Part3

17 February 2018 12:35

$$\mathbb{E}_{Y|X}(Y|X=x) = \sum_y y \cdot P_{Y|X}(y|X=x)$$

$$= \sum_y y \cdot \frac{P_{XY}(x,y)}{P_X(x)}$$

$$= \sum_y y \cdot \frac{\sum_z P_{XYZ}(x,y,z)}{P_X(x)}$$

$$= \sum_y y \cdot \frac{\sum_z P_{Y|XZ}(y|x,z) \cdot P_{XZ}(x,z)}{P_X(x)}$$

$$= \sum_z \frac{P_{XZ}(x,z)}{P_X(x)} \sum_y y \cdot P_{Y|X,Z}(y|x,z)$$

$$= \sum_z P_{Z|X}(z|x) \cdot \mathbb{E}_{Y|X,Z}(y|x,z)$$

$$= \mathbb{E}_{Z|X}\left\{ \mathbb{E}_{Y|X,Z}(y|x,z) \,\middle|\, x \right\}$$