

## STA 504 Homework #1

Due: Monday, September 9, 2024

Show your work to earn full credit. You are encouraged to work with your peers on assignments. The write-up must be your own.

**Caution:** The problem labels in the following may not be different from those used in the textbook.

- 2.14** A survey classified a large number of adults according to whether they were diagnosed as needing eyeglasses to correct their reading vision and whether they use eyeglasses when reading. The proportions falling into the four resulting categories are given in the following table:

Needs glasses	Uses Eyeglasses for Reading	
	Yes	No
Yes	.44	.14
No	.02	.40

If a single adult is selected from the large group, find the probabilities of the events defined below. The adult

- a needs glasses.
- b needs glasses but does not use them.
- c uses glasses whether the glasses are needed or not.

- 2.31** The Bureau of the Census reports that the median family income for all families in the United States during the year 2003 was \$43,318. That is, half of all American families had incomes exceeding this amount, and half had incomes equal to or below this amount. Suppose that four families are surveyed and that each one reveals whether its income exceeded \$43,318 in 2003

- a List the points in the sample space.
- b Identify the simple events in each of the following events:
  - A: At least two had incomes exceeding \$43,318.
  - B: Exactly two had incomes exceeding \$43,318.
  - C: Exactly one had income less than or equal to \$43,318.
- c Make use of the given interpretation for the median to assign probabilities to the simple events and find  $P(A)$ ,  $P(B)$ , and  $P(C)$ .

- 2.72** For a certain population of employees, the percentage passing or failing a job competency exam, listed according to sex, were as shown in the accompanying table. That is, of all the people taking the exam, 24% were in the male-pass category, 16% were in the male-fail category, and so forth. An employee is to be selected randomly from this population. Let  $A$  be the event that the employee scores a passing grade on the exam and let  $M$  be the event that a male is selected.

Outcome	Sex		Total
	Male ( $M$ )	Female ( $F$ )	
Pass ( $A$ )	24	36	60
Fail ( $\bar{A}$ )	16	24	40
Total	40	60	100

- a** Are the events  $A$  and  $M$  independent?
- b** Are the events  $\bar{A}$  and  $F$  independent?
- 2.86** Suppose that  $A$  and  $B$  are two events such that  $P(A) = .8$  and  $P(B) = .7$ .
- a** Is it possible that  $P(A \cap B) = .1$ ? Why or why not?
- b** What is the smallest possible value for  $P(A \cap B)$ ?
- c** Is it possible that  $P(A \cap B) = .77$ ? Why or why not?
- d** What is the largest possible value for  $P(A \cap B)$ ?
- 2.102** Diseases I and II are prevalent among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract disease II eventually, and 3% will contract both diseases.
- a** Find the probability that a randomly chosen person from this population will contract at least one disease.
- b** Find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.
- 2.114** A lie detector will show a positive reading (indicate a lie) 10% of the time when a person is telling the truth and 95% of the time when the person is lying. Suppose two people are suspects in a one-person crime and (for certain) one is guilty and will lie. Assume further that the lie detector operates independently for the truthful person and the liar. What is the probability that the detector
- a** shows a positive reading for both suspects?
- b** shows a positive reading for the guilty suspect and a negative reading for the innocent suspect?
- c** is completely wrong—that is, that it gives a positive reading for the innocent suspect and a negative reading for the guilty?
- d** gives a positive reading for either or both of the two suspects?