Visual Perception Lab 3 - Epipolar Geometry

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I. INTRODUCTION

In this report, we will explain how every step in the instruction about epipolar geometry is solved as well as analysing the results. During the implementation, we try to compute the fundamental matrix by using both 8-point method and SVD. Also, we will discuss some comparisons when we increase the noise level and number of 3D points.

Moreover, the whole epipolar geometry system is drawn in the last part.

II. ANALYZATION & RESULTS

A. Define parameters of camera 1

The first step is just defining all the intrinsic and extrinsic parameters of camera 1. Because the camera 1 frame is set to world frame, the rotation matrix is identity and translation is equal to zero.

B. Define parameters of camera 2

Just the same as the last step, here all the parameters of camera 2 are defined. It should be noted that the total rotation matrix is the multiplication of the rotation matrices in x, y, z 3 directions.

C. Get intrinsic matrices

We model intrinsic matrices of both cameras in this part. The intrinsic matrix can be written as following:

$$A = \begin{bmatrix} \alpha_u & 0 & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

where $\alpha_u, \alpha_v, u_0, v_0$ for camera 1 and 2 are given in II-A and II-B respectively.

D. Get the Fundamental matrix

The fundamental matrix can be obtained using the formulation:

$$F = A'^{-t} R^t [t]_x A^{-1}$$

where A and A' are the intrinsic matrices of camera 1 and 2 respectively. The $[t]_x$ is the translation vector in the form of its antisymmetrix matrix:

$$[t]_x = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$$

Using this formulation, ground truth of fundamental matrix can be obtained.

E. Define 3D points

We first define 20 3D points in the world frame. In the later step, we will compare different number of noisy points (from 10 to 200).

F. Project 3D points to the both image planes

When we have the extrinsic and intrinsic matrices of both camera, we can project the 3D points with respect to world frame, to the 2D points with respect to image frame of both cameras using the formulation:

$$\begin{bmatrix} s^I x \\ s^I y \\ s \end{bmatrix} = IN \times EX \times \begin{bmatrix} {}^W X \\ {}^W Y \\ {}^W Z \\ 1 \end{bmatrix}$$

where s is a scaling parameter.

Here, we should pay attention that here we transform 3D points from the world frame to the image frame. For the camera 2, the extrinsic transformation matrix should be:

$$EX_2 = inv(\begin{bmatrix} R_{3\times3} & t_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix}) = \begin{bmatrix} R^t_{3\times3} & -R^t t_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix}$$

G. Draw the 2D points

In this step we simply draw the 2D points in two camera windows. The code is within the function `epi_plot.m` and the results are shown in Fig. 1.

H. Compute Fundamental matrix by 8-point method

Because we know that the system of equation is in the following form:

$$m'^t F m = 0$$

where m' and m are the 2D points already acquired in the step II-F. What we want to do is to model the above formulation to a linear equation form:

$$U_n f = 0$$

where U_n is composed of the multiplication of m' and m elements, and f contains all the elements in the 3×3 fundamental matrix. When we divide f by the last element of f, only 8 unknown elements remain and that is why we need at least 8 points to solve the problem. Finally, we can use the least mean square method to solve the linear equation. In our code, the script `compute_F.m` corresponds to this step.

I. Compare the fundamental matrix acquired in II-H with ground truth

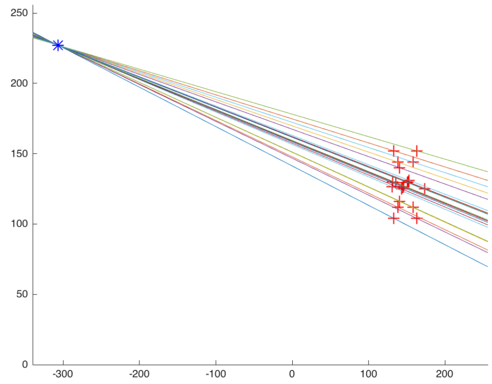
Here we compute the mean difference between two matrices and the result is 1.6528e-17, which can be ignored. Therefore, we can say that, when there is no noise existing in the 2D points, the fundamental matrix obtained in the last step is the same as the ground truth obtained in step II-F.

J. Draw the epipolar geometry

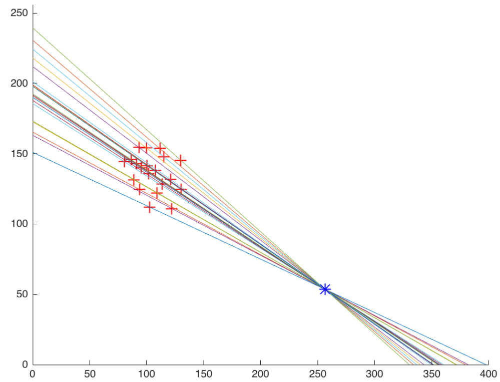
Here we want to get the epipolar lines in the camera. The first step is to get the cross product of two vectors in the π plane, whose final form can be written as:

$$l'_m = Fm = [u_1, u_2, u_3]^t$$

Based on this function, we know that any point that lies on the corresponding epipolar line should have $[x, y, 1][u_1, u_2, u_3]^t = 0$. So the m, d in $y = mx + d$ can be obtained. After that, we choose the proper boundaries to plot the epipolar lines in order to show the epipoles later.



(a) Camera 1



(b) Camera 2

Fig. 1. Epiploar geometries of two cameras (blue points are the epipoles)

After acquiring epipolar lines, we also want to plot the epipoles. There are 3 methods to get epipoles:

1) Get the intersections of two epipolar lines
2) The last column of U and V in SVD(F)
3) Project the focal point of each camera to the image plane of the other

In this case, all the 3 methods can be applied. However, when the noise is added in the next step, the epipolar lines cannot intersect in a single point so the first one cannot be used. Also, the rank of the fundamental matrix may not 2 anymore, epipoles acquired from the second method may be unreal. We choose to project the focal point, which always leads to the ground truth of epipoles. Be careful, we should add the intrinsic matrix to change epipoles to the pixels instead of metrics. The whole process is implemented in the function `epi_plot.m`. The blue points are the epipoles in the 1

K. Compute Fundamental matrix by SVD

Like in step II-F, we similarly model the system to $U_n f = 0$, but instead of dividing f by the last element, we directly get the nullspace of U_n using SVD. As we know that the last column of U_n is all 1, the rank of U_n should be 8 and the nullspace is the last column of the V of the SVD(U_n). In our code, function `compute_F_svd.m` implements this part.

L. Add noise and compare epipolar geometry

In this part, we add different levels of noise to 2D points on both image planes. Here we try different variation σ of Gaussian noise. Fig. 2 illustrates the image plane of camera 2 after adding different levels of noise.

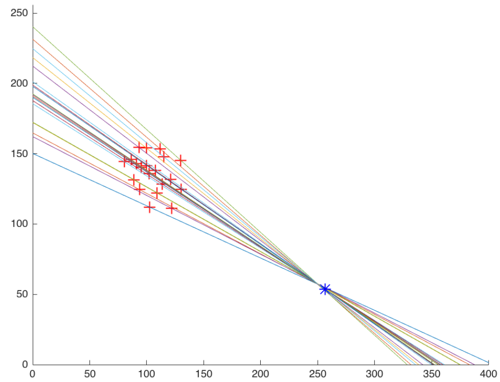
As we can see, with the increment of the noise, the 2D points become further to the corresponding epipolar lines. Also, the ground truth epipoles are not the intersections of the epipolar lines anymore. Also, along the columns in Table I,we can notice that the mean distances increase significantly when the noise level increases.

Moreover, we can notice something interesting in Fig. 2(c). Here we set the noisy F to rank 2, which leads to a unique epipole, but the epipolar lines spread in order to get an unique epipole (mean distance between points and corresponding epipolar lines increases almost 10 times). If we want to find the corresponding points of the other camera, we should keep noisy F rank 3 because the points are closer to the corresponding epipolar lines. If we want to get some geometry details, for example, get the R and t from the fundamental matrix, we need to set rank to 2 in order to get a unique epipole.

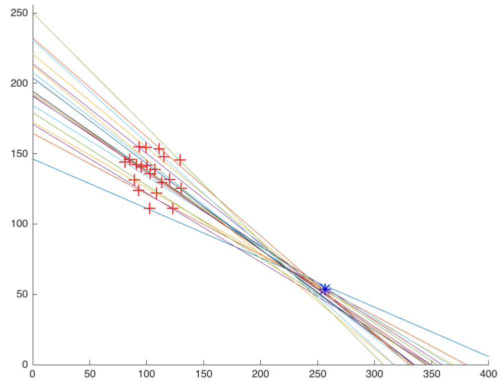
M. Compare the epipolar geometries using 8-point method and SVD

As shown in the Table I, the two methods have no much difference when the noise level is low. In the cell that two values are different, the results from SVD method slightly outperform the ones from the 8-point method.

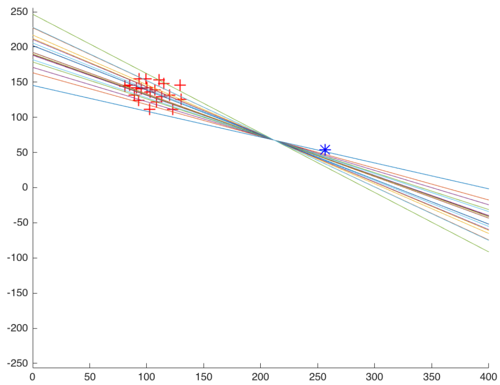
Also, when we increase the number of points, the mean distances decrease as expected. However,



(a) $\sigma = 0.05$, mean distance = 0.054



(b) $\sigma = 0.5$, mean distance = 0.4514



(c) $\sigma = 0.5$, mean distance = 4.0293 (set rank(F) = 2)

Fig. 2. Epiploar geometries of camera2 with different σ (blue points are the epipoles)

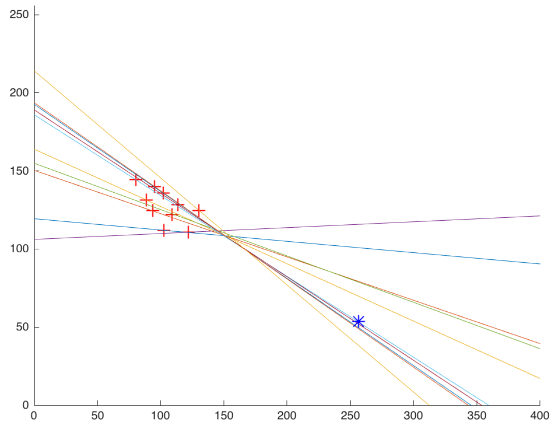


Fig. 3. $\sigma = 0.05$, 10 noisy points (blue points are epipoles)

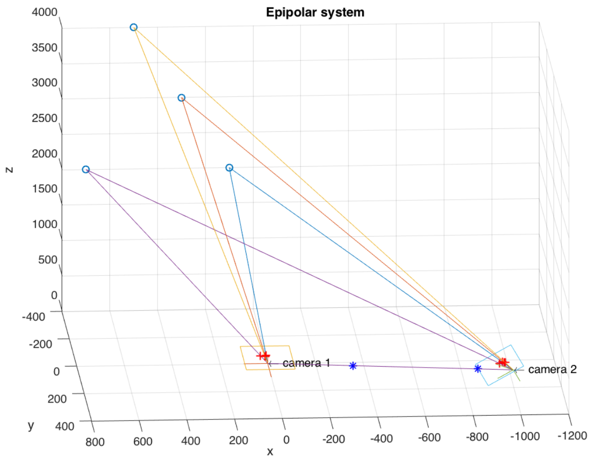


Fig. 4. Epipolar geometry system (blue points are epipoles)

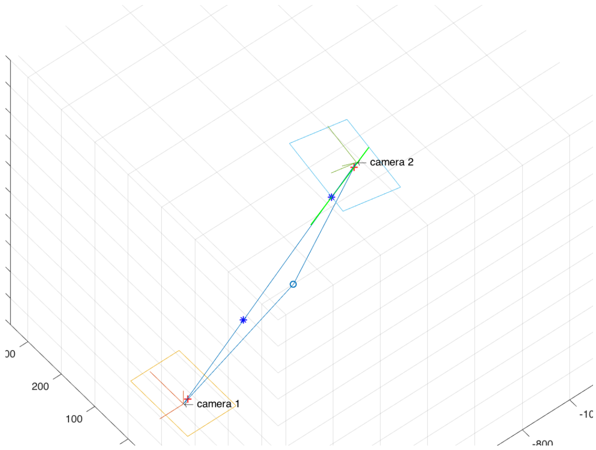
one thing that we want to mention here is: the data is weird when only 10 points are used to calculate the fundamental matrix. As we can see, almost all the mean distances of 10 points are smaller than the distances acquired when more points are involved.

When we compare the epipolar geometry of the 10 noisy points ($\sigma = 0.05$) in Fig. 3 with 20 noisy points system in Fig .2(a), we find out although the distances between points and the epipolar lines are very small, the epipolar lines actually have wrong slopes. This implies the obtained fundamental matrix is quite different from the ground truth one when only 10 noisy points are taken. As a result, we know that we should take as many points as possible when we want to calculate fundamental matrix.

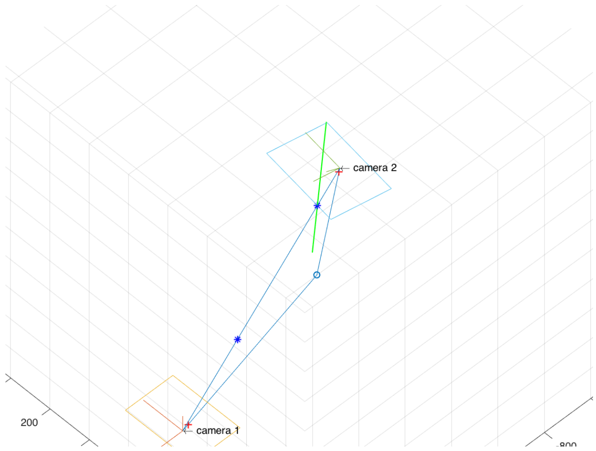
N. Draw the calibration system

The Fig. 4 illustrates the whole two camera epipolar geometry system. In the system, we need to transform all the points and lines to the 3D world coordinate (function `im2world.m`). The focal length is set to 80mm for both cameras. Here we also check if the epipolar lines are coplanar in plane π when there exists noise. As we compare in Fig. 5, the epipolar line is coplanar when there is no noise, while the coplanarity is lost in the presence of noise.

Moreover, when we plot more epipolar lines, they really intersect in the epipole (Fig. 6).



(a) Without noise



(b) With noise

Fig. 5. Coplanarity of the epipolar lines (green lines are the epipolar lines)

TABLE I

MEAN DISTANCE BETWEEN 2D POINTS AND THE CORRESPONDING EPIPOLAR LINES

Point number	**10**		**30**		**50**		**100**		**200**	
Noise σ	LMS	SVD	LMS	SVD	LMS	SVD	LMS	SVD	LMS	SVD
0.05	0.0496	0.0497	0.0545	0.0545	0.0557	0.0557	0.0564	0.0564	0.0537	0.0537
0.1	0.1046	0.1046	0.0727	0.0727	0.0748	0.0748	0.0878	0.0878	0.0739	0.0739
0.5	1.5049	1.5050	1.4078	1.4078	0.8280	0.8280	0.7652	0.7651	0.5556	0.5556
1	0.6980	0.6979	7.4358	7.4360	6.4812	6.4812	2.0376	2.0376	1.4037	1.4035

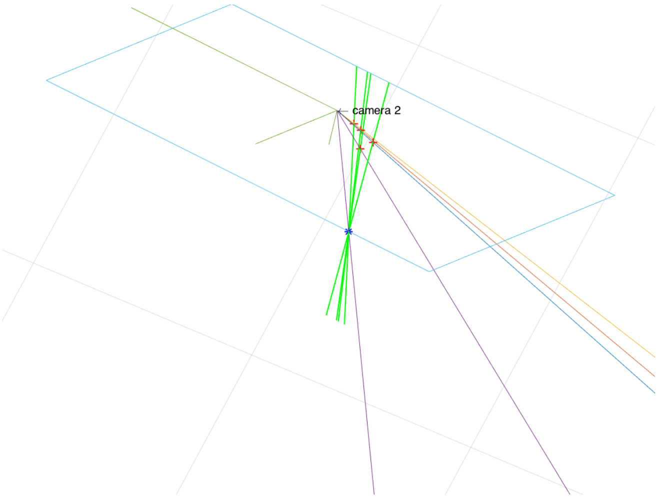


Fig. 6. Intersection of epipolar lines in the image 2 system

III. CONCLUSIONS

This report mainly discusses the implementation of calculating fundamental matrix using 8-point method and SVD respectively. When there is no noise, both of the methods can ideally acquire the ground truth fundamental matrix. When noise is involved like in real life, increasing the number of points used for calibration can lead to accurate results.

Besides, we also plot the two cameras epipolar geometry system, which vividly illustrates the coplanarity of epipolar lines and the epipoles are the intersection of any of two epipolar lines if there is no noise.