# **STA 504 Mathematical Statistics with Calculus Review**

## Midterm Exam #2

11/19/2022

Please Print:		
	(First Name)	(Last Name)

### Instructions

- This is an open-book test. Textbook and notes can be used. However, you must complete this exam independently. All forms of collaborations are NOT allowed.
- You may use a calculator for the exam.
- Please show your detailed work to earn full credit.
- Partial credit will be granted to the key steps that reflect your correct reasoning even if your numerical answer is incorrect.

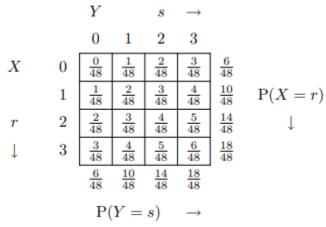
Problem 1

### Problem 1.

Consider two discrete random variables X and Y whose values are r and s respectively and suppose that the probability of the event  $\{X = r\} \cap \{Y = s\}$  is given by:

$$f(s,t) = \begin{cases} \frac{r+s}{48}, & 0 \le r,s \le 3\\ 0, & \text{elsewhere} \end{cases}$$

The above probability distribution can be tabulated in the following



Find the expectation of

- 1. Are *X* and *Y* independent?
- 2. E[X + Y]
- 3. E[XY]
- 4. COV(X,Y)

#### Problem 2.

Let X be the total time that a customer spends at a bank, and Y the time she spends waiting in line. Assume that X and Y have joint density

$$f(x,y) = \begin{cases} \lambda^2 e^{-\lambda x}, & 0 \le y \le x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Sketch the domain or related regions whenever appropriate

- 1. Find the marginal density functions of *X* and *Y*.
- 2. Are *X* and *Y* independent?
- 3. Find out the mean service time: E[T] = E[X Y].
- 4. Find the probability  $P[T > \lambda]$
- 5. Find the variance T.
- 6. Find the correlation coefficient between *X* and *Y*.
- 7. Given that waiting time  $Y = \lambda$ , what is  $E[X|Y = \lambda]$ ?