

## STA 504 Homework #3

Due: Monday, September 25

Show you're your work to earn full credit. You are encouraged to work with your peers on assignments. The write-up must be your own.

**Note:** The following questions are based on the definition of continuous random variables and their distribution functions [probability density function (pdf) and cumulative distribution function (CDF)]; conditions for a given function to be a valid pdf; the relationship between pdf and CDF, etc. You are expected to have a clear understanding of these basic concepts.

4.8 Suppose that  $Y$  has density function

$$f(y) = \begin{cases} ky(1-y), & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a Find the value of  $k$  that makes  $f(y)$  a probability density function.
- b Find  $P(.4 \leq Y \leq 1)$ .
- c Find  $P(.4 \leq Y < 1)$ .
- d Find  $P(Y \leq .4 | Y \leq .8)$ .
- e Find  $P(Y < .4 | Y < .8)$ .

4.17 The length of time required by students to complete a one-hour exam is a random variable with a density function given by

$$f(y) = \begin{cases} cy^2 + y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a Find  $c$ .
- b Find  $F(y)$ .
- c Use  $F(y)$  in part (b) to find  $F(-1)$ ,  $F(0)$ , and  $F(1)$ .
- d Find the probability that a randomly selected student will finish in less than half an hour.
- e Given that a particular student needs at least 15 minutes to complete the exam, find the probability that she will require at least 30 minutes to finish.

4.21 If, as in Exercise 4.17,  $Y$  has density function

$$f(y) = \begin{cases} (3/2)y^2 + y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere,} \end{cases}$$

find the mean and variance of  $Y$ .

- 4.32** Weekly CPU time used by an accounting firm has probability density function (measured in hours) given by

$$f(y) = \begin{cases} (3/64)y^2(4 - y), & 0 \leq y \leq 4, \\ 0, & \text{elsewhere.} \end{cases}$$

- a Find the expected value and variance of weekly CPU time.
  - b The CPU time costs the firm \$200 per hour. Find the expected value and variance of the weekly cost for CPU time.
  - c Would you expect the weekly cost to exceed \$600 very often? Why?
- 4.47** The failure of a circuit board interrupts work that utilizes a computing system until a new board is delivered. The delivery time,  $Y$ , is uniformly distributed on the interval one to five days. The cost of a board failure and interruption includes the fixed cost  $c_0$  of a new board and a cost that increases proportionally to  $Y^2$ . If  $C$  is the cost incurred,  $C = c_0 + c_1 Y^2$ .
- a Find the probability that the delivery time exceeds two days.
  - b In terms of  $c_0$  and  $c_1$ , find the expected cost associated with a single failed circuit board.
- 4.48** Beginning at 12:00 midnight, a computer center is up for one hour and then down for two hours on a regular cycle. A person who is unaware of this schedule dials the center at a random time between 12:00 midnight and 5:00 A.M. What is the probability that the center is up when the person's call comes in?