Comments on Fixed-point Methods

Cheng Peng

Lecture Note for MAT325 Numerical Analysis

Contents

This short note emphasizes some facts about the fixed-point method.

The idea of the fixed point method with an initial guess x_0 chosen computes a sequence

$$x_{n+1} = g(x_n), \quad n \geq 0$$

in the hope that $x_n \to \alpha$.

- For a given non-linear equation $f(x) = 0$, there different ways to formulate the fixed-point problems. That is, we can define different $g(x) = x$ based on $f(x) = 0$.

Example 1: Solve $f(x) = 5x^3 - 7x^2 - 40x + 100 = 0$. We can define different fixed-point problems:

(1). $g(x) = (5x^3 - 7x^2 + 100)/40 = x$

(2). $g(x) = \sqrt{(5x^3 - 40x + 100)/7} = x$

(3). $g(x) = \sqrt[3]{(7x^2 + 40x - 100)/5} = x$

(4). $g(x) = -\sqrt[3]{(-7x^2 - 40x + 100)/5} = x$

We write more fixed-point form.

- Whether the sequence generated from the fixed-point algorithm is dependent on the form of $g(x)$ and the starting value x_0.

Example 2 (continuation of example 1): Formulation (4) generates a convergent sequence (with $x_0 = -3$) and the rest of the listed fix-point formulations cannot generate a convergence sequence.

- In general, showing the convergence of the sequence (x_n) obtained from the iterative process is not easy. The contraction mapping theorem can be used to test whether the sequence is convergent.