

## Answer Key to Topic #6 Assignment

### Problem 1.

Find the derivative of the function of x:

$$y = 10^x$$

$$(a^x)' = a^x \cdot \ln a$$

Answers \*

☐

$$x \ln 10$$

☐

$$10^x \ln x$$

$$(10^x)' = 10^x \cdot \ln 10$$

☐

$$10^x$$

☒

$$10^x \ln 10$$

### Problem 2.

Find the derivative of the following function:

$$y = t^{5-e}$$

This is a power function!

Answers \*

☐

$$t^{5-e}$$

$$(t^{5-e})' = (5-e) \cdot t^{5-e-1}$$

☐

$$(4-e)t^{5-e}$$

$$= (5-e) t^{4-e}$$

☒

$$(5-e)t^{4-e}$$

**Problem 3.**

Find the derivative of the following function:

$$y = 2^{\ln(6t)}$$

Answers \*

☐

$$2 \ln 6t$$

☒

$$\frac{\ln 2}{t} 2^{\ln 6t}$$

☐

$$\frac{6 \ln 2}{t}$$

☐

$$\frac{6 \ln 2}{t} 2^{\ln 6t}$$

composite exponential function!

$$[2^{\ln(6t)}]' = 2^{\ln(6t)} \cdot \ln 2 \cdot [\ln 6t]'$$

$$= 2^{\ln(6t)} \ln 2 \cdot [\ln 6 + \ln t]'$$

$$= 2^{\ln(6t)} \cdot \ln 2 \cdot [0 + \frac{1}{t}]$$

$$= \frac{\ln 2}{t} \cdot 2^{\ln(6t)}$$

**Problem 4.**

Find the derivative of the following function:

$$y = 3^{\sqrt{t}}$$

Answers \*

☐

$$3^{\sqrt{t}} \ln 3$$

☐

$$\frac{\ln 3^{\sqrt{t}}}{2\sqrt{t}}$$

☐

$$\frac{1}{2\sqrt{t}} 3^{\sqrt{t}}$$

☒

$$\frac{\ln 3}{2\sqrt{t}} 3^{\sqrt{t}}$$

$$(3^{\sqrt{t}})' = 3^{\sqrt{t}} \cdot \ln 3 \cdot [\sqrt{t}]'$$

$$= 3^{\sqrt{t}} \cdot \ln 3 \cdot [t^{\frac{1}{2}}]'$$

$$= 3^{\sqrt{t}} \ln 3 \cdot \frac{1}{2} t^{\frac{1}{2}-1}$$

$$= 3^{\sqrt{t}} \ln 3 \cdot \frac{1}{2} t^{-\frac{1}{2}}$$

$$= \frac{\ln 3}{2\sqrt{t}} \cdot 3^{\sqrt{t}}$$

### Problem 5.

Find the derivative of the following function:

$$y = \log_4 \left( \frac{x^2}{6\sqrt{x+1}} \right)$$

[Hint: simplify the expression before taking the derivative]

Answers \*

☐

$$\frac{1}{\ln 4} \left( \frac{2}{x^2} - \frac{1}{2\sqrt{x+1}} \right)$$

☐

$$e^4 \left( \frac{6\sqrt{x+1}}{x^2} \right)$$

☒

$$\frac{1}{\ln 4} \left( \frac{2}{x} - \frac{1}{2(x+1)} \right)$$

☐

$$\frac{1}{\ln 4} \left( \frac{6\sqrt{x+1}}{x^2} \right)$$

$$\log_4(x^2) - \log_4 \left[ 6 \cdot (x+1)^{\frac{1}{2}} \right]$$

$$= 2 \log_4 x - \log_4 6 - \log_4 (x+1)^{\frac{1}{2}}$$

$$= 2 \log_4 x - \log_4 6 - \frac{1}{2} \log_4 (x+1)$$

$$\Rightarrow \left[ \log_4 \frac{x^2}{6\sqrt{x+1}} \right]' = 2 [\log_4 x]' - [\log_4 6]' - \frac{1}{2} [\log_4 (x+1)]'$$

$$= 2 \cdot \frac{1}{x \ln 4} - 0 - \frac{1}{2} \frac{(x+1)'}{(x+1) \ln 4}$$

$$= \frac{2}{x \ln 4} - \frac{1}{2(x+1) \ln 4}$$

### Problem 6.

Find the derivative of the following function.

$$y = (\pi + 3)^x$$

⚠ This is <sup>NOT</sup> a power function!

Answers \*

☐

$$\pi + \ln(3)3^x$$

$$[(\pi+3)^x]' = (\pi+3)^x \cdot \ln(\pi+3)$$

☐

$$\ln(x)(\pi + 3)^x$$

☐

$$x(\pi + 3)^{x-1}$$

☒

$$(\pi + 3)^x \ln(\pi + 3)$$

Problem 7.

Find the derivative of the following function:

$$y = x^e + e^x$$

Answers \*

☐

$$x^e + xe^{x-1}$$

☒

$$ex^{e-1} + e^x$$

☐

$$x^e + e^x$$

☐

$$ex^{e-1} + xe^{x-1}$$

$$[x^e + e^x]'$$

$$= [x^e]' + [e^x]'$$

$$= ex^{e-1} + e^x$$

Problem 8.

Use logarithmic differentiation to find the derivative of the following function.

$$y = a^{\ln x}$$

Answers \*

☐

$$\frac{a^{\ln(x)}}{x}$$

☒

$$\frac{a^{\ln(x)} \ln(a)}{x}$$

☐

$$(\ln(x) - 1)a^{\ln(x)}$$

☐

$$x^{\ln a}$$

$$[a^{\ln x}]'$$

$$= a^{\ln x} \cdot \ln a [\ln x]'$$

$$= a^{\ln x} \cdot \ln a \cdot \frac{1}{x}$$

$$= \frac{a^{\ln x} \cdot \ln a}{x}$$

**Problem 9.**

Use logarithmic differentiation to find the derivative of  $y$  with respect to the independent variable.

$$y = a^{6x+9} \quad \text{for } a > 0 \text{ and } a \neq 1.$$

*This is a composite exponential function!*

Answers \*

☐

$$(6x + 8)a^{6x+9}$$

$$[a^{6x+9}]'$$

☒

$$6 \ln(a) a^{6x+9}$$

$$= a^{6x+9} \cdot \ln a [6x+9]'$$

$$= a^{6x+9} \cdot \ln a \cdot 6$$

$$= 6 \cdot (\ln a) \cdot a^{6x+9}$$

☐

$$(6x + 9) \ln(a)$$

**Problem 10.**

Find the derivative of  $y$ .

$$y = 9e^{x^2}$$

$$[9e^{x^2}]' = 9[e^{x^2}]'$$

☐

$$9x^2 e^{x^2-1}$$

$$= 9 \cdot e^{x^2} \cdot [x^2]'$$

☒

$$18xe^{x^2}$$

$$= 9e^{x^2} \cdot (2x)$$

$$= 18xe^{x^2}$$

☐

$$\ln(9)e^{x^2} + 9xe^{x^2-1}$$

☐

$$9e^{x^2}$$

### Problem 11.

**Recycling glass.** In 2012, 34.1% of all glass containers were recycled. A beverage company used 400,000 lb of glass containers per year. After recycling, the amount of glass, in pounds, still in use after  $t$  years is given by

$$N(t) = 400000(0.341)^t$$

Find  $N'(t)$

Note that  $\ln(0.341) = -1.075873$ .

Answers \*



$$-430349.2(0.341)^t$$



$$430349.2(0.341)^t$$



$$-430349.2(0.651)^t$$



$$400000t(0.341)^{t-1}$$

$$[400000(0.341)^t]'$$

$$= 400000 [0.341^t]'$$

$$= 400000 \cdot 0.341^t \cdot \ln 0.341$$

$$= 400000 \times \ln 0.341 \cdot (0.341)^t$$

$$= 400000 \times (-1.07587) \times 0.341^t$$

$$\approx -430349.2 \times 0.341^t$$

### Problem 12.

**Agriculture.** Farmers wishing to avoid the use of nonheirloom seeds are increasingly concerned about inadvertently growing nonheirloom plants as a result of pollen drifting from nearby farms. Assuming that these farmers raise their own seeds, the fractional portion of their crop that remains free of nonheirloom plants  $t$  years later can be approximated by

$$P(t) = 1.1(0.985)^t$$

Find  $P'(t)$ .

Answers \*



$$1.1t(0.985)^t$$



$$1.0835(0.985)^t$$



$$-0.016625(0.985)^t$$



$$1.1(0.985)^t$$

$$[1.1 \times 0.985^t]'$$

$$= 1.1 [0.985^t]'$$

$$= 1.1 [0.985^t \cdot \ln 0.985]$$

$$= 1.1 \times \ln 0.985 \cdot 0.985^t$$

$$= 1.1 \times (-0.0151136) \times 0.985^t$$

$$\approx -0.016625 \times 0.985^t$$