

STA 504 Homework #3

Due: Monday, September 26

Show you're your work to earn full credit. You are encouraged to work with your peers on assignments. The write-up must be your own.

Note: Following questions are based on the definition of continuous random variables and their distribution functions [probability density function (pdf) and cumulative distribution function (CDF)]; conditions for a given function to be a valid pdf; the relationship between pdf and CDF, etc. You are expected to have a clear understanding of these basic concepts.

4.8 Suppose that Y has density function

$$f(y) = \begin{cases} ky(1-y), & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a Find the value of k that makes $f(y)$ a probability density function.
- b Find $P(.4 \leq Y \leq 1)$.
- c Find $P(.4 \leq Y < 1)$.
- d Find $P(Y \leq .4 | Y \leq .8)$.
- e Find $P(Y < .4 | Y < .8)$.

Solution

- a. The constant $k = 6$ is required so the density function integrates to 1.
- b. $P(.4 \leq Y \leq 1) = .648$.
- c. Same as part b. above.
- d. $P(Y \leq .4 | Y \leq .8) = P(Y \leq .4) / P(Y \leq .8) = .352 / .896 = 0.393$.
- e. Same as part d. above.

4.17 The length of time required by students to complete a one-hour exam is a random variable with a density function given by

$$f(y) = \begin{cases} cy^2 + y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a Find c .
- b Find $F(y)$.
- c Use $F(y)$ in part (b) to find $F(-1)$, $F(0)$, and $F(1)$.
- d Find the probability that a randomly selected student will finish in less than half an hour.
- e Given that a particular student needs at least 15 minutes to complete the exam, find the probability that she will require at least 30 minutes to finish.

Suggested Solution

- a. $\int_0^1 (cy^2 + y)dy = \left[cy^3/3 + y^2/2 \right]_0^1 = 1, c = 3/2.$
- b. $F(y) = y^3/2 + y^2/2$ for $0 \leq y \leq 1.$
- c. $F(-1) = 0, F(0) = 0, F(1) = 1.$
- d. $P(Y < .5) = F(.5) = 3/16.$
- e. $P(Y \geq .5 \mid Y \geq .25) = P(Y \geq .5)/P(Y \geq .25) = 104/123.$

4.21 If, as in Exercise 4.17, Y has density function

$$f(y) = \begin{cases} (3/2)y^2 + y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere,} \end{cases}$$

find the mean and variance of Y .

Suggested Solution

$$E(Y) = \int_0^1 1.5y^3 + y^2 dy = \left[\frac{3y^4}{8} + \frac{y^3}{3} \right]_0^1 = 17/24 = .708.$$

$$E(Y^2) = \int_0^1 1.5y^4 + y^3 dy = \left[\frac{3y^5}{10} + \frac{y^4}{4} \right]_0^1 = 3/10 + 1/4 = .55.$$

$$\text{So, } V(Y) = .55 - (.708)^2 = .0487.$$

4.32 Weekly CPU time used by an accounting firm has probability density function (measured in hours) given by

$$f(y) = \begin{cases} (3/64)y^2(4 - y), & 0 \leq y \leq 4, \\ 0, & \text{elsewhere.} \end{cases}$$

- a Find the expected value and variance of weekly CPU time.
- b The CPU time costs the firm \$200 per hour. Find the expected value and variance of the weekly cost for CPU time.
- c Would you expect the weekly cost to exceed \$600 very often? Why?

Suggested Solution

a. $E(Y) = \frac{3}{64} \int_0^4 y^3(4-y)dy = \frac{3}{64} \left[y^4 - \frac{y^5}{5} \right]_0^4 = 2.4. \quad V(Y) = .64.$

b. $E(200Y) = 200(2.4) = \$480, \quad V(200Y) = 200^2(.64) = 25,600.$

c. $P(200Y > 600) = P(Y > 3) = \frac{3}{64} \int_3^4 y^2(4-y)dy = .2616, \text{ or about 26\% of the time the cost will exceed \$600 (fairly common).}$

- 4.47** The failure of a circuit board interrupts work that utilizes a computing system until a new board is delivered. The delivery time, Y , is uniformly distributed on the interval one to five days. The cost of a board failure and interruption includes the fixed cost c_0 of a new board and a cost that increases proportionally to Y^2 . If C is the cost incurred, $C = c_0 + c_1 Y^2$.

- a Find the probability that the delivery time exceeds two days.
- b In terms of c_0 and c_1 , find the expected cost associated with a single failed circuit board.

Suggested Solutions

The density for Y = delivery time is $f(x) = 1/4, 1 \leq y \leq 5$. Also, $E(Y) = 3, V(Y) = 4/3$.

a. $P(Y > 2) = 3/4.$

b. $E(C) = E(c_0 + c_1 Y^2) = c_0 + c_1 E(Y^2) = c_0 + c_1 [V(Y) + (E(Y))^2] = c_0 + c_1 [4/3 + 9]$

- 4.48** Beginning at 12:00 midnight, a computer center is up for one hour and then down for two hours on a regular cycle. A person who is unaware of this schedule dials the center at a random time between 12:00 midnight and 5:00 A.M. What is the probability that the center is up when the person's call comes in?

Suggested Solutions

Let Y = time when the phone call comes in. Then, Y has a uniform distribution on the interval (0, 5) with density function

$$f(x) = \begin{cases} \frac{1}{5}, & x \in [0, 5] \\ 0, & \text{otherwise} \end{cases}.$$

The probability is $P(0 < Y < 1) + P(3 < Y < 4) = (1/5)(1-0) + (1/5)(4-3) = 2/5.$