Let $X_i \sim \text{Poi}(\lambda)$

The likelihood function is

$$L(\lambda \mid \underline{x}) = \prod_{i=1}^{n} \mathbb{P}(X_i = x_i ; \lambda) \quad \text{(independence)}$$

$$= \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

$$= \frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{\prod_{i=1}^{n} x_i!}$$

$\lceil g(t, \lambda) \quad t = \Sigma x_i \rfloor$

'No λ' $= h(\underline{x})$

$$= g(t(\underline{x}), \lambda)\, h(\underline{x})$$

with $t(\underline{x}) = \sum_{i=1}^{n} x_i$

$$g(t(\underline{x}), \lambda) = \lambda^{t(\underline{x})} e^{-n\lambda}$$

$$h(\underline{x}) = \frac{1}{\prod_{i=1}^{n} x_i!}$$

$\therefore$ By the factorisation criterion

$T(\underline{X}) = \sum_{i=1}^{n} X_i$ is sufficient for λ.

$X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

The likelihood function is:

$$L(\mu, \sigma^2 | \underline{x}) = \prod_{i=1}^{n} f(x_i ; \mu, \sigma^2)$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(x_i - \mu)^2\right\}$$

$$= (2\pi\sigma^2)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2\right\}$$

$$= (2\pi\sigma^2)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \bar{x} + \bar{x} - \mu)^2\right\}$$

$$= (2\pi\sigma^2)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2 + n(\bar{x} - \mu)^2\right]\right\}$$

$$= g(t(\underline{x}), \mu, \sigma^2)\, h(\underline{x})$$

$$t(\underline{x}) = \left(\sum_{i=1}^{n}(x_i - \bar{x})^2, \bar{x}\right)^T$$

$$g(t(\underline{x}), \mu, \sigma^2) = (2\pi\sigma^2)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2 + n(\bar{x} - \mu)^2\right]\right\}$$

$$h(\underline{x}) = 1$$

$\therefore$ By the factorisation criterion

$T(\underline{X}) = \left(\sum_{i=1}^{n}(X_i - \bar{X})^2, \bar{X}\right)^T$ is sufficient for $(\mu, \sigma^2)^T$.

$X_1, \dots, X_n \overset{iid}{\sim} Exp(\theta)$

The likelihood function is

$$\mathcal{L}(\theta \mid \underline{x}) = \prod_{i=1}^{n} \theta e^{-\theta x_i}$$

$$= \theta^n \exp\left(-\theta \sum_{i=1}^{n} x_i\right)$$

$$= g(t(\underline{x}), \theta)\, h(\underline{x})$$

$$t(\underline{x}) = \sum_{i=1}^{n} x_i$$

$$g(t(\underline{x}), \theta) = \theta^n \exp(-\theta t(\underline{x}))$$

$$h(\underline{x}) = 1$$

$\therefore$ By the factorisation criterion $T(\underline{X}) = \sum_{i=1}^{n} X_i$ is sufficient for θ.

Now, let $Y_1, \dots, Y_n$ be iid $Y_i \sim Exp(\theta)$.

The likelihood function is:

$$\mathcal{L}(\theta \mid \underline{y}) = \theta^n \exp\left(-\theta \sum_{i=1}^{n} \cancel{x}y_i\right)$$

Now,

$$\frac{f(\underline{x};\theta)}{f(\underline{y};\theta)} = \frac{\mathcal{L}(\theta \mid \underline{x})}{\mathcal{L}(\theta \mid \underline{y})} = \frac{\cancel{\theta^n} \exp(-\theta \sum_i x_i)}{\cancel{\theta^n} \exp(-\theta \sum_i y_i)}$$

$$= \exp\left[-\theta\left(\sum_i x_i - \sum_i y_i\right)\right]$$

which is independent of $\theta \iff \sum_i x_i = \sum_i y_i$

$\therefore$ $T(\underline{X}) = \sum_i X_i$ is minimal sufficient.