Mathematics 166, Sec. A3 Exam 4-Y December 7, 2010

Read all directions carefully. Do any 4 of the 5 problems. You must indicate CLEARLY the problem not to be counted for credit. If you do all five problems and do not indicate one to be omitted, then *your grade will be the sum of your scores on the first four problems.* Show all work; partial credit may be given but only if correct work and reasoning are shown and explained. Correct answers without work shown may not receive full credit.

1. For each of the following, determine, with reasons, whether the series converges or diverges.

a. $\displaystyle\sum_{k=0}^{\infty} \frac{3}{2k^2-1}$. Conv – Comparison

b. $\displaystyle\sum_{\ell=1}^{\infty} \frac{2\ell^2+1}{3^\ell}$. Conv. – Ratio

c. $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(\ln r+2)^r}$. Conv. – Comparison (to $\sum \frac{1}{2^r}$)

d. $\displaystyle\sum_{m=1}^{\infty} (-1)^m\sqrt{1+\frac{1}{m}}$. Div. – n^{th} term test

2. Find the convergence set for the power series $\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{n+3}$.

Rad of Conv $= \frac{1}{3}$

Conv. for $\frac{5}{3} \le x < \frac{7}{3}$

3. Let $f(x) = e^{2x}$.

a. Find $P_3(x)$, the Taylor polynomial of degree 3 for f about the point $c = 0$.

$$P_3(x) = 1 + (2x) + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} \quad (\text{sub } t = 2x \text{ in } e^t \text{ series})$$

$$= 1 + 2x + 2x^2 + \frac{4}{3}x^3$$

b. Estimate the absolute error $|R_3(x)| = |e^{2x} - P_3(x)|$ if we use the P_3 from part a. to approximate e^{2x} on the interal $-1 \le x \le 1$.
(You may use the estimate

$$|R_n(x)| \le M_{n+1} \frac{|x-c|^{n+1}}{(n+1)!},$$

where M_{n+1} is the maximum value of $|f^{(n+1)}(x)|$ on the interval in question.)

$$|e^{2x} - P_3(x)| \le \frac{M_4 |x|^4}{4!} \le \frac{(16e^2) \cdot 1^4}{4!}$$

$$M_4 = \text{any } \# \ge 16e^2$$

4. a. Find a power series for $g(x) = \frac{2}{3+4x}$ in powers of $(x-2)$.

$$\boxed{\frac{2}{11} \sum_{k=0}^{\infty} (-1)^k \frac{4^k (x-2)^k}{11^k}}$$

b. Find a formula $f(x)$ for the sum of the power series $\sum_{k=1}^{\infty} k(x-2)^k$.

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$$\frac{1}{1-t} = 1 + t + t^2 + \cdots$$

So $\frac{1}{(1-t)^2} = 1 + 2t + 3t^2 + 4t^3 + \cdots \Rightarrow \frac{t}{(1-t)^2} = t + 2t^2 + 3t^3 + \cdots = \sum_{k=1}^{\infty} k t^k$

~~Sub t = x-2~~

Let x−2 replace t to see

$$\boxed{\sum_{1}^{\infty} k(x-2)^k = \frac{x-2}{(1-(x-2))^2} = \frac{x-2}{(3-x)^2}}$$

5. Consider the power series $\sum_{k=0}^{\infty} a_k(x-3)^k$, and assume the series has convergence set $-1 < x \leq 7$.

a. What is the radius of convergence of this series?

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b. Consider the infinite series $\sum_{k=0}^{\infty} a_k(x+4)^k$, where the a_ks are the same as for the above series. What is the convergence set for this series?

$-8 \leq x < 0$

$-8 < x \leq 0$

c. Consider the infinite series $\sum_{k=0}^{\infty} a_k \frac{(x-3)^k}{3^k}$, where the a_ks are the same as for the above series. What is the convergence set for this series?

$-9 < x \leq 12$