STA 504 Homework #10

Due: Monday, November 18

Problem 1.

Let Y_1 and Y_2 be continuous random variables with pdf:

$$f_{Y_1Y_2}(y_1, y_2) = \begin{cases} \dfrac{3y_1^2}{4}, \text{if } 0 \le y_1 \le y_2 \le 2. \\ 0, \quad \text{otherwise.} \end{cases}$$

(a) Compute the conditional expectation of Y_2 given $Y_1 = y_1$.

(b) Given that $Y_1 = 1/2$, what is the expectation of Y_2

(c). Compute the conditional expectation of Y_2 given Y_1

Problem 2.

For the daily output of an industrial operation, let Y_1 denote the amount of sales and Y_2, the costs, in thousands of dollars. Assume that the density functions for Y_1 and Y_2 are given by

$$f_1(y_1) = \begin{cases} \left(\dfrac{1}{6}\right) y_1^3 e^{-y_1}, & y_1 > 0; \\ 0, & y_1 < 0. \end{cases}$$

and

$$f_2(y_2) = \begin{cases} \left(\dfrac{1}{2}\right) e^{-y_2/2}, & y_2 > 0; \\ 0, & y_2 < 0. \end{cases}$$

The daily profit is given by $U = Y_1 - Y_2$.

(1) Find $E(U)$.

(2) Assuming that Y_1 and Y_2 are independent, find $V(U)$.

(3) Would you expect the daily profit to drop below zero very often? Why?

Problem 3.

Y_1 and Y_2 denoted the lengths of life, in hundreds of hours, for components of types I and II, respectively, in an electronic system. The joint density of Y_1 and Y_2 is

$$f(y_1, y_2) = \begin{cases} \left(\frac{1}{8}\right) y_1 e^{-(y_1+y_2)/2}, & y_1 > 0, y_1 > 0; \\ 0, & \text{elsewhere.} \end{cases}$$

The cost C of replacing the two components depends upon their length of life at failure and is given by $C = 50 + 2Y_1 + 4Y_2$.

(1). Find $E(C)$ and $V(C)$.

(2). Let $U = Y_1 - Y_2$ and $W = Y_1 + Y_2$. Find $COV(U, W)$