Midterm Exam One

MAT325 Numerical Analysis

3/10/2023

Instructions

General Information: This is an open book and open note exam. You can use the textbook, lecture notes, R code/function in lecture/lab notes, and internet resources for the exam. However, you must complete the exam independently. No collaboration is allowed.

Specific Requirements:

1. Use RMarkdown or other typesetting software programs to prepare your solution.
2. The code must be commented on and included in the solution. If you use a typesetting program other than RMarkdown, please put the code in the appendix.
3. Please upload a copy of your solution in PDF format and a copy source file to the D2L drop box.
4. Please feel free to use/modify the code from the lectures and those you developed in the assignments.

Problem Set

Problem 1: Finding the root of a non-linear equation

Consider the function $f(x) = \tan(\sin(x^3))$ over interval $[0, \sqrt{\frac{5}{2}}]$.

1. Choose 500 equally-spaced base points in the interval $[0, \sqrt{\frac{5}{2}}]$ and plot the curve of the function.
2. Find all roots of equation $\tan(\sin x^3) = 0$ mathematically over interval $\left[0, \sqrt{\frac{5}{2}}\right]$.
3. Use the Newton method to find these roots (use $TOL = 10^{-6}$) and compare these approximate roots with their corresponding true roots.

Problem 2. Approximation with interpolated Polynomials

The Laguerre polynomial that has applications in differential equations and physics (particularly in quantum mechanics) that is is defined by

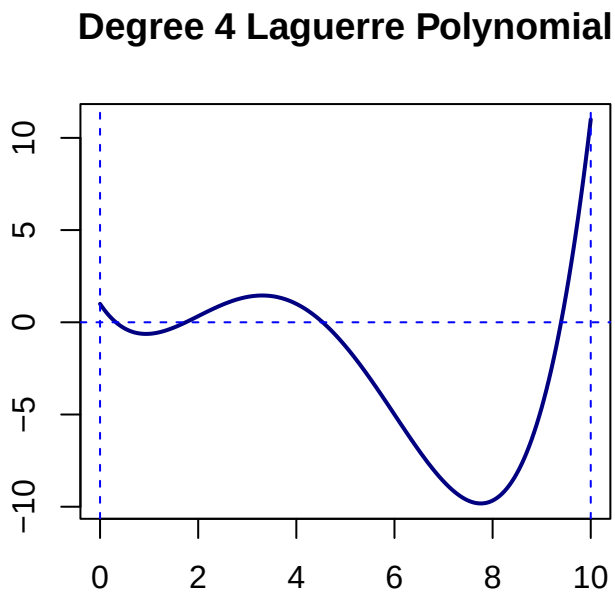
$$L_n(x) = \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{k!} x^k$$

In this problem, we only consider the degree 4 Laguerre polynomial

$$L_4(x) = \frac{x^4}{24} - \frac{2x^3}{3} + 3x^2 - 4x + 1$$

over interval $[0, 10]$.

```
xx= seq(0, 10, length = 200)
yy = xx^4/24 - (2/3)*xx^3 + 3*xx^2 - 4*xx + 1
plot(xx, yy, type = "l", lwd = 2, col = "navy",
     xlab = "", ylab = "",
     main = "Degree 4 Laguerre Polynomial")
abline(v=c(0, 10), h = 0, lty = 2, col = "blue")
```



The purpose of this problem is to assess the performance of the following three different approximated polynomials:

1. The degree 4 Taylor polynomial expanded at $x = 7.75$.
2. The Newton interpolated polynomial using unequally-spaced nodes $x = 0.3, 0.95, 3.3, 7.75, 9.4$

3. The Newton interpolated polynomial using equally-spaced nodes $x = 1, 3, 5, 7, 9$

Evaluate the above three approximated polynomials and the original Laguerre polynomial at $x = 3$ and compare the three approximated values with the true value obtained from the original Laguerre polynomial. Which polynomial gives the best approximation?

Problem 3. Application - Approximating Unknown Function and Making Prediction

The table below gives the actual thermal conductivity data for the element mercury. The objective is to find the analytic expression that approximates the relationship between temperature and pressure.

Temperature (Fahrenheit), T	220	230	240	250	260	270	280	290
Pressure (Pound), P	17.19	20.78	24.97	29.82	35.42	41.85	49.18	57.53

Use the Newton interpolation and all given data points to construct a polynomial of degree 8 and use this interpolating polynomial to **predict** the corresponding pressures for temperatures $T = 235, 255, 279, 295$.