Visual Perception Lab 2 - Camera Calibration

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I. INTRODUCTION

In this report, we will explain how every calibration step is solved and analyse the results of each step. Also, we will discuss the some details of two camera calibration methods, which are the method of Hall and the method of Faugeras.

II. RESULTS & ANALYZATION

A. Define parameters

The first step is just defining all the intrinsic and extrinsic parameters, including pixel conversion parameters, principal point in the image plane, focal length, rotation and translation parameters. They will be set as the ground truth and the transformation matrix is constructed from them. The goal of calibration is to acquire the calculated matrix as similar to the ground truth as possible.

B. Get transformation matrices

We model intrinsic and extrinsic transformation matrices in this part. The intrinsic matrix can be written as following:

$$IN = \begin{bmatrix} \alpha_u & 0 & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

where $\alpha_u, \alpha_v, u_0, v_0$ are all given in II-A.

Also, the extrinsic matrix can be obtained easily:

$$EX = \begin{bmatrix} R_{3\times 3} & T_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix}$$

where R is the rotation matrix and T is the translation vector. They can be calculated based on the rotation angles and translation parameters given in Step A as well.

C. Define 3D points

We randomly define *pn* points in the 3D space, where *pn* is the number of points. *pn* is a tuning parameter which can be modified to see the difference in Step I. However, *pn* should not be smaller than 6 because there are 11 unknown parameters in the transformation matrix and we need at least 11 known measurements to calculate the parameters.

D. Project 3D points to the image plane

When we have the extrinsic and intrinsic matrix, we can project the 3D points with respect to world coordinate to the 2D points with respect to image coordinate using the formulation:

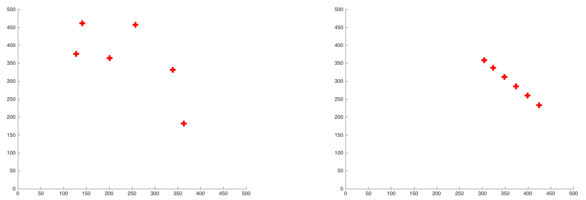
$$\begin{bmatrix} s^I x \\ s^I y \\ s \end{bmatrix} = IN \times EX \times \begin{bmatrix} {}^W X \\ {}^W Y \\ {}^W Z \\ 1 \end{bmatrix}$$

where s is a scaling parameter.

In our code, the function `project3Dto2D.m` is the implementation of the projection when the transformation matrix is given.

E. Check the distribution of the 2D points

After acquiring the 2D points from the last step, we can plot them to check whether they are well spread or not. The Fig. 1 represents the image plane and show the positions of 2D points. The 2D points in Fig. 1(a) are projected from random 3D points while the ones in Fig. 1(b) form a line, which is a special case of point distribution.



(a) Random distribution (b) Special distribution

Fig. 1. 2D points projected from 3D points with different distribution

When we compare the transformation matrices obtained from these two sets of 2D points, we find out the matrix obtained from random 2D points is exactly the same as the ground truth matrix.

$$A_{rand} = \begin{bmatrix} -0.3324 & 0.0624 & -0.2662 & 363.5214 \\ -0.1207 & 0.4903 & 0.1028 & 298.6679 \\ 0.0001 & 0.0004 & -0.0005 & 1.0000 \end{bmatrix}$$

However, as for the points set with the line distribution, the matrix is:

$$A_{line} = \begin{bmatrix} 0.0000 & -0.5363 & 0.0000 & 363.5214 \\ -0.4725 & 0.0000 & 0.0000 & 298.6679 \\ -0.0001 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

The calculation of the matrix is even not correct. This offers us a fact: when we calibrate the camera, we should not just choose the points with special distributions in a small region, for example, points lying on an edge or a corner. The good and simple way is to pick points randomly.

It should be noted that not all the 3D points can be properly projected to the 640×480 image plane defined in Step A because of the intrinsic and extrinsic parameters as well as the range of 3D points. It is possible that some 2D projected points cannot be seen in the image plane, but we can still use it for calibration as long as we know their coordinates in the camera space.

F. Compute the transformation matrix using the method of Hall

The method of Hall simply models the calibration problem as the following formulation without considering intrinsic and extrinsic details.

$$\begin{bmatrix} s^I x \\ s^I y \\ s \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & 1 \end{bmatrix} \begin{bmatrix} {}^W X \\ {}^W Y \\ {}^W Z \\ 1 \end{bmatrix}$$

What we need to do is: first expand and list the formulations using all the points, and then re-arrange to $QA = B$, where $A = [A_{11}, A_{12}, \dots, A_{33}]^t$, which is unknown. And then we can simply use the least mean square method to get the unique solution $A = (Q^t Q)^{-1} Q^t B$. Finally, we reshape the solution vectors back to the 3×4 matrix, which is exactly the transformation matrix we want.

G. Compare the obtained matrix to ground truth

As we mentioned in Step E, we found out the transformation matrix obtained from random points is the same as the ground truth matrix. For the situation of 6 random points, the l_2 norm error between these two matrices is around 10^{-13}, which can actually be ignored.

H. Add noise and compute matrix again

We know that if we want the condition that 95% points should be within the range of $[-1, 1]$ to be satisfied, the standard deviation σ of Gaussian noise should be 0.5. After adding the Gaussian noise to the 2D points, we use the method of Hall to compute the transformation matrix again, and then use the "noisy" matrix to project the 3D points to the image plane.

The way we calculate the accuracy is *MAE*, which means the "mean absolute error" between ground truth 2D points and projected 2D points. The lower the MAE is, the better calibration result we have. As we can see in the Table I, when we increase the standard deviation of Gaussian noise, the calibration result becomes worse.

Actual	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 1.5$
240.9479 263.5417	241.1672 263.0684	241.6434 263.1796	242.1714 261.6311
433.8784 52.1000	433.5787 52.1785	434.1598 52.7256	434.0066 51.5236
321.8114 191.6598	321.6901 191.6195	321.8398 193.3308	322.1803 188.8082
420.3831 283.7253	420.6172 284.2456	419.8706 283.0033	422.1656 284.5664
234.9335 271.7905	234.9955 272.0984	232.9458 272.4651	239.0865 274.6097
427.5865 336.8418	427.5103 336.9572	428.3891 336.7553	427.9469 335.4140
MAE	0.2124	0.7042	1.5369

TABLE I

DIFFERENT STANDARD DEVIATION OF NOISE USING THE METHOD OF HALL

I. Increase the number of 3D points

When we try to increase the number of 3D points, the MAE decreases. Assuming the standard deviation of noise is 0.5, for the 6-point situation, the MAE is around 0.3 while it is at most 0.15 if there are 50 noisy 2D points. If we desire to calibrate camera accurately, more points are suggested.

Number of Points	MAE (average)
6	0.3
50	0.15
500	0.05

TABLE II
ACCURACY OF DIFFERENT NUMBER OF POINTS

Actual	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 1.5$
240.9479 263.5417	241.1672 263.0684	241.6434 263.1796	242.1714 261.6311
433.8784 52.1000	433.5787 52.1785	434.1598 52.7256	434.0066 51.5236
321.8114 191.6598	321.6901 191.6195	321.8398 193.3308	322.1803 188.8082
420.3831 283.7253	420.6172 284.2456	419.8706 283.0033	422.1656 284.5664
234.9335 271.7905	234.9955 272.0984	232.9458 272.4651	239.0865 274.6097
427.5865 336.8418	427.5103 336.9572	428.3891 336.7553	427.9469 335.4140
MAE	0.2124	0.7042	1.5369

TABLE III
DIFFERENT STANDARD DEVIATION OF NOISE USING THE METHOD OF FAUGERAS

J. Compute the parameters obtained from the method of Faugeras

In this part, we implement the calibration using the method of Faugeras-Toscani. All the formulations used for calculation come from [1]. Our function `Faugeras.m` applies the least mean square method to calculate vector X, while `computeTrans_F.m` extracts both intrinsic and extrinsic parameters from X and reconstructs the transformation matrix .

The method of Faugeras is able to obtain exactly the same transformation matrix as the ground truth when the 2D points have no noise. Moreover, the method of Faugeras can be used to calculate not only the transformation matrix but also the intrinsic and extrinsic parameters, which has an advantage over the method of Hall.

K. Add noise and compute parameters again

The same as the Step H, we add various noise to the 2D points which are used to calculate the transformation matrix and finally to compare the results acquired from 2D points with different levels of noise, shown in Table III.

Comparing Table I with III, their discrepancies are really insignificant but we can say that the method of Hall can give a slightly better result than the method of Faugeras. The reason is: the method of Hall simply models the calibration problem considering neither intrinsic nor extrinsic details, so the transformation matrix is directly related to the process of calibration. In contrast, the idea of the method of Faugeras is first to extract the intrinsic and extrinsic parameters and then reconstruct the transformation matrix, which may introduce some unexpected noise during the calculation.

L. Draw the calibration system

The Fig. 2 illustrates the whole calibration system. In the system, we set the origin of the camera coordinate in the position of (0,0,0), so correspondingly, the position of the origin of the world coordinate with respect to the camera coordinate is equal to $(t_x, t_y, t_z) = (100, 0, 1500)$

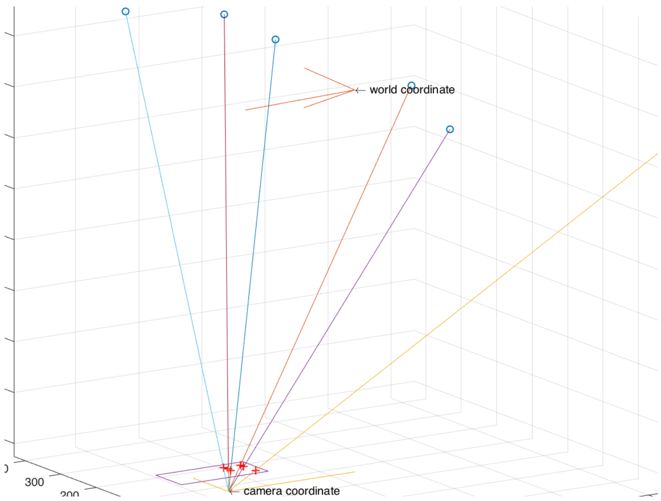


Fig. 2. calibration system

We can notice from the image that all the 2D points on the image plane lie in the line between the corresponding 3D points and the focal point. Therefore, we are also able to say that all the optical rays cross at the focal point. This shows our calibration is correct.

III. Conclusions

This report mainly discusses the implementation and results of the two well-known calibration techniques. When there is no noise, both of the methods can ideally acquire the ground truth transformation matrix. However, there exist more or less some noise in the measurement in real life. It turns out that increasing the number of points used for calibration can lead to accurate results.

One final comment about the lab is: although we know that the result acquired from the method of Hall should be a bit better than the method of Faugeras if some noise has been added to the 2D points, actually the difference between the two results is really insignificant. We really hope the professor can give some feedback on this part.

References

[1] Salvi, Joaquim, Xavier Armangu, and Joan Batlle. ”A comparative review of camera calibrating methods with accuracy evaluation.” Pattern recognition 35, no. 7 (2002): 1617-1635.