Joshua Kim CSCI 104 4th

Homework 5 Counting Problems

1. Unusual 3's = 5 unique letters unique subsets = subsets w/ 1 '4'

+2 '4's +3 '4's = $1 + \binom{4}{3} + \binom{4}{2} = 1+4+6 = 11$

dif. strings = $5! + \left(4 \cdot \frac{5!}{2!}\right) + \left(6 \cdot \frac{5!}{3!}\right) = 480$

2. 52 cards

pair combos = $\binom{13}{2}$ suit combos = $\binom{4}{2}$

2 pairs = $\binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2}$ 1 extra = $\binom{11}{1} \cdot \binom{4}{1}$ (remaining)

ways = $\frac{\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$ ← all possible

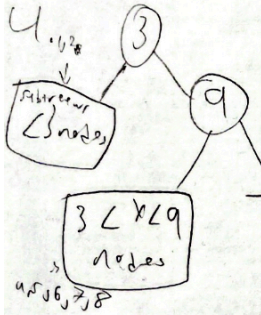
= 123,552

3. 16 songs 7 couples 1 couple ≥ 1 song = 0 or 1 song total

rest of couples split 16 songs (0 songs – couple – 1 song)

$\binom{16+6-1}{6-1} + \binom{15+6-1}{6-1} = \binom{21}{5} + \binom{20}{5} = 35,853$

4. 168



BST of 2 nodes → (1,2) = 2! = 2

BST of 3 nodes → left: 2 right: 2! = 5

BST of 4 nodes → left: 3 right: 0 / 1 / 2 / 3 etc etc → 14 + 5 + 4 + 5 + 14 = 42

42 · 2 · 5 = 420

5. 10 friends 4 identical nurses 1 maybe break

on break

8 1 1

7 1 2

7 2 1 identical

6 1 3

etc ↓ 8 ways

+

no break

7 1 1 1

6 1 2 1

etc ↓ 9 ways

= 17