# STA 504 Homework #6

## Due: Monday, November 3

This set of homework problems focuses on setting up the integral limits of double integral. You are expected to draw the integral region on the $y_1$-$y_2$ coordinate plane and set up the limits of the iterative based on the shape of the region.

The following linked page helps set up integral limits of double integrals.

http://tutorial.math.lamar.edu/Classes/CalcIII/DIGeneralRegion.aspx

The level of detail (graphs and algebra) in your work should be similar to that in the working examples in the lecture note.

**Problem 1.**

Let $Y_1$ and $Y_2$ denote the proportions of two different types of components in a sample from a mixture of chemicals used as an insecticide. Suppose that $Y_1$ and $Y_2$ have the joint density function given by

$$f(y_1, y_2) = \begin{cases} 2, & 0 \le y_1 \le 1, 0 \le y_2 \le 1, 0 \le y_1 + y_2 \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(Notice that $Y_1 + Y_2 \le 1$ because the random variables denote proportions within the same sample.) Find

a  $P(Y_1 \le 1/2, Y_2 \le 1/2)$.

**Problem 2.**

The joint density function of $Y_1$ and $Y_2$ is given by

$$f(y_1, y_2) = \begin{cases} 30y_1 y_2^2, & y_1 - 1 \le y_2 \le 1 - y_1, 0 \le y_1 \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a  Find $F(1/2, 1/2)$.

b  Find $F(1/2, 2)$.

c  Find $P(Y_1 > Y_2)$.

d. Find the marginal distributions of $y_1$ and $y_2$, respectively.

**Problem 3.**

Suppose that the random variables $Y_1$ and $Y_2$ have joint probability density function $f(y_1, y_2)$ given by

$$f(y_1, y_2) = \begin{cases} 6y_1^2 y_2, & 0 \le y_1 \le y_2, y_1 + y_2 \le 2, \\ 0, & \text{elsewhere.} \end{cases}$$

a  Verify that this is a valid joint density function.

b  What is the probability that $Y_1 + Y_2$ is less than 1?

c. Find the marginal distributions of $y_1$ and $y_2$, respectively.