Problem 1.

Find the average rate of change of the following function over the given interval.

$y = x^2 + x$, [1, 9]

Solution :

Answers *

- 10
- 88/9
- 45/4
- ✓ 11

$$\frac{f(9)-f(1)}{9-1} = \frac{(9^2+9)-(1^2+1)}{8}$$

$$= \frac{81+9-2}{8} = \frac{88}{8} = 11$$

Problem2.

Find the average rate of change of the function over the given interval.

f(x) = 3/(x-2), [4, 7]

Answers *

- 2
- 7
- 1/3
- ✓ 3/10

$$\frac{f(7)-f(4)}{7-4} = \frac{\left(\frac{3}{7-2}\right)-\left(\frac{3}{4-2}\right)}{3}$$

$$= \frac{\frac{3}{5}-\frac{3}{2}}{3} = \frac{\frac{3\times 2 - 3\times 5}{10}}{3}$$

$$= \frac{\frac{6-15}{10}}{3} = \frac{-9}{30} = -\frac{3}{10}$$

Problem 3.

Find the slope of the tangent line of the curve at given point P.

$y = x^2 + 5x$, P(4, 36)

Answers *

- 1/20
- ✓ 13
- -4/25
- -39

The slope of a tangent line is the Instantaneous rate of change at $x=4$.

$$\lim_{h\to 0} \frac{(4+h)^2 + 5(4+h) - [4^2+5\times 4]}{h}$$

$$= \lim_{h\to 0} \frac{16 + 8h + h^2 + 20 + 5h - 36}{h}$$

$$= \lim_{h\to 0} \frac{13h + h^2}{h} = \lim_{h\to 0} (13+h) = 13$$

Problem 4.

Evaluate

$$\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

for the given x_0 and function f(x):

$f(x) = 5x^2$ for $x_0 = 3$.

Answers *

- ✓ 30
- 15
- does not exist
- 45

$$\lim_{h\to 0} \frac{5(x_0+h)^2 - 5x_0^2}{h} = \lim_{h\to 0} \frac{5(x_0^2+2x_0h+h^2) - 5x_0^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{5x_0^2} + 10x_0h + 5h^2 - \cancel{5x_0^2}}{h}$$

$$= \lim_{h\to 0} (10x_0 + 5h) = 10x_0$$

$$= 10 \times 3 = 30$$

(since $x_0 = 3$).

Problem 5

Find limit

$$\lim_{x \to 1^-} \sqrt{\frac{x+1}{x+5}}$$

solution: using substitution, we have

$$\lim_{x \to 1} \sqrt{\frac{x+1}{x+5}} = \sqrt{\frac{1+1}{1+5}} = \sqrt{\frac{2}{6}} = \sqrt{\frac{1}{3}}$$

$$= \frac{1}{\sqrt{3}}$$

Answers *

- 0
- does not exist
- 1/3
- ✓ $1/\sqrt{3}$

Problem 6

Find the intervals on which the function is continuous.

$$f(x) = \frac{2}{x+5} - 3x$$

Solution:

Since $\frac{2}{x+5}$ is undefined if $x+5=0$

Therefore, $f(x)$ is discontinous at $x=-5$.

Answers *

- discontinuous only when x =5
- discontinuous everywhere
- ✓ discontinuous only when x = -5
- discontinuous only when x = -8

Problem 7

Find the value of x on which the function is discontinuous.

$$f(x) = \frac{2}{x^2 - 9}$$

Answers *

- discontinuous only when x = 9
- discontinuous only when x = -3
- discontinuous only when x = -9 or x = 9
- ✓ discontinuous when x = 3 or x = -3

Solution:

f(x) is undefined when $x^2 - 9 = 0$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$.

Problem 8

Find limit

$$\lim_{x \to \infty} \frac{6}{7 - (5/x^2)}$$

Answers *

- 3
- 6
- ✓ 6/7
- negative infinity

Solution: by substitution method

$$\lim_{x \to \infty} \frac{6}{7 - \frac{5}{x^2}} = \frac{6}{7 - \frac{5}{\infty^2}} = \frac{6}{7 - 0} = \frac{6}{7}.$$

Problem 9

Find the limit

$$\lim_{x\to\infty} \frac{-14x^2 - 3x + 17}{-6x^2 + 8x + 13}$$

Answers *

- 1
- 17/13
- infinity
- 7/3 ✓

$$\frac{(-14x^2 - 3x + 17)/x^2}{(-6x^2 + 8x + 13)/x^2} = \frac{-14 - \frac{3}{x} + \frac{17}{x^2}}{-6 + \frac{8}{x} + \frac{13}{x^2}}$$

$$\frac{x \to \infty}{\quad} \quad \frac{-14 - \frac{3}{\infty} + \frac{17}{\infty^2}}{-6 + \frac{8}{\infty} + \frac{13}{\infty^2}} = \frac{-14}{-6} = \frac{7}{3}$$

Problem 10

Divide numerator and denominator by the highest power of x in the denominator to find the limit.

$$\lim_{x\to\infty} \sqrt{\frac{25x^2}{3 + 9x^2}}$$

Answers *

- 25/3
- 5/3 ✓
- 25/9
- does not exist

Solution: direct substitution will end up with an indeterminate form $\frac{\infty}{\infty}$

We divide both numerator and the denominator by x^2 (in the square root sign).

$$\lim_{x\to\infty} \sqrt{\frac{25x^2}{3+9x^2}} = \lim_{x\to\infty} \sqrt{\frac{\frac{25x^2}{x^2}}{\frac{3+9x^2}{x^2}}}$$

$$= \lim_{x \to \infty} \sqrt{\frac{25}{\frac{3}{x^2} + 9}} = \sqrt{\frac{25}{\frac{3}{\infty} + 9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}.$$

Problem 11

Find the limit

$$\lim_{x \to \sqrt[3]{3}} \left(\frac{x^2}{3} - \frac{1}{x} \right)$$

Solution:

We do direct substitution

Answers *

- negative infinity
- 0 ✓
- 2/3
- positive infinity

$$\lim_{x \to \sqrt[3]{3}} \left(\frac{x^2}{3} - \frac{1}{x} \right) = \frac{(\sqrt[3]{3})^2}{3} - \frac{1}{\sqrt[3]{3}} = 3^{\frac{2}{3} - 1} - 3^{-\frac{1}{3}} = 0.$$

Problem 12

Find the derivative of function

$$f(x) = 2023$$

Answers *

- [ ] 2022
- [ ] 2023
- [x] 0
- [ ] ∞

Solution :

$$f'(x) = (2023)' = 0$$

Problem 13

Find the derivative of function

$$f(x) = x^2 + 6x$$

Answers *

- [x] $2x + 6$
- [ ] $x^2 + 6$
- [ ] $2x$
- [ ] $2x^2 + 6$

Solution :

$$f'(x) = (x^2 + 6x)'$$

$$= (x^2)' + (6x)'$$

$$= 2x + 6$$