Section A

A1 (a) $Gamma(\alpha + 5, \beta + 17.3)$

(b) The mean is 0.29

(c) Similar to derivation seen in Lecture 6 pp.4-6

(d)

$$\int_0^5 \frac{17.3}{\Gamma(5)} \frac{\Gamma(6)}{(17.3 + \tilde{Y})^6} d\tilde{Y}$$

(e) Trapezoid rule will approximate it by:

$$(5-0)\frac{f(5) + f(0)}{2} = 5\frac{(0.063 + 0.289)}{2} = 0.88$$

A2 (a) The 99% Value-at-risk is a number (or percentage) X such that the probability of loosing more than X over specified time period is 1%.

(b) See Exercise 6

(c) See Exercise 6, the only difference is the notation for parameters: α_0, α_1, and α_2.

(d) See Exercise 6, the only difference is the notation for parameters: α_0, α_1, and α_2.

A3 (a) See Exercise 4, Question 2, part (a)

$$\frac{\rho(\pi^*, a_0|Y)}{\rho(\pi^*, a_1|Y)} = \frac{0.7^4 0.3^1}{0.5^4 0.5^1} = \frac{0.072}{0.031} = 2.32$$

Action a_1 has lower risk, so we decide that the coin is biased.

(b) Similar to Exercise 4, Question 2, (b).

$$\frac{\rho(\pi^*, a_0|Y)}{\rho(\pi^*, a_1|Y)} = \frac{c_0 0.7^4 0.3^1}{k c_0 0.5^4 0.5^1} = \frac{0.072}{k 0.031}$$

It is optimal to classify the coin as unbiased if $k > 2.33$.

Section B

B1 (a) $\alpha = 2$, $\beta = 1$

(b) Similar to Exercise 3, Question 1, part 4).
Posterior is $Gamma(\tilde{\alpha}, \tilde{\beta})$, where $\tilde{\alpha} = 15$, and $\tilde{\beta} = 7$.

$$p(\tilde{Y}|Y) = \frac{\tilde{\beta}^{\tilde{\alpha}}}{\Gamma(\tilde{\alpha})} \frac{1}{\tilde{Y}!} \frac{\Gamma(\tilde{\alpha} + \tilde{Y})}{(\tilde{\beta} + 1)^{\tilde{\alpha} + \tilde{Y}}}$$

Substituting in the values:

$$p(\tilde{Y}|Y) = \frac{1}{\tilde{Y}!} \frac{7^{15}}{\Gamma(15)} \frac{\Gamma(15 + \tilde{Y})}{8^{15+\tilde{Y}}}$$

(c) $p(\tilde{Y} > 2) = 0.359$

(d) $p(Y|M_0) = 1.06 \times 10^{-5}$
$p(Y|M_1) = 3.23 \times 10^{-5}$
Then the posterior probabilities for models M_0 and M_1 given data Y are:
$p(M_0|Y) = 0.25$
$p(M_1|Y) = 0.75$

B2 (a)

$$p(\sigma^2|Y) = \frac{p(Y|\sigma^2) \cdot p(\sigma^2)}{p(Y)} = \frac{\tilde{\beta}^{\tilde{\alpha}}}{\Gamma(\tilde{\alpha})} (\sigma^2)^{-\tilde{\alpha}-1} e^{-\frac{\tilde{\beta}}{\sigma^2}}$$

where $\tilde{\alpha} = \alpha + \frac{n}{2}$ and $\tilde{\beta} = \beta + \frac{\sum Y_i^2}{2}$

(b) $\tilde{\alpha} = \alpha + \frac{n}{2} = \alpha + 3$ and $\tilde{\beta} = \beta + \frac{\sum Y_i^2}{2} = \beta + 0.17$

(c) In this case $\tilde{\alpha} = 3$ and $\tilde{\beta} = 0.17$.

$$\int_{-1}^{-0.5} p(\tilde{Y}|\mathbf{Y}) d\tilde{Y} = \int_{-1}^{-0.5} \frac{1}{\sqrt{2\pi}} \frac{\tilde{\beta}^{\tilde{\alpha}}}{\Gamma(\tilde{\alpha})} \frac{\Gamma(\tilde{\alpha} + \frac{1}{2})}{(\tilde{\beta} + \frac{\tilde{Y}^2}{2})^{\tilde{\alpha} + \frac{1}{2}}} d\tilde{Y}$$

Trapezoid rule will approximate it by:

$$(-0.5 - (-1)) \frac{f(-0.5) + f(-1)}{2} = 0.5 \frac{(0.237 + 0.013)}{2} = 0.06$$

(d) The PBH theorem states that for any distribution $p(Y)$ that satisfies certain regularity conditions, we have that $p(Y \leq D|Y > u)$ is asymptotically a Generalised Pareto Distribution as $u \to \infty$.

(e) $\hat{k} = 0.045$, $\hat{\sigma} = 0.115$

$$p(-1 \leq Y \leq -0.5) = 0.031$$

END OF PAPER