STA 504 Homework #13

Due: Monday, December 9

Using transformation methods to find the following distributions.

Problem 1.

Let continuous random vector (X_1, X_2) have joint probability density function

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} \frac{1}{4} e^{-(x_1+x_2)/2}, 0 < x_1 < \infty, 0 < x_2 < \infty; \\ 0, \quad \text{elsewhere.} \end{cases}$$

Define $Y_1 = \frac{(X_1 - X_2)}{2}$ and $Y_2 = \frac{(X_1 + X_2)}{2}$. Find the joint probability distribution of Y_1 and Y_2.

Problem 2.

Let continuous random vector (X_1, X_2) have the joint probability density function

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} 10 x_1 x_2^2, 0 < x_1 < x_2 < 1; \\ 0, \quad \text{elsewhere.} \end{cases}$$

Let $Y_1 = X_1/X_2$ and $Y_2 = X_2$. Find the joint probability distribution of Y_1 and Y_2.