googleai-gemini-2.5-pro

To address this question, I must first establish the physical situation from the correct frame of reference. The problem asks for the magnetic field at the position of the electron, so I must place myself in the electron's reference frame. In this frame, the electron is stationary at the origin, and the proton, with charge (+e), is the object that is moving. Due to the relative motion, the proton appears to be in a circular orbit of radius (r_1) around the electron.

A moving electric charge constitutes an electric current, and as such, it must generate a magnetic field in the space around it. My goal is to determine the magnitude of this field at the center of the orbit, where the electron is located.

The magnitude of this magnetic field should depend on the properties of the moving particle—its charge (e) and its speed (v_1)—and on the geometry of its motion, specifically the radius of its orbit, (r_1). Intuitively, a faster-moving or more highly charged particle should generate a stronger field. Conversely, as the field is measured at the center, a larger orbital radius (r_1) should result in a weaker field.

To formalize this, I can model the orbiting proton as a current loop. The time it takes for the proton to complete one orbit, the period (T), is the circumference of the orbit divided by its speed: [T = \frac{2\pi r_1}{v_1}] An electric current (I) is defined as the amount of charge passing a point per unit time. In this case, the charge (e) completes a full circuit in time (T), so the equivalent current is: [I = \frac{e}{T} = \frac{e}{\left(\frac{2\pi r_1}{v_1}\right)} = \frac{e v_1}{2\pi r_1}] The fundamental law for the magnetic field at the center of a circular current loop of radius (r) carrying current (I) gives the field magnitude (B) as: [B = \frac{\mu_0 I}{2r}] Applying this to our specific situation with radius (r_1): [B_1 = \frac{\mu_0 I}{2r_1}] Now, I can substitute the expression for the equivalent current (I) into this equation: [B_1 = \frac{\mu_0}{2r_1} \left(\frac{e v_1}{2\pi r_1} \right)] Simplifying this expression gives the magnitude of the magnetic field: [B_1 = \frac{\mu_0 e v_1}{4\pi r_1^2}] This result is consistent with the Biot-Savart law for a point charge, which would yield the same outcome. The velocity of the proton in this frame, (v_1), is the same as the velocity of the electron in the ground state orbit, as determined in question A.2. From the problem statement for A.2, we are guided to express this velocity in terms of the fine-structure constant, (a), and the speed of light, (c). The result from that section is (v_1 = ac).

The question requires the final answer for (B_1) to be in terms of (\mu_0), (e), (a), (c), and (r_1). I will substitute (v_1 = ac) into my derived expression for (B_1): [B_1 = \frac{\mu_0 e (ac)}{4\pi r_1^2}] Re-arranging for clarity: [B_1 = \frac{\mu_0 a c e}{4\pi r_1^2}] This is the final expression for the magnitude of the magnetic field produced by the orbiting proton at the location of the electron. This model assumes a classical circular orbit, which is a simplification of the true quantum mechanical nature of the atom, but it captures the essential physics of how the proton's motion generates a magnetic field that the electron experiences.