1. Naloga: Sferna trigonometrija

1. Napiši program, ki za objekt z znano rektascenzijo in deklinacijo (α in δ) izračuna položaj na nebu ob določeni uri na na določenem mestu na Zemlji (ϕ in λ). Položaj izrazi kot azimut in višino nad obzorjem (A in h). Preveri, ali se tvoj izračun sklada s podatki, ki jih najdeš na nebu ali prebereš iz kakega programa.

2. Nariši, kako se tekom noči iz 19. na 20. februar spreminjata azimut in višina za zvezdi Procyon in β UMi, če ju opazujemo z Astronomskega observatorija na Golovcu ($\lambda = 14.5277°$, $\phi = 46.0439°$).

3. Nasprotno od zvezd, Sonce nima konstantne pozicije v nebesnem ekvatorialnem koordinatnem sistemu. Spreminjanje rektascenzije in deklinacije Sonca opišemo z enačbama:

$$\tan\alpha_{\odot} = \tan\lambda\cos\epsilon, \qquad \sin\delta_{\odot} = \sin\lambda\sin\epsilon,$$

kjer je $\epsilon = 23.44°$ nagnjenost Zemljine osi glede na ekliptiko, λ pa je ekliptična dolžina Sonca. Prvi pribljižek je, če rečemo, da je λ sorazmerna s časom od zadnjega pomladanskega enakonočja (Δt):

$$\lambda = \frac{\Delta t/\mathrm{dan}}{365.2422}2\pi.$$

Izračunaj, kako se spreminjata višina in azimut Sonca tekom leta za različne kraje na Zemlji, če ju merimo vsak dan ob istem času. Komentiraj obliko analeme.

4. Boljši približek je, če upoštevamo, da Zemljina orbita okoli Sonca ni okrogla. Izračunamo lahko srednjo anomalijo (M) Zemljine orbite ob julijanskem datumu J:

$$M = (M_0 + M_1\,(J - J_{2000})) \bmod 360°,$$

kjer je $J_{2000} = 2451545$. Konstanti M_0 in M_1 podaja tabela spodaj.

Ekliptično dolžino Sonca izračunamo po formuli

$$\lambda = M + \Pi + C + 180°,$$

kjer je Π ekliptična dolžina perihelija, C pa dobimo po formuli

$$C \simeq C_1\sin(M) + C_2\sin(2M) + C_3\sin(3M) + C_4\sin(4M) + C_5\sin(5M) + C_6\sin(6M).$$

Ekliptično dolžino perihelija in konstante C_1 do C_6 podaja spodnja tabela. Primerjaj dobljeno analemo s tisto iz prejšnjega vprašanja.

Planet	M_0	M_1	Π	C_1	C_2	C_3	C_4	C_5	C_6
Merkur	174.7948	4.09233445	230.3265	23.4400	2.9818	0.5255	0.1058	0.0241	0.0055
Venera	50.4161	1.60213034	73.7576	0.7758	0.0033				
Zemlja	357.5291	0.98560028	102.9373	1.9148	0.0200	0.0003			
Mars	19.3730	0.52402068	71.0041	10.6912	0.6228	0.0503	0.0046	0.0005	
Jupiter	20.0202	0.08308529	237.1015	5.5549	0.1683	0.0071	0.0003		
Saturn	317.0207	0.03344414	99.4587	6.3585	0.2204	0.0106	0.0006		
Uran	141.0498	0.01172834	5.4634	5.3042	0.1534	0.0062	0.0003		
Neptun	256.2250	0.00598103	182.2100	1.0302	0.0058				

vir: `https://www.aa.quae.nl/en/reken/zonpositie.html`

5. (*Dodatno*) Izračunaj analemo za Mars in še kakšen planet.

1. Homework: Spherical trigonometry

1. Write a software that calculates the position in the sky for an object with given coordinates (right ascension α and declination δ) for an observer on Earth (at longitude λ and latitude ϕ). Give the position as the azimuth and altitude above the horizon (A and h). Check whether your results match the data from the internet or from some other program.

2. Plot how the azimuth and altitude change throughout the night of 19. to 20. February for stars Procyon and β UMi. Assume we are observing them from the Astronomical observatory at Golovec ($\lambda = 14.5277°$, $\phi = 46.0439°$).

3. Unlike stars, the Sun does not have a constant position in the celestial coordinate system. Right ascension and declination of the Sun can be described by the following equations:

$$\tan\alpha_\odot = \tan\lambda\cos\epsilon, \qquad \sin\delta_\odot = \sin\lambda\sin\epsilon,$$

where $\epsilon = 23.44°$ is the obliquity of the Earth's orbit, and λ is the ecliptic longitude of the Sun. In the first approximation you can say that λ is proportional to the time since the last vernal equinox (time Δt):

$$\lambda = \frac{\Delta t/\mathrm{dan}}{365.2422} 2\pi.$$

Calculate how the azimuth and altitude of the Sun change throughout the year if measured at the same time of the day every day. Discuss the sahpe of the analema curve.

4. A better approximation is to take the elliptical shape of the Earth's orbit into the account. We can calculate the mean anomaly (M) of the Earth's orbit at julian date J:

$$M = (M_0 + M_1 (J - J_{2000})) \bmod 360°,$$

where $J_{2000} = 2451545$. Constants M_0 and M_1 are given in the table below. Ecliptic longitude of the Sun is then calculated as

$$\lambda = M + \Pi + C + 180°,$$

where Π is the ecliptic longitude of the perihelion, and C is calculated from the formula

$$C \simeq C_1\sin(M) + C_2\sin(2M) + C_3\sin(3M) + C_4\sin(4M) + C_5\sin(5M) + C_6\sin(6M).$$

Ecliptic longitude of the perihelion and constants C_1 to C_6 are given in the table below. Compare this analema with the one from the previous question.

Planet	M_0	M_1	Π	C_1	C_2	C_3	C_4	C_5	C_6
Mercury	174.7948	4.09233445	230.3265	23.4400	2.9818	0.5255	0.1058	0.0241	0.0055
Venus	50.4161	1.60213034	73.7576	0.7758	0.0033				
Earth	357.5291	0.98560028	102.9373	1.9148	0.0200	0.0003			
Mars	19.3730	0.52402068	71.0041	10.6912	0.6228	0.0503	0.0046	0.0005	
Jupiter	20.0202	0.08308529	237.1015	5.5549	0.1683	0.0071	0.0003		
Saturn	317.0207	0.03344414	99.4587	6.3585	0.2204	0.0106	0.0006		
Uranus	141.0498	0.01172834	5.4634	5.3042	0.1534	0.0062	0.0003		
Neptune	256.2250	0.00598103	182.2100	1.0302	0.0058				

source: `https://www.aa.quae.nl/en/reken/zonpositie.html`

5. (*Extra credits*) Calculate the analema for Mars and some other planet.