STA 504 Mathematical Statistics with Calculus Review

Midterm Exam #2

11/19/2022

Please Print: _____ _____
(First Name) (Last Name)

Instructions

- This is an open-book test. Textbook and notes can be used. However, you must complete this exam independently. All forms of collaborations are NOT allowed.
- You may use a calculator for the exam.
- Please show your detailed work to earn full credit.
- Partial credit will be granted to the key steps that reflect your correct reasoning even if your numerical answer is incorrect.

Problem 1

Problem 1.

Consider two discrete random variables X and Y whose values are r and s respectively and suppose that the probability of the event {X = r} ∩ {Y = s} is given by:

$$f(s,t) = \begin{cases} \frac{\mathrm{r}+\mathrm{s}}{48}, & 0 \leq r,s \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

The above probability distribution can be tabulated in the following

		Y		s	$\rightarrow$	
		0	1	2	3	
X	0	$\frac{0}{48}$	$\frac{1}{48}$	$\frac{2}{48}$	$\frac{3}{48}$	$\frac{6}{48}$
	1	$\frac{1}{48}$	$\frac{2}{48}$	$\frac{3}{48}$	$\frac{4}{48}$	$\frac{10}{48}$
r	2	$\frac{2}{48}$	$\frac{3}{48}$	$\frac{4}{48}$	$\frac{5}{48}$	$\frac{14}{48}$
$\downarrow$	3	$\frac{3}{48}$	$\frac{4}{48}$	$\frac{5}{48}$	$\frac{6}{48}$	$\frac{18}{48}$
		$\frac{6}{48}$	$\frac{10}{48}$	$\frac{14}{48}$	$\frac{18}{48}$	
		$\mathrm{P}(Y=s)$		$\rightarrow$		

$\mathrm{P}(X=r)$ $\downarrow$

Find the expectation of

1. Are X and Y independent?
2. $E[X + Y]$
3. $E[XY]$
4. $COV(X,Y)$

Problem 2.

Let X be the total time that a customer spends at a bank, and Y the time she spends waiting in line. Assume that X and Y have joint density

$$f(x,y) = \begin{cases} \lambda^2 \mathrm{e}^{-\lambda \mathrm{x}}, & 0 \leq y \leq x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Sketch the domain or related regions whenever appropriate.

1. Find the marginal density functions of X and Y.
2. Are X and Y independent?
3. Find out the mean service time: $E[T] = E[X - Y]$.
4. Find the probability $P[T > \lambda]$
5. Find the variance T.
6. Find the correlation coefficient between X and Y.
7. Given that waiting time $Y = \lambda$, what is $E[X|\, Y = \lambda]$?