Review of Double Integrals.

Problem: Evaluate the integral $\iint_R f(x,y)dxdy$.

Steps:

1. Sketch the region R.
2. Sketch the region where $f(x,y) > 0$. In the integral the region where $f(x,y) = 0$ does not matter.
3. Sweep the region Horizontally or Vertically.
4. Get bounds for x and y, depending on the order of sweeping.
5. Set up the double integral as a repeated integral.
6. In some cases, might need multiple intervals.

Note that

- for horizontal sweeping, y is fixed with constant bounds, but bounds on x might depend on y.
- for vertical sweeping, x is fixed with constant bounds, but bounds on y might depend on x.

Some examples to solve:

1. Set up a double integral of $f(x,y)$ over the region given by $0 < x < 1, x < y < x + 1$.
2. Set up a double integral of $f(x,y)$ over the part of the unit square $0 \le x \le 1, 0 \le y \le 1$, on which $y \le x/2$.
3. Set up a double integral of $f(x,y)$ over the part of the unit square $0 \le x \le 1, 0 \le y \le 1$, on which $x + y > 1/2$.
4. Set up a double integral of $f(x,y)$ over the part of the unit square $0 \le x \le 1, 0 \le y \le 1$, on which both x and y are greater than $1/2$.
5. Set up a double integral of $f(x,y)$ over the part of the unit square $0 \le x \le 1, 0 \le y \le 1$, on which at least one of x and y is greater than $1/2$.
6. Set up a double integral of $f(x,y)$ over the part of the region given by $0 < x < 50 - y < 50$ on which both x and y are greater than 20.
7. Set up a double integral of $f(x,y)$ over the set of all points (x,y) in the first quadrant with $|x-y| \le 1$.
8. Evaluate $\iint_R e^{-x-y}dxdy$, where R is the region in the first quadrant with $x + y \le 1$.
9. Evaluate $\iint_R e^{-x-2y}dxdy$, where R is the region in the first quadrant with $x \le y$.
10. Evaluate $\iint_R (x^2 + y^2)dxdy$, where R is the region $0 \le x \le y \le L$.
11. Setup the integral $\iint_R f(x,y)dxdy$, where R is the region inside the unit square in which both the coordinated x and y are greater than $1/2$.
12. Evaluate $\iint_R (x - y + 1)dxdy$, where R is the region inside the unit square in which both the coordinated $x + y \ge 1/2$.
13. Evaluate $\int_0^1 \int_0^1 x \max(x,y)dxdy$.