

Formulas for Differentiation

- 1. Power Rule.** For any real number k , $\frac{d}{dx} x^k = kx^{k-1}$.
- 2. Derivative of a Constant Function.** If $F(x) = c$, then $F'(x) = 0$.
- 3. Derivative of a Constant Times a Function.** If $F(x) = cf(x)$,
then $F'(x) = cf'(x)$.
- 4. Derivative of a Sum.** If $F(x) = f(x) + g(x)$, then
 $F'(x) = f'(x) + g'(x)$.
- 5. Derivative of a Difference.** If $F(x) = f(x) - g(x)$, then
 $F'(x) = f'(x) - g'(x)$.
- 6. Derivative of a Product.** If $F(x) = f(x)g(x)$, then
 $F'(x) = f(x)g'(x) + g(x)f'(x)$.
- 7. Derivative of a Quotient.** If $F(x) = \frac{f(x)}{g(x)}$, then
 $F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$.
- 8. Extended Power Rule.** If $F(x) = [g(x)]^k$, then
 $F'(x) = k[g(x)]^{k-1}g'(x)$.
- 9. Chain Rule.** If $F(x) = f[g(x)]$, then $F'(x) = f'[g(x)]g'(x)$.
Or, if $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Formulas for Differentiation

(continued)

10. $\frac{d}{dx} e^x = e^x$

11. $\frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot f'(x)$

12. $\frac{d}{dx} \ln x = \frac{1}{x}, \quad x > 0$

13. $\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}, \quad f(x) > 0$

14. $\frac{d}{dx} \ln |x| = \frac{1}{x}, \quad x \neq 0$

15. $\frac{d}{dx} \ln |f(x)| = \frac{f'(x)}{f(x)}, \quad f(x) \neq 0$

16. $\frac{d}{dx} a^x = (\ln a)a^x$

17. $\frac{d}{dx} \log_a x = \frac{1}{\ln a} \cdot \frac{1}{x}, \quad x > 0$

18. $\frac{d}{dx} \log_a |x| = \frac{1}{\ln a} \cdot \frac{1}{x}, \quad x \neq 0$

Table of Integrals

- Antiderivative of a constant: $\int k \, dx = kx + C$
- Antiderivative of a constant times a function: $\int k \cdot f(x) \, dx = k \int f(x) \, dx$
- Sum/difference property of antidifferentiation:
 $\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
- $\int \frac{dx}{x} = \ln x + C, \quad x > 0$
- $\int u \, dv = uv - \int v \, du$
- $\int e^x \, dx = e^x + C$
- $\int e^{ax} \, dx = \frac{1}{a} \cdot e^{ax} + C$
- $\int x e^{ax} \, dx = \frac{1}{a^2} \cdot e^{ax}(ax - 1) + C$
- $\int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx + C$
- $\int \ln x \, dx = x \ln x - x + C$
- $\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx + C, \quad n \neq -1$
- $\int x^n \ln x \, dx = x^{n+1} \left[\frac{\ln x}{n+1} - \frac{1}{(n+1)^2} \right] + C, \quad n \neq -1$
- $\int a^x \, dx = \frac{a^x}{\ln a} + C, \quad a > 0, a \neq 1$
- $\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln |x + \sqrt{x^2 + a^2}| + C$
- $\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln |x + \sqrt{x^2 - a^2}| + C$
- $\int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$
- $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$
- $\int \frac{1}{x\sqrt{a^2 + x^2}} \, dx = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$
- $\int \frac{1}{x\sqrt{a^2 - x^2}} \, dx = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$
- $\int \frac{x}{a + bx} \, dx = \frac{a}{b^2} + \frac{x}{b} - \frac{a}{b^2} \ln |a + bx| + C$
- $\int \frac{x}{(a + bx)^2} \, dx = \frac{a}{b^2(a + bx)} + \frac{1}{b^2} \ln |a + bx| + C$
- $\int \frac{1}{x(a + bx)} \, dx = \frac{1}{a} \ln \left| \frac{x}{a + bx} \right| + C$
- $\int \frac{1}{x(a + bx)^2} \, dx = \frac{1}{a(a + bx)} + \frac{1}{a^2} \ln \left| \frac{x}{a + bx} \right| + C$
- $\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln |x + \sqrt{x^2 \pm a^2}|] + C$
- $\int x\sqrt{a + bx} \, dx = \frac{2}{15b^2} (3bx - 2a)(a + bx)^{3/2} + C$
- $\int x^2 \sqrt{a + bx} \, dx = \frac{2}{105b^3} (15b^2x^2 - 12abx + 8a^2)(a + bx)^{3/2} + C$
- $\int \frac{x \, dx}{\sqrt{a + bx}} = \frac{2}{3b^2} (bx - 2a)\sqrt{a + bx} + C$
- $\int \frac{x^2 \, dx}{\sqrt{a + bx}} = \frac{2}{15b^3} (3b^2x^2 - 4abx + 8a^2)\sqrt{a + bx} + C$