Name: Answer Key Section: Group #:

Exam 3 Review

1. Suppose on day 1, three people know a secret. The next day (day 2), they each tell five new people. The following day, those five people tell five additional people. If this pattern were to continue, how many people will learn the secret on day 5? Define a function whose input is the day, t, and whose output is the number of people who learned the secret on day t.

1875 people learn the secret on day 5

$f(t) = 3 \cdot 5^{t-1}$

2. Consider the following tables of values. Determine if they represent a linear function, an exponential function, or neither. If it represents a linear or expenential function, write out the function.

(a)

x	-4	-3	0	2	4
$f(x)$	0.125	0.5	1	4	32

Neither

(b)

x	-4	-3	0	2	4
$g(x)$	3	3.5	5	6	7

Linear, $g(x) = \frac{1}{2}x + 5$

(c)

x	-4	-3	0	2	4
$h(x)$	16	8	1	0.25	0.0625

Exponential, $h(x) = \left(\frac{1}{2}\right)^x$

3. Suppose you are investing \$1000 into a bank account that is compounded continuously. What interest rate is needed to triple your investment in 50 years?

$r = \frac{\ln 3}{50}$

4. Suppose $f(t) = 756(1.03)^t$ is a function that gives the population of a city (in thousands of people) after t years.

   (a) What is the initial population?

   $756{,}000$

   (b) What is the annual percent change of the population?

   3% increase

   (c) What is the one-year growth factor?

   1.03

   (d) What is the one-month growth factor?

   $(1.03)^{1/12}$

   (e) What is the ten-year growth factor?

   $(1.03)^{10}$

   (f) How many years will it take for the initial population to double?

   $t = \frac{\ln 2}{\ln(1.03)}$

5. Solve the following for x:

   (a) $\log_5(25x^2) = 6$

   $x = \pm 25$

   (b) $\log_2(2+x) + \log_2(7) = 3$

   $x = \frac{-6}{7}$

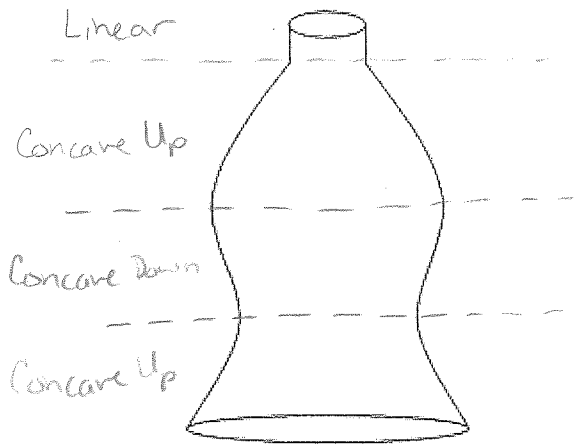
   (c) $\ln(e^{2x}) = 16$

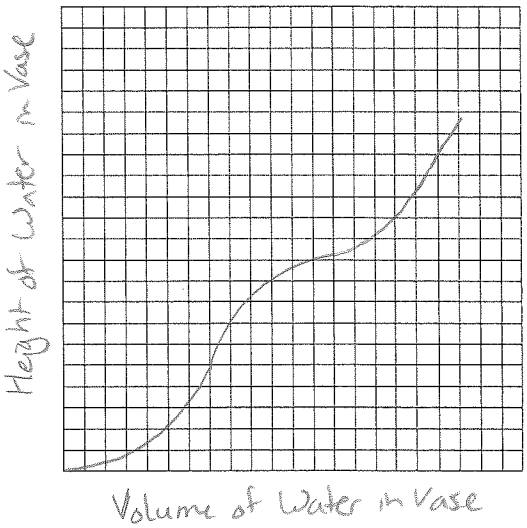
   $x = 8$

   (d) $3^{\log_3(6x)} = 12$

   $x = 2$

6. Suppose water is poured into the vase below at a constant rate. That is, the volume is increasing at a constant rate. Give a rough sketch of the graph that represents the height of the water in the vase as a function of the volume of the water in the vase. On your graph, indicate on which intervals the graph is linear, concave up, and concave down.





7. Consider the table of values for the function $f(x)$:

x	0	3	5	6
$f(x)$	1	1.5	1.9	2.2

Based on this information, is the graph of $f(x)$ concave up, concave down, both concave up and concave down, or neither? Why?

Concave Up

8. Without graphing, determine if the function $m(x) = \frac{6}{x^2-1}$ has any vertical asymptotes. If so, where?

Vertical Asymptotes at $x=-1$ and $x=1$

9. Let $P(t)$ be the population of a country, in millions, t years after 1990, with $P(5) = 3.33$ and $P(13) = 3.77$.

   (a) Find a formula for $P(t)$ assuming that it is linear.

   $P(t) = .055t + 3.055$

   (b) Find a formula for $P(t)$ assuming that it is exponential.

   $P(t) = 3.0815(1.0156)^t$

10. If you need \$20,000 eight years from now, what is the minimum amount of money you need to deposit into a bank account that pays 7% annual interest, compounded

    (a) annually?

    \$ 11640.18

    (b) monthly?

    \$ 11442.78

    (c) continuously?

    \$ 11424.18

11. Find the exact solution to the equation below.

$$ae^{2x+1} = b$$

$x = \frac{\ln(\frac{b}{a}) - 1}{2}$

12. Suppose that $u = \log(4)$ and $v = \log(5)$. Find a possible formula for $\log(0.025)$ in terms of u and v.

$2v - 3$

*Note: There is more than one correct answer to this problem

13. After a cup of hot chocolate is poured, the temperature cools off very rapidly at first, and then cools off more slowly, until the temperature of the hot choclate eventually reaches room temperature. Let $F(t)$ denote the temperature of the hot chocolate t minutes after being poured.

    (a) Based on this description, is the function $F(t)$ increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing? Why?

Decreasing

    (b) Based on this description, will the graph of the function $F(t)$ be concave up, concave down, both concave up and concave down, or linear? Why?

Concave Up

14. Let $g(x) = \dfrac{-1}{(x-3)^3}$.

(a) Find $\lim_{x \to 3^+} g(x)$. $= -\infty$

(b) Find $\lim_{x \to 3^-} g(x)$. $= +\infty$

15. For each of the following functions, determine the polynomial or constant that the rational function behaves like as $x \to \pm\infty$

(a) $f(x) = \dfrac{2x^2+9}{x^3-8}$

Looks like $y = 0$

(b) $g(x) = \dfrac{7x^4+9}{x^2+2}$

Looks like $y = 7x^2$

(c) $h(x) = \dfrac{x^3+9}{2x^3+5}$

Looks like $y = \frac{1}{2}$

16. What is the length of an arc cut off by an angle of $\frac{7\pi}{15}$ radians on a circle of radius 3 inches?

$\frac{7\pi}{5}$

17. What angle (measured in radians) corresponds to 0.75 rotations around a circle of radius 10 centimeters?

$\frac{3\pi}{2}$

18. A circle of radius 5 is centered at the point $(-6, 7)$. Let P be the point on the circle where the terminal ray for the angle θ intersects the circle. *Hint:* Sketch a picture.

    (a) Find a formula for $f(\theta)$, the x-coordinate of the point P.

$f(\theta) = 5\cos(\theta) - 6$

    (b) Find a formula for $g(\theta)$, the y-coordinate of the point P.

$g(\theta) = 5\sin(\theta) + 7$