STA 504 Homework #6

Due: Monday, 10/31

This set of homework problems focuses on setting up the integral limits of double integral. You are expected to draw the integral region on the y_1-y_2 coordinate plane and set up the limits of the iterative based on the shape of the region.

The following linked page helps set up integral limits of double integrals.

http://tutorial.math.lamar.edu/Classes/CalcIII/DIGeneralRegion.aspx

The level of detail (graphs and algebra) in your work should be similar to that in the working examples in the lecture note.

Problem 1.

Let Y_1 and Y_2 denote the proportions of two different types of components in a sample from a mixture of chemicals used as an insecticide. Suppose that Y_1 and Y_2 have the joint density function given by

$$f(y_1, y_2) = \begin{cases} 2, & 0 \le y_1 \le 1, 0 \le y_2 \le 1, 0 \le y_1 + y_2 \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(Notice that $Y_1 + Y_2 \le 1$ because the random variables denote proportions within the same sample.) Find

a $P(Y_1 \le 1/2, Y_2 \le 1/2)$.

Problem 2.

The joint density function of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 30y_1y_2^2, & y_1 - 1 \le y_2 \le 1 - y_1, 0 \le y_1 \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find $F(1/2, 1/2)$.

b Find $F(1/2, 2)$.

c Find $P(Y_1 > Y_2)$.

d. Find the marginal distributions of y_1 and y_2, respectively.

[Hints: (1). You need to draw the region on which the density was defined;
(2). F(a,b) = P[Y_1<a, Y_2<b] gives you the additional constraints to define the sub-region on which the probability will be calculated -i.e., you need the sub-region to set up the integral limits. You should draw the sub-region before setting up the integral limits]

Problem 3.

Suppose that the random variables Y_1 and Y_2 have joint probability density function $f(y_1, y_2)$ given by

$$f(y_1, y_2) = \begin{cases} 6y_1^2 y_2, & 0 \le y_1 \le y_2,\ y_1 + y_2 \le 2, \\ 0, & \text{elsewhere.} \end{cases}$$

a Verify that this is a valid joint density function.

b What is the probability that $Y_1 + Y_2$ is less than 1?

c. Find the marginal distributions of y_1 and y_2, respectively.

[Hints: (1). You need the region (domain) to set up the limits to check whether the double integral is equal to 1.

(2). The additional constraint is given by $P[Y_1 + Y_2 < 1]$. Please draw the sub-region defined by the additional constraint.]