googleai-gemini-2.5-pro

Here are the solutions for questions C.4 and C.5.

C.4 Solution

Objective: Determine the relative radial velocity ($v_{r,E/S}$) and the galactic radius (R) for the three sources observed in Figure 3.

Given Data:

- Sun's orbital velocity, ($v_\odot = 220 \, \text{km/s}$)
- Sun's orbital radius, ($R_\odot = 8.00 \, \text{kpc}$)
- Galactic longitude of observation, ($l = 30^\circ$)
- Rest frequency of the 21 cm line, ($f_0 = 1.42 \, \text{GHz} = 1.42 \times 10^9 \, \text{Hz}$)
- Speed of light, ($c \approx 3.00 \times 10^5 \, \text{km/s}$)

Step 1: Determine frequency shifts from Figure 3.

We identify the three peaks in the flux shown in Figure 3 and read their corresponding frequency shifts, ($\Delta f = f - f_0$).

- **Source 1:** The first peak is at ($\Delta f_1 = +0.16 \, \text{MHz}$).
- **Source 2:** The second, most prominent peak is at ($\Delta f_2 = +0.28 \, \text{MHz}$).
- **Source 3:** The third peak is at ($\Delta f_3 = +0.35 \, \text{MHz}$).

Step 2: Calculate the relative radial velocities (($v_{r,E/S}$)).

The relative radial velocity is found using the non-relativistic Doppler shift formula: $$v_{r,E/S} = -c \frac{\Delta f}{f_0}$$ A positive (Δf) (blueshift) corresponds to a negative radial velocity (the source is moving towards the observer).

- **Source 1:** $$v_{r1} = -(3.00 \times 10^5 \, \text{km/s}) \times \frac{0.16 \times 10^6 \, \text{Hz}}{1.42 \times 10^9 \, \text{Hz}} = -33.80 \, \text{km/s}$$ To 3 significant digits, ($\boldsymbol{v_{r1} = -33.8 \, \text{km/s}}$).

- **Source 2:** $$v_{r2} = -(3.00 \times 10^5 \, \text{km/s}) \times \frac{0.28 \times 10^6 \, \text{Hz}}{1.42 \times 10^9 \, \text{Hz}} = -59.15 \, \text{km/s}$$ To 3 significant digits, ($\boldsymbol{v_{r2} = -59.2 \, \text{km/s}}$).

- **Source 3:** $$v_{r3} = -(3.00 \times 10^5 \, \text{km/s}) \times \frac{0.35 \times 10^6 \, \text{Hz}}{1.42 \times 10^9 \, \text{Hz}} = -73.94 \, \text{km/s}$$ To 3 significant digits, ($\boldsymbol{v_{r3} = -73.9 \, \text{km/s}}$).

Step 3: Calculate the distances from the galactic center ((R)).

From question C.3, the relationship between the relative radial velocity and the orbital radii is given by: $$v_{r,E/S} = v_\odot \sin l \left(\frac{R_\odot}{R} - 1 \right)$$ We rearrange this formula to solve for (R): $$R = \frac{R_\odot}{1 + \frac{v_{r,E/S}}{v_\odot \sin l}}$$ We are given ($l=30^\circ$), so ($\sin l = 0.5$). The denominator term is ($v_\odot \sin l = 220 \, \text{km/s} \times 0.5 = 110 \, \text{km/s}$).

- **Source 1:** $$R_1 = \frac{R_\odot}{1 + \frac{-33.8}{110}} = \frac{R_\odot}{1 - 0.3073} = \frac{R_\odot}{0.6927} \approx 1.444 \, R_\odot$$ To 2 significant digits, ($\boldsymbol{R_1 = 1.4 \, R_\odot}$).

- **Source 2:** $$R_2 = \frac{R_\odot}{1 + \frac{-59.2}{110}} = \frac{R_\odot}{1 - 0.5382} = \frac{R_\odot}{0.4618} \approx 2.165 \, R_\odot$$ To 2 significant digits, ($\boldsymbol{R_2 = 2.2 \, R_\odot}$).

- **Source 3:** $$R_3 = \frac{R_\odot}{1 + \frac{-73.9}{110}} = \frac{R_\odot}{1 - 0.6718} = \frac{R_\odot}{0.3282} \approx 3.047 \, R_\odot$$ To 2 significant digits, ($\boldsymbol{R_3 = 3.0 \, R_\odot}$).

C.5 Solution

Objective: Indicate the positions of the sources on a top-down view of the galaxy and state what can be deduced from repeated measurements at different longitudes (l).

1. Position of the Sources:

To indicate the positions, one would draw a top-down diagram of the galactic plane:

1. Place the Galactic Center (C) at the origin.
2. Draw a circle of radius ($R_\odot$) centered at C to represent the Sun's orbit. Place the Sun (S) at a point on this circle.
3. Draw a ray starting from C and passing through S.
4. Draw a second ray starting from S (the line of sight) such that the angle between the ray CS and the line of sight is ($l=30^\circ$).
5. Draw three more circles centered at C with radii ($R_1 = 1.4 R_\odot$), ($R_2 = 2.2 R_\odot$), and ($R_3 = 3.0 R_\odot$).
6. The positions of the three sources (E1, E2, E3) are the intersection points of their respective orbital circles with the line of sight. All three sources will lie on the line of sight, at increasing distances from the Sun.

2. Deduction from Repeated Measurements:

By repeating these measurements for a wide range of galactic longitudes ((l)), astronomers can perform a comprehensive survey of the galaxy. This allows them to deduce fundamental properties of the Milky Way:

- **Mapping the Galactic Structure:** The measurements reveal the location of neutral hydrogen gas clouds. Since this gas is a primary constituent of the spiral arms, plotting the positions of these sources for many different longitudes allows astronomers to trace the spiral structure of our own galaxy, something that is impossible to do with a single visual image from our position inside it.

- **Determining the Galactic Rotation Curve:** By analyzing the full set of radial velocity data across different longitudes, especially by using methods like the "tangent point method" (which measures the maximum velocity along each line of sight), it is possible to determine the orbital velocity ($v_c(R)$) as a function of the distance (R) from the galactic center. This produces the galaxy's rotation curve.

- **Inferring Mass Distribution and Dark Matter:** The rotation curve ($v_c(R)$) is directly related to the mass ($M(R)$) enclosed within radius (R) by the formula ($M(R) = v_c(R)^2 R / G$). By measuring the rotation curve, one can calculate the mass distribution throughout the galaxy. The discrepancy between this dynamically calculated mass and the mass of visible matter (stars, gas, dust) provides the primary evidence for the existence and distribution of dark matter in our galaxy.