Pogosto se zapiše: $P = \underbrace{P_{ion} + P_e}_{P_{gas}} + P_{rad} = P_{gas} + P_{rad}$

$$P_{gas} = \beta P$$

β... delež tlaka, ki ga prispeva plin

$$P_{rad} = (1-\beta) P$$

Pri manjših (lahkih) zvezdah pride blizu ~~d~~ e NR, pri masivnih zvezdah pa postane sevalni tlak važen.

Adiabatni eksponent

u... notranja energija na enoto mase

$$du + p\, d\left(\frac{1}{\rho}\right) = 0$$

$$u = \phi \frac{p}{\rho}$$

$$\Rightarrow \quad \phi \frac{1}{\rho} dp + \phi p\, d\left(\frac{1}{\rho}\right) + p\, d\left(\frac{1}{\rho}\right) = 0$$

$$\phi \frac{1}{\rho} dp + (1+\phi) p\, d\left(\frac{1}{\rho}\right) = 0$$

$$(1+\phi) p\, d\left(\frac{1}{\rho}\right) = -\phi \frac{1}{\rho} dp$$

$$\frac{(1+\phi)}{-\phi} \frac{d(1/\rho)}{1/\rho} = \frac{dp}{p}$$

$$\frac{(1+\phi)}{-\phi} d\left(\ln \frac{1}{\rho}\right) = d(\ln p)$$

$$\Rightarrow \quad \ln \frac{1}{\rho} \left(\frac{\phi+1}{-\phi}\right) = \ln p + C$$

obrnemo ln

Tako dobimo:

$$\ln \rho \left(\frac{\phi+1}{\phi}\right) = \ln p + C$$

$$p \propto \rho^{\frac{\phi+1}{\phi}}$$ Adiabatni eksponent γ_a

Primeri:

$\gamma_a = 5/3$ nerelativistični idealni plin, popolnoma degeneriran plin

$\gamma_a = 4/3$ Sevanje, relativistični degeneriran plin

Sahova enačba

Z njeno pomočjo razvijemo gostoto ionov/nevtralnih gradnikov.

n_0 (nevtralni atomi)

n_+ (ioni)

n_e (prosti elektroni)

Stopnja ionizacije x: $x = \frac{n_+}{n_0 + n_+}$ št. ioniziranih / vsi delci

$$\boxed{\frac{n_+ n_e}{n_0} = \frac{g}{h^3}(2\pi m_e kT)^{3/2} e^{-\chi/kT}}$$ Sahova enačba

χ ... ionizacijska energija (en. da odstranimo elektron)

$$p = nkT = (n_0 + n_+ + n_e)kT = \left(\frac{n_0 + n_+}{n_0 + n_+} + \frac{n_e}{n_0 + n_+}\right)kT(n_0 + n_+) =$$

$n_e = n_+$

$$= (1 + x)kT(n_0 + n_+) = p$$

Mogoče uporabno pri kaki nalogi:

$$\frac{x^2}{1 - x^2} = \frac{g}{h^3}(2\pi m_e)^{3/2}\frac{(kT)^{5/2}}{p} e^{-\chi/kT}$$

V primeru delne ionizacije:

$$\chi\frac{n_+}{\rho} = \chi\frac{n_+}{(n_0+n_+)m_H} = \chi\frac{x}{m_H}$$

$$u = \frac{3}{2}\frac{p}{\rho} + \chi\frac{x}{m_H}$$

Zanima nas povezava med adiabatnim eksponentom in stopnjo ionizacije:

$$\frac{\phi+1}{\phi}\ln\rho \propto \ln p$$

$$\gamma_a \ln\rho \propto \ln p \Rightarrow \gamma_a \propto \frac{\ln p}{\ln \rho}$$

$$du + p\,d\left(\frac{1}{\rho}\right) = 0$$

Prvi zakon termodinamike (za adiabatne procese)

$$\frac{3}{2}\frac{1}{\rho}dp + \frac{3}{2}p\,d\left(\frac{1}{\rho}\right) + \frac{\chi}{m_H}\frac{\partial x}{\partial p}dp + \frac{\chi}{m_H}\frac{\partial x}{\partial \rho}d\rho + p\,d\left(\frac{1}{\rho}\right) = 0 \quad / \cdot \frac{\rho}{p}$$

$$d(1/\rho) = -1/\rho^2 \cdot d\rho$$

$$\frac{3}{2}\frac{dp}{p} + \frac{3}{2}\rho\,d\left(\frac{1}{\rho}\right) + \frac{\chi}{m_H}\rho\frac{\partial x}{\partial p}\frac{dp}{p} + \frac{\chi}{m_H}\frac{\rho^2}{p}\frac{\partial x}{\partial \rho}\frac{d\rho}{\rho} + \rho\,d\left(\frac{1}{\rho}\right) = 0$$

$$\left[\frac{3}{2} + \frac{\chi}{m_H}\rho\frac{\partial x}{\partial p}\right]\frac{dp}{p} - \frac{3}{2}\frac{d\rho}{\rho} + \frac{\chi}{m_H}\frac{\rho^2}{p}\frac{\partial x}{\partial \rho}\frac{d\rho}{\rho} - \frac{d\rho}{\rho} = 0$$

$$\left[\frac{3}{2} + \frac{\chi}{m_H}\rho\frac{\partial x}{\partial p}\right]\frac{dp}{p} - \left[\frac{5}{2} - \frac{\chi}{m_H}\frac{\rho^2}{p}\frac{\partial x}{\partial p}\right]\frac{d\rho}{\rho} = 0 \quad (*)$$

Od preš:

$$p m_H = (1+x)\overbrace{(n_0+n_+)kT\,m_H}^{\rho} = (1+x)\rho kT$$

$$\Rightarrow \frac{\rho}{m_H} = \frac{p}{(1+x)kT} \qquad \frac{\rho}{p} = \frac{m_H}{(1+x)kT}$$

$$(*) \Rightarrow \left[\frac{3}{2} + \chi\frac{p}{(1+x)kT}\frac{\partial x}{\partial p}\right]\underbrace{\frac{dp}{p}}_{d(\ln p)} = \left[\frac{5}{2} - \chi\frac{\rho}{(1+x)kT}\frac{\partial x}{\partial \rho}\right]\underbrace{\frac{d\rho}{\rho}}_{d(\ln\rho)} \qquad \gamma_a \propto \frac{\ln p}{\ln\rho}$$

Se lahko pokaže

$$\gamma_a(x) = \frac{5 + \left(\frac{5}{2} + \frac{\chi}{kT}\right)^2 x(1-x)}{3 + \left[\frac{3}{2} + \left(\frac{3}{2} + \frac{\chi}{kT}\right)^2\right]x(1-x)}$$

Malo primeri:

$$\left.\begin{array}{l} x = 0 \\ x = 1 \end{array}\right\} \gamma_a = 5/3$$

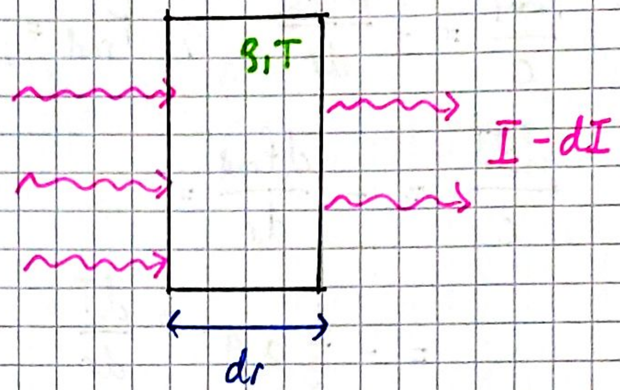
$$\gamma_{a,min} = 0,5$$

$$\frac{\chi}{kT} = 1 \rightarrow 1,63$$

$$\frac{\chi}{kT} = 10 \rightarrow 1,21$$

Sevalni prenos

I... intenziteta $[I] = \frac{\text{Energija}}{m^2 \cdot t}$



$$dI = -\varkappa I \rho \, dr$$

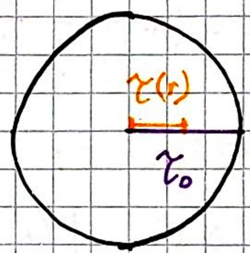
↳ Koeficient neprozornosti

$$\frac{dI}{I} = -\varkappa \rho \, dr \Rightarrow I(r) = I_0 e^{-\varkappa \rho r}$$

$$\varkappa = \varkappa_0 \rho^a T^b$$

$$\frac{1}{\varkappa \rho} = \text{karakteristična dolžina absorpcije/sipanja}$$

Optična globina



$$d\tau = -\varkappa \rho \, dr$$

$$I(r) = I_0 e^{-(\tau_0 - \tau(r))} \quad ; \quad \tau_0 > \tau(r)$$

Na podlagi optične globine določamo tudi zvezdni radij (kje je meja):

$$-\int_{\infty}^{R} \varkappa \rho \, dr \simeq 1$$ (snov izven zvezde je prozorna)

Interakcije:

Delež vodika

- Sipanje elektronov (Compton, Tomson) $\varkappa \simeq \frac{1}{2} \varkappa_0 (1 + X)$
- abs. prosto-prosto $\varkappa \simeq \frac{1}{2} \varkappa_0 (1 + X) \left\langle \frac{Z^2}{A} \right\rangle \rho T^{-7/2}$

  Dominantni znotraj zvezd
- fotoionizacija
- abs. vezano-vezano

$$\frac{|dI|}{c} = -\frac{dP_{rad}}{dr} \quad ; \quad P_{rad} = \frac{1}{3} a T^4$$

$$\frac{\kappa \rho I}{c} = -\frac{dP_{rad}}{dr} =$$

$$= -\frac{1}{3} a 4 T^3 \frac{dT}{dr}$$

$$I = -\frac{c a 4 T^3}{\kappa \rho 3} \frac{dT}{dr}$$

$$L(r) = -4\pi r^2 \frac{4}{3} \frac{a c T^3}{\kappa \rho} \frac{dT}{dr}$$

Izsev pri nekem radiju

$$L(r) = -4\pi r^2 \frac{4}{3} \frac{a c T^3}{\kappa \rho} \frac{dT}{dr}$$

Zanima nas opis temperature:

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa \rho}{T^3} \frac{L(r)}{4\pi (r^2)}$$

Oz. ker smo rekli, da bi gledali na enoto mase:

$$\frac{dT}{dm} = -\frac{3}{4ac} \frac{\kappa}{T^3} \frac{L}{(4\pi r^2)^2}$$

Še ostale:

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho} \qquad \frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \qquad \frac{dL}{dm} = q$$

Če dodamo še neko enačbo

$$P \propto \rho^x$$

lahko rešimo ločeno

Politropni modeli

$$\frac{dp}{dr} = -\rho \frac{Gm}{r^2} \Big/ \cdot \frac{r^2}{\rho} \qquad \frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dp}{dr}\right) = -G\frac{dm}{dr}$$

$$\frac{r^2}{\rho}\frac{dp}{dr} = -Gm$$

$$\Rightarrow \frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dp}{dr}\right) = -G\,4\pi r^2 \rho \Big/ : r^2$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dp}{dr}\right) = -4\pi G\rho \quad (*)$$

Politropna enačba stanja:

$$p = K \cdot \rho^{\gamma}$$

K → Konstanta.

Politropni indeks n:

$$\gamma = 1 + \frac{1}{n}$$

Tako ρ:

$$\frac{dp}{dr} = K\gamma\rho^{\gamma-1}\frac{d\rho}{dr}$$

in:

$$(*) \Rightarrow K\gamma\frac{1}{r^2}\left(\frac{r^2}{\rho}\rho^{\gamma-1}\frac{d\rho}{dr}\right) = -4\pi G\rho$$

$$\cancel{K\gamma\frac{1}{r^2}} \quad K\,\frac{1+n}{n}\,\frac{1}{r^2}\left(r^2\rho^{-1}\rho^{1/n}\frac{d\rho}{dr}\right) = -4\pi G\rho$$

Politropni zvezdni modeli (znova/ponoviter prejšnje ure)

Povežemo kontinuitetno in hidrostatično enačbo.

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{dp}{dm} = -\frac{Gm}{4\pi r^4}$$

Dodamo še enačbo stanja.

$$P = K\rho^{\gamma} \quad \text{(ni odvisnosti od temperature)}$$

Politropni indeks n:

$$\gamma = 1 + 1/n$$

deg. plin el. $\gamma = 5/3$ (nerel.)

$\gamma = 4/3$ (relativističen)

Torej:

$$\frac{dp}{dr} = -\rho\frac{Gm}{r^2} \Big/ \cdot \frac{r^2}{\rho}$$

$$\frac{r^2}{\rho}\frac{dp}{dr} = -Gm \quad \to \quad \frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dp}{dr}\right) = -G\frac{dm}{dr}$$

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dp}{dr}\right) = -G4\pi r^2\rho \qquad \frac{dp}{dr} = K\gamma\rho^{\gamma-1}\frac{d\rho}{dr}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dp}{dr}\right) = -G4\pi\rho$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}K\gamma\rho^{\gamma-1}\frac{d\rho}{dr}\right) = -G4\pi\rho$$

$$K\left(\frac{1+n}{n}\right)\frac{1}{r^2}\frac{d}{dr}\left(r^2\rho^{\frac{1}{n}-1}\frac{d\rho}{dr}\right) = -4\pi G\rho$$

$$K\left(\frac{1+n}{n}\right)\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho^{\frac{n-1}{n}}}\frac{d\rho}{dr}\right) = -4\pi G\rho \quad (*)$$

Rešitev $\rho(r)$ in ki bo veljala na območju $0 \leq r \leq R$ imenujemo Politrop.
Potrebujemo še dva robna pogoja:

- $r = R \rightarrow \rho(R) = 0$
- $r = 0 \rightarrow \frac{d\rho}{dr} = 0$

Nova spremenljivka:

$$\rho = \rho_c\theta^n;\ 0 \leq \theta \leq 1$$

$$\rho^{\frac{n-1}{n}} = \rho_c^{\frac{n-1}{n}}\theta^{n-1} \quad \text{in} \quad \frac{d\rho}{dr} = \rho_c n\theta^{n-1}\frac{d\theta}{dr}$$

$$(*) \Rightarrow K\left(\frac{1+n}{\cancel{n}}\right)\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho_c^{\frac{n-1}{n}}\cancel{\theta^{n-1}}}\cancel{\rho_c}\cancel{n}\cancel{\theta^{n-1}}\frac{d\theta}{dr}\right) = -4\pi G\cancel{\rho_c}\theta^n$$

$$\frac{(n+1)K}{r^2\rho_c^{\frac{n-1}{n}}}\frac{d}{dr}\left(r^2\frac{d\theta}{dr}\right) = -4\pi G\theta^n$$

Nova spremenljivka:

$$r = \alpha\xi$$
$$dr = \alpha\, d\xi$$

$$\Rightarrow \underbrace{\frac{(n+1)K}{4\pi G\rho_c^{\frac{n-1}{n}}}}_{=\alpha^2\,[m^2]}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\theta}{dr}\right) = -\theta^n$$

$$\cancel{\alpha^2}\frac{1}{\cancel{\alpha^2}\xi^2}\frac{1}{\cancel{\alpha}}\frac{d}{d\xi}\left(\frac{\cancel{\alpha^2}\xi^2}{\cancel{\alpha}}\frac{d\theta}{d\xi}\right) = -\theta^n$$

Dobimo:

$$\boxed{\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n}$$

Lane-Emdenova enačba za politropni indeks n

- $\theta = 1$ pri $r = 0$
- $\frac{d\theta}{d\xi} = 0$ pri $\xi = 0$

Analitično je možno rešiti za $n = 0, 1, 5$

Ostale numerično

$n = 5 \rightarrow$ neskončna zvezda

$$R = \alpha \xi_1$$

Prva ničla dobljene funkcije

Masa politropne zvezde

Uporabimo L-E enačbo.

$$M = \int_0^R 4\pi r^2 \rho(r)\,dr = \int_0^{\xi_1} 4\pi\alpha^2\xi^2\rho_c\theta^n \alpha\, d\xi =$$

$$= -\int_0^{\xi_1} 4\pi\alpha^3\rho_c \frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right)d\xi =$$

$$= -4\pi\alpha^3\rho_c\left[\xi^2\frac{d\theta}{d\xi}\right]\Bigg|_{\xi=\xi_1}$$

Povezave med lastnostmi

~~Središčna in povprečna gostota~~

- Središčna in povprečna gostota

$$\rho_c = D_n\bar{\rho} = D_n\frac{M}{\frac{4}{3}\pi R^3}\ ;\quad D_n = -\left[\frac{3}{\xi_1}\left(\frac{d\theta}{dr}\right)\Bigg|_{\xi=\xi_1}\right]^{-1}$$

- Masa in radij

$$\frac{M}{-\xi_1^2\left(\frac{d\theta}{d\xi}\right)_{\xi_1}} = -4\pi\rho_c\left(\frac{R}{\xi_1}\right)^3$$

$$R^2 = \alpha^2\xi_1^2$$

$$\frac{R^2}{\xi_1^2} = \frac{(n+1)K}{4\pi G\rho_c^{\frac{n-1}{n}}}$$

$$\rho_c = \left[\frac{(n+1)K}{4\pi G}\right]^{\frac{n}{n-1}}\left(\frac{\xi_1}{R}\right)^{\frac{2n}{n-1}}$$

$$\left(\frac{GM}{M_n}\right)^{n-1}\left(\frac{R}{R_n}\right)^{3-n} = \frac{[(n+1)K]^n}{4\pi G} \quad ; \quad M_n = -\xi_1^2\left(\frac{d\theta}{d\xi}\right)\Big|_{\xi_1}$$

$$R_n = \xi_1$$

Pri $n=3$:

$$M = 4\pi M_3 \left(\frac{K}{\pi G}\right)^{3/2}$$

Pri $n=1$

$$R = R_1\left(\frac{K}{2\pi G}\right)^{1/2}$$

Za $1<n<3$:

$$R^{3-n} \propto \frac{1}{M^{n-1}}$$

Primer bele pritlikavke (nerelativistični degenerirani elektroni)

$$\gamma = 5/3 \rightarrow n = 1{,}5 \Rightarrow R \propto M^{-1/3}$$

Ponovitev od zadnjič [Politropni modeli]

① Središčna gostota - povprečna gostota

② masa - radij

$$\left(\frac{GM}{M_n}\right)^{n-1}\left(\frac{R}{R_n}\right)^{3-n} = \left[\frac{(n+1)K}{4\pi G}\right]^n \quad ; \quad M_n = -\xi_1^2\left(\frac{d\theta}{d\xi}\right)_{\xi_1}$$

$$R_n = \xi_1$$

$n=3$ $\quad M$

$n=1$ $\quad R$

$1<n<3$ $\quad R^{3-n} \propto \frac{1}{M^{n-1}}$

③ ~~Središčni tlak in središčna gostota~~

Nazaj:

• Središčni tlak in središčna gostota

$$P_c = \frac{(4\pi G)^{1/n}}{n+1}\left(\frac{GM}{M_n}\right)^{\frac{n-1}{n}}\left(\frac{R}{R_n}\right)^{\frac{3-n}{n}} \rho_c^{\frac{n+1}{n}}$$

Izrazimo R z enačbo od prej. Vse kar ima indeks n spravimo v konstanto B_n =>

$$P_c = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3}$$!

Primer: [Tlak nerelativističnih elektronov v Beli pritlikavki]

$$P_{e,NR} = K_1 \rho^{5/3} \qquad \gamma = \frac{5}{3} \rightarrow n = 1{,}5 \qquad \Rightarrow R \propto M^{-1/3}$$

$$\bar{\rho} \propto M^2$$

Masa se veča, ko se radij manjša. Dokler ne ratajo elektroni relativistični, takrat se spremeni enačba stanja v $\propto \rho^{4/3}$ in imamo ravno $n=3$. Takrat lahko izračunamo maso. Masa, ki jo izračunamo je Chandrasekharjeva masa!

$$M_{Ch} = \frac{M_3 \sqrt{1{,}5}}{4\pi}\left(\frac{hc}{G m_H^{4/3}}\right)^{3/2} \mu_e^{-2}$$

$$= 5{,}83\, \mu_e^{-2} M_\odot \xrightarrow[\substack{\mu_e = 2 \\ \text{(elektroni)}}]{} M_{Ch} = 1{,}46\, M_\odot$$

Eddingtonov izsev

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2} \qquad P_{rad} = \frac{1}{3}aT^4$$

$$\frac{dP_{rad}}{dr} = \frac{4}{3}aT^3 \frac{dT}{dr}$$

$$\Rightarrow \frac{\cancel{3}}{\cancel{4}aT^3}\frac{dP_{rad}}{dr} = -\frac{\cancel{3}\kappa\rho}{\cancel{4}ac\cancel{T^3}}\frac{L}{4\pi r^2} \Rightarrow \frac{dP_{rad}}{dr} = -\frac{\kappa\rho}{c}\frac{L}{4\pi r^2}$$

Uporabimo še enačbo hidrostatičnega ravnovesja: $\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$

To lahko zdelimo in zapišemo:

$$\Rightarrow \quad \frac{dP_{rad}}{dP} = \frac{\varkappa L(m)}{4\pi G m c} < 1$$

Oboje pada, ko gremo k večjemu radiju

Tako dobimo pogoj za sevalno ravnovesje:

$$\varkappa L < 4\pi G m c$$

V bližini zvezdnega središča nastaja energija: $\frac{dL}{dm} = q$

Specifična energija, torej en. na enoto mase:

$$\frac{L}{m} \rightarrow q_c$$

Tako lahko pogoj zapišemo še na drug način:

$$\frac{L}{m} < \frac{4\pi c G}{\varkappa}$$

$$\downarrow$$

$$q_c < \frac{4\pi c G}{\varkappa}$$

Omejitev za specifično energijo. Določa največji možen nastanek energije na enoto mase, da bo zvezda v sevalnem ravnovesju (prenaša energijo s sevanjem)

Če velja sevalno ravnovesje tudi na površju pri $L(M) = L$ lahko zapišemo:

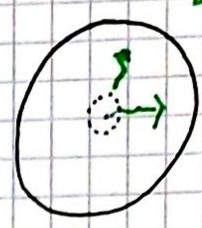
$$L < \underbrace{\frac{4\pi c G M}{\varkappa}}_{L_{edd}}$$

Electron scattering

$$L_{edd} = 3{,}2 \cdot 10^4 \left(\frac{M}{M_\odot}\right)\left(\frac{\varkappa_{es}}{\varkappa}\right) L_\odot$$

Standadni model (Eddingtonov model)

L=konst



Najprej definiramo:

$$\frac{L(m)}{m} = \eta \frac{L}{\mathcal{M}} \qquad \eta = 1 \quad (m = \mathcal{M})$$

$$\Rightarrow L(m) = \eta m \frac{L}{\mathcal{M}}$$

konst.

η se veča, ko se m manjša

$\varkappa$ se veča, ko gremo navzven

To razmerje bomo lahko zapisali kot:

$$\frac{dP_{rad}}{dP} = \frac{L}{4\pi c G \mathcal{M}} \varkappa \eta$$

Predpostavimo, da se kompenzirata:

$$\varkappa \eta = \text{konst.} = \varkappa_s$$

Prozornost na površju (s = standard)

$$\Rightarrow P_{rad} = \frac{L}{4\pi c G \mathcal{M}} \varkappa_s P$$

Vidimo, da se ubistu razmerje tlakov ohranja: $\frac{P_{rad}}{P} = -\beta + 1 = 1 - \beta$; $\beta = \frac{P_{gas}}{P}$

Zapišimo še izsev:

$$L = \frac{4\pi c G \mathcal{M}}{\varkappa_s}(1-\beta) = L_{Edd}(1-\beta)$$

$$L = L_{Edd}(1-\beta)$$

$$\frac{P_{rad}}{1-\beta} = P = \frac{P_{gas}}{\beta} \Rightarrow \frac{1}{3}\frac{aT^{\cancel{4}\,3}}{(1-\beta)} = \frac{\rho k_B \cancel{T}}{\beta \mu m_H}$$

$$\Rightarrow T = \left[\frac{k_B 3(1-\beta)}{\beta \mu m_H a}\right]^{1/3} \rho^{1/3} \; ; \; P \propto \rho T \qquad \frac{4}{3} \Rightarrow n = 3$$

$$\Rightarrow P = K\rho^{4/3}$$

$$\hookrightarrow K = \left[\frac{3 k_B^4}{a(\mu m)^4 \beta^4}(1-\beta)\right]^{1/3}$$

n=3

$$\boxed{1 - \beta = 0{,}003\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^2 \mu^4 \beta^4}$$

Polinom 4. stopnje za β

Masa se zgleda prepodobno kot mu.

Torej če zapišemo $(1-\beta)$ in damo v relacijo za izsevo:

$$\frac{L}{L_{Edd}} = 0{,}003\left(\frac{M}{M_\odot}\right)^2 \mu^4 \beta^4$$

Če vstavimo L_{Edd} dobimo Mass-Luminosity relation za zvezde glavne

Veje:
Če bi imeli samo sevalni pritisk! (tak ni res)

$$\frac{L}{L_\odot} = \frac{4\pi G c}{\varkappa} \frac{M_\odot}{L_\odot} 0{,}003\, \mu^4 \beta^4 \left(\frac{M}{M_\odot}\right)^3$$

Prispeva k razpršenosti glavne veje HR.

$$\frac{L}{L_\odot} \propto \left(\frac{M}{M_\odot}\right)^3$$

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^\alpha \qquad \alpha \in [3, 3{,}5]$$

Hitrost jedrskih reakcij

sipalni presek delcev $\leftarrow \sigma$

$v \rightarrow$ Hitrost delcev, ki bojo interagirali (Ubistvu relativna ... in potem ...)

$\hookrightarrow$ Maxwell-Boltzman distribution

$$\propto e^{-\frac{mv^2}{2k_BT}}$$

Razdalja na katero se lahko zbližata nabita delca (dobljeno iz el. pot. energije in kinetične) je:

$$d = \frac{1}{4\pi\varepsilon_0} \frac{Z_i Z_j e^2}{\frac{1}{2} m v^2}$$

Drugo verjetnost za pojav sipanja v odvisnosti od v pa je izmeril Gamow

$$\propto e^{-\frac{\pi Z_i Z_j e^2}{\varepsilon_0 h v}}$$

Končno moramo produkt teh dveh porazdelitev. Zanima nas pravzaprav odvisnost od temperature.

graf na .ppt

$$q = q_0 \rho T^n$$