Topic #4: Natural-base Exponential and Logarithmic Functions and Equations

1. Express $3\ln a + \ln b - \ln c$ as a single logarithm.

A) $\ln\left(\frac{a^3 b}{c}\right)$
B) $\ln(a^3 + b - c)$
C) $3\ln\left(\frac{ab}{c}\right)$
D) $\ln\left(\frac{3ab}{c}\right)$

$3\ln a + \ln b - \ln c = \ln a^3 + \ln b - \ln c$
$= \ln(a^3 b) - \ln c = \ln\left(\frac{a^3 b}{c}\right)$

Answer: A

2. Solve for x: $\ln x = \frac{3}{2}$

A) $e^{3/2}$
B) $\ln 3/2$
C) $e^3 - e^2$
D) e^{3-2}

$\ln x = \frac{3}{2}\ln e = \ln e^{\frac{3}{2}}$
$\Rightarrow x = e^{3/2}$.

Answer: A

3. Solve for x: $\ln(x+2) - \ln 5 = \ln x$.

A) $-8/5$
B) $1/2$
C) $5/2$
D) $4/3$

$\ln(x+2) - \ln x = \ln 5$.
$\Rightarrow \ln\frac{x+2}{x} = \ln 5 \Rightarrow \frac{x+2}{x} = 5$
$\Rightarrow x + 2 = 5x \Rightarrow x = \frac{1}{2}$

Answer B

4. What is the inverse of $y = e^x$

A) $y = e\ln 2$
B) $y = e^{-x}$
C) $y = \ln x$
D) $y = 1/e^x$

by definition
$f^{-1}(x) = \ln x$

Answer C

5. Solve for x: $e^x = 4$.

A) $\ln(4/e)$
B) $e/4$
C) 2
D) $\ln 4$

taking natural base log on both sides of the equation, we have

$\ln e^x = \ln 4 \Rightarrow x \ln e = \ln 4$

$\Rightarrow x = \ln 4$

Answer D

6. The equation $2 = \ln(y + 1)$ can also be written as __.

A) $y = \frac{2}{\ln(x+1)} - 2$
B) $y = e^2 - 1$
C) $y = 2(x+1) - 1$
D) $y = \ln 2^{x+1} - 1$

$2 = \ln(y+1) \Rightarrow 2\ln e = \ln(y+1)$

$\Rightarrow \ln e^2 = \ln(y+1)$

$\Rightarrow e^2 = y+1 \Rightarrow y = e^2 - 1$

Answer B

7. Determine an expression for x if $\ln x = \ln a - 2\ln b$.

A) $\frac{a}{b^2}$
B) $\frac{a}{2b}$
C) $\frac{a}{\sqrt{b}}$
D) $\frac{a}{2\sqrt{b}}$

$\ln x = \ln a - 2\ln b = \ln a - \ln b^2 = \ln \frac{a}{b^2}$

$\Rightarrow x = \frac{a}{b^2}$

Answer A

8. Solve for x: $\ln x^2 = \ln \frac{1}{9}$

A) ± 3
B) $\pm \frac{1}{3}$
C) $\frac{1}{3}$
D) 3

$\Rightarrow x^2 = \frac{1}{9} = \left(\pm\frac{1}{3}\right)^2$

$\Rightarrow x = \pm\frac{1}{3}$

Answer B

9. If $5000 is invested in a bank where the amount is compounded **continuously** at a rate of 7% per year, then what is the resultant amount after 3 years?

A) $5001.6
B) $5117.5
C) $6168.4
D) $6164.6

Answer C

$A = 5000e^{0.07 \times 3} = 5000e^{0.21}$

≈ 6168.4

10. For an initial balance of $100 and an annual interest rate of 5% compounded **continuously** over 10 years, calculate the closing balance rounded to the nearest penny.

A) $101.64
B) $117.51
C) $168.47
D) $164.87

Answer D

$A = 100 \times e^{0.05 \times 10} = 100e^{0.5}$

≈ 164.87