This is a review submitted to Mathematical Reviews/MathSciNet.

Reviewer Name: Peng, Cheng

Mathematical Reviews/MathSciNet Reviewer Number: 82361

Address:

Department of Mathematics
West Chester University
West Chester, PA 19383
cpeng@wcupa.edu

Author: Einmahl, John H. J.; He, Yi

Title: Extreme value inference for heterogeneous power law data.

MR Number: MR4630951

Primary classification:

Secondary classification(s):

Review text:

This article extended the extreme value statistic to independent data with possibly very different distributions and derived a novel asymptotic normality result for the well-known Hill estimator of the tail index of the underlying distribution. Consider independent random variables $\{X_1^{(p)}, \cdots, X_p^{(p)}\}$, for $p \in N$, that are not necessarily identically distributed. Define their empirical distribution function by

$$F_{\mathrm{emp}}(x) = \frac{1}{p}\sum_{i=1}^{p} \mathbb{1}[X_i^{(p)} \leq x]$$

and their average distribution faction by

$$F_p(x) = \mathbb{E}F_{\mathrm{emp}}(x) = \frac{1}{p}\sum_{i=1}^{p} F_{pi}(x), \qquad F_{pi}(x) = \mathbb{P}(X_i^{(p)} \leq x)$$

Let $T_p = 1 - F_p$, the average survival function, and

$$\lim_{t\to\infty} \frac{T(tx)}{T(t)} = x^{-1/\gamma}, \quad x > 0.$$

$\gamma > 0$ is a natural extension on the tail index of F. The well-known Hill estimator of γ is given by

$$\hat{\gamma} = \frac{1}{k}\sum_{i=0}^{k-1} \log X_{p-i:p} - \log X_{p-k:p}$$

where $X_{p-k:p} \leq X_{p-i+1:p} \leq \cdots \leq X_{p:p}$ are the $k+1$ upper-order statistics defined based on the random sample.

Under some regularity conditions, the authors derived a new asymptotic normal distribution of $\hat{\gamma}$ with an explicit expression of variance that depended on a tail empirical process.

From the application perspective, the authors also proposed an extreme quantile estimator and derived its asymptotic distribution as well.