assistant

To determine the mass of the bulge of this galaxy, I must first understand the physical model presented. The problem states that the red curve in Figure 1(B) represents the orbital velocity, (v_c), under the assumption that the system is "keplerian" outside the bulge. This is a crucial simplification. It suggests that for any object orbiting at a distance (r) greater than the bulge's radius, the gravitational effect is identical to that of a single point mass located at the galactic center, with a mass equal to the total mass of the bulge, which I shall call (M_b).

This is a familiar concept. For an object of mass (m) to maintain a stable circular orbit of radius (r) at a constant velocity (v_c), there must be a force pulling it towards the center. This centripetal force is given by the expression: [F_c = \frac{m v_c^2}{r}] In this astronomical system, the force responsible for this motion is gravity. According to Newton's law of universal gravitation, the gravitational force exerted by the central bulge mass (M_b) on the orbiting object is: [F_g = \frac{G M_b m}{r^2}] where (G) is the gravitational constant.

For a stable orbit to exist, these two forces must be equal. The gravitational pull must provide precisely the required centripetal force. [F_g = F_c] [\frac{G M_b m}{r^2} = \frac{m v_c^2}{r}] A remarkable feature of this equality is that the mass of the orbiting object, (m), cancels from both sides. This means the orbital velocity at a given radius depends only on the central mass, not the mass of the star or nebula we are observing.

Rearranging the equation to solve for the mass of the bulge, (M_b), I find: [M_b = \frac{v_c^2 r}{G}] This equation provides a direct method to calculate the bulge's mass using observable quantities from the provided graph. I must now extract a pair of values ((r, v_c)) from the red curve in Figure 1(B), ensuring I select a point in the keplerian region (outside the bulge, which is defined as (r > 1) kpc).

To minimize reading errors from the graph, I will select a point that lies on a grid line if possible. The red curve appears to pass near the peak at (r \approx 2) kpc, where the velocity is approximately (v_c \approx 220) km/s. Let's use this point for the calculation.

Data Extraction:

- Radius: (r = 2 \text{ kpc})
- Orbital velocity: (v_c = 220 \text{ km/s})

Constants and Unit Conversion: The problem provides the necessary conversion factors. I must work in standard SI units (meters, kilograms, seconds) for the calculation.

- (r = 2 \text{ kpc} = 2 \times (3.09 \times 10^{19} \text{ m}) = 6.18 \times 10^{19} \text{ m})
- (v_c = 220 \text{ km/s} = 220 \times 10^3 \text{ m/s} = 2.2 \times 10^5 \text{ m/s})
- Gravitational constant: (G \approx 6.674 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2})
- Solar mass: (1 M_\odot = 1.99 \times 10^{30} \text{ kg})

Calculation: Now I substitute these values into my derived formula for (M_b): [M_b = \frac{(2.2 \times 10^5 \text{ m/s})^2 (6.18 \times 10^{19} \text{ m})}{6.674 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}}] [M_b = \frac{(4.84 \times 10^{10} \text{ m}^2/\text{s}^2) (6.18 \times 10^{19} \text{ m})}{6.674 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}}] [M_b = \frac{2.991 \times 10^{30} \text{ m}^3/\text{s}^2}{6.674 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}}] [M_b \approx 4.48 \times 10^{40} \text{ kg}] The question asks for the mass in solar mass units. I will perform this final conversion: [M_b (\text{in } M_\odot) = \frac{4.48 \times 10^{40} \text{ kg}}{1.99 \times 10^{30} \text{ kg}/M_\odot} \approx 2.25 \times 10^{10} M_\odot]

The mass of the bulge of NGC 6946, as deduced from the keplerian model curve, is approximately (2.25 \times 10^{10}) solar masses.