STA 504 Homework #2

Due: Tuesday, September 10

Show you're your work to earn full credit. You are encouraged to work with your peers on assignments. The write-up must be your own.

Caution: The problem labels in the following may not be different from those used in the textbook.

3.9 In order to verify the accuracy of their financial accounts, companies use auditors on a regular basis to verify accounting entries. The company's employees make erroneous entries 5% of the time. Suppose that an auditor randomly checks three entries.

a Find the probability distribution for Y, the number of errors detected by the auditor.

b Construct a probability histogram for $p(y)$.

c Find the probability that the auditor will detect more than one error.

Suggested Solution

The random variable *Y* takes on values 0, 1, 2, and 3.

a. Let *E* denote an error on a single entry and let *N* denote no error. There are 8 sample points: *EEE*, *EEN*, *ENE*, *NEE*, *ENN*, *NEN*, *NNE*, *NNN*. With $P(E) = .05$ and $P(N) = .95$ and assuming independence, we can construct the probability distribution table.

$P(Y = 3) = 1(.05)^3\,(0.95)^0 = 0.000125$

$P(Y = 2) = 3(.05)^2(.95)^1 = 0.007125$

$P(Y = 1) = 3(.05)^1(.95)^2 = 0.135375$

$P(Y = 0) = 1(0.05)^0\,(.95)^3 = 0.857375.$

b. The graph is omitted.

c. $P(Y > 1) = P(Y = 2) + P(Y = 3) = 0.00725.$

3.12 Let Y be a random variable with $p(y)$ given in the accompanying table. Find $E(Y)$, $E(1/Y)$, $E(Y^2 - 1)$, and $V(Y)$.

y	1	2	3	4
$p(y)$	.4	.3	.2	.1

Suggested Solution:

Using the definition of the expectation of a random variable and the expectation of functions of a random variable to find the above quantities.

$E(Y) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = 2.0$
$E(1/Y) = 1(.4) + 1/2(.3) + 1/3(.2) + 1/4(.1) = 0.6417$

$E(Y^2 – 1) = E(Y^2) – 1 = [1(.4) + 2^2(.3) + 3^2(.2) + 4^2(.1)] – 1 = 5 – 1 = 4.$

$V(Y) = E(Y^2) = [E(Y)]^2 = 5 – 22 = 1.$

3.31 Suppose that Y is a discrete random variable with mean μ and variance σ^2 and let $W = 2Y$.

a Do you expect the mean of W to be larger than, smaller than, or equal to $\mu = E(Y)$? Why?

b Use Theorem 3.4 to express $E(W) = E(2Y)$ in terms of $\mu = E(Y)$. Does this result agree with your answer to part (a)?

c Recalling that the variance is a measure of spread or dispersion, do you expect the variance of W to be larger than, smaller than, or equal to $\sigma^2 = V(Y)$? Why?

d Use Definition 3.5 and the result in part (b) to show that

$$V(W) = E\{[W - E(W)]^2\} = E[4(Y - \mu)^2] = 4\sigma^2;$$

that is, $W = 2Y$ has variance four times that of Y.

THEOREM **3.4**

Let Y be a discrete random variable with probability function $p(y)$, $g(Y)$ be a function of Y, and c be a constant. Then

$$E[cg(Y)] = cE[g(Y)].$$

DEFINITION **3.5**

If Y is a random variable with mean $E(Y) = \mu$, the variance of a random variable Y is defined to be the expected value of $(Y - \mu)^2$. That is,

$$V(Y) = E[(Y - \mu)^2].$$

The *standard deviation* of Y is the positive square root of $V(Y)$.

Suggested Solution:

a. The mean of W will be larger than the mean of Y if $\mu > 0$. If $\mu < 0$, the mean of W will be smaller than μ. If $\mu = 0$, the mean of W will equal μ.

b. $E(W) = E(2Y) = 2E(Y) = 2\mu$.

c. The variance of W will be larger than σ^2, since the spread of values of W has increased.

d. $V(X) = E[(X – E(X))^2] = E[(2Y – 2\mu)^2] = 4E[(Y – \mu)^2] = 4\sigma^2$.

3.49 Tay-Sachs disease is a genetic disorder that is usually fatal in young children. If both parents are carriers of the disease, the probability that their offspring will develop the disease is approximately .25. Suppose that a husband and wife are both carriers and that they have three children. If the outcomes of the three pregnancies are mutually independent, what are the probabilities of the following events?

a All three children develop Tay-Sachs.
b Only one child develops Tay-Sachs.
c The third child develops Tay-Sachs, given that the first two did not.

Suggested Solution

There is a 25% chance the offspring of the parents will develop the disease. Then, Y = # of offspring that develop the disease is binomial with $n = 3$ and $p = .25$.

a. $P(Y = 3) = (.25)^3 = 0.015625$.
b. $P(Y = 1) = 3(.25)(.75)^2 = 0.421875$
c. Since the pregnancies are mutually independent, the probability is simply 25%.

3.122 Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities that

a no more than three customers arrive?
b at least two customers arrive?
c exactly five customers arrive?

Suggested Solution:

Let Y = # of customers that arrive during the hour. Then, Y is Poisson with $\lambda = 7$.
a. $P(Y \leq 3) = .0818$.
b. $P(Y \geq 2) = .9927$.
c. $P(Y = 5) = .1277$

3.125 Refer to Exercise 3.122. If it takes approximately ten minutes to serve each customer, find the mean and variance of the total service time for customers arriving during a 1-hour period. (Assume that a sufficient number of servers are available so that no customer must wait for service.) Is it likely that the total service time will exceed 2.5 hours?

Suggested Solution:

Let S = total service time = $10Y$. From Ex. 3.122, Y is Poisson with $\lambda = 7$.
Therefore, $E(S) = 10E(Y) = 10 \times \mathbf{7} = \mathbf{70}$ and $V(S) = 100V(Y) = 700$.
Also,
$P(S > 150) = P(Y > 15) = 1 - P(Y \leq 15) = 1 - .998 = .002$, and unlikely event.