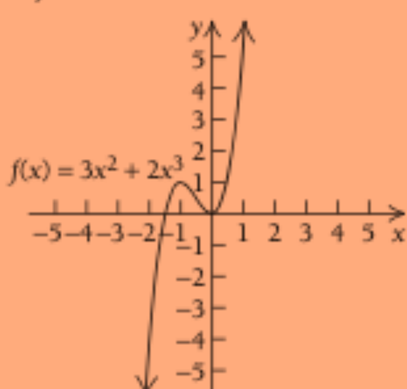
$e^{-6x}(1 - 6x) = 0$, $x = 1/6$.

30. **(a)** $x = \frac{1}{6}$; **(b)** relative maximum minimum $\left(-\frac{1}{4}, -0.092\right)$

find any relative extrema of each function. List each extremum along with the x-value at which it occurs. Identify intervals over which the function is increasing and over which it is decreasing. Then sketch a graph of the function.

40. $f(x) = 3x^2 + 2x^3$

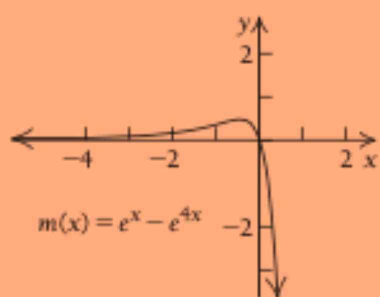
40. Relative minimum $(0, 0)$, relative maximum $(-1, 1)$, decreasing on $(-1, 0)$, increasing on $(-\infty, -1)$ and on $(0, \infty)$



Setting f '(x) = 0 yields $6x + 6x^2 = 0$, $6x(1+x) = 0$. Solving for x, we have x = 0 or x = -1.

46. $m(x) = e^x - e^{4x}$

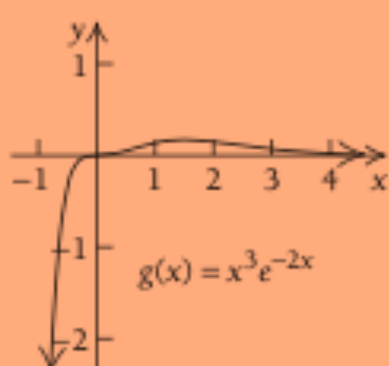
46. Relative maximum $\left(\frac{1}{3}\ln\left(\frac{1}{4}\right), 0.472\right)$, increasing on $\left(-\infty, \frac{1}{3}\ln\left(\frac{1}{4}\right)\right)$, decreasing on $\left(\frac{1}{3}\ln\left(\frac{1}{4}\right), \infty\right)$



Set m'(x) = 0, $e^x - 4e^{4x} = 0$, that is, $1 = 4e^{3x}$. $\Leftrightarrow$ $\frac{1}{4} = e^{3x}$ $\Leftrightarrow$ $3x = \ln(1/4) = -\ln 4$ $\Leftrightarrow$ $x = -(1/3)\ln(1/4)$

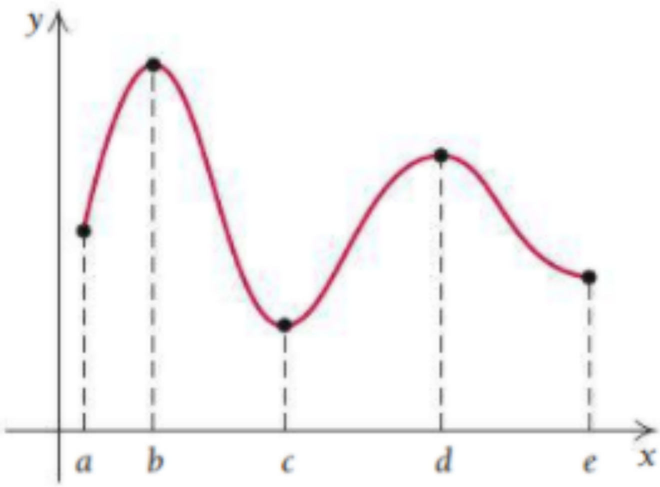
62. $g(x) = x^3e^{-2x}$

62. Relative maximum $\left(\frac{3}{2}, \frac{27}{8}e^{-3}\right)$



Setting $g'(x) = 0$ yields. $3x^2e^{-2x} – 2x^3e^{-2x} = 0$. This equivalent to $3x^2 - 2x^3 = 0$
Solving the above equation, we have $x = 0$ or $x = 3/2$.

66. Consider this graph.



Using the graph and the intervals noted, explain how a function being increasing or decreasing relates to the first derivative.

f ′(x) is increasing on (a, b) and (c, d) and decreasing on (b, c) and (d, e).

68. Optimizing revenue. A software developer notices that the number y of downloads of an app (in thousands) is related to the price x (in dollars) of the app by $y = 2.6 - 0.4x$.

a) Find $R(x)$, the total revenue generated when the price of the app is x dollars.
b) Find the relative extremum of R, and interpret this result.

68. **(a)** $R(x) = 2.6x - 0.4x^2$; **(b)** relative maximum (3.25,4.225), which means that when the price of the app is \$3.25, a maximum total revenue of 4.225 thousand, or \$4225, is generated

Note that the revenue (*R*) = # downloads (*y*) x price (*x*).

$$R(x) = 2.6\,x - 0.4\,x^2$$