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Part A

Question 1 (a)

From the question we can see:

$$F(x) = P(X \leq x) = \begin{cases} 1 - e^{1/x}, & \text{for } x < 0 \text{ .} \\ 1, & \text{for } x \geq 0 \text{ .} \end{cases} \tag{1}$$

$$M_n = max\{x_1, ..., x_n\}$$

Hazard Function:

$$h(x) = \frac{1 - F(x)}{f(x)} \tag{1}$$

$$= \frac{e^{1/x}}{e^{1/x} \times (-x^{-2})} \tag{2}$$

$$= -x^2 \tag{3}$$

Derivative of h(x): $h^{'} = -2x$

$$\lim_{x \to x^F} h^{'}(x) \to \xi \tag{4}$$

$$Let\ \xi = -2M_n \tag{5}$$

$$We\ can\ see:\ x \sim GEV(0, 1, -2M_n) \tag{6}$$

From the slides we have:

$$\begin{cases} 1 - F(b_n) = 1/n \\ a_n = h(b_n) \end{cases}$$

solve it we have:

$$\begin{cases} b_n = -\frac{1}{log(n)} \\ a_n = -\frac{1}{log^2(n)} \end{cases}$$

Question 1 (b)

When $\xi < 0$ the GEV distribution has light upper tail with the finite upper limit which is $\mu - \sigma/\xi$.

Let $\mu - \sigma/\xi = 0$ then this distribution with a finite upper end point of 0 has an upper end point of infinity.

Question 1 (c)

We assume $A = \{x : 0 < F(x) < 1\}$, and $x^* = sup_{x \in A} A$.

Here F(x) can be any function including the question's function(1).

For $\forall x, x < x^*$, we have $Pr(M_n \leq x) = F^n(x) \to 0, as\ n \to \infty$.

For $\forall x, x \geq x^*$, we have $Pr(M_n \leq x) = F^n(x) \to 1, as\ n \to \infty$.

This is mean whatever x or F(x), $M_n = 0 or 1$ when $n \to \infty$.

The M_n is Degenerate distribution, which is useless.

That is why we need fit a GEV model.

Question 1 (d)

The log-likelihood:

$$l(\phi) = l(\mu, \sigma, \xi) = logL(\mu, \sigma, \xi) = log \prod_{i=1}^{m} g(z_i; \mu, \sigma, \xi)$$

Part B

Task 1

```r
setwd("/Users/hongwei/Documents/GitHub/STAT/STAT0017_ICA1")
raw_wm <- load("wm.Rdata")
raw_pot <- load("pot.Rdata")
```

Task 2

Task 3.1

Task 3.2

Task 3.3

Task 4.1

Task 4.2