STA 504 Mathematical Statistics with Calculus Review

Midterm Exam #2

11/20/2023

Due: **9:00 AM, 11/21/2023** (Tuesday)

Please Print: _____ _____
(First Name) (Last Name)

Instructions

- This is an open-book test. Textbooks and notes can be used. However, you must complete this exam independently. All forms of collaboration are NOT allowed.
- Please show your detailed work to earn full credit.
- Partial credit will be granted to the key steps that reflect your correct reasoning even if your numerical answer is incorrect.

Problem 1.

Consider two discrete random variables X and Y whose values are r and s respectively and suppose that the joint probability distribution is given by:

		Y			s $\rightarrow$	
		0	1	2	3	
X	0	$\frac{0}{48}$	$\frac{1}{48}$	$\frac{2}{48}$	$\frac{3}{48}$	$\frac{6}{48}$
	1	$\frac{1}{48}$	$\frac{2}{48}$	$\frac{3}{48}$	$\frac{4}{48}$	$\frac{10}{48}$
r	2	$\frac{2}{48}$	$\frac{3}{48}$	$\frac{4}{48}$	$\frac{5}{48}$	$\frac{14}{48}$
$\downarrow$	3	$\frac{3}{48}$	$\frac{4}{48}$	$\frac{5}{48}$	$\frac{6}{48}$	$\frac{18}{48}$
		$\frac{6}{48}$	$\frac{10}{48}$	$\frac{14}{48}$	$\frac{18}{48}$	
		$\mathrm{P}(Y = s)$ $\rightarrow$				

$\mathrm{P}(X = r)$ $\downarrow$

Answer the following questions based on the above distribution table.

1. Are X and Y independent?

2. $E[X + Y]$

3. $E[XY]$

4. $\mathrm{COV}(\mathrm{X}, \mathrm{Y})$

Problem 2.

Let X be the total time that a customer spends at a bank, and Y the time she spends waiting in line. Assume that X and Y have joint density

$$f(x,y) = \begin{cases} 4\mathrm{e}^{-2\mathrm{x}}, & 0 \leq y \leq x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Sketch the domain or related regions whenever appropriate.

1. Find the marginal density functions of X and Y.

2. Are X and Y independent?

3. Find out the mean service time: $E[X - Y]$.

4. Find the probability $P[X - Y > 2]$

5. Find the variance of $X - Y$.

6. Find the correlation coefficient between X and Y.

7. Given that waiting time $Y = 2$, what is $E[X|\, Y = 2]$?