Linearna napoved

Ostajamo pri časovnih vrstah (signalih) $s_n = s(t_n)$ vzorčenih ob $t_n = n\Delta t$:

Običajno $b_0 = 1$

izhod iz sistema ob zdajšnjem času $= s_n = -\sum_{k=1}^{p} a_k s_{n-k} + G \sum_{l=0}^{q} b_l u_{n-l}$

vhodni v sistem ob $t_n, t_{n-1}, \dots, t_{n-q}$

to enačbo Z transformiramo:

$$\mathcal{Z}[x] = X(z) = \sum_{n=-\infty}^{\infty} x_n z^{-n}$$

$$\Downarrow$$

če je z na enotski krožnici je to F.T.

$$\sum_{n=-\infty}^{\infty} x_n e^{-i 2\pi nm/N} = \sum_{n} x_n e^{-inw}$$

$$\underbrace{\sum_n S_n z^{-n}}_{S(z)} = -\underbrace{\sum_{k=1}^{p} a_k \sum_n s_{n-k} z^{-n}}_{z^{-n+k-k}} + G \underset{1}{\underset{\shortparallel}{b_0}} \underbrace{\sum_n u_n z^{-n}}_{U(z)} + G \underbrace{\sum_{l=1}^{q} b_l \sum_n u_{n-l} z^{-n}}_{z^{-n} = z^{-n+l-l}}$$

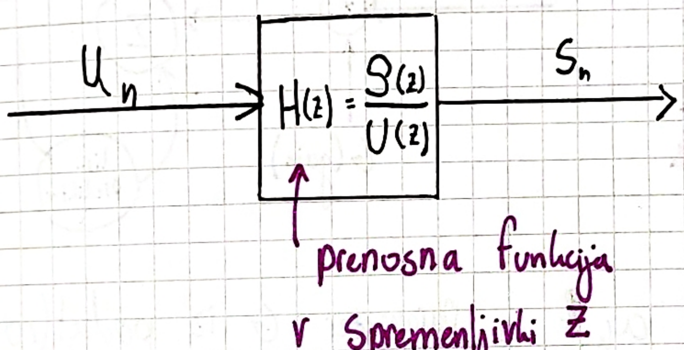
$$-\sum_{k=1}^{p} a_k z^{-k} \underbrace{\sum_n s_{n-k} z^{-(n-k)}}_{S(z)}$$

$$G \sum_{l=1}^{p} b_l z^{-l} \underbrace{\sum_n u_{n-l} z^{-(n-l)}}_{U(z)}$$

Torej:

$$S(z)\left[1 + \sum_{k=1}^{p} a_k z^{-k}\right] = G U(z)\left[1 + \sum_{l=1}^{q} b_l z^{-l}\right]$$

To opisuje sistem:



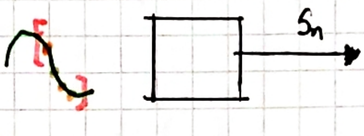
Digitalen filter

$$H(z) = \frac{S(z)}{U(z)} = \frac{G\left[1 + \sum_{l=1}^{q} b_l z^{-l}\right]}{\left[1 + \sum_{k=1}^{p} a_k z^{-k}\right]}$$

Tri možnosti:

• Model s samimi ničlami (števca) : Moving average

delamo samo => $S_n = G \sum_{l=0}^{q} b_l u_{n-l}$ ... samo lin. komb. vhodov



• Model s samimi poli (ničle imenovalca): Auto regression

$$b_l = 0 \ \forall l \quad \text{razen } b_0 = 1$$

$$s_n = -\sum_{k=1}^{p} a_k s_{n-k}$$

Najbolj zanimivo za nas.

Potrebujemo ničle kompleksnega polinoma

Ko

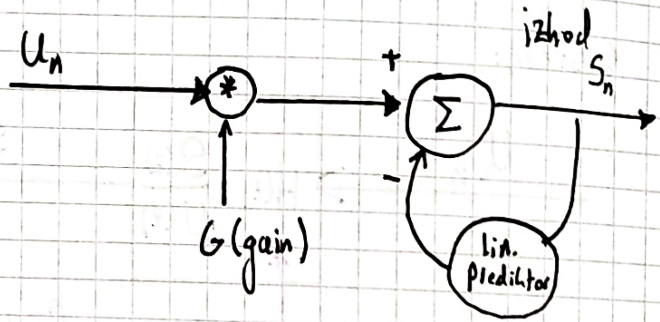
• Model z ničlami in poli: ARMA

Torej pri nas bo zdaj:

$$S_n = -\sum_{k} a_k S_{n-k} + G u_n$$

red filtra

$$H(z) = \frac{G}{1 + \sum_{k=1}^{p} a_k z^{-k}}$$



Kako določiti koef. a_k in parameter G iz podatkov?

Včasih u_n (input) sploh ni znan. Recimo, da so u_n popolnoma neznani:

$$\text{dobimo samo } \tilde{S}_n = -\sum_{k=1}^{p} a_k S_{k-n}$$

$$\Rightarrow \text{napaka (residual)} = e_n = S_n - \tilde{S}_n$$

$$\mathcal{E} = E[e_n^2] = E\left[\left(S_n + \sum_{k=1}^{p} a_k S_{k-n}\right)^2\right]$$

Pogoj za določitev koeficientov a_k:

$$\frac{\partial \mathcal{E}}{\partial k} = 0$$

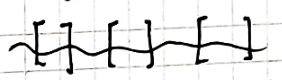
$$\sum_{k=1}^{p} a_k E[S_{n-k} S_{n-i}] = -E[S_n S_{n-i}] \qquad 1 \leq i \leq p$$

$$\mathcal{E}_{min}^{(p)} = E[S_n^2] + \sum_{k=1}^{p} a_k E[S_n S_{n-k}]$$

Kakšen je naključni proces, ki ga gledamo? $(= S_n)$

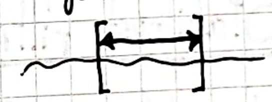
Nestacionaren $\Rightarrow E[S_{n-k} S_{n-i}] = R(n-k, n-i)$

↳ avtokorelacijska funkcija



Stacionaren $\Rightarrow$ $= R(k-i)$!

če je vse to enako je važno samo



$$R(k) = \underbrace{\frac{1}{N-k+1}}_{} \sum_n S_n S_{n-k}$$

Utež zaradi nepristranskega povprečja

Od tod dobimo enačbo za a_k:

$$a_k: \quad R(i) = -\sum_{k=1}^{p} a_k R(i-k) \qquad 1 < i < \infty$$

$$R(0) = -\sum_{k=1}^{p} a_k R(k) + G^2$$

Yule-Walker

Matrično:

$$\begin{pmatrix} R(0) & R(1) & \cdots & R(p-1) \\ R(1) & R(0) & \cdots & R(p-2) \\ \vdots & \vdots & \ddots & \vdots \\ R(p-1) & R(p-2) & \cdots & R(0) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix} = -\begin{pmatrix} R(1) \\ R(2) \\ \vdots \\ R(p) \end{pmatrix}$$

Recept:

Signali S_n

$\downarrow$

avtokorelacije $R(k)$ $\quad k \in [1, p]$

$\downarrow$

rešimo YW sistem

$\downarrow$

$a_k \; (k = 1, 2, \dots, p)$

To ni hokus-pokus: $\quad \ddot{x} + \beta \dot{x} + x = a \cos \omega t$

$$\frac{x_{n+1} - 2x_n + x_{n-1}}{(\Delta t)^2} + \beta \frac{x_{n+1} - x_n}{\Delta t} + x_n = \dots$$

$$x_{n+1} = \square\, x_n + \square\, x_{n-1} + \dots$$

$\uparrow$ nekaj

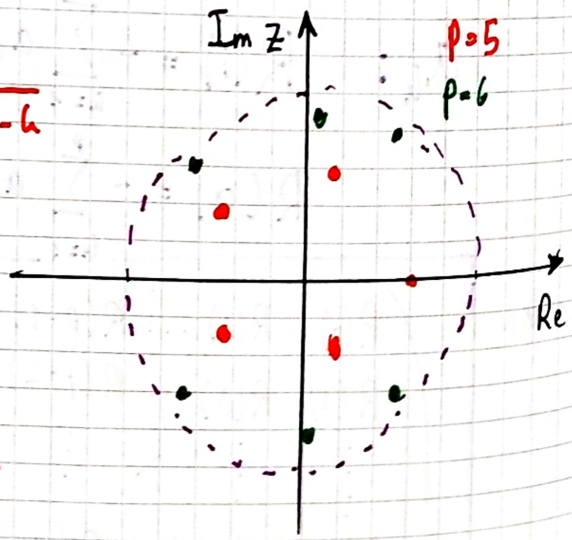
Stabilnost filtra:

Ko dobimo $\{a_k\}_{k=1}^{p}$ moramo preveriti, ali je filter stabilen:

(δ- na vhodu $\Rightarrow$ odziv $\to 0$, ko $n \to \infty$)

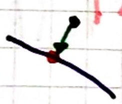
To je zagotovljeno, če so vsi poli filtra znotraj enotske krožnice v $\mathbb{C}$:

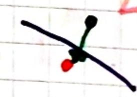
$$\frac{1}{1 + \sum_{k=1}^{p} a_k z^{-k}}$$



Če kateri od z-jev ni tak, da bi padel znotraj ⊕, ($|z_i| > 1$)

$$z_i \to \hat{z}_i = \frac{z_i}{|z_i|} \qquad \hat{z}_i = \frac{1}{z_i^*}$$

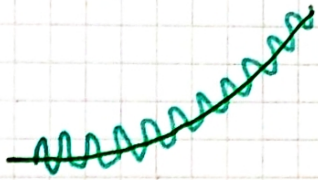


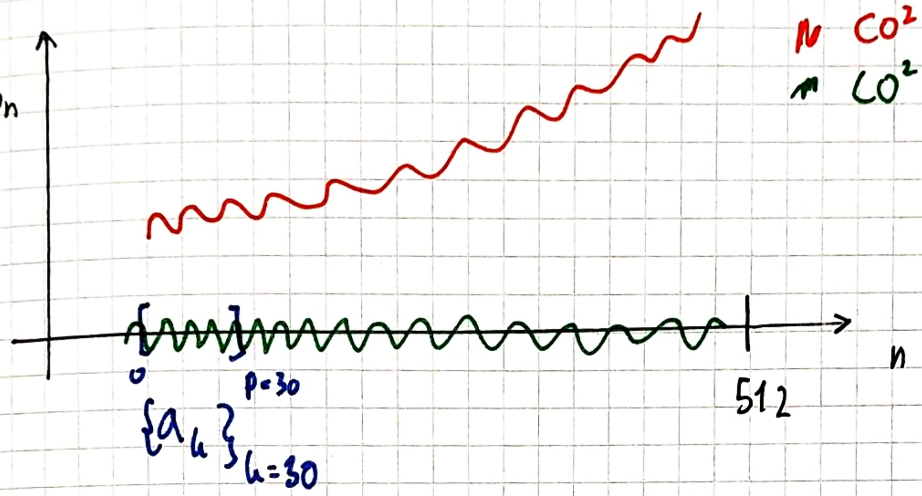


Pozor: linearna napoved = linearna v a_k ne v signalu.

Kdaj stabilnost propade?

- premajhen p (prekratka sekvenca s_n)
- signal z izrazitimi trendi (te raje odštejemo)
- zaokrožena natančnost rešitve YW sistema (ki je kar občutljiv).





Ne splača se pretiravati z p. Dobra praksa je:

$$p_{max} \approx 2K$$

→ Število frekvenčnih komponent ki jih pričakujemo v signalu.

Zakaj je to "metoda max. entropije"?

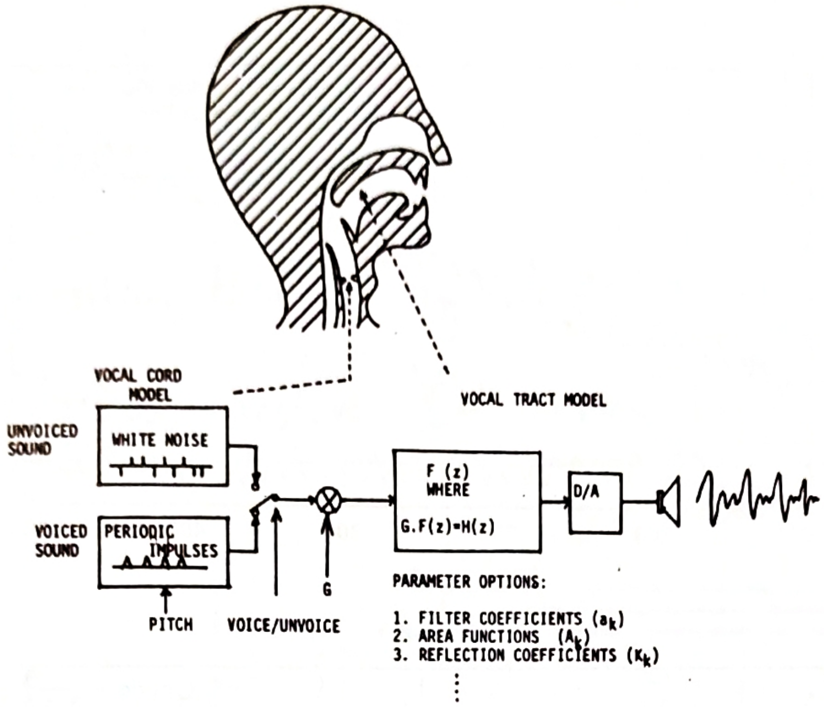
→ glej Verjetnost v fiziki § 11.5 "CUTE"

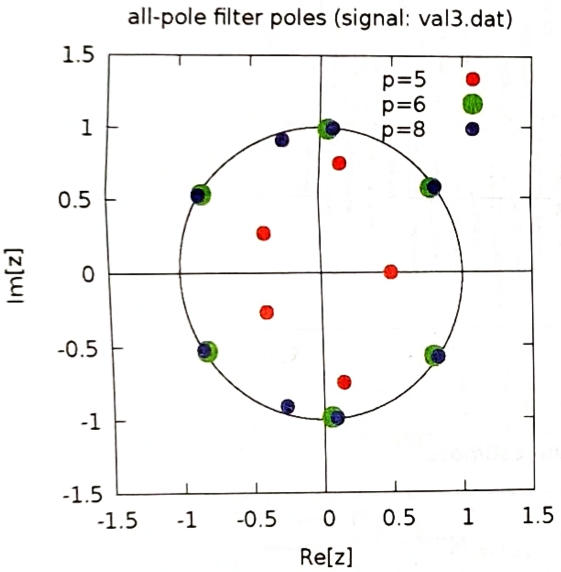
↳ pravi Simon Širca

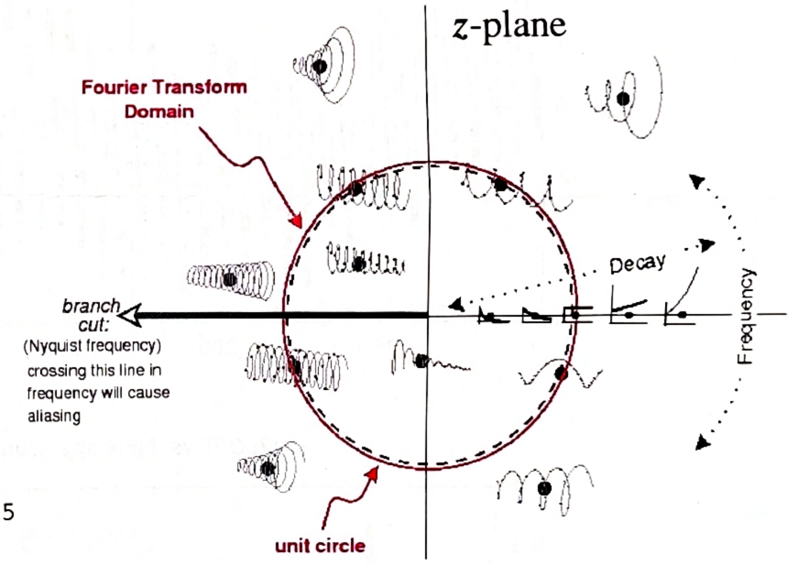
Spekter

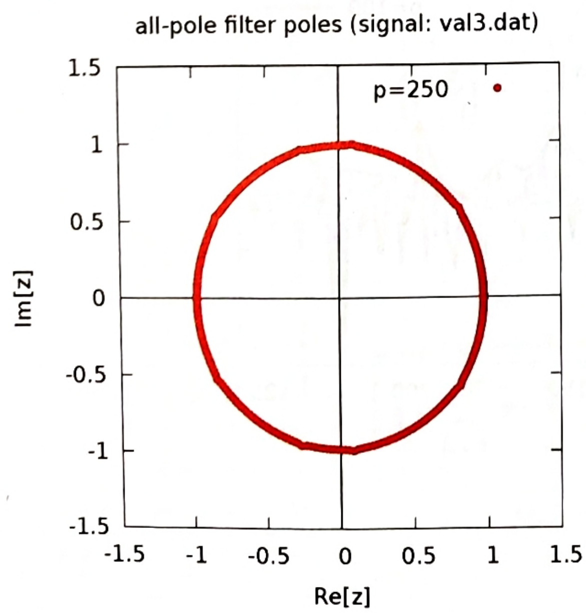
$$P(\omega) = |H(e^{i\omega})|^2 = \frac{G^2}{\left|1+\sum_{k=1}^{p} a_k e^{-i\omega k}\right|^2}$$

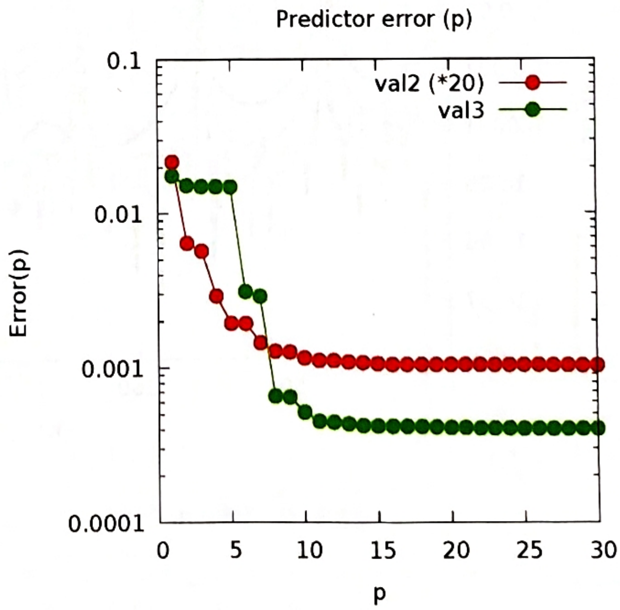
Nakukaj da sta dva blizu in kakšna ločljivost.

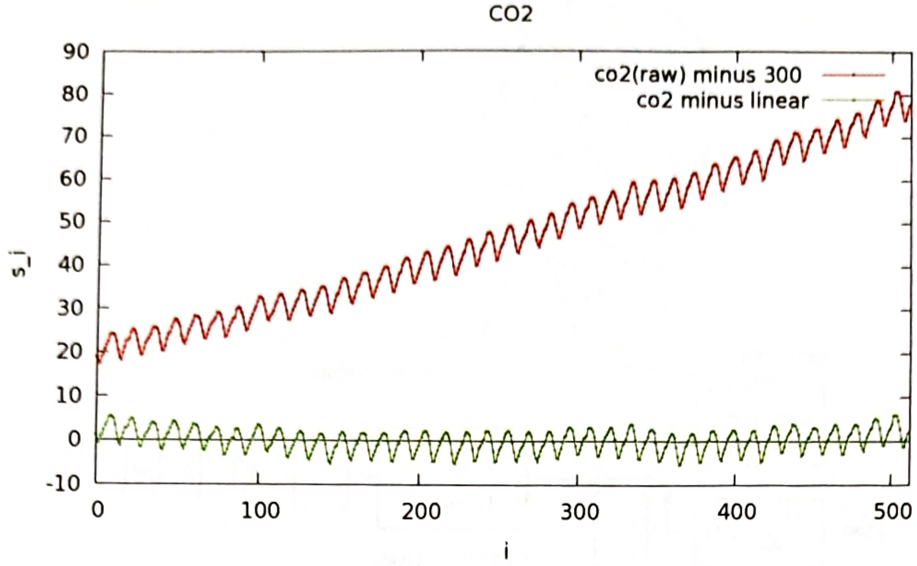

VOCAL CORD MODEL
VOCAL TRACT MODEL
UNVOICED SOUND
WHITE NOISE
VOICED SOUND
PERIODIC IMPULSES
PITCH
VOICE/UNVOICE
G
F (z) WHERE
G.F(z)=H(z)
D/A
PARAMETER OPTIONS:
1. FILTER COEFFICIENTS (a_k)
2. AREA FUNCTIONS (A_k)
3. REFLECTION COEFFICIENTS (K_k)
⋮

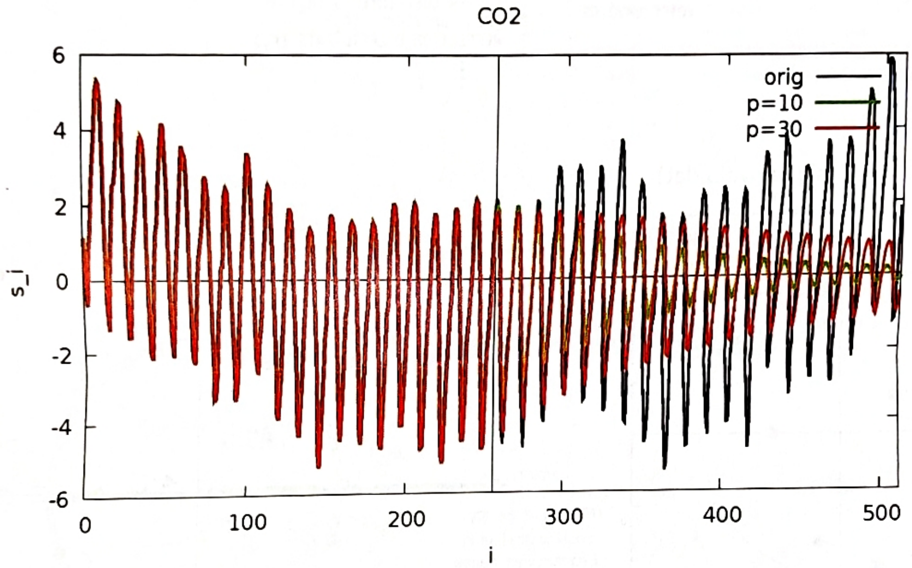

all-pole filter poles (signal: val3.dat)
p=5
p=6
p=8
Im[z]
Re[z]
1.5
1
0.5
0
-0.5
-1
-1.5

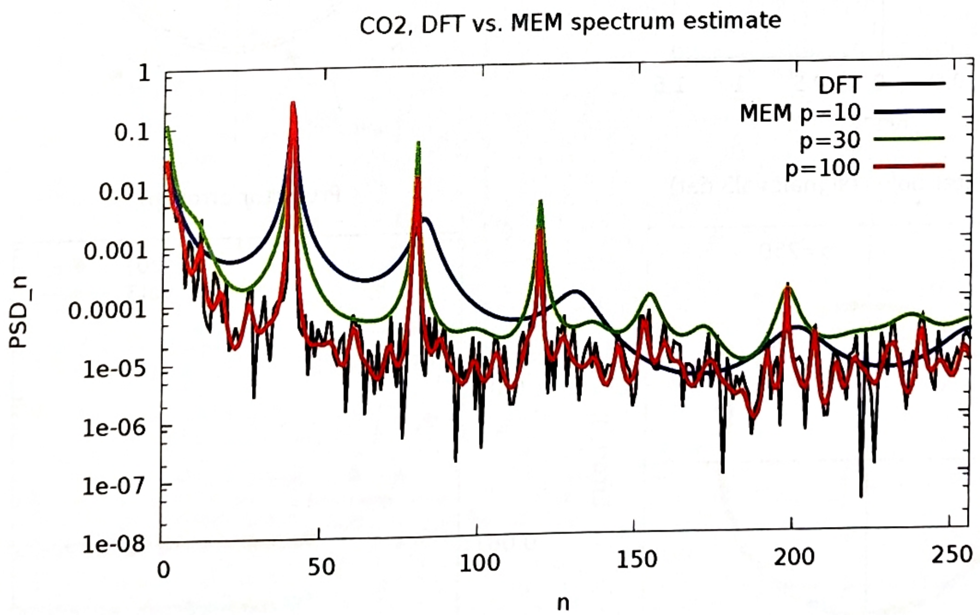

z-plane
Fourier Transform Domain
branch cut:
(Nyquist frequency)
crossing this line in frequency will cause aliasing
Decay
Frequency
unit circle


all-pole filter poles (signal: val3.dat)
p=250
Im[z]
Re[z]
1.5
1
0.5
0
-0.5
-1
-1.5


Predictor error (p)
val2 (*20)
val3
Error(p)
p
0.1
0.01
0.001
0.0001
0
5
10
15
20
25
30


CO2
co2(raw) minus 300
co2 minus linear
s_j
i
90
80
70
60
50
40
30
20
10
0
-10
0
100
200
300
400
500


CO2
orig
p=10
p=30
s_j
i
6
4
2
0
-2
-4
-6
0
100
200
300
400
500


CO2, DFT vs. MEM spectrum estimate
DFT
MEM p=10
p=30
p=100
PSD_n
n
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
0
50
100
150
200
250