Learning for Logistic Regression Model in Parallel

Haoruo Peng
HTC Research Lab
Beijing, China
penghaoruo@hotmail.com

Ding Liang
HTC Research Lab
Beijing, China
Ding_Liang@htc.com

Deli Zhao
HTC Research Lab
Beijing, China
zhaodeli@gmail.com

Cyrus Choi
HTC Research Lab
Beijing, China
Cyrus_Choi@htc.com

Edward Y. Chang
HTC Research Lab
Beijing, China
Edward_Chang@htc.com

***Abstract*—Logistic regression (LR) model has various applications in machine learning. It is a supervised learning model for pattern classification and enjoys the benefits of statistical principles. This paper addresses the issue of computational efficiency for solving LR in big data scenario. We compare between different computing platforms and different parallel algorithms. Our goal is to offer valuable suggestions on choosing computing platforms and learning algorithms according to different application situations for specific datasets and machine resources. Three different parallel computing platforms along with three different types of parallel algorithms are carefully studied in this paper. In such process, we present a novel parallel sublinear method based on its sequential version as another contribution. To enhance practicality, we consider tradeoff between efficiency and precision. We also provide fault tolerance for parallel distributed computations.**

***Keywords*-Logistic Regression Model; Parallel Computing; Sublinear Method; Big Data;**

I. Introduction

Logistic regression model [11] plays a vital role in machine learning. It is a widely used model in algorithms like PageRank [20] and anti-spam filtering [1]. The model serves for classification problems, and is supported by a substantial body of statistical theories and algorithms. A binary classification problem modeled by LR can be easily extended to a multi-class classification problem. Also, due to the strong statistical foundation of the LR model, we can extend achieved results into a deeper analysis which may apply to other classification models. We will focus on the binary LR model in this paper.

In recent years, many modern datasets are increasingly larger and larger. A most identifiable characteristic of these datasets is that data scale is very large and data dimensionality is very high. Large data volume and high data dimensionality bring unavoidable computational challenges to machine learning problems. For example, in social networks, we have datasets [17] covering multiple dimensions like friendship relationship and share of common interests. They generally include millions of records over millions of attributes. More evidently, for the task of text categorization, we usually have to handle datasets with data volumes in billion scales and each data instance is characterized by thousands of feature dimensions [16]. We tackle these challenges on both algorithm level and computing platform level via parallelization methods. For many applications, in order to attain speedup in performance, we can employ not only task parallelism, but also data parallelism [23]. In this paper, we mainly discuss issues in task parallelism.

When developing parallel algorithms for LR, it is inevitable to come across the question of choosing which parallel computing platform to employ. After a thorough survey, we choose three unique and best-known computing platforms to test: Hadoop [25], Mahout [19] and Spark [27]. Hadoop employs HDFS [2] and MapReduce [6]. Mahout runs on Apache Hadoop and makes improvements in low level code. Spark promotes efficiency of iterative algorithms and also supports HDFS. We will compare design features of these three computing platforms separately in Section II-B.

In sequential optimization algorithms for LR model, a classical way is to resort to stochastic approximation methods. Stochastic approximation methods, such as stochastic gradient descent [28] and stochastic dual averaging [26], obtain optimal generalization guarantees with a small number of passes over data with runtime linear to the dataset size. The stochastic gradient descent method can work in an online fashion, as we view sampled data points as data stream over algorithmic iteration. It is hard to be changed to task-parallelism, but it is blazingly fast. If we take a step back to the general gradient descent to solve LR model, we can find that it can be actually embarrassingly parallel. We can take in all of the data at the same iteration and just compute the gradient in a MapReduce fashion. We can further speed up the runtime by employing sublinear algorithms [21] via the application of the stochastic approximation idea. These methods access a single feature of feature vectors instead of entire feature vectors at each iteration. We propose a parallel version of this algorithm and achieve the result of comparable accuracy while being faster convergent.

II. Related Work

A. Scalability Solutions

In big data scenarios, large datasets require us to develop machine learning algorithms towards a more efficient and more parallelized end. Researchers have already done much work to cater for the needs of massive datasets. Early work like Parallel Support Vector Machines (PSVM) [3] employed an approximation matrix decomposition method based on row selections to reduce memory usage. It can then increase the number of parallel computational nodes to several hundreds. Later, the work of Parallel Latent Dirichlet Allocation (PLDA) [24] improves the computational efficiency of the LDA model by means of sampling methods. The parallel algorithm in Hadoop is robust, for it has fault tolerance of machine failures, thus taking advantage of Hadoop features. Recently, Dean *et al.* [15] claimed that the advantage of parallelization is fully taken in deep learning algorithms. It pushed the limit of parallel computational nodes to a hundred-million level, and in the meantime, achieved the best learning performance ever. GraphLab [14] is a recently developed tool for large-scale machine learning. It tackles efficiency and scalability problems of machine leaning algorithms by graph-theoretical models.

B. *Computing Platforms*

Three computing platforms that we are working on have their own advantages to implement machine learning algorithms. The Apache Hadoop [25] software library allows for distributed processing of large datasets across clusters of computers using simple programming models. Hadoop utilizes MapReduce [6] as its computational paradigm. MapReduce paradigm takes a set of key-value pairs as its input. Then a user-defined *map* function turns the input into some intermediate key-value pairs. Another user-defined *reduce* function merges values for the same intermediate pairs to produce output. All of the operations are independent, thereby making MapReduce paradigm easy to be parallelized. In addition, Hadoop provides a Distributed File System (HDFS). Both MapReduce and HDFS are designed to handle node failures in an automatic way, making Hadoop support large clusters that are built on commodity hardware. The goal of Apache Mahout [19] is to build scalable machine learning libraries. Their core algorithms for clustering, classification and collaborative filtering are implemented on top of Apache Hadoop using the MapReduce paradigm. The core libraries are highly optimized so that Apache Mahout also has good performance for non-distributed algorithms. Spark [27] is a cluster computing system that aims to make data analysis fast. Spark supports in-memory cluster computing. A job can load data into memory and query it repeatedly by creating and caching resilient distributed datasets (RDDs). Moreover, RDDs achieve fault tolerance through lineage: information about how RDDs are derived from other RDDs is stored in reliable storage, thus making RDDs easy to be rebuilt if a certain partition is lost. Spark can run up to 100 times faster than Hadoop for iterative algorithms.

In Table I, we compare different features between Hadoop and Spark computing platforms. As Mahout is mainly built on Hadoop, it inherits most of Hadoop features. For simplicity, we use short abbreviations in the table. Spark supports two types of operations on RDDs: Actions and Transformations (As and Ts). Actions include functions like *count*, *collect* and *save*. They usually return a result from input RDDs. Transformations include functions like *map*, *filter* and *join*. They normally build new RDDs from other RDDs, which comply with the lineage rule. In the table, we use ”NFS” to represent ”Normal File System” and ”Fault Tolerance” here is specifically for node failure situations.

Table I
PLATFORM COMPARISON: HADOOP VS. SPARK

	Hadoop	Spark
Computational Paradigm	MapReduce	As and Ts
File System Supported	HDFS	HDFS and NFS
Design Concept	Key-value Pairs	RDD
Fault Tolerance Technique	Redundancy	Lineage

C. *Sublinear Methods*

Sublinear methods are recently proposed by many researchers. Clarkson *et al.* [4] first presented the solution of approximation algorithms in sublinear time. It is applied to optimization problems for linear classifiers and minimum enclosing balls. The algorithm employs a novel sampling technique along with a new multiplicative update procedure. Hazan *et al.* [13] exploited the approach to linear SVM. Cotter *et al.* [5] extended the method to kernelized SVM. In [12], Hazan *et al.* applied the sublinear approximation approach to solve linear regression with penalties. Garber and Hazan [9] also developed it in Semi-Definite Programming (SDP). Peng *et al.* [21] utilized the method in the LR model with penalties and developed sequential sublinear algorithms for both ℓ_1-penalty and ℓ_2-penalty.

III. LOGISTIC REGRESSION MODEL AND SEQUENTIAL SUBLINEAR ALGORITHM

A. *Logistic Regression Model*

In this paper, we are mainly concerned with the binary classification problem. We define the training dataset as $\mathcal{X} = \{(\mathbf{x}_i, y_i) : i = 1, \ldots, n\}$, where $\mathbf{x}_i \in \mathbb{R}^d$ are input samples and $y_i \in \{-1, 1\}$ are the corresponding labels. For simplicity, we will use the notation $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n]^T$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)^T$ to represent the training dataset in the following. To fit in the logistic regression model, the expected value of y_i is given by

$$P(y_i|\mathbf{x}_i) = \frac{1}{1 + \exp(-y_i(\mathbf{x}_i^T\mathbf{w} + b))} \triangleq g_i(y_i),$$

where $\mathbf{w} = (w_1, \ldots, w_d)^T \in \mathbb{R}^d$ is a regression vector and $b \in \mathbb{R}$ is an offset term.

The learning process aims to optimize $\mathbf{w}$ and b. To make the optimization problem more practical to solve, a common practice is to add penalties to LR model. Typically, for ℓ_2-penalty, we solve the following optimization problem:

$$\max_{\mathbf{w},b} \Big\{F(\mathbf{w}, b|\mathcal{X}) - \frac{\lambda}{2}\|\mathbf{w}\|_2^2\Big\}. \quad (1)$$

For ℓ_1-penalty, we optimize the following criterion:

$$\max_{\mathbf{w},b} \big\{F(\mathbf{w}, b|\mathcal{X}) - \gamma\|\mathbf{w}\|_1\big\}. \quad (2)$$

Here, we define $F(\mathbf{w}, b|\mathcal{X}) = \sum_{i=1}^n \log g_i(y_i)$ in both (1) and (2). To be brief, we omit the derivation here.

B. Sequential Sublinear Algorithm

We use the following notations to define sequential sublinear algorithm for penalized logistic regression. $clip(\cdot)$ is a projection function defined as $clip(a, b) \triangleq \max(\min(a, b), -b)$, $a, b \in \mathbb{R}$. $\mathrm{sgn}(\cdot)$ is the sign function; namely, $\mathrm{sgn}(\cdot) \in \{-1, 0, 1\}$. $g(\cdot)$ is the logistic function $g(x) = 1/(1 + e^{-x})$.

In Algorithm 1, we give the sequential sublinear approximation procedure for logistic regression. The algorithm is mainly from [21]. Symbols we use here comply with the definition in Section III-A. ε controls learning rate, T determines iteration number, and η defines approximation level. Each iteration of the SLLR algorithm has two phases: *stochastic primal update* and *stochastic dual update*. Steps from line 4 to line 11 consist of the primal part while the dual part is composed of steps from line 12 to line 16. We give the sublinear algorithm in a unified way for ℓ_2-penalty and ℓ_1-penalty. If we are dealing with ℓ_2-penalty, we ignore line 9 and accomplish the computation in line 8 by a simple greedy algorithm. Here, Λ represents a Euclidean space with conditions $\Lambda = \{\xi \in \mathbb{R}_n \,|\, \forall i,\, 0 \le \xi_i \le 2,\, \|\xi\|_1 \le \nu n\}$. If we are faced with ℓ_1-penalty, we ignore line 8 and expand the procedure of line 9 as following.

```
Algorithm 1: Sub-Linear Logistic Regression (SLLR)
1: Input parameters: $\varepsilon, \nu\ or\ \gamma, X, Y$
2: Initialize parameters: $T, \eta, \mathbf{u}_0, \mathbf{w}_1, \mathbf{q}_1, b_1$
3: Iterations: $t = 1 \sim T$
4:     $\mathbf{p}_t \leftarrow \mathbf{q}_t / \|\mathbf{q}_t\|_1$
5:     Choose $i_t \leftarrow i$ with probability $\mathbf{p}(i)$
6:     $coef \leftarrow y_{i_t} g\left(-y_{i_t}\left(\mathbf{w}_t{}^T \mathbf{x}_{i_t} + b_t\right)\right)$
7:     $\mathbf{u}_t \leftarrow \mathbf{u}_{t-1} + \frac{coef}{\sqrt{2T}} \mathbf{x}_{i_t}$
8:     $\xi_t \leftarrow \mathrm{argmax}_{\xi \in \Lambda}\left(\mathbf{p}_t{}^T \xi\right)$, if input $\nu$ for $\ell_2$-penalty
9:     $\mathbf{u}_t \leftarrow$ soft-thresholding operations, if input $\gamma$ for $\ell_1$-penalty
10:    $\mathbf{w}_t \leftarrow \mathbf{u}_t / \max\{1, \|\mathbf{u}_t\|_2\}$
11:    $b_t \leftarrow \mathrm{sgn}\left(\mathbf{p}_t{}^T \mathbf{y}\right)$
12:    Choose $j_t \leftarrow j$ with probability $\mathbf{w}_t(j)^2 / \|\mathbf{w}_t\|_2{}^2$
13:    Iterations: $i = 1 \sim n$
14:        $\sigma \leftarrow \mathbf{x}_i(j_t)\|\mathbf{w}_t\|_2{}^2/\mathbf{w}_t(j_t) + \xi_t(i) + y_i b_t$
15:        $\hat{\sigma} \leftarrow clip(\sigma, 1/\eta)$
16:        $\mathbf{q}_{t+1}(i) \leftarrow \mathbf{p}_t(i)\left(1 - \eta\hat{\sigma} + \eta^2\hat{\sigma}^2\right)$
17: Output: $\bar{\mathbf{w}} = \frac{1}{T}\sum_t \mathbf{w}_t, \bar{b} = \frac{1}{T}\sum_t b_t$
```

```
Procedure: Line 9 in Algorithm 1
Iterations: $j = 1 \sim d$
    if $\mathbf{uprev}_t(j) > 0$ and $\mathbf{u}_t(j) > 0$
        $\mathbf{u}_t(j) = \max(\mathbf{u}_t(j) - \gamma, 0)$
    if $\mathbf{uprev}_t(j) < 0$ and $\mathbf{u}_t(j) < 0$
        $\mathbf{u}_t(j) = \min(\mathbf{u}_t(j) + \gamma, 0)$
$\mathbf{uprev}_{t+1} \leftarrow \mathbf{u}_t$
```

In this sequential mode, each iteration takes $O(n + d)$ time, which is sublinear to the dataset size.

IV. Parallel Sublinear Logistic Regression

In this section, we will describe our parallel sublinear algorithms implemented in Hadoop and Spark. We will also formally introduce the traditional parallel gradient algorithm used in Spark and the online stochastic gradient descent method used in Mahout. In all pseudo-code, we follow symbol notations defined in Section III-A.

A. Parallel Sublinear Algorithms in Hadoop

We develop an algorithm to sublinear learning for logistic regression using the architecture of MapReduce. The pseudo-code of Algorithm 2, Procedure for Primal-Map, Procedure for Primal-Reduce, Procedure for Primal-Update, Procedure for Dual-Map, and Procedure for Dual-Update explain the critical parts of this algorithm.

```
Algorithm 2: PSUBPLR-MR
Input parameters: $\varepsilon, \nu\ or\ \gamma, X, Y, n, d$
Initialize parameters: $T, \eta, \mathbf{u}_0, \mathbf{w}_1, \mathbf{q}_1, b_1$
Iterations: $t = 1 \sim T$
    $\mathbf{w}_t \rightarrow$ storeInHdfsFile("hdfs://paraw").addToDistributedCache()
    $\mathbf{p}_t \rightarrow$ storeInHdfsFile("hdfs://parap").addToDistributedCache()
    conf_primal $\leftarrow$ new Configuration()
    job_primal $\leftarrow$ new MapReduce-Job(conf_primal)
    conf_primal.passParameters($T, n, d, b_t$)
    job_primal.setInputPath("...")
    job_primal.setOutputPath("tmp/primal$t$")
    job_primal.run()
    $(\mathbf{w}_{t+1}, b_{t+1})$←Primal-Update($\mathbf{w}_t, b_t$)
    Choose $j_t \leftarrow j$ with probability $\mathbf{w}_{t+1}(j)^2 / \|\mathbf{w}_{t+1}\|_2{}^2$
    $\mathbf{w}_{t+1} \rightarrow$ storeInHdfsFile("hdfs://paraw").addToDistributedCache()
    conf_dual $\leftarrow$ new Configuration()
    job_dual $\leftarrow$ new MapReduce-Job(conf_dual)
    conf_dual.passParameters($d, j_t, b_{t+1}, \eta$)
    job_primal.setInputPath("...")
    job_dual.setOutputPath("tmp/dual$t$")
    job_dual.run()
    $\mathbf{p}_{t+1}$←Dual-Update($\mathbf{p}_t$)
Output: $\bar{\mathbf{w}} = \frac{1}{T}\sum_t \mathbf{w}_t, \bar{b} = \frac{1}{T}\sum_t b_t$
```

```
Procedure: Primal-Map(inputfile)
Configuration.getParameters($T, n, d, b_t$)
$\mathbf{w}_t \leftarrow$ readCachedHdfsFile("paraw")
$\mathbf{p}_t \leftarrow$ readCachedHdfsFile("parap")
$i_t \leftarrow$ parseRowIndx(inputfile)
$\mathbf{x}_{i_t} \leftarrow$ parseRowVector(inputfile)
$y_{i_t} \leftarrow$ parseRowLabel(inputfile)
$r \leftarrow$ random(seed)
if $\mathbf{p}_t(i_t) > \frac{r}{n}$
    $tmp\_coef = \mathbf{p}_t(i_t) y_{i_t} g\left(-y_{i_t}\left(\mathbf{w}_t{}^T \mathbf{x}_{i_t} + b_t\right)\right)$
else
    $tmp\_coef = 0$
Iterations: $j = 1 \sim d$
    Set $key \leftarrow j$
    Set $value \leftarrow \frac{tmp\_coef}{\sqrt{2T}} \mathbf{x}_{i_t}(j)$
    Output(key, value)
```

```
Procedure: Primal-Reduce(key_in, value_in)
$key\_out \leftarrow key\_in$
$value\_out \leftarrow \sum_{for\ same\ key\_in} value\_in$
Output(key_out, value_out)
```

```
Procedure: Primal-Update($\mathbf{w}_t, b_t$)
$\Delta\mathbf{w}_t \leftarrow$ readFromHdfsFile("tmp/primal$t$")
$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t + \Delta\mathbf{w}_t$
$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_{t+1} / \max\{1, \|\mathbf{w}_{t+1}\|_2\}$
$b_{t+1} \leftarrow \mathrm{sgn}\left(\mathbf{p}_t{}^T\mathbf{y}\right)$
```

```
Procedure: Dual-Map(inputfile)
Configuration.getParameters($d, j_t, b_{t+1}, \eta$)
$\mathbf{w}_{t+1} \leftarrow$ readCachedHdfsFile("paraw")
$i_t \leftarrow$ parseRowIndx(inputfile)
$\mathbf{x}_{i_t} \leftarrow$ parseRowVector(inputfile)
$y_{i_t} \leftarrow$ parseRowLabel(inputfile)
$\sigma \leftarrow \mathbf{x}_{i_t}(j_t)\,\|\mathbf{w}_{t+1}\|_2{}^2 / \mathbf{w}_{t+1}(j_t) + y_{i_t} b_{t+1}$
$\hat{\sigma} \leftarrow clip\,(\sigma, 1/\eta)$
$res \leftarrow 1 - \eta\hat{\sigma} + \eta^2\hat{\sigma}^2$
$key \leftarrow i_t$
$value \leftarrow res$
Output(key, value)
```

This parallel design generally follows the framework of sequential sublinear algorithm. It remains to have two computational components in each iteration: the primal update part from line 4 to line 12 and the dual update part from line 13 to line 21. Within the primal update part, there is a parallel implementation period from line 4 to line 11, and also an unavoidable sequential period as illustrated in line 12. Within the dual part, it is the same situation that steps from line 14 to 20 employ parallel implementation while sequential operation is implemented in line 13 and line 21. As the primal part and dual part are decoupled in the sense of the parameters that they affect, these two separate MapReduce jobs can be performed simultaneously for each iteration, which brings us further parallelization and makes the algorithm more efficient. This framework is shown explicitly in Fig. 1.

For the parallelization period in the *primal mapreduce job*, we take advantage of the MapReduce design that takes in every data instance. Instead of the fashion in the sequential algorithm that we only sample one data instance in the primal update step, we compute gradients from "almost" all data instances and make the weighted average value according to vector $\mathbf{p}$ as the output gradient for update. The details of this algorithm design is shown in Procedure Primal-Map and Procedure Primal-Reduce. Here, we employ a randomization strategy when we compute those "almost" all gradients. From line 7 to line 8 in Procedure Primal-Map, all data instances are computed if $r = 0$. As the expectation value for $\mathbf{p}_t(i_t)$ is $\frac{1}{n}$, we normally set r to range from 0 to 1. For the parallelization period in the *dual mapreduce job*, we actually implement an embarrassingly parallel operation. It simply computes an individual value for each data instance in a parallel mode. It does not even need a reduce session.

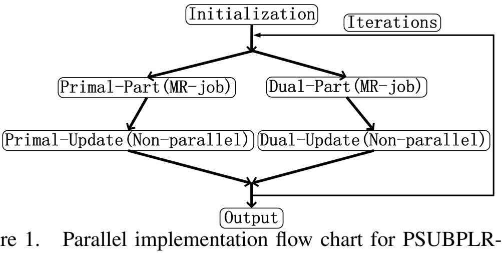


Figure 1. Parallel implementation flow chart for PSUBPLR-MR

```
Procedure: Dual-Update($\mathbf{p}_t$)
$\mathbf{var} \leftarrow$ readFromHdfsFile("tmp/dual$t$")
Iterations: $j = 1 \sim n$
    $\mathbf{p}_{t+1}(j) \leftarrow \mathbf{p}_t(j) * \mathbf{var}(j)$
```

There are also three more things that need to be handled here. First is the parameter passing issue. It is critical to choose an efficient way to pass the updated parameters between iterations and even between different MapReduce jobs. It is even more challenging when we have to deal with HDFS. It is clear that the fastest communication way is to pass parameters by *Configuration()*. However, it always suffers from memory buffer limits, and will greatly impair scalability when the number of parameters is huge. Another way is to compress the parameter sequence in a string when sending and uncompress them when receiving. It has computational overhead and it still cannot avoid the issue of memory buffer limit. Considering the philosophy of the design of Hadoop, the natural thing to do is to pass parameters by file, the same way to pass all the data. Although such an operation is insufficiently efficient, it is the feasible way to support large datasets.

The second issue is the small changes that we make to cater for the ℓ_2-penalty and ℓ_1-penalty. The changes are only made in Procedure Primal-Update. It affects little for the implementation, and the extension is just the same as that in Algorithm 1. To make the framework brief and standard, we omit the explanation here.

The datasets are generally sparse when data dimensionally is high. This characteristic makes us focus on dealing with data sparsity issue when writing code. This is the third important thing for the algorithm, although it is not reflected in the pseudo-code. Instead of naively writing the simple code in data intensive situations, we assign the *index* for each data value in the sparse vector, and all computations are changed accordingly. This sparse format brings us great efficiency improvement, and all results in Section VI are obtained from the code for sparsity.

B. Parallel Sublinear algorithms in Spark

The algorithm to solve sublinear learning for penalized logistic regression in Spark is shown below. In the pseudo-code of Algorithm 3, the procedure for Primal-Update and the procedure for Dual-Map are the same as those in Algorithm PSUBPLR-MR.

This parallel design is very similar to that of Algorithm PSUBPLR-MR. The most important difference is the *cache()* operation in line 3. To make it work in Spark, we follow

Algorithm 3: PSUBPLR-SPARK

```
Input parameters: $\varepsilon, \nu$ or $\gamma, X, Y, n, d$
Initialize parameters: $T, \eta, \mathbf{u}_0, \mathbf{w}_1, \mathbf{q}_1, b_1$
points $\leftarrow$ spark.textFile(inputfile).map(parsePoint()).cache()
Iterations: $t = 1 \sim T$
    gradient $\leftarrow$ points.map(($\frac{1}{1+e^{-y((\mathbf{w}_t^T\mathbf{x})+b)}} - 1) * y * \mathbf{p}[index]$)
        .reduce(sum())
    $(\mathbf{w}_{t+1}, b_{t+1})$←Primal-Update$(\mathbf{w}_t, b_t)$
    Choose $j_t \leftarrow j$ with probability $\mathbf{w}_{t+1}(j)^2/\|\mathbf{w}_{t+1}\|_2{}^2$
    pAdjust $\leftarrow$ points.map(MW-Update()).reduce(copy())
    $\mathbf{p}_{t+1}$←Dual-Update$(\mathbf{p}_t)$
Output$(\mathbf{w}, b)$
```

the rule to construct an RDD for each data instance. Also to cater for data sparsity, the design is that every data value correspond to its individual *index*. And the *index* is also involved in the computation along with the value. We omit the changes for ℓ_2-penalty and ℓ_1-penalty here to make the algorithm easier to be understood.

In parallel mode, the primal update contains the update of $\mathbf{w}_t$, which takes $O(n)$ time. The dual update contains the ℓ_2-sampling process for the choice of j_t in $O(d)$ time, and the update of $\mathbf{p}$ in $O(1)$ time. Altogether, each iteration takes $O(n+d)$ time. Compared to the analysis of sequential algorithm, parallelization does not necessarily change computational complexity. Theoretically, by deeper analysis of constant coefficients of the obtained computational complexity, parallel sublinear algorithm can be two times faster than the sequential version. Moreover, by starting two separate MapReduce jobs in one iteration simultaneously, the running time can be reduced to $O(max\{n, d\})$.

C. Parallel Gradient Descent in Spark

The parallel gradient descent method to solve LR in Spark is shown below.

Algorithm 4: PGDPLR-SPARK

```
Input parameters: $\varepsilon, \nu$ or $\gamma, X, Y, n, d$
Initialize parameters: $T, \eta, \mathbf{u}_0, \mathbf{w}_1, \mathbf{q}_1, b_1$
points $\leftarrow$ spark.textFile(inputfile).map(parsePoint()).cache()
Iterations: $t = 1 \sim T$
    gradient $\leftarrow$ points.map(($\frac{1}{1+e^{-y((\mathbf{w}_t^T\mathbf{x})+b)}} - 1) * y$)
        .reduce(sum())
    $\mathbf{w}_{t+1} = \mathbf{w}_t - gradient * \mathbf{x}$
    $b = b - gradient$
Output$(\mathbf{w}, b)$
```

In the pseudo-code of Algorithm 4, it is implemented in an embarrassingly parallel mode. We can take in all data at the same iteration and just compute the gradient in a MapReduce fashion. As for the *cache()* operation and RDD design for data sparsity, it is the same with Algorithm PSUBPLR-SPARK.

D. Online Stochastic Gradient Descent in Mahout

Though SGD is an inherently sequential algorithm, it is blazingly fast. Thus Mahout's implementation can handle training sets of tens of millions of examples. The SGD system in Mahout is an online learning algorithm, implying that we can learn models in an incremental fashion and perform testing when the system runs. In addition, we can halt training when a model reaches a target level of performance. The SGD framework includes classes to do online evaluation using cross validation (the *CrossFoldLearner*) and an evolutionary system to hyper-parameter optimization on the fly (the *AdaptiveLogisticRegression*). The *AdaptiveLogisticRegression* system makes heavy use of threads to increase machine utilization. To do so, it runs 20 *CrossFoldLearners* in separate threads, each with slightly different learning parameters. As better settings are found, these new settings propagate to the other learners.

Because the SGD algorithms need feature vectors of fixed length and it is a pain to build a dictionary ahead of time, most SGD applications use the encoding system that is rooted at *FeatureVectorEncoder* to derive hashed feature vectors. The basic idea is that you create a vector, typically a *RandomAccessSparseVector*, and then you use various feature encoders to progressively add features to that vector. The size of the vector should be large enough to avoid feature collisions as features are hashed. There are specialized encoders for a variety of data types. You can normally encode either a string representation of the value you want to encode or you can encode a byte representation to avoid string conversion. In the case of *ContinuousValueEncoder* and *ConstantValueEncoder*, it is also possible to encode a null value and pass the real value as a weight.

In our implementation, we use *RandomAccessSparseVector* for data sparsity and the function call by *OnlineLogisticRegression* to train the LR classifier. We also perform cross validation. However, to keep consistent with other methods, we write our own code to do cross validation.

V. Experiments Setup

In this section, we explains details of the datasets information, cluster information, and the test programs of our experiments.

A. Datasets Information

We choose five open datasets to run all six test programs: The Simulated **2d** dataset has 2 features and 200 instances. The **20NewsGroup** dataset has 16428 features and 1988 instances. We split it into a training set of 1800 instances and a test set of 188 instances. The third test dataset is the **Gisette** [10] dataset, which has 5000 features and 7000 instances. We split it into a training set of 6000 instances and a test set of 1000 instances. The fourth test dataset is the **ECUESpam** [7] dataset, which has 100249 features and 10678 instances (after proper preprocessing). We split it into a training set of 9000 instances and a test set of 1687 instances. Finally, we have the **URL-Reputation** [18] dataset, which has 3231961 features and 2376130 instances.

We split it into a training set of 2356130 instances and a test set of 20000 instances. Apart from the Simulated **2d** dataset, other four datasets are all sparse. Details are shown in Table II. We randomly repeat such split 20 times and our analysis is based on the average performance of 20 repetitions.

In Table II, *Density* is computed as

$$Density(Dataset) \;=\; \frac{\#\ of\ Nonzeros}{Total\ \#\ of\ entries},$$

which is adopted from [22]. *Balance* describes the binary distribution of labels. We compute it as following:

$$Balance(Dataset) \;=\; \frac{\#\ of\ Positive\ Instances}{\#\ of\ Negative\ Instances}.$$

We carefully select these five datasets. The Simulated **2d** dataset can be viewed for visual result and serves for initial test of correctness of implementation. The **20NewsGroup** dataset is best-known for the test of LR model, which has a balanced distribution between positive and negative data instances. The **Gisette** dataset is relatively larger, and data is less sparse. Thus it requires more computation theoretically. The **ECUESpam** dataset features for its imbalanced distribution between positive and negative data instances. It has a higher data dimensionality than **Gisette** dataset. However, because of data sparsity, it has fewer nonzero values involved in the computation. Finally, the **URL-Reputation** dataset contains millions of data instances and features. The raw data is stored in the svmlight format, which has the size of more than 2GB. It achieves the current level for massive dataset, and exceed the scalability of Liblinear, which will be shown below.

Table II
DATASETS

Name	n	d	Density	Balance
2d	2	200	1.0	1.000
20NewsGroup	16428	1988	7.384×10^{-3}	1.006
Gisette	5000	7000	0.12998	1.000
ECUESpam	100249	10687	2.563×10^{-3}	5.882
URL-Reputation	3231961	2376130	3.608×10^{-5}	0.500

B. Cluster Information

The cluster that we implement on has the following configurations shown in Table III. This kind of cluster configuration is now more and more common in research labs. Hence our results reported in the following can have a real impact in the research community.

Table III
CLUSTER INFORMATION

CPU Model	Intel Xeon E5-1410: 2.80GHz
Number of node	6
Number of CPU per node	4 Cores, 8 Threads
RAM per node	16G
Disk per node	4T HDD
Interconnection Method	Gigabyte Ethernet

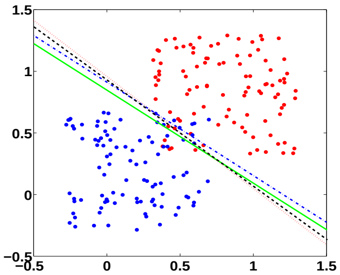


Figure 2. Visual Result on Simulated **2d** Dataset

C. Test Programs

There are all together 6 test programs for comparison. Online stochastic gradient descent method is run on Mahout in a sequential mode. Liblinear [8] is the baseline test program we choose. It is sequential and it outperforms many other programs for LR on a single machine. SLLR performs sequential sublinear algorithm on a single machine. PSUBPLR-MR performs parallel sublinear algorithm on Hadoop on cluster. PGDPLR-SPARK is the test program for parallel gradient descent run on Spark. PSUBPLR-SPARK implements parallel sublinear algorithm run on Spark.

VI. EXPERIMENTAL RESULTS

In this section, we conduct a thorough empirical analysis.

A. Results on Simulated 2d Dataset

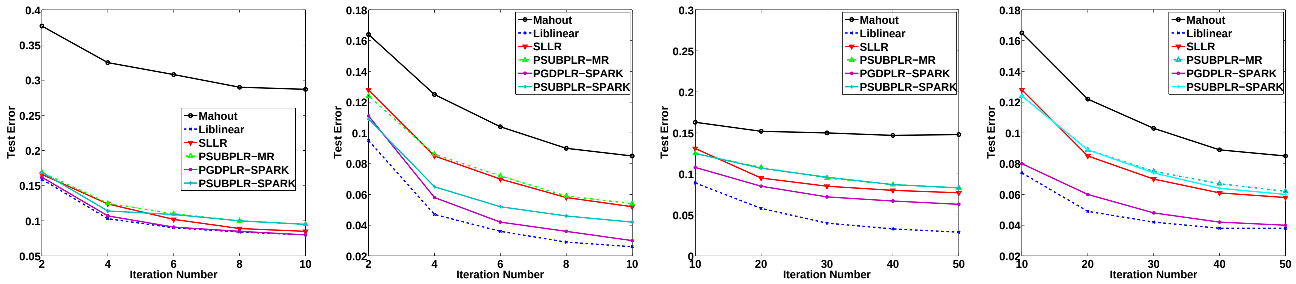
Fig. VI-A shows the visual result for Simulated **2d** Dataset. Here we use all data for training and testing, thus ”Test Error” here can be interpreted as ”Training Error”. The blue line represents the learned separation vector of Mahout. The black line indicates the result for Liblinear. The green line shows the result for all sublinear methods, including SLLR, PSUBPLR-MR and PSUBPLR-SPARK. The red line is the result for PGDPLR-SPARK. The results on the Simulated **2d** Dataset proves the correctness of all test programs.

B. Results on Precision

Accuracy of six test programs achieved on five datasets are shown in Table IV. These are averaged results of cross validation. In Figures 3 (a)-(d), we show the test error as a function of iteration number on each dataset for all six test programs. Note that, Liblinear can not be implemented with the full **URL-Reputation** dataset on our machine due to

Table IV
ACCURACY RESULTS. THE MEANING OF ABBREVIATIONS ARE AS FOLLOWS: 20-N-G, 20 NEWS GROUP; URL-R, URL-REPUTATION.

	20-N-G	Gisette	ECUESpam	URL-R
Mahout	71.3%	91.5%	85.2%	91.5%
Liblinear	92.0%	97.4%	97.1%	—
SLLR	91.5%	94.8%	92.3%	94.2%
PSUBPLR-MR	90.5%	94.6%	91.7%	93.8%
PGDPLR-SPARK	92.0%	97.0%	93.7%	96.0%
PSUBPLR-SPARK	90.5%	95.8%	91.7%	94.0%



(a) **20NewsGroup**.

(b) **Gisette**.

(c) **ECUESpam**.

(d) **URL-Reputation**.

Figure 3. Test error, as a function of iteration number.

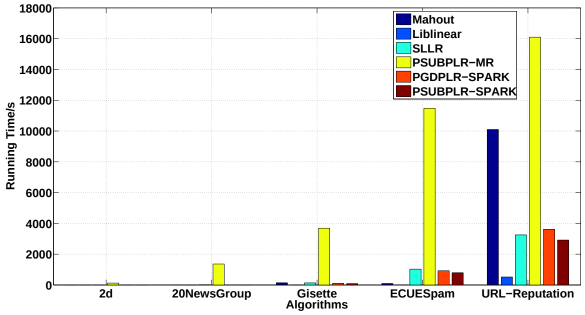


Figure 4. All running times

memory limit. This proves the scalability issue of Liblinear. In the tests, Liblinear can handle 33.3% of the whole data volume at most, with a test accuracy of 96.2% and running time of 519s.

C. *Results on Running Time*

Running Time of six test programs used on five datasets are shown in Table V. These are averaged results corresponding to the accuracy we achieved in Table IV.

Table V
RUNNING TIME RESULTS. THE MEANING OF ABBREVIATIONS ARE AS FOLLOWS: 20-N-G, 20 NEWS GROUP; URL-R, URL-REPUTATION.

	20-N-G	Gisette	ECUESpam	URL-R
Mahout	9.83s	131.8s	96s	10100s
Liblinear	0.79s	2.4s	13s	—
SLLR	20.05s	130.5s	1028s	3248s
PSUBPLR-MR	1360.85s	3687.9s	11478s	16098s
PGDPLR-SPARK	10.52s	99.2s	924s	3615s
PSUBPLR-SPARK	8.57s	89.1s	796s	2918s

In another way, we show the running time results in Fig. VI-C for comparison.

From the accuracy results in section VI-B and running time results in section VI-C. We can come up with following conclusions.

1) Liblinear always performs the best. It fully utilizes memory and single machine implementation does not require any communication between machines, which causes overhead. However, its scalability is limited by the memory size of the single machine that it runs on. This prevents its use in massive datasets. Nevertheless, if the dataset can be fully loaded into a single machine's memory, this efficient and direct sequential way of implementation is still recommended.

2) Mahout's precision is not good, especially for those datasets in which positive and negative instances are not balanced. However, it is a representative of sequential algorithm that can train massive data in acceptable time. When we monitor its memory when running, it is much lower than Liblinear. This is the advantage brought by both online algorithm and hashing operations on features. Thus, it provides good scalability guarantee. If you are in the situation of single machine and limited memory, sequential online training like Mahout is recommended.

3) The precision of all sublinear methods is acceptable. The developed parallel sublinear algorithm only has a small drop in precision.

4) Hadoop system has a great drawback for running LR optimization methods. Its cluster programming model is based on acyclic data flow from stable storage to stable storage. Though it has the benefits of deciding where to run tasks and can automatically recover from failures in runtime, acyclic data flow is inefficient for iterative algorithms. All six test programs contains a number of iterations, thus making PSUBPLR-MR performs poorly, even worse than sequential algorithms. When we study the details of Hadoop implementation, we find the MapReduce job starting time in PSUBPLR-MR is about 20s. It consists of task configuration time and parameter passing time. This running time overhead is not negligible, especially when dataset is relatively small. We also identify the running of ”Primal-Map” dominates the whole iteration time (more than 66%). It is the same situation for PSUBPLR-SPARK.

5) Spark employs the "all-in-memory" strategy, and it constructs RDD on demand. We implement PGDPLR-SPARK and PSUBPLR-SPARK in normal file system instead of HDFS. Current results on Spark show that it is much more efficient than Hadoop, and performs better than Mahout. In the parallel situation, and have massive data to process, we recommend using Spark, and can choose PSUBPLR-SPARK for less time in exchange for a little bit precision loss. As the platform is premature and still under developing process, we expect our running time results for PGDPLR-SPARK

and PSUBPLR-SPARK can still be improved. For example, Spark currently only supports for one reducer, thus it does not use full CPU power when doing reduce jobs.

6) Another interesting point we would like to raise is about dataset. If we compare **ECUESpam** dataset and **GISETTE** dataset. The former has has higher data dimensionality but more sparse. We can find that algorithms on **ECUESpam** dataset enjoy less running time per iteration as the dataset has fewer nonzero values involved in the computation. However, algorithms on **ECUESpam** dataset need more iteration number, which is related to its higher data dimensionality. This even causes more running time in total. To sum up, to get a general sense of running time for individual dataset, we have to consider both data volume and sparsity.

D. Results on Changing Cluster

In Figures 5 (a)-(d), we show the running time as a function of used node number on each dataset for all three parallel test programs.

As we can see from the results, fewer computation resources provided, longer the implementation process will take. However, this trend is not in a consistent fashion. When number of machines gets higher, the less impact it will make. We may reach a point for every algorithm when running time stops decreasing as machine number increases. This shows the true scalability of the designed algorithm, which we can study in our future work.

E. Fault Tolerance

There are both system level techniques and algorithm level techniques to provide fault tolerance in parallel computation. Results are shown in Table VI. Here, randomization is employed in all sublinear methods, thus provides algorithm level fault tolerance for PSUBPLR-MR and PSUBPLR-SPARK. Hadoop and Spark can both provide system level fault tolerance, but in different ways. Hadoop employs HDFS while Spark has RDDs. Of all six test programs, PSUBPLR-MR and PSUBPLR-SPARK are fault tolerant in both system level and algorithm level.

Table VI
FAULT TOLERANCE ANALYSIS

	System Level	Algorithm Level
PSUBPLR-MR	✓	✓
PGDPLR-SPARK	✓	×
PSUBPLR-SPARK	✓	✓

We show the iteration time as a function of percentage of failed maps on **URL-Reputation** dataset for all three parallel test programs in Fig. VI-E. This is tested on 6 nodes. As the outcome shows, PSUBPLR-MR suffers no running time increase per iteration when maps failed. But it takes the cost of precision loss if we enforce no additional iterations are implemented. PGDPLR-SPARK and PSUBPLR-SPARK will increase iteration time right after the maps failed. It is due to RDD reconstruction. The increase is not significant. This difference also proves the different mechanism of fault tolerance between Hadoop and Spark.

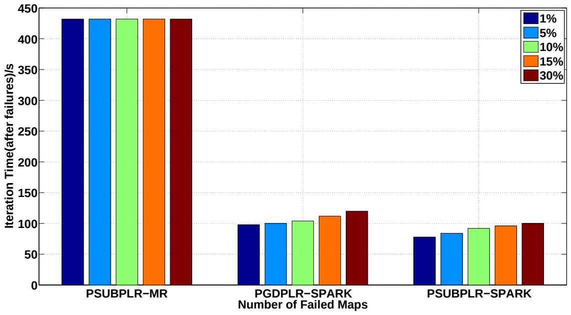


Figure 6. Iteration time, as a function of percentage of failed maps, on **URL-Reputation** Dataset, run on 6 nodes

VII. Conclusion

In this paper we analyzed three optimization approaches along with three computing platforms to train LR model with large-volume, high-dimensional dataset for classification. Test results show that different algorithms and different computing platforms have their own advantages. We can intelligently choose between them according our needs and resources. Another contribution of this paper is to present a novel parallel sublinear algorithm for LR implemented on both Hadoop and Spark. We realize this paper has its limitation in both machine resources and datasets. It is exactly we are heading for: More machines and larger datasets will bring more convincing results. We also plan to employ the research angle of combining algorithm design and computing platform to study other machine learning models, thus giving more valuable suggestions in practical applications.

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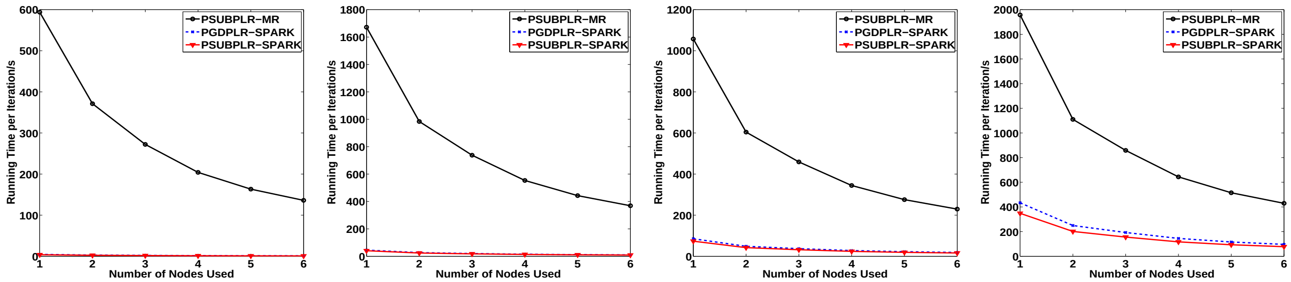

(a) **20NewsGroup**. (b) **Gisette**. (c) **ECUESpam**. (d) **URL-Reputation**.

Figure 5. Running time, as a function of used node number.

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