Daily Quiz #13

1. Let $u = 3x^2 + x$, then

   A. $du = 6xdx + x,$
   B. $du = 6xdx + 1,$
   C. $du = (6x + 1)dx$
   D. $du = dx$

$$du = d(3x^2 + x) = d(3x^2) + dx$$
$$= 3dx^2 + dx = 3 \cdot 2x\,dx + dx$$
$$= 6x\,dx + dx = (6x+1)\,dx$$

   Answer: C

2. Let $u = x^3 - 1$, then
   A. $du = 3x^2dx - 1,$
   B. $du = (3x^2 - 1)dx,$
   C. $du = 3x^2dx$
   D. $du = dx$

$$du = d(x^3 - 1) = dx^3 - d1$$
$$= 3x^2\,dx - 0 = 3x^2\,dx$$

   Answer: C

3. Find the indefinite integral $\int \sqrt{x+1}\,dx$

   $A. \sqrt{3(x+1)/2}$

   B. $\left(\frac{2}{3}\right)(x+1)\sqrt{x+1} + C$

   C. $(\frac{2}{3})x\sqrt{x} + C$

   D. $\left(\frac{3}{2}\right)\sqrt{x}\sqrt{x+1} + C$

$$\int \sqrt{x+1}\,dx = \int \underbrace{(x+1)}_{u}^{\frac{1}{2}}\,dx$$

$u = x+1$

$du = d(x+1) = dx$

$$\int u^{\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$
$$= \frac{u^{3/2}}{3/2} + C = \frac{2}{3}(x+1)^{\frac{3}{2}} + C$$

   Answer: B.

4. $\int \frac{2t}{(3+t^2)^3}\,dt = ?$

   A. $\frac{2}{(3+t^2)^3}$
   B. $\frac{1}{2(3+t^2)^3} + C$
   C. $\frac{2t}{(3+t^2)^3} + C$
   D. $-\frac{1}{2(3+t^2)^2} + C$

$$\int \frac{2t}{(3+t^2)^3}\,dt$$

$u = 3+t^2$

$du = d(3+t^2)$

$= dt^2 = 2t\,dt$

$$\int \frac{1}{u^3}\,2t\,dt = \int \frac{1}{u^3}\,du$$
$$= \int u^{-3}\,du = \frac{u^{-3+1}}{-3+1} + C = -\frac{u^{-2}}{2} + C = -\frac{1}{2u^2} + C$$
$$= -\frac{1}{2(3+t^2)^2} + C$$

   Answer: D

5. $\int_{-1}^{0} \frac{2t}{(1+t^2)^3}\,dt = ?$

$$F(t) = \int \frac{2t}{\underbrace{(1+t^2)}_{u}{}^3}\,dt \qquad \frac{u = 1+t^2}{du = d(1+t^2) = 2t\,dt} \qquad \int \frac{1}{u^3}\underbrace{2t\,dt}_{du}$$

$$= \int u^{-3}\,du = \frac{u^{-3+1}}{-3+1} = -\frac{1}{2u^2} = -\frac{1}{2(1+t^2)^2}$$

$$\int_{-1}^{0} \frac{2t}{(1+t^2)^3}\,dt = F(0) - F(-1) = -\frac{1}{2(1+0)^2} - \left[-\frac{1}{2[1+(-1)^2]}\right]$$

$$= -\frac{1}{2} + \frac{1}{2\times 2^2} = -\frac{1}{2} + \frac{1}{8} = -\frac{3}{8}$$

C. $\frac{1}{2}$

D. $\frac{3}{8}$

C. $-\frac{1}{2}$

D. $-\frac{3}{8}$

Answer: D.

6. $\int_0^1 \sqrt{x+1}\,dx = ?$

$$F(x) = \int \sqrt{x+1}\,dx = \int \underbrace{(x+1)}_{u}{}^{\frac{1}{2}}\,dx$$

$$\frac{u = x+1}{du = d(x+1) = dx} \qquad \int u^{\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{2}{3}u^{\frac{3}{2}}$$

$$= \frac{2}{3}(x+1)^{3/2}$$

$$\int_0^1 \sqrt{x+1}\,dx = F(1) - F(0) = \frac{2}{3}(1+1)^{\frac{3}{2}} - \frac{2}{3}(0+1)^{3/2} = \frac{2}{3}\cdot 2^{\frac{3}{2}} - \frac{2}{3}\cdot 1$$

$$= \frac{2}{3}(2\sqrt{2} - 1) = \frac{4}{3}\sqrt{2} - \frac{2}{3}.$$

A. $\frac{4}{3}\sqrt{2} - \frac{2}{3}$

B. $\frac{4}{3}\sqrt{2}$

C. $2\sqrt{2} - 1$

D. $3\sqrt{2} - 3$

Answer: A.

7. $\int \frac{6x^2}{\sqrt{16+3x^3}}\,dx = ?$

$$\int \frac{6x^2}{\sqrt{\underbrace{16+3x^3}_{u}}}\,dx \qquad \frac{u = 16+3x^3}{du = d(16+3x^3)} \qquad \int \frac{6}{\sqrt{u}}\cdot x^2\,dx = \int \frac{6}{\sqrt{u}}\cdot\frac{1}{9}\cdot\underbrace{9x^2\,dx}_{du}$$

$$= d(16) + 3\,dx^3$$

$$= 3\cdot 3x^2\,dx = 9x^2\,dx$$

$$= \frac{6}{9}\int \frac{1}{\sqrt{u}}\,du = \frac{2}{3}\int u^{-\frac{1}{2}}\,du = \frac{2}{3}\cdot\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{2}{3}\cdot\frac{u^{\frac{1}{2}}}{1/2} + C = \frac{4}{3}\sqrt{u} + C = \frac{4}{3}\sqrt{16+3x^2} + C$$

A. $\sqrt{16+3x^3} + C$

B. $4\sqrt{16+3x^3} + C$

C. $\frac{4}{3}\sqrt{16+3x^3} + C$

D. $\frac{3}{4}\sqrt{16+3x^3} + C$

Answer: C

8. $\int \frac{9-\sqrt{x}}{\sqrt{x}}\,dx = ?$

$$\int \frac{9-\sqrt{x}}{\sqrt{x}}\,dx = \int \left(\frac{9}{\sqrt{x}} - 1\right)dx = \int \frac{9}{\sqrt{x}}\,dx - \int 1\,dx$$

$$= 9\int x^{-\frac{1}{2}}\,dx - x = 9\cdot\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - x + C$$

$$= 18\sqrt{x} - x + C$$

A. $18\sqrt{x} - x + C$

B. $\frac{\sqrt{x}}{9} - x + C$

C. $\frac{9-x}{x} + 1$

D. $\frac{\sqrt{x}}{9} - 1 + C$

Answer: A

9. $\int \frac{5}{5x+7} dx = ?$ for $x \neq 7/5$.

$$\int \frac{5}{\underbrace{5x+7}_{u}} dx \qquad \frac{u = 5x+7}{\begin{aligned} du &= d(5x+7) \\ &= d(5x) + d(7) \\ &= 5dx \end{aligned}} \qquad \int \frac{1}{u} 5dx$$

$$= \int \frac{1}{u} du = \ln|u| + C = \ln|5x+7| + C.$$

A. $\ln|5x+7| + C$
B. $5\ln|5x+7| + C$
C. $\frac{1}{5}\ln|5x+7| + C$
D. $\ln|5x| + 7 + C$

Answer: A

10. $\int \frac{(\ln x)^7}{x} dx = ?,\ for\ x > 0$

$$\int \frac{[\overbrace{\ln x}^{u}]^7}{x} dx \qquad \frac{u = \ln x}{du = \frac{1}{x} dx} \qquad \int u^7 \cdot \frac{1}{x} dx$$

$$= \int u^7 du = \frac{u^{7+1}}{7+1} + C = \frac{[\ln x]^8}{8} + C$$

A. $(\ln x)^8 + C$
B. $\frac{1}{8}(\ln x)^8 + C$
C. $\frac{1}{7}(\ln x)^7 + C$
D. $(\ln x)^7 + C$

Answer: B.