

Answer Key to Topic #5 Assignment

Problem 1.

Find the derivative of the following exponential function of x .

$$y = e^{7-10x}$$

Answers *

☐

$$e^{-10}$$

☒

$$-10e^{7-10x}$$

☐

$$-10 \ln(7-10x)$$

☐

$$7e^{7-10x}$$

This is a composite function defined based on the natural base exponential function. need chain rule:

$$\begin{aligned} y' &= e^{7-10x} \cdot (7-10x)' \\ &= e^{7-10x} \cdot (-10) = -10e^{7-10x} \end{aligned}$$

Problem 2.

Find the derivative of the following function of x .

$$y = 8xe^x - 8e^x$$

Answers *

☐

$$8e^x$$

☒

$$8xe^x$$

☐

$$8x$$

☐

$$8xe^x + 16e^x$$

We need to use several rules to find the derivative, simplification first,

$$\begin{aligned} y' &= 8[e^x(x-1)]' = 8[(e^x)'(x-1) + e^x(x-1)'] \\ &= 8[e^x(x-1) + e^x] = 8e^x[x-1+1] \\ &= 8xe^x \end{aligned}$$

Problem 3.

Find the derivative of the following function of x .

$$y = (x^2 - 2x + 4) e^x$$

Answers *

☐

$$(x^2 + 4x + 2) e^x$$

☒

$$(x^2 + 2) e^x$$

☐

$$(2x - 2) e^x$$

☐

$$\left(\frac{x^3}{3} + 2x + 4 \right) e^x$$

$$\begin{aligned} y' &= (x^2 - 2x + 4)' e^x + (x^2 - 2x + 4) (e^x)' \\ &= (2x - 2) e^x + (x^2 - 2x + 4) e^x \\ &= e^x [2x - 2 + x^2 - 2x + 4] \\ &= e^x [x^2 + 2] \end{aligned}$$

Problem 4.

Find the derivative of the following function.

$$y = \ln(x - 3)$$

Answers *

☐

$$\frac{1}{3 - x}$$

☒

$$\frac{1}{x - 3}$$

☐

$$\frac{1}{x + 3}$$

☐

$$-\frac{1}{x + 3}$$

Need to use the chain Rule:

$$\begin{aligned} y' &= [\ln(x-3)]' = \frac{1}{x-3} \cdot (x-3)' \\ &= \frac{1}{x-3} \end{aligned}$$

Problem 5.

Find the derivative of the following function.

$$y = \frac{\ln x}{x^6}$$

$$y' = \frac{(\ln x)' \cdot x^6 - \ln x \cdot (x^6)'}{(x^6)^2}$$

Answers *

☐

$$\frac{1 - 6 \ln x}{x^{12}}$$

$$= \frac{\frac{1}{x} \cdot x^6 - \ln x \cdot 6x^5}{x^{12}} = \frac{x^5 - 6x^5 \ln x}{x^{12}}$$

☐

$$\frac{1 + 6 \ln x}{x^{12}}$$

$$= \frac{x^5(1 - 6 \ln x)}{x^{12}} = \frac{1 - 6 \ln x}{x^7}$$

☐

$$\frac{6 \ln x - 1}{x^7}$$

☒

$$\frac{1 - 6 \ln x}{x^7}$$

Problem 6.

Find the derivative of the function.

$$y = \ln 9x^2$$

$$y = \ln 9 + \ln x^2$$

Answers *

☐

$$\frac{1}{2x + 9}$$

$$= \ln 9 + 2 \ln x$$

☒

$$\frac{2}{x}$$

$$y' = [\ln 9 + 2 \ln x]'$$

$$= [\ln 9]' + 2[\ln x]'$$

☐

$$\frac{18}{x}$$

$$= 0 + 2 \cdot \frac{1}{x} = \frac{2}{x}$$

Problem 7.

Find the derivative of the function.

$$y = \ln \frac{1-x}{(x+5)^3}$$

do simplification first.

Answers *



$$\frac{2x-8}{(x+5)(1-x)}$$

$$\checkmark y = \ln \frac{1-x}{(x+5)^3} = \ln(1-x) - \ln(x+5)^3$$

$$= \ln(1-x) - 3 \ln(x+5)$$



$$\frac{(x+5)^3}{1-x}$$

$$\Rightarrow y' = [\ln(1-x)]' - 3 [\ln(x+5)]'$$



$$\frac{2x-8}{(x+5)^4}$$

$$= \frac{1}{1-x} \cdot [1-x]' - 3 \cdot \frac{1}{x+5} (x+5)'$$

$$= -\frac{1}{(1-x)^2} - \frac{3}{x+5} = -\frac{1}{(1-x)^2} - \frac{3}{x+5}$$

$$= -\frac{1-x-3(1-x)}{(1-x)^2(x+5)} = \frac{2x-8}{(1-x)^2(x+5)}$$

Problem 8.

Find the derivative of the function of

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$$y = \ln(10\theta e^{-\theta})$$

$$y = \ln(10 \cdot \theta e^{-\theta})$$

$$= \ln 10 + \ln \theta + \ln e^{-\theta}$$

$$= \ln 10 + \ln \theta - \theta \ln e$$

$$= \ln 10 + \ln \theta - \theta$$

Answers *



$$\frac{1}{\theta} - 1$$



$$\frac{1}{10\theta e^{\theta}}$$



$$e^{\theta} \left(\frac{1}{\theta} + 1 \right)$$

$$\Rightarrow y' = [\ln 10]' + [\ln \theta]' - \theta'$$

$$= 0 + \frac{1}{\theta} - 1 = \frac{1}{\theta} - 1$$



$$\ln(10e^{-\theta}(1-\theta))$$

Problem 9.

Find the derivative of

$$y = \ln(x^2)$$

$$y = \ln x^2 = 2 \ln x$$

Answers *



$$\frac{2}{x}$$

$$y' = [2 \ln x]'$$



$$\frac{1}{x^2}$$

$$= 2 [\ln x]'$$



$$2 + \frac{1}{x}$$

$$= 2 \cdot \frac{1}{x} = \frac{2}{x}.$$



$$\frac{1}{2x}$$

power
functionexp.
function**Problem 10.**

Find the derivative of

$$y = x^e + e^x$$

$$y' = [x^e + e^x]'$$

Answers *



$$x^e + e^x$$

$$= [x^e]' + [e^x]'$$



$$ex^{e-1} + xe^{x-1}$$

$$= ex^{e-1} + e^x$$



$$x^e + xe^{x-1}$$



$$ex^{e-1} + e^x$$

Problem 11.

It is reasonable for a manufacturer to expect the daily output of a new worker to be low at first, increase over time, and then level off. A manufacturer of LED flashlights determines that after t workdays, the number of flashlights produced per day by the average worker can be modeled by

$$N(t) = 80 - 70e^{-0.1t}$$

Find the derivative of $N(t)$.

Answers *

☐

$$-70e^{-0.1t}$$

☒

$$7e^{-0.1t}$$

☐

$$-0.7e^{-0.1t}$$

☐

$$7te^{-0.1t}$$

$$N'(t) = [80 - 70e^{-0.1t}]'$$

$$= (80)' - 70(e^{-0.1t})'$$

$$= 0 - 70 \cdot e^{-0.1t} \cdot (-0.1t)'$$

$$= -70e^{-0.1t} \cdot (-0.1)$$

$$= 7e^{-0.1t}$$

Problem 12.

Marginal cost. The total cost, in millions of dollars, for Greenleaf Construction is given by

$$C(x) = 100 - 50e^{-x}$$

where x is the number of houses built. Find the **marginal cost function** (i.e., the derivative of $C(x)$).

Answers *

☒

$$50e^{-x}$$

☐

$$50xe^{-x}$$

☐

$$50xe^{-x-1}$$

☐

$$-50e^{-x}$$

$$C'(x) = [100 - 50e^{-x}]'$$

$$= (100)' - 50(e^{-x})'$$

$$= 0 - 50 \cdot e^{-x} \cdot (-x)'$$

$$= -50e^{-x} \cdot (-1)$$

$$= 50e^{-x}$$

