Explicit Copulas

Before: Frank copula density:

$$c(u_1, u_2) = \frac{\delta^2 C(u_1, u_2)}{\delta u \delta v}$$
$$= \theta\left(1 - e^{-\theta}\right) e^{-\theta(u_1+u_2)} \left[\left(1 - e^{-\theta}\right) - \left(1 - e^{-\theta u_1}\right)\left(1 - e^{-\theta u_2}\right)\right]^{-2}$$

After: Frank copula density:

$$c(u_1, u_2) = \frac{\partial^2 C(u_1, u_2)}{\partial u_1 \partial u_2}$$
$$= \theta\left(1 - e^{-\theta}\right) e^{-\theta(u_1+u_2)} \left[\left(1 - e^{-\theta}\right) - \left(1 - e^{-\theta u_1}\right)\left(1 - e^{-\theta u_2}\right)\right]^{-2}$$

Explicit Copulas

Before: Gumbel copula density:

$$
\begin{aligned}
c(u,v) &= \frac{\delta^2 C(u,v)}{\delta u \delta v} \\
&= C(u,v)(uv)^{-1}\left((-\log u)^{\delta} + (-\log v)^{\delta}\right)^{-2+2/\delta}(\log u \log v)^{\delta-1} \\
&\times \left\{1 + (\delta-1)\left((-\log u)^{\delta} + (-\log v)^{\delta}\right)^{-1/\delta}\right\}
\end{aligned}
$$

After: Gumbel copula density:

$$
\begin{aligned}
c(u_1,u_1) &= \frac{\partial^2 C(u_1,u_2)}{\partial u_1 \partial u_2} \\
&= C(u_1,u_2)(u_1u_2)^{-1}\left((-\log u_1)^{\theta} + (-\log u_2)^{\theta}\right)^{-2+2/\theta}(\log u_1 \log u_2)^{\theta-1} \\
&\times \left\{1 + (\theta-1)\left((-\log u_1)^{\theta} + (-\log u_2)^{\theta}\right)^{-1/\theta}\right\}
\end{aligned}
$$

Explicit Copulas

Before: Clayton copula density:

$$c(u,v) = \frac{\delta^2 C(u,v)}{\delta u \delta v} = (1+\delta)(uv)^{-1-\delta}\left(u^{-\delta}+v^{-\delta}-1\right)^{-1/\delta-2}$$

After: Clayton copula density:

$$c(u_1,u_2) = \frac{\partial^2 C(u_1,u_2)}{\partial u_1 \partial u_2} = (1+\theta)(u_1 u_2)^{-1-\theta}\left(u_1^{-\theta}+u_2^{-\theta}-1\right)^{-1/\theta-2}$$