

STA 504 Homework #6

Due: Monday, 10/30

This set of homework problems focuses on setting up the integral limits of a double integral. You are expected to draw the integral region on the y_1 - y_2 coordinate plane and set up the limits of the iterative based on the shape of the region.

The following linked page helps set up integral limits of double integrals.

<http://tutorial.math.lamar.edu/Classes/CalcIII/DIGeneralRegion.aspx>

The level of detail (graphs and algebra) in your work should be similar to that in the working examples in the lecture notes.

Problem 1.

Let Y_1 and Y_2 denote the proportions of two different types of components in a sample from a mixture of chemicals used as an insecticide. Suppose that Y_1 and Y_2 have the joint density function given by

$$f(y_1, y_2) = \begin{cases} 2, & 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1, 0 \leq y_1 + y_2 \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(Notice that $Y_1 + Y_2 \leq 1$ because the random variables denote proportions within the same sample.) Find

a $P(Y_1 \leq 1/2, Y_2 \leq 1/2)$.

Problem 2.

The joint density function of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 30y_1y_2^2, & y_1 - 1 \leq y_2 \leq 1 - y_1, 0 \leq y_1 \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find $F(1/2, 1/2)$.

b Find $F(1/2, 2)$.

c Find $P(Y_1 > Y_2)$.

d. Find the marginal distributions of y_1 and y_2 , respectively.

[Hints: (1). You need to draw the region on which the density was defined;

(2). $F(a,b) = P[Y_1 < a, Y_2 < b]$ gives you the additional constraints to define the sub-region on which the probability will be calculated -i.e., you need the sub-region to set up the integral limits. You should draw the sub-region before setting up the integral limits]

Problem 3.

Suppose that the random variables Y_1 and Y_2 have joint probability density function $f(y_1, y_2)$ given by

$$f(y_1, y_2) = \begin{cases} 6y_1^2 y_2, & 0 \leq y_1 \leq y_2, y_1 + y_2 \leq 2, \\ 0, & \text{elsewhere.} \end{cases}$$

- a Verify that this is a valid joint density function.
- b What is the probability that $Y_1 + Y_2$ is less than 1?
- c Find the marginal distributions of y_1 and y_2 , respectively.

[Hints: (1). You need the region (domain) to set up the limits to check whether the double integral is equal to 1.
(2). The additional constraint is given by $P[Y_1 + Y_2 < 1]$. Please draw the sub-region defined by the additional constraint.]