STA 504 Homework #8

Due: Monday, November 4

Answer the following questions based on the given joint density function in problems 1 -4.

(i) Find the marginal densities $f(x)$ and $f(y)$.

(ii) Check if the variables X and Y are independent.

(iii) find the conditional probability distribution function of X, given $Y = y$

[Hint: sketch the domain of the joint density function before doing any calculation.]

Problem 1.

Let the continuous random vector (X, Y) have a joint probability density function.

$$f(x,y) = e^{-y}, 0 < x < y < \infty.$$

In addition to the (i)-(iii), compute the $P(X \geq 2;\ Y \geq 4)$.

Problem 2.

Let X and Y be a random vector with a joint probability density function.

$$f(x,y) = \begin{cases} 6xy(2 - x - y), & \text{if } 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then find the conditional probability distribution function of X, given $Y = y$, where $0 < y < 1$.

Problem 3.

Let *X* and *Y* be a random vector with a joint probability density function.

$$f(x,y) = \begin{cases} 1/x, & \text{if } 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Problem 4.

Let X and Y have density.

$$f(x,y) = \begin{cases} xe^{-x(1+y)}, & \text{if } y > 0, x > 0 \\ 0, & \text{otherwise} \end{cases}$$

In addition to the (i)-(iii), compute the $P(X \geq 2;\ Y \geq 4)$.