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Data Flow in a Randomized Computer Network

Final Assignment for Model Analysis 1, 2023/24

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1 Introduction

The scope of computer networks has been expanding rapidly in the past few decades. What once were simple networks of interconnected computers have now evolved into complex systems that are used for a wide range of applications. It makes sense then for one to plan and analyze the flow of data in such networks to ensure that they are efficient and reliable and thus cheaper to maintain.

As my final assignment for Model Analysis 1, I opted to simulate the flow of data in a randomized computer network. The network consists of a $N \times N$ grid, where each unit of the grid can be a server, a user or a wire used to connect the two. Each of these units is connected to its four nearest neighbors and has a value associated with it that specifies it's bandwidth. For wires this is the maximum data throughput, for servers it is the maximum data processing rate and for users it is the maximum data consumption rate. We'd like to find the distribution of server-loads for a grid of wires with random bandwidths. It is possible to study many different types of server and user placements but the most interesting one to us will be where we have the users and servers placed on the top and bottom rows of the grid, respectively.

We can solve for the flow rates through the network by solving a set of constrained linear equations. This field of study is known as linear programming and is a powerful tool for solving optimization problems. We've already seen some linear programming use in the second task of this course `mod102` where we created a dietary plan from a set of given foods based on their costs and nutritional values. For the sake of completeness it makes sense to quickly go over the basics of linear programming before we proceed with the simulation. Linear programming uses linear optimization to find the maximum or minimum of a linear function subject to a set of linear constraints. This function is known as the objective function and is quite analogous to the cost function we've seen mentioned in the world of Machine Learning. The constraints are linear inequalities or equations that define the feasible region of the problem. Mathematically formulated we can consider a cost function $f(x)$ and a set of constraints defined as:

$$\begin{gathered}
f(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + \cdots + c_nx_n \ , \\
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \ , \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2 \ , \\
\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \ .
\end{gathered}$$

The goal is then to find the values of $x_1, x_2, \ldots, x_n$ that maximize or minimize the cost function $f(x)$ while satisfying the constraints. The feasible region is the set of all points that satisfy the constraints and the optimal solution is the point in the feasible region that maximizes or minimizes the cost function. That is all that is necessary from a mathematical perspective. Of course the actual implementation of solvers for such problems are much more complex and involve a lot of optimization techniques but that is not the focus of this assignment.

2 Task

The original text of the assignment reads as follows:

> **Razporejanje prenosa podatkov po omrežju:** Za model omrežja vzemi $N \times N$ kvadratno mrežo, vsak rob pa ima naključno maksimalno hitrost povezave med 0 in 1. Vozlišča na zgornjem robu so internetni odjemalci, spodnji rob pa so strežniki. V notranjih vozliščih velja 1. Kirchhoffov zakon. S pomočjo linearnega programiranja določi, kolikšne hitrosti prenosa imajo strežniki in odjemalci, ko je skupna hitrost prenosa največja. Ker gre za naključna omrežja, si oglej tudi statistično porazdelitev zanimivih količin.

3 Solution Overview

4 Results

5 Conclusion and Comments