

## STA 504 Homework #6

**Due: Monday, 10/31**

This set of homework problems focuses on setting up the integral limits of double integral. You are expected to draw the integral region on the  $y_1$ - $y_2$  coordinate plane and set up the limits of the iterative based on the shape of the region.

The following linked page helps set up integral limits of double integrals.

<http://tutorial.math.lamar.edu/Classes/CalcIII/DIGeneralRegion.aspx>

The level of detail (graphs and algebra) in your work should be similar to that in the working examples in the lecture note.

### Problem 1.

Let  $Y_1$  and  $Y_2$  denote the proportions of two different types of components in a sample from a mixture of chemicals used as an insecticide. Suppose that  $Y_1$  and  $Y_2$  have the joint density function given by

$$f(y_1, y_2) = \begin{cases} 2, & 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1, 0 \leq y_1 + y_2 \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(Notice that  $Y_1 + Y_2 \leq 1$  because the random variables denote proportions within the same sample.) Find

a  $P(Y_1 \leq 1/2, Y_2 \leq 1/2)$ .

### Problem 2.

The joint density function of  $Y_1$  and  $Y_2$  is given by

$$f(y_1, y_2) = \begin{cases} 30y_1y_2^2, & y_1 - 1 \leq y_2 \leq 1 - y_1, 0 \leq y_1 \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find  $F(1/2, 1/2)$ .

b Find  $F(1/2, 2)$ .

c Find  $P(Y_1 > Y_2)$ .

d. Find the marginal distributions of  $y_1$  and  $y_2$ , respectively.

### Problem 3.

Suppose that the random variables  $Y_1$  and  $Y_2$  have joint probability density function  $f(y_1, y_2)$  given by

$$f(y_1, y_2) = \begin{cases} 6y_1^2y_2, & 0 \leq y_1 \leq y_2, y_1 + y_2 \leq 2, \\ 0, & \text{elsewhere.} \end{cases}$$

a Verify that this is a valid joint density function.

b What is the probability that  $Y_1 + Y_2$  is less than 1?

c. Find the marginal distributions of  $y_1$  and  $y_2$ , respectively.