

## STA 504 Homework #10

Due: Monday, November 18

### Problem 1.

Let  $Y_1$  and  $Y_2$  be continuous random variables with pdf:

$$f_{Y_1 Y_2}(y_1, y_2) = \begin{cases} \frac{3y_1^2}{4}, & \text{if } 0 \leq y_1 \leq y_2 \leq 2. \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Compute the conditional expectation of  $Y_2$  given  $Y_1 = y_1$ .
- (b) Given that  $Y_1 = 1/2$ , what is the expectation of  $Y_2$
- (c). Compute the conditional expectation of  $Y_2$  given  $Y_1$

### Problem 2.

For the daily output of an industrial operation, let  $Y_1$  denote the amount of sales and  $Y_2$ , the costs, in thousands of dollars. Assume that the density functions for  $Y_1$  and  $Y_2$  are given by

$$f_1(y_1) = \begin{cases} \left(\frac{1}{6}\right) y_1^3 e^{-y_1}, & y_1 > 0; \\ 0, & y_1 < 0. \end{cases}$$

and

$$f_2(y_2) = \begin{cases} \left(\frac{1}{2}\right) e^{-y_1/2}, & y_2 > 0; \\ 0, & y_2 < 0. \end{cases}$$

The daily profit is given by  $U = Y_1 - Y_2$ .

- (1) Find  $E(U)$ .
- (2) Assuming that  $Y_1$  and  $Y_2$  are independent, find  $V(U)$ .
- (3) Would you expect the daily profit to drop below zero very often? Why?

### Problem 3.

$Y_1$  and  $Y_2$  denoted the lengths of life, in hundreds of hours, for components of types I and II, respectively, in an electronic system. The joint density of  $Y_1$  and  $Y_2$  is

$$f(y_1, y_2) = \begin{cases} \left(\frac{1}{8}\right) y_1 e^{-(y_1+y_2)/2}, & y_1 > 0, y_2 > 0; \\ 0, & \text{elsewhere.} \end{cases}$$

The cost  $C$  of replacing the two components depends upon their length of life at failure and is given by  $C = 50 + 2Y_1 + 4Y_2$ .

**(1).** Find  $E(C)$  and  $V(C)$ .

**(2).** Let  $U = Y_1 - Y_2$  and  $W = Y_1 + Y_2$ . Find  $COV(U, W)$