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# Metropolis-Hastings Algorithm

8. Task for Model Analysis I, 2023/24

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Ljubljana, July 2024

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## 1 Introduction

We're continuing our exploration into random numbers and their applications. Previously we had a look at Monte Carlo sampling. Today we're going to delve into the Metropolis-Hastings algorithm, which can be thought of as Monte Carlo sampling with a few extra steps. Since our end goal is to simulate the relaxation of a lattice of spins in a magnetic field, we'll take the Ising model as our physical context.

We know from statistical physics that the 2D Ising model relaxes to a state of minimum energy. We can simulate this relaxation by flipping spins at random and accepting or rejecting the new state based on the change in energy. We can have a negative change of energy which we can call a *good move* or a positive change of energy which we can call a *bad move*. The added twist with this algorithm is that while we always accept the new state if we make a good move, we also sometimes accept a new state after a bad move. This is the key to the Metropolis-Hastings algorithm. Given our system the probability of accepting a bad move is given by the Boltzmann factor and the temperature of the system:

$$P_{\text{bad accept}} = \exp\left(-\frac{\Delta E}{kT}\right), \quad (1)$$

where  $\Delta E$  is the change in energy,  $k$  is the Boltzmann constant and  $T$  is the temperature. Why exactly this works is a bit more involved and probably out of the scope of this report however a dedicated reader can find more information in this well written blog post by Gregory Gundersen of Princeton University [\[1\]](#).

## 2 Task

## 3 Solution Overview

## 4 Results

## 5 Conclusion and Comments

## References

- [1] Gregory Gundersen. Why Metropolis-Hastings Works. <https://gregorygundersen.com/blog/2019/11/02/metropolis-hastings/#bishop2006pattern>, Nov 2019. [Accessed 24-07-2024].