`MAT143 Brief Calculus
Midterm Exam #3

Friday, 6/20/2023

Time: 50 minutes

Name: ____________________________________ WCU-ID ___________________
(Please print)

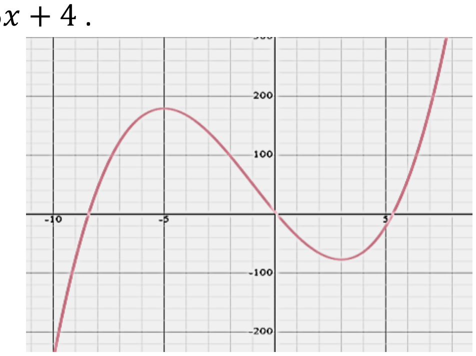
Instructions: This is a closed-book exam. No notes, books, or a computer should be used for the exam. However, you can use a calculator (TI or scientific calculator) for the exam. The first part consists of 6 multiple-choice problems and the 2nd part consists of 3 show-your-work problems.

Part I: Multiple-choice problems. (50 points)

1. Which of the following functions is increasing $(0, \infty)$?

   A) $f(x) = 13 - x^3$ $f'(x) = -3x^2 < 0$
   B) $f(x) = -4x + 1$ $f'(x) = -4 < 0$
   C) $f(x) = -4x^2$ $f'(x) = -8x < 0$
   D) $f(x) = x^2$ $f'(x) = 2x > 0$

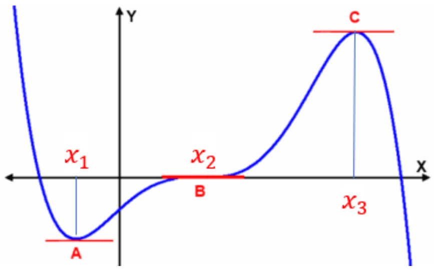
2. Let $f(x) = x^3 + 3x^2 - 45x + 4$.



   Then the local extrema of $f(x)$ are

   A). A local minimum of -179 at $x = 5$ and a local maximum of 77 at $x = -3$.
   B). A local minimum of -77 at $x = 3$ and a local maximum of 179 at $x = -5$.
   C). A local minimum of -179 at $x = -5$ and a local maximum of -77 at $x = 3$.
   D). A local minimum of -77 at $x = 3$ and a local maximum of 77 at $x = 5$.

3. Which of the following statements is true based on the given figure?



A) A, B, and C are critical points.
B) C and A are inflection points.
C) A, B, and C are relative extrema.
D) Only A and C are critical points.

4. Find the relative extrema for the following function by (1) determining the critical value(s) and (2) determining whether at these critical values, the function is a relative maximum or minimum (or possible inflection point).

$$f(x) = (x - 1)^3$$

A) x = 1, an inflection point
B) x = 1, relative minimum
C) x = 1, relative maximum
D) x = 1 is not a critical value

5. For what value of x does the function $y = x^3 - 6x$ have a local minimum?

$y' = 3x^2 - 6$, $y'' = 6x$

$\Rightarrow x^2 = 2$

$x = \pm\sqrt{2}$, $y''(\sqrt{2}) > 0 \rightarrow$ min , $y''(-\sqrt{2}) < 0 \rightarrow$ max

A) 0
B) $\sqrt{2}$
C) $-\sqrt{2}$
D) 6

6. The function $f\ (x) = x^2\ + 2x^3$ has

$f'(x) = 2x + 6x^2$

$2x(1+3x^2) \Rightarrow x = 0$

$f''(x) = 2 + 12x \Rightarrow$

$f''(0) = 2 > 0 \rightarrow$ min

A) no relative extrema
B) two relative extrema and one inflection point
C) one relative extremum and two inflection points
D) one relative extremum and one inflection point
E) three relative extrema and two inflection point

7. Find the x-coordinate(s) of the inflection point(s) of the curve of the following function

$$y = \frac{x^3}{3} - x^2$$

$y' = x^2 - 2x$

$y'' = 2x - 2 = 2(x-1)$

set $2(x-1) = 0$

$\Rightarrow x = 1$

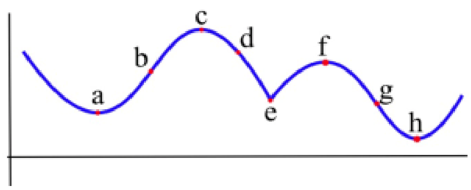
A) 1
B) 0, 2
C) 0, 1
D) 1, 2

8. Given revenue function $R(x) = -0.03x - 3x^3$, find the marginal revenue function.

$R'(x) = -0.03 - 9x^2$

A). $-0.03 - 3x^2$
B). $-0.03 - 3x^2$
C). $-0.03 - 9x^2$
D). $-0.03x + 9x^2$

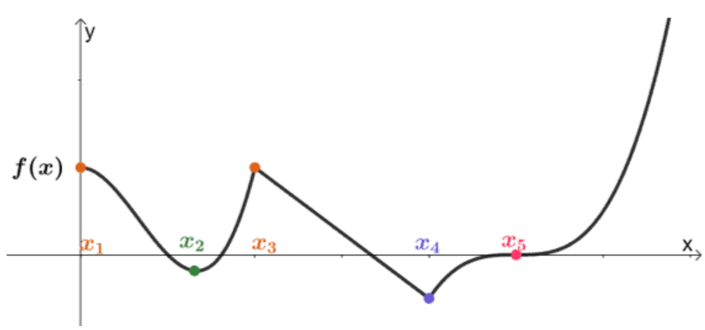
9. Assume that the graph of function f(x) is given below.



Which of the following statement is true about the sign of $f''(x)$ at $x = a$?

A) Positive because it is a relative minimum.
B). Negative because it is a relative minimum.
C). Positive because it is an absolute minimum.
D). Negative because it is an inflection point.

10. Which of the following x values can be found from $f'(x) = 0$?



A) x_2, x_3, x_4, x_5
B) x_3, x_4
C) x_2, x_5
D) x_2

Part I: Show your work to receive credit. *(50 points).*

Problem 1. (**20 points**)
Using the following steps to find the absolute extreme values of the function g(x) $=x^3 - 3$ on the interval [-2, 2].

1. Find all critical value(s) of $g(x)$.

$g'(x) = (x^3-3)' = 3x^2$

Set $g'(x) = 0$, $3x^2 = 0 \Rightarrow x = 0$

$\Rightarrow$ There is only one critical value.

2. Identify relative maxima and minima.

$g''(x) = 6x$

$g''(0) = 6(0) = 0$, $x = 0$ is neither local maximum nor minimum.

3. Find the inflection point.

set $g''(x) = 0 \rightarrow 6x = 0$

$\Rightarrow x = 0$, $g(0) = 0^3 - 3 \times 0 = 0$

$\Rightarrow$ the inflection point is $(0, -3)$.

4. Find the absolute maximum and minimum of g(x) on [-2, 2]

$g(-2) = (-2)^3 - 3 = -11$

$g(2) = (2)^3 - 3 = 5$

The absolute maximum is 5 when $x = 2$.
The absolute minimum is -11 when $x = -2$.

Problem 2. (15 points) The cost of materials, C, in dollars, to produce x dozen birthday cakes at Yum's Bakery is given by

$$C(x) = x^2 + 40x + 200$$

1. Find the average cost function $\overline{C(x)} = \frac{C(x)}{x}$.

$\bar{C}(x) = \frac{x^2 + 40x + 200}{x} = x + 40 + \frac{200}{x} = x + 40 + 200x^{-1}$.

2. Find the minimum average cost based on $\overline{C(x)}$ obtained in the above part 1.

$[\bar{C}(x)]' = [x + 40 + 200x^{-1}]' = 1 + 0 + 200(-1) \cdot x^{-2} = \frac{x^2 - 200}{x^2}$

Since $x > 0 \Rightarrow [\bar{C}(x)]' = 0$ has only one root: $x = \sqrt{200} = 10\sqrt{2}$

3. Find the marginal cost function $C'(x)$

$C'(x) = [x^2 + 40x + 200]' = 2x + 40$

Problem 3. (5 points). Find the inflection point of $f(x) = \frac{x^3}{3} + 5x^2 + 24$.

$f'(x) = x^2 + 10x$, $f''(x) = 2x + 10$

$f''(x) = 0 \Rightarrow 2x + 10 = 0 \Rightarrow x = -5$

$f(-5) = \frac{(-5)^3}{3} + 5 \times (-5)^2 + 24$

$= -\frac{125}{3} + 125 + 24 = \frac{2}{3} \times 125 + 24 = \frac{250}{3} + 24 = \frac{250 + 72}{3}$

$= \frac{322}{3} \approx 107.3$

$\Rightarrow$ inflection point is $(-5, 107.3)$.