Spektralna analiza in filtriranje

Oz. (hitra) diskretna fourierova transformacija: $\mathbb{C}^N \rightleftharpoons \mathbb{C}^N$

$$S = \mathcal{F}_N[s]$$

S – Fourier transformiranka; $\mathcal{F}_N$ – FT od N točk.

$$S_k = \frac{1}{\ldots} \sum_{j=0}^{N-1} s_j e^{-i2\pi jk/N}$$

ali $\frac{1}{\sqrt{N}}$

$$s_k = \frac{1}{N} \sum_{k=0}^{N-1} S_k e^{+i2\pi jk/N}$$

ali $\frac{1}{\sqrt{N}}$

ali $k = N/2+1$ do $k = N/2$

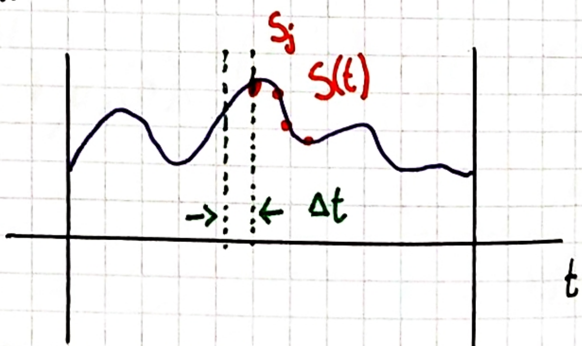
$$f_{krit} = \frac{1}{2\Delta t}$$ ← I think

Spekter (moč) = power spectral density (PSD)

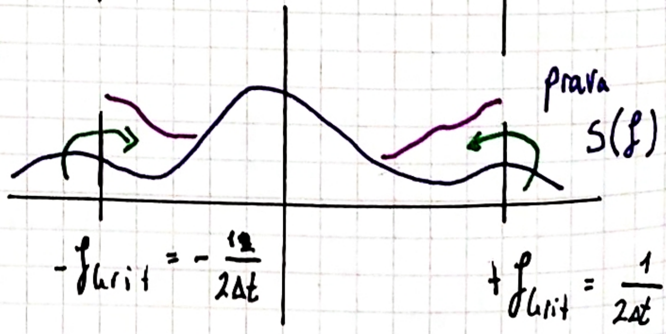
$$P_0 = |S_0|^2$$

$$2P_k = |S_k|^2 + |S_{N-k}|^2$$

$$P_{N/2} = |S_{N/2}|^2$$

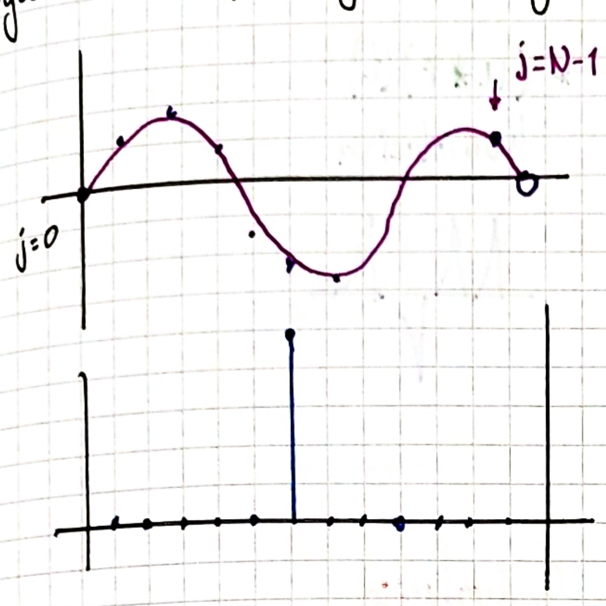


"Aliasing" (potujitev)

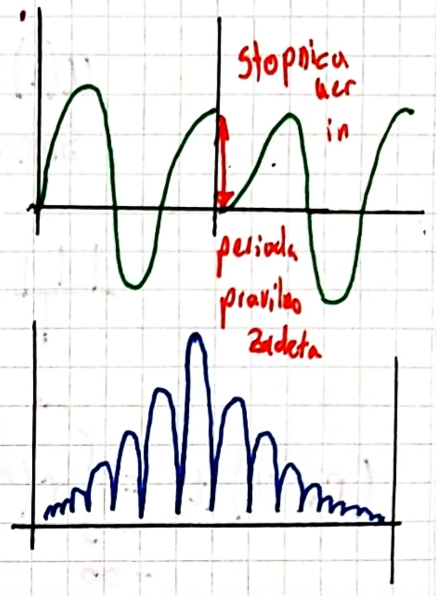


Druga težava: puščanje (leakage)

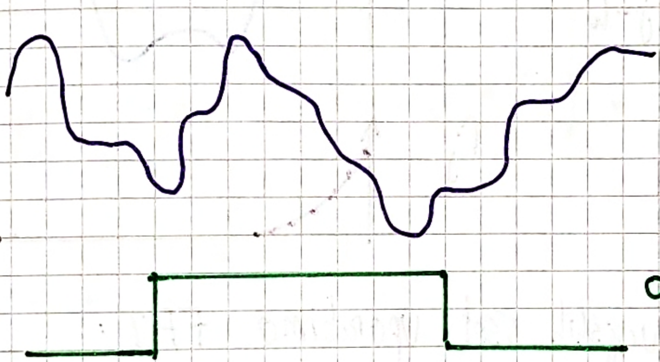
tu moramo nehati!



indeks j (= čas)



Ta efekt lahko zmanjšamo preko okenskih funkcij.



okno "1"

Več vrst okenskih funkcij:

$$\tilde{S}_k = \sum_j w_j s_j e^{-i 2\pi j k / N}$$

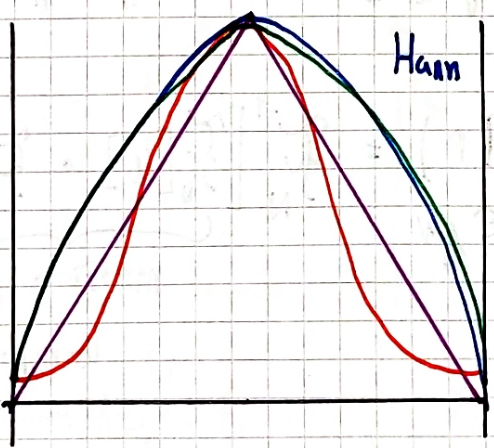
$$P_0 = \frac{1}{W} |\tilde{S}_0|^2$$

$$2P_k = \frac{1}{W} \left[|\tilde{S}_k|^2 + |\tilde{S}_{N-k}|^2 \right]$$

$$P_{N/2} = 1/W \, |\tilde{S}_{N/2}|^2$$

$$W = \sum_j w_j^2$$

Priljubljena okna:



Bartlett $w_j = 1 - \frac{|j - N/2|}{N/2}$

Hann $w_j = \frac{1}{2}\left[1 - \cos\frac{2\pi j}{N}\right]$

Ideja: Steganography

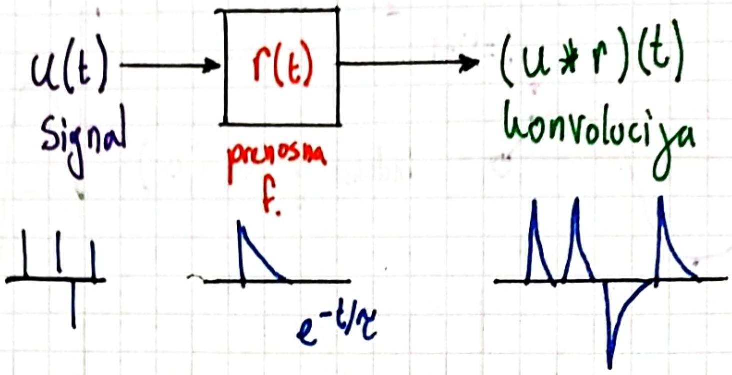
Welch $w_j = 1 - \left(\frac{j - N/2}{N/2}\right)^2$

Gauss

FWHM (transf.) (#binov) $\approx \frac{N}{\text{širina okna}}$

leakage (#binov) $\approx \frac{N}{\text{čas vzpona/padca okna}}$

2. del: konvolucija in dekonvolucija (v prisotnosti šuma)

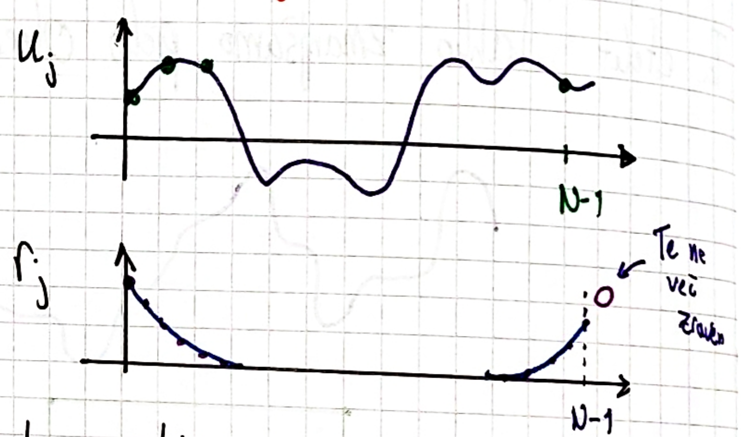


$$(r * u)(t) = \int_{-\infty}^{\infty} r(\tau)\, u(t-\tau)\, d\tau$$

↑ parameter konvolucije

$$(r * u)_{rj} = \bigcirc \sum_k r_k u_{j-k}$$

$\bigcirc \to \frac{1}{N-j}$

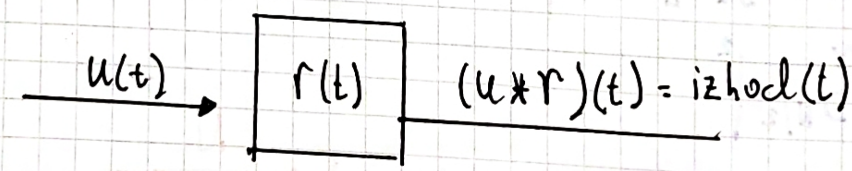


Namesto računanja konvolucijskih vsot uporabimo FFT:

$$\sum_k u_{j-k} r_k \iff U_n R_n \quad \text{"Fourierov par"}$$

$$\underset{\in \mathbb{R}^N}{(u * r)} = \mathcal{F}_N^{-1}\Big[\underbrace{\mathcal{F}_N[u] \cdot \mathcal{F}_N[r]}_{\text{množimo po komponentah (element-wise)}}\Big]$$

Dekonvolucija:



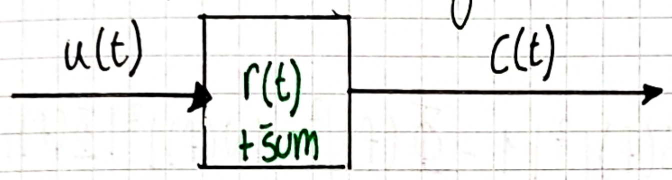
radi bi **Vhod**:

$$u = \mathcal{F}_N^{-1}\left[\frac{\mathcal{F}_N[\text{izhod}]}{\mathcal{F}_N[r]}\right]$$

deljenje po komponentah

Idea: make audio example

Šum lahko zelo pokvari dekonvolucijo



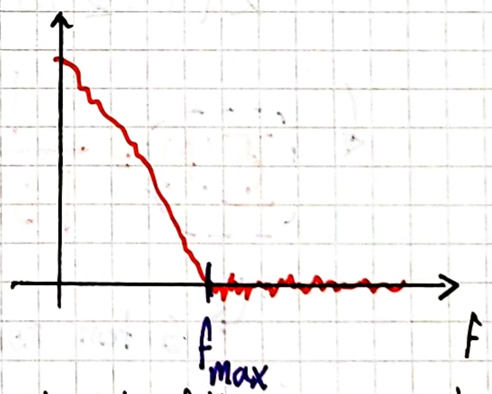
$$c(t) = (u * r)(t) + n(t) = s(t) + n(t)$$

signal, noise

$X(f)$ ... transformiranka $x(t)$

Brez šuma: $$U(f) = \frac{C(f)}{R(f)} \checkmark$$

Tipično obnašanje:



S šumom:

$$\underset{\text{ocena}}{\tilde{U}(f)} = \frac{U(f)R(f) + N(f)}{R(f)}$$

lahko pride 0

Ena sama ničla $R(f)$ uniči dekonvolucijo. Rešitev je, da filtriramo signal

-> Odrežemo visoke frekvence (=> signal ni več isti) } ni preveč dobro

-> stabiliziramo dekonvolucijo

Optimalni (Wienerjev) filter:

$$\tilde{U}(f) = \frac{C(f)\,\Phi(f)}{R(f)}$$

približek za FT vhodnega signala

Zahtevamo:

$$\int_{-\infty}^{\infty} |u(t) - \tilde{u}(t)|^2 dt = \int_{-\infty}^{\infty} |U(f) - \tilde{U}(f)|^2 df = \min.$$

$$I = \int_{-\infty}^{\infty} \left| \frac{S(f)}{R(f)} - \frac{S(f)+N(f)}{R(f)}\,\Phi(f)\right|^2 df =$$

$c(t) = s(t) + n(t)$

$C(f) = S(f) + N(f)$

$$= \int_{-\infty}^{\infty} \frac{1}{|R(f)|^2} \Big\{ |S(f)|^2 |1 - \Phi(f)|^2 + |N(f)|^2 |\Phi(f)|^2 + \text{mešani členi} \Big\} df = \min$$

Se izpovprečijo v 0 pri integraciji

iščemo samo min. Variacijsko po Φ

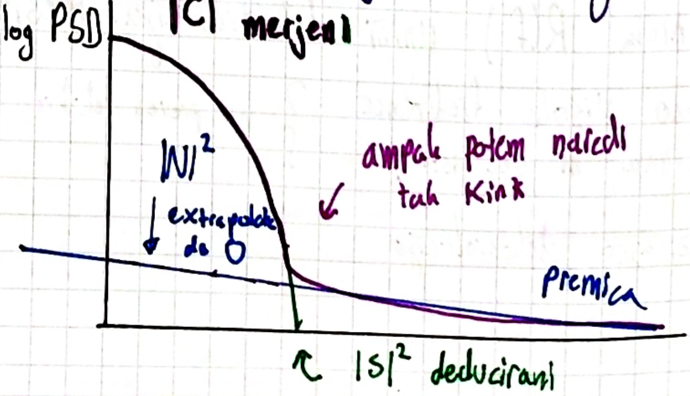
$$-2(1-\Phi)S^2 + 2\Phi N^2 = 0$$

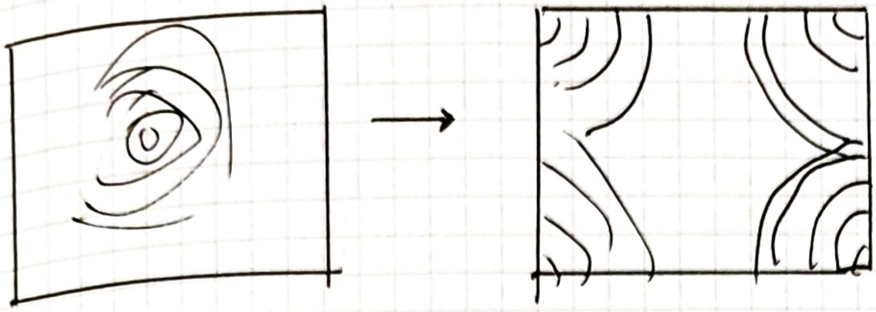
$$\Phi(f) = \frac{|S(f)|^2}{|S(f)|^2 + |N(f)|^2} = \frac{|C(f)|^2 - |N(f)|^2}{|C(f)|^2}$$

Ideja: Skrij leno popolnoma in jo nazaj dobi

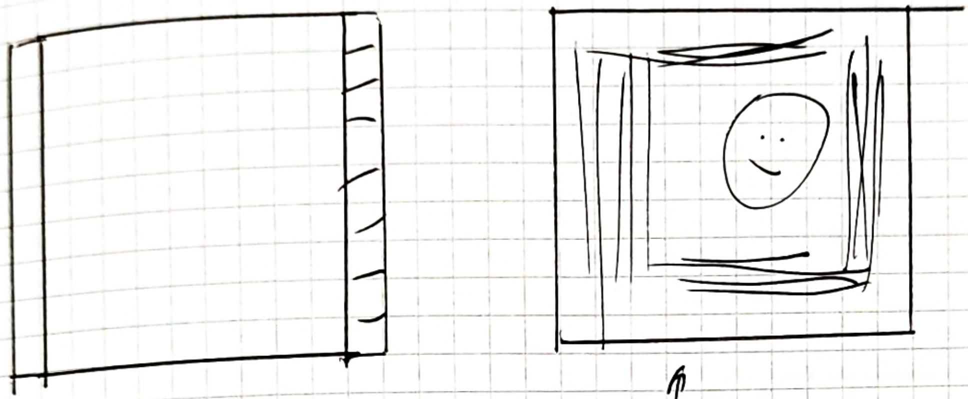
↳ S tem moramo pomnožiti F.T. signala, preden ga dekonvoluiramo.

Običajna situacija →



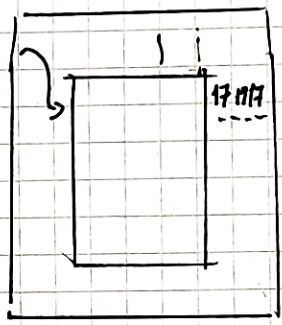


gauss blur



Artifacts zaradi robnih pogojev

NxN



razširi robne vrednosti

večje