STA 504 Homework #1

Due: Monday, September 11, 2023

Brief Solutions from Publisher's Solution Manual

Show your work to earn full credit. You are encouraged to work with your peers on assignments. The write-up must be your own.

Caution: The problem labels in the following may not be different from those used in the textbook.

2.14 A survey classified a large number of adults according to whether they were diagnosed as needing eyeglasses to correct their reading vision and whether they use eyeglasses when reading. The proportions falling into the four resulting categories are given in the following table:

	Uses Eyeglasses for Reading	
Needs glasses	Yes	No
Yes	.44	.14
No	.02	.40

If a single adult is selected from the large group, find the probabilities of the events defined below. The adult

a needs glasses.

b needs glasses but does not use them.

c uses glasses whether the glasses are needed or not.

Suggested Solution:

a. $P(\text{needs glasses}) = .44 + .14 =$ 0.58

b. $P(\text{needs glasses but doesn't use them}) = .14$

c. $P(\text{uses glasses}) = .44 + .02 = .46$

2.31 The Bureau of the Census reports that the median family income for all families in the United States during the year 2003 was \$43,318. That is, half of all American families had incomes exceeding this amount, and half had incomes equal to or below this amount. Suppose that four families are surveyed and that each one reveals whether its income exceeded \$43,318 in 2003.

a List the points in the sample space.

b Identify the simple events in each of the following events:

A: At least two had incomes exceeding \$43,318.

B: Exactly two had incomes exceeding \$43,318.

C: Exactly one had income less than or equal to \$43,318.

c Make use of the given interpretation for the median to assign probabilities to the simple events and find $P(A)$, $P(B)$, and $P(C)$.

Suggested Solution:

This problem is essentially a binomial distribution problem.

a. Define the events: G = family income is greater than \$43,318, N otherwise. The points are

E1: GGGG	E2: GGGN	E3: GGNG	E4: GNGG
E5: NGGG	E6: GGNN	E7: GNGN	E8: NGGN
E9: GNNG	E10: NGNG	E11: NNGG	E12: GNNN
E13: NGNN	E14: NNGN	E15: NNNG	E16: NNNN

b. $A = \{E1, E2, \ldots, E11\}$ $B = \{E6, E7, \ldots, E11\}$ $C = \{E2, E3, E4, E5\}$

c. If $P(E) = P(N) = .5$, each element in the sample space has probability 1/16. Thus, $P(A) = 11/16$, $P(B) = 6/16$, and $P(C) = 4/16$.

2.72 For a certain population of employees, the percentage passing or failing a job competency exam, listed according to sex, were as shown in the accompanying table. That is, of all the people taking the exam, 24% were in the male-pass category, 16% were in the male-fail category, and so forth. An employee is to be selected randomly from this population. Let A be the event that the employee scores a passing grade on the exam and let M be the event that a male is selected.

	Sex		
Outcome	Male (M)	Female (F)	*Total*
Pass (A)	24	36	60
Fail ($\overline{A}$)	16	24	40
Total	40	60	100

a Are the events A and M independent?

b Are the events $\overline{A}$ and F independent?

Suggested Solution:

Caution: $P(A \cap B) = P(A)P(B)$ if and only if A and B are independent! We should never assume this is correct without checking whether A and B are independent.

Note that $P(A) = 0.6$ and $P(A|M) = .24/.4 = 0.6$. So, A and M are independent. Similarly, $P(\overline{A} \mid F) = .24/.6 = 0.4 = P(\overline{A})$, so $\overline{A}$ and F are independent.

2.86 Suppose that A and B are two events such that $P(A) = .8$ and $P(B) = .7$.

- **a** Is it possible that $P(A \cap B) = .1$? Why or why not?
- **b** What is the smallest possible value for $P(A \cap B)$?
- **c** Is it possible that $P(A \cap B) = .77$? Why or why not?
- **d** What is the largest possible value for $P(A \cap B)$?

Suggested Solution

a. No. It follows from $P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq 1$.
b. $P(A \cap B) \geq 0.5$ The smallest value is 0.5.
c. No. P(A and B) < min{P(A), P(B)} = 0.7.
d. $P(A \cap B) \leq 0.70$. The largest possible value is 0.7.

2.102 Diseases I and II are prevalent among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract disease II eventually, and 3% will contract both diseases.

- **a** Find the probability that a randomly chosen person from this population will contract at least one disease.
- **b** Find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.

Suggested Solution:

Define the events: I: disease I us contracted II: disease II is contracted. Then,
$P(\text{I}) = 0.1$, $P(\text{II}) = 0.15$, and $P(\text{I} \cap \text{II}) = 0.03$.
a. $P(\text{I} \cup \text{II}) = .1 + .15 - .03 = 0.22$
b. $P(\text{I} \cap \text{II} | \text{I} \cup \text{II}) = .03/.22 = 3/22$.

Part (b) uses the fact that $(A \cap B) \cap (A \cup B) = A \cap B$.

Therefore, $P[A \cap B \mid A \cup B] = P[(A \cap B) \cap (A \cup B)] / P(A \cup B) = P(A \cap B) / P(A \cup B)$.

2.114 A lie detector will show a positive reading (indicate a lie) 10% of the time when a person is telling the truth and 95% of the time when the person is lying. Suppose two people are suspects in a one-person crime and (for certain) one is guilty and will lie. Assume further that the lie detector operates independently for the truthful person and the liar. What is the probability that the detector

- **a** shows a positive reading for both suspects?
- **b** shows a positive reading for the guilty suspect and a negative reading for the innocent suspect?
- **c** is completely wrong—that is, that it gives a positive reading for the innocent suspect and a negative reading for the guilty?
- **d** gives a positive reading for either or both of the two suspects?

Suggested Solution:

Pay attention to the assumption of independence. We can use the special multiplicative rule $P[A \cap B] = P(A)P(B)$

Let T = {detects truth} and L = {detects lie}. The sample space is TT, TL, LT, LL. Since one suspect is guilty, assume the guilty suspect is questioned first:

a. $P(LL) = .95(.10) = 0.095$ **b.** $P(LT) = ..95(.9) = 0.885$

b. $P(TL) = .05(.10) = 0.005$ **d.** $1 - (.05)(.90) = 0.955$

You may want to use better notation to reformulate the questions. The following are examples of my new notations.

L	T
P(Positive|innocent) = 0.1,	P(Negative|innocent) = 0.9,
P(Positive|guilty) = 0.95.	P(Negative|guilty) = 0.05,

(a). P[LL] = P(Positive|guilty) x P(Positive|innocent) = 0.95 x 0.1 = 0.095

(b). P[LT] = P(Positive|guilty) x P(Negative|innocent) = 0.95 x 0.90 = 0.885

(c). P[TL] = P(Negative|guilty)x P(Positive|innocent) = 0.05 x 0.10 = 0.005

(d). A little bit tricky question!

The **opposite** of “given a positive reading for either or both” is “given both negative readings”

By definition of the complementary event, we can find the answer to part (d) in the following

P[LT or TL or LL] = 1 – P(TT) = 1 - P(Negative|innocent) x P(Negative|guilty)
= 1 – 0.90 x 0.05 = 1-0.045 = 0.955.