Week #12 Quiz

Problem 1.

Find the definite integral $\int_{-2}^{5}(x+1)dx$

A). -17.5

B). 0

C). 27/2

D). 35/2

$F(x) = \int (x+1)dx = \int x\,dx + \int 1\,dx = \frac{x^{1+1}}{1+1} + x = \frac{x^2}{2} + x$

Ans: D

$\int_{-2}^{5}(x+1)\,dx = F(x)\Big|_2^5 = F(5) - F(2)$

$= \left(\frac{5^2}{2}+5\right) - \left(\frac{2^2}{2}+2\right) = \frac{5^2-2^2}{2} + 5-2 = \frac{21}{2} + 3 = \frac{27}{2}$

Problem 2.

Find the definite integral $\int_{1}^{3}(x^2+1)dx$

A). 32/3

B). 8

C). 2

D). 26

$F(x) = \int (x^2+1)dx = \int x^2 dx + \int 1\,dx = \frac{x^{2+1}}{2+1} + x = \frac{x^3}{3} + x$

Ans: A

$\Rightarrow \int_1^3 (x^2+1)\,dx = F(x)\Big|_1^3 = \left(\frac{3^3}{3}+3\right) - \left(\frac{1^3}{3}+1\right) = \frac{3^3-1}{3} + 2 = \frac{26}{3} + 2 = \frac{32}{3}$

Problem 3.

Find the integral $\int x\sqrt{x}d\,x$

A). $\frac{5}{2}x^{5/2}$

B). $\frac{2}{5}x^{1/2}$

C). $\frac{2}{5}x^{5/2}$

D). $\frac{5}{2}x^{1/2}$

$\Rightarrow \int x \cdot x^{\frac{1}{2}} dx = \int x^{1+\frac{1}{2}} dx = \int x^{\frac{3}{2}} dx$

$= \frac{x^{3/2+1}}{3/2+1} + C = \frac{x^{5/2}}{5/2} + C = \frac{2}{5} x^{\frac{5}{2}} + C$

Ans: C

Problem 4.

Find one antiderivative of $f(x) = x^2 - x + 2$

A). $2x - 1 + C$

B). $\frac{x^3}{3} - \frac{x^2}{2} + 2x + C$

C). $x^3 - x^2 + 2x$

D). $\frac{x^3}{3} - \frac{x^2}{2} + 2 + C$

Ans: B

$\int (x^2 - x + 2)dx$

$= \int x^2 dx - \int x\,dx + 2\int 1\,dx$

$= \frac{x^{2+1}}{2+1} - \frac{x^{1+1}}{1+1} + 2x + C$

$= \frac{x^3}{3} - \frac{x^2}{2} + 2x + C$

Problem 5.

Find the antiderivative of $f(x) = 1 - e^x$

$\int (1-e^x)\,dx$

A). $1 - e^x$ + C

B). $1 - e^{-x}$+ C

C). $x - e^x$+ C

D). $x + e^x$+ C

$= \int 1\,dx - \int e^x\,dx = x - e^x + C$

Ans C.

Problem 6.

Find the antiderivative of $f(x) = (1+x)/x$

A). $1 + \ln|x| + c$

B). $\ln|x| + c$

C). $1 + \frac{1}{x^2} + C$

D). $x + \ln|x| + c$

$\int \frac{1+x}{x}\,dx = \int \left(\frac{1}{x} + \frac{x}{x}\right) dx = \int \frac{1}{x}\,dx + \int 1\,dx$

$= \ln|x| + x + C$

Ans: D

Problem 7.

Compute the definite integral $\int_1^3 \frac{3}{x^3}\,dx$

$F(x) = \int \frac{3}{x^2}\,dx = 3\int x^{-3}\,dx$

$= 3 \cdot \frac{x^{-3+1}}{-3+1} = 3 \cdot \frac{x^{-2}}{-2} = -\frac{3}{2x^2}$

A). 1/2

B). 1/3

C). 3

D). 4/3

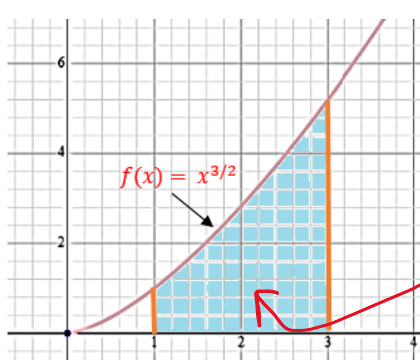
$\int_1^3 \frac{3}{x^3}\,dx = F(x)\Big|_1^3 = \left(-\frac{3}{2\cdot 3^2}\right) - \left(-\frac{3}{2\cdot 1^2}\right) = -\frac{3}{18} + \frac{3}{2}$

Ans: D

$= -\frac{1}{6} + \frac{3}{2} = \frac{-1}{6} + \frac{9}{6} = \frac{8}{6} = \frac{4}{3}$

Problem 8.

Which of the intergral gives the area of the shaded region in the following figure.



$\int_1^3 x^{\frac{3}{2}}\,dx$

A). $\int_1^3 \frac{3}{x^2} dx$
B). $\int_1^3 x^{3/2} dx$ ✓
C). $\int x\sqrt{x} d x$
D). $\int \frac{3}{x^2} d x$

Ans: B

Problem 9.

$F(x) = \int 5\sqrt{x}\, dx = 5\int x^{\frac{1}{2}} dx$

$= 5 \cdot \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = 5 \cdot \frac{x^{3/2}}{3/2} = \frac{10}{3} x^{\frac{3}{2}}$

Find the integral $\int_0^9 5\sqrt{x}dx$
A). 135
B). 90
C). 405/2
D). 45/2

$\int_0^9 5\sqrt{x}\, dx = F(x)\Big|_0^9 = F(9) - F(0)$

$= \frac{10}{3} 9^{\frac{3}{2}} - \frac{10}{3} \times 0 = \frac{10}{3} \cdot 3^{2 \times \frac{3}{2}} = \frac{10}{3} \times 3^3$

Ans: B

$= 10 \times 3^2 = 90.$

Problem 10.

Find the definite integral $\int_0^2 5x^4 dx$.

$F(x) = \int 5x^4 dx = 5 \cdot \int x^4 dx = 5 \cdot \frac{x^{4+1}}{4+1} = 5 \cdot \frac{x^5}{5} = x^5$

A). 120
B). 32
C). 80
D). 160

$\int_0^2 5x^2 dx = F(2) - F(0) = 2^5 - 0 = 32$

Ans: B.

Problem 11.
Find definite integral $\int_2^2 5e^x dx$

A). 5
B). $5e^2$
C). 0
D). $5e^0$

Since the lower and the upper integral limits are equal $\Rightarrow$ the resulting integral is 0.

Ans: C.

Problem 12

Which of the following is correct?

A). $\int_1^3 e^x dx = \int_3^1 e^x dx$

B). $\int_1^3 e^x dx = -\int_3^1 e^x dx$ ✓ by the property.

C). $\int_3^1 e^x dx = \int_3^1 e^x dx + \int_2^1 e^x dx$

D). $\int_3^1 e^x dx = \int_3^1 e^x dx - \int_2^1 e^x dx$

Ans: B