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Distributed Dual Objective Control of Energy Storage Systems

He Cai[1,2] and Guoqiang Hu[1]

[1]School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore
[2]School of Automation Science and Engineering, South China University of Technology, Guangzhou, China
(E-mail: caihe@scut.edu.cn, gqhu@ntu.edu.sg)

Abstract: In this paper, a dual objective control problem is considered for energy storage systems. On one hand, the power output of the overall energy storage system should meet its reference. On the other hand, the state-of-energy of all the energy storage units should be balanced so as to maintain the maximum system power capacity. To achieve these two control objectives simultaneously, two distributed control approaches have been proposed depending on the uniformity of the energy capacities of the energy storage units. Case studies are provided to validate the effectiveness of the proposed control approaches.

Keywords: Distributed control, energy storage system, power tracking, energy balancing.

1. INTRODUCTION

Microgrids have gained increasing interest in modern power system, whose benefits have been demonstrated in many aspects, such as integration of renewable energy sources, reduction of transmission and distribution cost, improvement of power quality, and so on [7, 11-13]. Taking into account the multitude of the distributed generators within a microgrid, proper cooperation among different distributed generators is of great significance. Depending on the mechanism of cooperation, the control schemes for microgrids can be roughly classified into three categories. The first control scheme is centralized control [18], which utilizes point-to-point communication between a control center and each distributed generator to exchange information. In addition, the control center should also possess advanced computational ability to handle large scale data stream. As a result, the centralized control schemes are usually costly to implement. The second control scheme is decentralized control [15, 16], which is mainly applied to islanded microgrids requiring no communication but using the system frequency and bus voltage as references for the local droop characteristics. While, though the decentralized control is less expensive than the centralized control as no communication is needed, the system performance, e.g., reactive power sharing, is less satisfactory due to the lack of proper adjustment which relies on communications. The third control scheme is distributed control [1, 2, 21], which lies inbetween the centralized control and the decentralized control. On one hand, like centralized control, it requires a communication network to enhance the system performance, but the communication network is neighbor-based and sparse, and thus is more flexible, scalable, and cost-effective. On the other hand, like decentralized control, control decisions for distributed generators are made locally, which avoids the need for any computational center. These features make the distributed control a promising solution for the microgrid control problem.

Energy storage system plays an important role in microgrids to mitigate the intermittence of renewable energy sources, such as solar and wind [5, 6, 20]. There are two fundamental issues regarding the control of energy storage systems consisting of multiple energy storage units. First, the power output of the overall energy storage system should meet its reference scheduled by some upper level control. Second, the power capacity of the energy storage system should be kept maximum so that it can be fully functional. To this end, the energy levels of the energy storage units should be balanced since if an energy storage unit reaches critical high or low energy level, it will be forced off-line for protection, which will in turn reduce the power capacity of the overall energy storage system. For example, for battery energy storage systems, the balancing of state-of-charge, which is defined as the ratio of the stored charge and the charge capacity of a battery cell or pack, has attracted a lot of attention [3, 8, 9]. Both [8] and [9] considered the state-of-charge balancing of an islanded DC microgrid. Adaptive droop control approaches were proposed in the way that the droop coefficient is set inversely proportional to the state-of-charge so as to drive the state-of-charges to the balanced state. [3] considered the state-of-charge balancing of a grid-connected AC battery energy storage system. For each battery pack, the reference power output is determined jointly by a global reference signal and by the state-of-charge of its neighboring battery packs. However, the concept of state-of-charge may not be applicable to other types of energy storage units, such as flywheel or fuel cell. In [10], a general energy balancing problem was investigated, which was formulated as a leaderless consensus problem and solved by the simultaneous eigenvalue placement technique [19].

In this paper, we further consider a dual objective control problem for energy storage systems. For each energy storage unit, we define the state-of-energy as the ratio of the stored energy and the energy capacity. Clearly, the concept of state-of-energy is a natural extension of state-of-charge, which not only applies to batteries, but also to other types of energy storage units. The control objectives of this paper are two-fold. First, the power output of the overall energy storage system should meet its ref-

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erence. Second, the state-of-energy of all the energy storage units should be balanced to maintain the maximum system power capacity. To the best of our knowledge, such a problem has not been studied in the literature so far. To solve this problem, two cases are considered depending on the uniformity of the energy capacities of the energy storage units. First, we consider the special case where the energy capacities of all the energy storage units are identical. It is shown that the dual objective control problem in this case can be converted into a hybrid leaderless and leader-following consensus problem. Furthermore, we consider the general case where the energy capacities of the energy storage units are non-identical. This general case is much more complicated than the special case. By designing a delicate command generator, the dual objective control problem in this case is converted into a hybrid leaderless consensus problem and a tracking problem for an interconnected leader-follower multi-agent system. Two distributed control laws are synthesized to solve the dual objective control problem for these two cases, respectively.

2. NOTATION

For $x_i \in R^{n_i}$, $i = 1, \ldots, m$, $\mathrm{col}(x_1, \ldots, x_m) \triangleq (x_1^T, \ldots, x_m^T)^T$. $1_n \triangleq \mathrm{col}(1, \ldots, 1) \in R^n$. If A is positive semi-definite (definite), let $A^{1/2}$ denote the square root of A. A graph G is defined as $G = (V, E)$, which consists of a finite set of nodes $V = \{1, \ldots, N\}$ and an edge set $E = \{(i,j), i, j \in V, i \neq j\}$. An edge from node i to node j is denoted by (i,j), and node i is called the neighbor of node j. If the graph G contains a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})$, then, the set $\{(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})\}$ is called a path of G from node i_1 to node i_{k+1}, and node i_{k+1} is said to be reachable from node i_1. A graph is said to contain a spanning tree if there exists a node such that any other node is reachable from it and it is called the root of the spanning tree. A graph is said to be connected if it contains a spanning tree. The edge (i,j) is called undirected if $(i,j) \in E$ implies $(j,i) \in E$. A graph is called undirected if all the edges of it are undirected. The weighted adjacency matrix $A = [a_{ij}] \in R^{N \times N}$ of G is defined as $a_{ii} = 0$, and for $i \neq j$, $a_{ij} > 0 \Leftrightarrow (j,i) \in E$ and $a_{ij} = 0$ otherwise. Moreover, $a_{ij} = a_{ji}$ if (j,i) is an undirected edge. The Laplacian matrix $L = [l_{ij}] \in R^{N \times N}$ of G is defined as $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$.

3. PROBLEM FORMULATION

Consider an energy storage system consisting of N energy storage units as shown in Fig. 1. Each energy storage unit is connected to the common bus through a converter to adjust its power output. For the ith energy storage unit, $i = 1, \ldots, N$, let E_i, E_{ci}, P_i denote the stored energy, energy capacity and output power, respectively, where $P_i > 0$ ($P_i < 0$) during the energy release

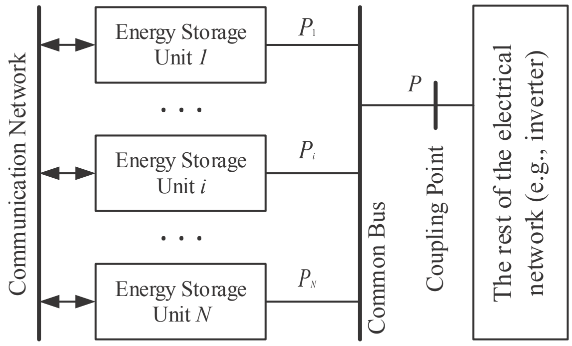

Fig. 1 Diagram of the energy storage system.

(storage) operation. We have

$$\dot{E}_i = -P_i. \tag{1}$$

Dividing E_{ci} on both sides of (1) gives

$$\dot{x}_i = -\gamma_i P_i \tag{2}$$

where $x_i \in R$ is the state-of-energy of the ith energy storage unit and $\gamma_i = 1/E_{ci}$.

Let $P_{ESS} = \sum_{i=1}^{N} P_i$ denote the power output of the overall energy storage system, which can be measured directly at the coupling point as shown in Fig. 1. In addition, let $P_{REF} \in R$ denote the reference for P_{ESS}, which is assumed to be a piecewise constant signal scheduled by some upper level control at a slower time scale. To achieve the reference power tracking, we introduce a command generator in the following generic form

$$\dot{x}_0 = f_0(t) \tag{3}$$

whose specific dynamics will be detailed in Section 4.

The energy storage system (2) can be viewed as a multi-agent system Σ_N with each energy storage unit being viewed as an agent. An extended multi-agent system $\Sigma_{(N+1)}$, with $N+1$ agents, is defined by adding the command generator (3) to Σ_N. The communication network for $\Sigma_{(N+1)}$ is represented by a graph $\bar{G} = (\bar{V}, \bar{E})$ with $\bar{V} = \{0, 1, \ldots, N\}$ and $\bar{E} \subseteq \bar{V} \times \bar{V}$. Here, the node 0 is associated with the command generator and the node $i, i = 1, \ldots, N$, is associated with the ith energy storage unit. For $i = 0, 1, \ldots, N$, $j = 1, \ldots, N$, $(i,j) \in \bar{E}$ if and only if the jth energy storage unit can receive the information from the command generator or the ith energy storage unit. Furthermore, we define a graph $G = (V, E)$ for Σ_N where $V = \{1, 2, \ldots, N\}$ and $E = \{V \times V\} \cap \bar{E}$. In the following, let $\bar{A} = [a_{ij}] \in R^{(N+1)\times(N+1)}$ be the weighted adjacency matrix of $\bar{G}$, $L \in R^{N \times N}$ be the Laplacian of G and $H = L + \mathrm{diag}\{a_{10}, \ldots, a_{N0}\}$.

Some assumptions are given as follows.

Assumption 1: The graph $\bar{G}$ contains a spanning tree with the node 0 as its root and the graph G is connected.

Assumption 2: The graph $\bar{G}$ contains a spanning tree with the node 0 as its root and the graph G is undirected and connected.

The problem considered in this paper is given as follows.

Problem 1: Given systems (2), (3) and the communication network $\bar{G}$, find P_i such that

$$\lim_{t\to\infty}(P_{ESS}(t) - P_{REF}) = 0 \tag{4}$$

and

$$\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0 \tag{5}$$

for $i, j = 1, \ldots, N$.

4. MAIN RESULTS

In this section, two distributed control approaches will be shown to solve Problem 1 depending on the uniformity of E_{ci}, i.e., whether E_{ci}'s are identical for energy storage units.

4.1. Identical E_{ci}

Since E_{ci}'s are identical, γ_i's are also identical. For simplicity, we let $\gamma_i = \gamma$ for $i = 1, \ldots, N$. First, we introduce the following command generator

$$\dot{\zeta}_0 = 0,\ \zeta_0(0) = P_{REF}/N \tag{6}$$

where $\zeta_0 \in R$. System (6) means that we let $\zeta_0(t) = P_{REF}/N$ for all $t \geq 0$. Then, for $i = 1, \ldots, N$, the control law is given as follows

$$\dot{\zeta}_i = \mu_\zeta\left(\sum_{j=1}^{N} a_{ij}(\zeta_j - \zeta_i) + a_{i0}(\zeta_0 - \zeta_i)\right) \tag{7a}$$

$$P_i = -k_x \sum_{j=1}^{N} a_{ij}(x_j - x_i) + \zeta_i \tag{7b}$$

where $\zeta_i \in R$, $\mu_\zeta, k_x > 0$. We have the following result.

Theorem 1: Given systems (2), (6) and the communication network $\bar{G}$, under Assumption 1, if $\gamma_i = \gamma$, for $i = 1, \ldots, N$, then for any system initial condition and any $\mu_\zeta, k_x > 0$, Problem 1 is solvable by the control law (7).

Proof: For $i = 1, \ldots, N$, let $\bar{\zeta}_i = \zeta_i - \zeta_0$ and $\bar{\zeta} = \mathrm{col}(\bar{\zeta}_1, \ldots, \bar{\zeta}_N)$. Then by (6) and (7a), we have

$$\dot{\bar{\zeta}} = -\mu_\zeta H\bar{\zeta}. \tag{8}$$

Under Assumption 1, by Lemma 1 of [17], all the eigenvalues of H have positive real parts. Therefore,

$$\lim_{t\to\infty}\bar{\zeta}_i(t) = 0. \tag{9}$$

Substituting (7b) into (2) gives

$$\dot{x}_i = \gamma k_x \sum_{j=1}^{N} a_{ij}(x_j - x_i) - \gamma\zeta_i. \tag{10}$$

Let $\zeta = \mathrm{col}(\zeta_1, \ldots, \zeta_N)$. Then $\bar{\zeta} = \zeta - \zeta_0 1_N$. Let $x = \mathrm{col}(x_1, \ldots, x_N)$ and then we have

$$\begin{aligned}\dot{x} &= -\gamma k_x Lx - \gamma\zeta \\ &= -\gamma k_x Lx - \gamma\zeta_0 1_N - \gamma\bar{\zeta}.\end{aligned} \tag{11}$$

Since $L1_N = 0$, let $T \in R^{N\times N}$ be such that

$$T = \begin{pmatrix} 1_N & T_\alpha \end{pmatrix},\ T^{-1} = \begin{pmatrix} t_\beta^T \\ T_\theta \end{pmatrix} \tag{12}$$

and

$$T^{-1}LT = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & J & \\ 0 & & & \end{pmatrix} \tag{13}$$

where $T_\alpha \in R^{N\times(N-1)}$, $t_\beta \in R^N$, $T_\theta \in R^{(N-1)\times N}$, $J \in R^{(N-1)\times(N-1)}$ is in Jordan form. Under Assumption 1, by Lemma 1.1 of [14], all eigenvalues of J have positive real parts. Moreover, since $T^{-1}T = I_N$, it follows that $t_\beta^T 1_N = 1$ and $T_\theta 1_N = 0$. Thus, $T^{-1}1_N = \mathrm{col}(1, 0, \ldots, 0) \in R^N$. Let $\bar{x} = T^{-1}x$. We have

$$\dot{\bar{x}} = -\gamma k_x T^{-1}LT\bar{x} - \gamma\zeta_0 T^{-1}1_N - \gamma T^{-1}\bar{\zeta}. \tag{14}$$

Divide $\bar{x}$ into $\bar{x} = \mathrm{col}(\bar{x}_s, \bar{x}_v)$ with $\bar{x}_s \in R$ and $\bar{x}_v \in R^{N-1}$. By (14), we have

$$\dot{\bar{x}}_s = -\gamma\zeta_0 - \gamma t_\beta^T\bar{\zeta} \tag{15a}$$

$$\dot{\bar{x}}_v = -\gamma k_x J\bar{x}_v - \gamma T_\theta\bar{\zeta}. \tag{15b}$$

Since $-\gamma k_x J$ is Hurwitz and $\lim_{t\to\infty}\bar{\zeta}(t) = 0$, $\lim_{t\to\infty}\bar{x}_v(t) = 0$. As a result, noting that $x = T\bar{x} = \bar{x}_s 1_N + T_\alpha\bar{x}_v$ gives $\lim_{t\to\infty}(x(t) - \bar{x}_s(t)1_N) = 0$, which implies, for $i, j = 1, \ldots, N$,

$$\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0. \tag{16}$$

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Then, by (9) and (16), noticing that

$$\begin{aligned} P_{ESS} - P_{REF} &= \sum_{i=1}^{N} P_i - N\zeta_0 \\ &= -k_x \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_j - x_i) + \sum_{i=1}^{N}\zeta_i - N\zeta_0 \\ &= -k_x \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_j - x_i) + \sum_{i=1}^{N}\bar{\zeta}_i \end{aligned} \tag{17}$$

gives $\lim_{t\to\infty}(P_{ESS}(t) - P_{REF}) = 0$. □

Remark 1: For the special case where E_{ci}'s are identical, from the proof of Theorem 1, it can be seen that Problem 1 has been converted into a coupled leaderless consensus problem aiming at $x_i = x_j$ and leader-following consensus problem aiming at $\zeta_i = \zeta_0$. In particular, if $\zeta_i = 0$, then (7b) solves the leaderless consensus problem, and if $x_i = x_j$, then (7) solves the leader-following consensus problem. Theorem 1 shows that these two control objectives can be achieved simultaneously using the combined control law (7).

4.2. Non-identical E_{ci}

First, we introduce the following command generator

$$\dot{\eta}_0 = \alpha(P_{REF} - P_{ESS}) \tag{18}$$

where $\eta_0 \in R$, $\alpha > 0$. Note that (18) can also be written into the following form

$$\dot{\eta}_0 = \alpha \left(P_{REF} - \sum_{i=1}^{N} P_i \right). \tag{19}$$

For $i = 1, \ldots, N$, the control law is given as follows.

$$\dot{\xi}_i = \sum_{j=1}^{N} a_{ij}(x_j - x_i) \tag{20a}$$

$$\dot{\eta}_i = \mu_\eta \left(\sum_{j=1}^{N} a_{ij}(\eta_j - \eta_i) + a_{i0}(\eta_0 - \eta_i) \right) \tag{20b}$$

$$P_i = -\kappa_x \sum_{j=1}^{N} a_{ij}(x_j - x_i) - \kappa_\xi \xi_i + \kappa_\eta \eta_i \tag{20c}$$

where $\mu_\eta, \kappa_x, \kappa_\xi, \kappa_\eta > 0$. We have the following result.

Theorem 2: Given systems (2), (18) and the communication network $\bar{G}$, under Assumption 2, for any system initial condition and any $\mu_\eta, \kappa_x, \kappa_\xi, \kappa_\eta > 0$, Problem 1 is solvable by the control law (20).

Proof: Let $\xi_{sum} = \sum_{i=1}^{N} \xi_i$. Since G is undirected under Assumption 2, it follows that

$$\dot{\xi}_{sum} = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(x_j - x_i) = 0 \tag{21}$$

and

$$\begin{aligned} \sum_{i=1}^{N} P_i &= \sum_{i=1}^{N} \left(-\kappa_x \sum_{j=1}^{N} a_{ij}(x_j - x_i) - \kappa_\xi \xi_i + \kappa_\eta \eta_i \right) \\ &= -\kappa_\xi \xi_{sum} + \kappa_\eta \sum_{i=1}^{N} \eta_i. \end{aligned} \tag{22}$$

Therefore, $\xi_{sum}(t) = \xi_{sum}(0)$ for all $t \geq 0$. Substituting (22) into (19) gives

$$\dot{\eta}_0 = \alpha \left(P_{REF} + \kappa_\xi \xi_{sum} - \kappa_\eta \sum_{i=1}^{N} \eta_i \right). \tag{23}$$

Let $\bar{P}_{REF} = P_{REF} + \kappa_\xi \xi_{sum}$, $\bar{\eta}_0 = \eta_0 - \bar{P}_{REF}/(\kappa_\eta N)$, $\bar{\eta}_i = \eta_i - \eta_0$, $\tilde{\eta}_i = \bar{\eta}_i + \bar{\eta}_0 = \eta_i - \bar{P}_{REF}/(\kappa_\eta N)$. Then it follows

$$\begin{aligned} \dot{\bar{\eta}}_0 &= \alpha \left(\bar{P}_{REF} - \kappa_\eta \sum_{i=1}^{N} \eta_i \right) \\ &= \alpha \left(\bar{P}_{REF} - \kappa_\eta \sum_{i=1}^{N} (\eta_0 + \bar{\eta}_i) \right) \\ &= \alpha \left(\bar{P}_{REF} - \kappa_\eta N \eta_0 - \kappa_\eta \sum_{i=1}^{N} \bar{\eta}_i \right) \\ &= -\alpha \kappa_\eta N \bar{\eta}_0 - \alpha \kappa_\eta \sum_{i=1}^{N} \bar{\eta}_i. \end{aligned} \tag{24}$$

Moreover, we have

$$\begin{aligned} \dot{\bar{\eta}}_i &= \mu_\eta \left(\sum_{j=1}^{N} a_{ij}(\eta_j - \eta_i) + a_{i0}(\eta_0 - \eta_i) \right) \\ &\quad + \alpha \kappa_\eta N \bar{\eta}_0 + \alpha \kappa_\eta \sum_{i=1}^{N} \bar{\eta}_i \\ &= \mu_\eta \left(\sum_{j=1}^{N} a_{ij}(\bar{\eta}_j - \bar{\eta}_i) - a_{i0} \bar{\eta}_i \right) \\ &\quad + \alpha \kappa_\eta N \bar{\eta}_0 + \alpha \kappa_\eta \sum_{i=1}^{N} \bar{\eta}_i. \end{aligned} \tag{25}$$

Let $\bar{\eta} = \mathrm{col}(\bar{\eta}_1, \ldots, \bar{\eta}_N)$. Then

$$\begin{aligned} \begin{pmatrix} \dot{\bar{\eta}} \\ \dot{\bar{\eta}}_0 \end{pmatrix} &= \begin{pmatrix} -\mu_\eta H + \alpha \kappa_\eta 1_N 1_N^T & \alpha \kappa_\eta N 1_N \\ -\alpha \kappa_\eta 1_N^T & -\alpha \kappa_\eta N \end{pmatrix} \\ &\quad \cdot \begin{pmatrix} \bar{\eta} \\ \bar{\eta}_0 \end{pmatrix} \triangleq A \begin{pmatrix} \bar{\eta} \\ \bar{\eta}_0 \end{pmatrix}. \end{aligned} \tag{26}$$

By Lemma 5 of [4], A is Hurwitz. As a result, for $i = 0, 1, \ldots, N$, $\lim_{t\to\infty} \bar{\eta}_i(t) = 0$, exponentially, and hence for $i = 1, \ldots, N$, $\lim_{t\to\infty} \tilde{\eta}_i(t) = 0$ exponentially. Noting that

$$\begin{aligned} P_{ESS} - P_{REF} &= \sum_{i=1}^{N} P_i - P_{REF} \\ &= -\kappa_\xi \xi_{sum} + \kappa_\eta \sum_{i=1}^{N} \eta_i - P_{REF} \\ &= -\bar{P}_{REF} + \kappa_\eta \sum_{i=1}^{N} \eta_i \\ &= k_\eta \left(\sum_{i=1}^{N} \eta_i - N \frac{\bar{P}_{REF}}{k_\eta N} \right) = k_\eta \sum_{i=1}^{N} \tilde{\eta}_i \end{aligned} \tag{27}$$

gives $\lim_{t\to\infty}(P_{ESS}(t) - P_{REF}) = 0$.

Next, submitting (20c) into (2) gives

$$\dot{x}_i = \gamma_i \kappa_x \sum_{j=1}^{N} a_{ij}(x_j - x_i) + \gamma_i \kappa_\xi \xi_i - \gamma_i \kappa_\eta \eta_i. \tag{28}$$

Let $x = \mathrm{col}(x_1, \ldots, x_N)$, $\xi = \mathrm{col}(\xi_1, \ldots, \xi_N)$, $\eta = \mathrm{col}(\eta_1, \ldots, \eta_N)$, $\tilde{\eta} = \mathrm{col}(\tilde{\eta}_1, \ldots, \tilde{\eta}_N)$ and $\Gamma = \mathrm{diag}\{\gamma_1, \ldots, \gamma_N\}$. Then

$$\dot{x} = -\kappa_x \Gamma L x + \kappa_\xi \Gamma \xi - \kappa_\eta \Gamma \eta. \tag{29}$$

Furthermore, let $\bar{\xi} = \xi - \frac{\bar{P}_{REF}}{\kappa_\xi N} 1_N$. Then we have

$$\begin{aligned}
\dot{x} &= -\kappa_x \Gamma L x + \kappa_\xi \Gamma \xi - \kappa_\eta \Gamma \eta \\
&= -\kappa_x \Gamma L x + \kappa_\xi \Gamma \left(\bar{\xi} + \frac{\bar{P}_{REF}}{\kappa_\xi N} 1_N \right) - \kappa_\eta \Gamma \eta \\
&= -\kappa_x \Gamma L x + \kappa_\xi \Gamma \bar{\xi} + \Gamma \left(\frac{\bar{P}_{REF}}{N} 1_N \right) - \kappa_\eta \Gamma \eta \\
&= -\kappa_x \Gamma L x + \kappa_\xi \Gamma \bar{\xi} - \kappa_\eta \Gamma \left(\eta - \frac{\bar{P}_{REF}}{\kappa_\eta N} 1_N \right) \\
&= -\kappa_x \Gamma L x + \kappa_\xi \Gamma \bar{\xi} - \kappa_\eta \Gamma \tilde{\eta}
\end{aligned} \tag{30}$$

and

$$\dot{\bar{\xi}} = \dot{\xi} = -Lx. \tag{31}$$

Again, by Lemma 1.1 of [14], L is semi-positive definite under Assumption 2. Let

$$V = \frac{1}{2} x^T L x + \frac{\kappa_\xi}{2} \bar{\xi}^T \Gamma \bar{\xi}. \tag{32}$$

Then noting that both Γ and L are symmetric gives

$$\begin{aligned}
\dot{V} &= -\kappa_x x^T L \Gamma L x + \kappa_\xi x^T L \Gamma \bar{\xi} \\
&\quad - \kappa_\eta x^T L \Gamma \tilde{\eta} - \kappa_\xi \bar{\xi}^T \Gamma L x \\
&= -\kappa_x x^T L \Gamma L x - \kappa_\eta x^T L \Gamma \tilde{\eta}.
\end{aligned} \tag{33}$$

Let $\gamma_{\min} = \min\{\gamma_1, \ldots, \gamma_N\}$ and $\gamma_{\max} = \max\{\gamma_1, \ldots, \gamma_N\}$. Then we have

$$\dot{V} \leq -\kappa_x \gamma_{\min} ||Lx||^2 + \kappa_\eta \gamma_{\max} ||Lx|| \cdot ||\tilde{\eta}||. \tag{34}$$

Since $\lim_{t \to \infty} \tilde{\eta}_i(t) = 0$ exponentially, suppose $||\tilde{\eta}|| \leq \delta_\eta e^{-\beta_\eta t}$ for some $\delta_\eta, \beta_\eta > 0$. Then

$$\begin{aligned}
\dot{V} &\leq -\kappa_x \gamma_{\min} ||Lx||^2 + \kappa_\eta \gamma_{\max} ||Lx|| \cdot \delta_\eta e^{-\beta_\eta t} \\
&= -\kappa_x \gamma_{\min} ||Lx|| \left(||Lx|| - \frac{\kappa_\eta \gamma_{\max} \delta_\eta}{\kappa_x \gamma_{\min}} e^{-\beta_\eta t} \right) \\
&\triangleq -\kappa_x \gamma_{\min} ||Lx|| \left(||Lx|| - \tilde{\delta}_\eta e^{-\beta_\eta t} \right)
\end{aligned} \tag{35}$$

where $\tilde{\delta}_\eta = \frac{\kappa_\eta \gamma_{\max} \delta_\eta}{\kappa_x \gamma_{\min}}$. Note that for all $t \geq 0$, we have

$$-||Lx|| \left(||Lx|| - \tilde{\delta}_\eta e^{-\beta_\eta t} \right) \leq \frac{\tilde{\delta}_\eta^2}{4} e^{-2\beta_\eta t} \tag{36}$$

where the equality holds if and only if $||Lx|| = \frac{\tilde{\delta}_\eta e^{-\beta_\eta t}}{2}$. Therefore, we have

$$\dot{V} \leq \kappa_x \gamma_{\min} \frac{\tilde{\delta}_\eta^2}{4} e^{-2\beta_\eta t} \triangleq \hat{\delta}_\eta e^{-2\beta_\eta t} \tag{37}$$

where $\hat{\delta}_\eta = \kappa_x \gamma_{\min} \tilde{\delta}_\eta^2 / 4$. Thus

$$V(\infty) - V(0) \leq \frac{\hat{\delta}_\eta}{2\beta_\eta} \tag{38}$$

which in turn implies that V is bounded. Since $x^T L x \geq 0$, $\bar{\xi}^T \Gamma \bar{\xi} \geq 0$, we have $L^{1/2} x$ and $\bar{\xi}$ are both bounded. As a result, Lx is bounded. Suppose $||Lx|| \leq \rho$ for all $t \geq 0$. Let

$$U = \int_0^t \kappa_\eta x(\tau)^T L \Gamma \tilde{\eta}(\tau) d\tau. \tag{39}$$

Then

$$\begin{aligned}
||U|| &\leq || \int_0^t \kappa_\eta x(\tau)^T L \Gamma \tilde{\eta}(\tau) d\tau || \\
&\leq \kappa_\eta \int_0^t ||x(\tau)^T L \Gamma \tilde{\eta}(\tau)|| d\tau \\
&\leq \kappa_\eta \gamma_{\max} \int_0^t ||Lx(\tau)|| \cdot ||\tilde{\eta}(\tau)|| d\tau \\
&\leq \kappa_\eta \gamma_{\max} \rho \int_0^t \delta_\eta e^{-\beta_\eta \tau} d\tau.
\end{aligned} \tag{40}$$

Therefore, U is bounded. Let

$$W = V + U. \tag{41}$$

Then W is lower bounded. Moreover, we have

$$\dot{W} = \dot{V} + \dot{U} = -\kappa_x x^T L \Gamma L x \leq 0. \tag{42}$$

Since Lx, $\bar{\xi}$ and $\tilde{\eta}$ are all bounded, by (30), $\dot{x}$ is bounded. Therefore, $\ddot{W}$ is bounded. Then by Barbalat's Lemma, $\lim_{t \to \infty} \dot{W}(t) = 0$. Since Γ is positive definite, we have $\lim_{t \to \infty} Lx(t) = 0$, which by Remark 1.1 of [14] implies that $\lim_{t \to \infty} (x_i(t) - x_j(t)) = 0$.

□

Remark 2: The integration controller (20a) is incorporated to compensate for the heterogeneity of E_{ci}'s. If $x_i = x_j$, then control law (20) solves the tracking problem for an interconnected leader-follower multi-agent system in the sense that the leader system (18) is interconnected with the followers as indicated by equation (19). From equation (27), it can be seen that if $\eta_i = \eta_0$, then it follows that $P_{ESS} = P_{REF}$. In all, for the general case where E_{ci}'s are non-identical, Problem 1 has been converted into a coupled leaderless consensus problem aiming at $x_i = x_j$ and a tracking problem for an interconnected leader-follower multi-agent system aiming at $\eta_i = \eta_0$.

Remark 3: Note that in the design of the control law (7), the number of the energy storage units N should be exactly known. In contrast, N is no longer needed in the design of the control law (20).

5. SIMULATION

In this section, we consider an energy storage system consisting of eighteen energy storage units. The communication network for the energy storage system is shown in Fig. 2. The initial state-of-energy for the ith energy storage unit is given by $x_i(0) = 0.9 - 0.01 * (i - 1)$. For the case where E_{ci}'s are identical, suppose $E_{ci} = 10$ kwh, and let $k_x = 10^5$, $\mu_\eta = 100$ and $\zeta_i(0) = 0$,

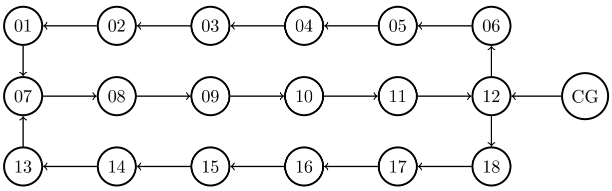

Fig. 2 Communication network.

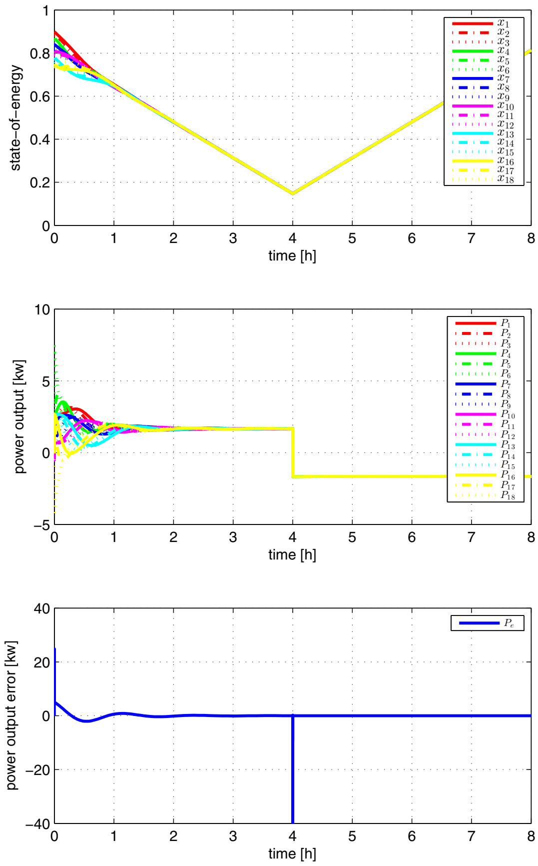

Fig. 3 The case of identical E_{ci}.

$i = 1, \ldots, 18$. For the the case where E_{ci}'s are non-identical, suppose $E_{ci} = 10 - 0.1 * (i - 1)$ kwh, and reset the directed edges of G to be undirected. Let $\kappa_x = 10^5$, $\kappa_\xi = 50$, $\kappa_\eta = 1$, $\eta_0(0) = 0$, $\xi_i(0) = 0$, $\eta_i(0) = 0$, $i = 1, \ldots, 18$. Suppose at $t = 0$ h, $P_{REF} = 30$ kw, and at $t = 4$ h, $P_{REF} = -30$ kw. The profiles of the state-of-energy and power outputs of the energy storage units, and the power output error ($P_e = P_{REF} - P_{ESS}$) of the overall energy storage system under control law (7) and (20) are shown in Fig. 3 and Fig. 4, respectively. It can be seen that, in both cases, reference power tracking and state-of-energy balancing have been achieved successfully.

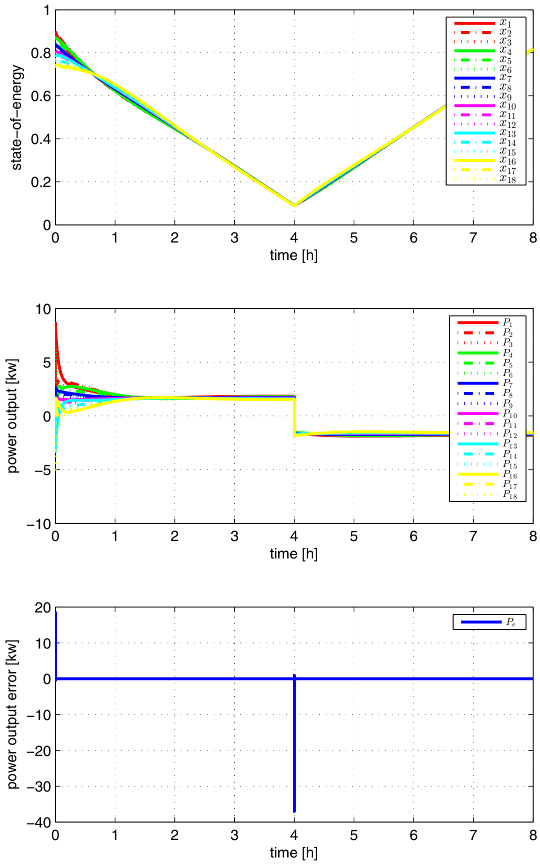

Fig. 4 The case of non-identical E_{ci}.

6. CONCLUSION

In this paper, a dual objective control problem is investigated for energy storage systems. To achieve simultaneous reference power tracking and state-of-energy balancing, two distributed control approaches have been proposed depending on the uniformity of the energy capacities of the energy storage units. Undirected communication network is required for the case where the energy capacities are non-identical. Such an assumption might be conservative, and it would be interesting to consider how to remove this assumption.

7. ACKNOWLEDGEMENT

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