

STA 504 Homework #4

Due: Monday, September 30

Show your work to earn full credit. You are encouraged to work with your peers on assignments. However, the write-up must be your own.

4.60 A normally distributed random variable has density function

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)}, \quad -\infty < y < \infty.$$

Using the fundamental properties associated with any density function, argue that the parameter σ must be such that $\sigma > 0$.

4.61 What is the median of a normally distributed random variable with mean μ and standard deviation σ ?

4.62 If Z is a standard normal random variable, what is

a $P(Z^2 < 1)$?

b $P(Z^2 < 3.84146)$?

Subtitle

Problem 4.

The lifetime (in hours) Y of an electronic component is a random variable with density function given by

$$f(y) = \begin{cases} \frac{1}{100} e^{-y/100}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Based on the above distribution answer the following questions.

1. Show that $f(y)$ is a valid density function.
2. Derive the cumulative distribution function (CDF) of the lifetime variable.
3. Derive the expectation and variance of Y .
4. Find $P(Y > 150)$ using the CDF derived in part 2.
5. Find $P(Y > 150 \mid Y > 100)$ using CDF.