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Advanced Particle Detectors and Data Analysis

Notes for Exercises

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1 Interactions of Particles with Photons

1.1 Bethe-Bloch Equation

The Bethe-Bloch equation describes the mean energy loss per distance traveled while traversing through matter. We generally use the Bethe-Bloch equation when we are dealing with **thick absorbers**, such as the ones in calorimeters. Do note that the Bethe-Bloch equation does not accurately describe the energy loss of **electrons** and **positrons** due to their small mass and the fact that they suffer from much larger energy losses due to bremsstrahlung and pair production. For a particle with charge z and velocity $\beta = v/c$, the Bethe-Bloch equation is given as:

$$-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e c^2\beta^2\gamma^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right] , \tag{1.1}$$

where δ is the **density effect correction** and C is the **shell correction**. The rest is as follows:

$$N_a = 6.022\times 10^{23}\,\mathrm{mol}^{-1} , \quad r_e = 2.818\times 10^{-15}\,\mathrm{m} , \quad m_e = 9.11\times 10^{-31}\,\mathrm{kg} , \quad c = 3\times 10^{8}\,\mathrm{m/s} ,$$
$$\rho = \text{density of the material} , \quad A = \text{atomic mass of the material} , \quad Z = \text{atomic number of the material} ,$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} , \quad W_{\max} = \text{maximum energy transfer in a single collision} , \quad I = \text{mean excitation energy} .$$

The constant factor in the equation can be written as:

$$\boldsymbol{\Xi = 2\pi N_a r_e^2 m_e c^2 = 0.1535\,\mathrm{MeV\ cm^2 mol^{-1}}} , \tag{1.2}$$

where I've chosen to mark this constant factor as Ξ for easier reference in further calculations. We can find the mean excitation energy I from the following experimentally determined formula:

$$I = \begin{cases} Z(12 + \frac{7}{Z}) & \text{for } Z < 13 , \\ Z(9.76 + 58.8Z^{-1.19}) & \text{for } Z \geq 13 . \end{cases} \tag{1.3}$$

The maximum energy transfer in a single collision $W_{\max}$ can be calculated as:

$$W_{\max} = \frac{2m_e c^2\beta^2\gamma^2}{1 + 2\gamma\left({}^{m_e}\!/\!{}_{M}\right) + \left({}^{m_e}\!/\!{}_{M}\right)^2} \approx 2m_e c^2\beta^2\gamma^2 . \tag{1.4}$$

For our purposes we will ignore the density effect correction δ and the shell correction C.

1.1.1 Energy Loss of Charged Kaons

Let us calculate the energy losses for charged kaons K^+ and K^- with a rest mass of $0.493\,\mathrm{MeV}$ and momentum of $2.5\,\mathrm{GeV}$ in copper which has the following properties:

$$\begin{aligned} \rho &= 8.92\,\mathrm{g/cm^3} , \\ Z &= 29 , \\ A &= 63.5\,\mathrm{g/mol} . \end{aligned}$$

First let us calculate the velocity β and the Lorentz factor γ. We know that

$$\beta = \frac{pc}{E} = \frac{pc}{\sqrt{(pc)^2 + (Mc^2)^2}} , \tag{1.5}$$

where M is the mass of the particle. Thus:

$$\beta = \frac{2.5 \, \dfrac{\text{GeV}}{c} \, c}{\sqrt{\left(2.5 \, \dfrac{\text{GeV}}{c} \, c\right)^2 + \left(0.493 \, \dfrac{\text{MeV}}{c^2} \, c^2\right)^2}} \approx 0.981031 \, . \tag{1.6}$$

Remember to take at least 4 significant digits for the velocity β! This is due to the logarithm in the Bethe-Bloch equation. The Lorentz factor γ is then:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \approx 5.159 \, . \tag{1.7}$$

Next let us calculate the maximum energy transfer in a single collision W_{max}:

$$W_{\text{max}} = 2m_e c^2 \beta^2 \gamma^2 = 2 \cdot 0.511 \, \frac{\text{MeV}}{c^2} \, c^2 \cdot (0.981031)^2 (5.159)^2 = 26.7 \, \text{MeV} \, . \tag{1.8}$$

Last prerequisite is the mean excitation energy I which we can calculate using the formula (1.3) for $Z \geq 13$:

$$I = 29(9.76 + 58.8 \cdot 29^{-1.19}) \, \text{eV} = 313.9 \, \text{eV} \, . \tag{1.9}$$

Now all that is left is to plug in the values into the Bethe-Bloch equation (1.1):

$$\begin{aligned}
-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle &= \Xi \, \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{W_{\text{max}}^2}{I^2} \right) - 2\beta^2 \right] \\
&= 0.1535 \, \frac{\text{MeV cm}^2}{\text{mol}} \cdot 8.92 \, \frac{\text{g}}{\text{cm}^3} \cdot \frac{29}{63.5} \, \frac{\text{mol}}{\text{g}} \cdot \frac{1}{0.981031^2} \cdot \left[\ln \left(\frac{(26.7 \cdot 10^6 \, \text{eV})^2}{(313.9 \, \text{eV})^2} \right) - 2 \cdot (0.981031)^2 \right] \\
&= 13.47 \, \frac{\text{MeV}}{\text{cm}} \, .
\end{aligned} \tag{1.10}$$

Thus the energy loss of charged kaons K^+ and K^- with a momentum of 2.5 GeV in copper is 13.47 MeV/cm.

1.1.2 What is the Energy Resolution of the Detector from the Previous Example?

Let's calculate the energy resolution of the detector from the previous example, assuming that the length of the particle track through the detector is $d = 5 \, \text{cm}$ and that energy is measured based on all deposited energy without any additional losses. Using the result from the previous example (1.10), we can calculate the average energy deposited in the detector as:

$$\Delta E = \bar{\Delta} = -\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle \cdot d = 13.47 \, \frac{\text{MeV}}{\text{cm}} \cdot 5 \, \text{cm} = 67.35 \, \text{MeV} \, . \tag{1.11}$$

This is an approximation since we are assuming that β is constant throughout the detector, which is not true. In reality we'd have to integrate the energy loss over the path of the particle, however at $p \sim \text{GeV}$ additional losses of $\sim \text{MeV}$ are negligible. Measurements of energy are dependant on the energy resolution R which is defined as:

$$R = \frac{\sigma_E}{\bar{\Delta}} \, , \tag{1.12}$$

where σ_E is the standard deviation of the energy measurement which we assume to have a Gaussian distribution like such:

$$p(\Delta) = \frac{1}{\sqrt{2\pi}\sigma_E} \exp\left(-\frac{\Delta - \bar{\Delta}^2}{2\sigma_E^2} \right) \, . \tag{1.13}$$

σ_E is determined empirically. For **non-relativistic** particles it can be calculated as the variance of the Bethe-Bloch equation as:

$$\sigma_0^2 = 4\pi N_a r_e^2 (m_e c^2)^2 \rho \frac{Z}{A} \Delta x \, . \tag{1.14}$$

For **relativistic** particles we can correct the variance from (1.14) as such:

$$\sigma_E^2 = \sigma_0^2 \, \frac{1 - \frac{1}{2}\beta^2}{1 - \beta^2} \, . \tag{1.15}$$

In our case this gives us:

$$
\begin{aligned}
\sigma_E^2 &= 2 \cdot 0.511 \, \frac{\mathrm{MeV}}{c^2} \, c^2 \cdot 0.1535 \, \frac{\mathrm{MeVcm^2}}{\mathrm{mol}} \cdot 8.92 \, \frac{\mathrm{g}}{\mathrm{cm^3}} \cdot \frac{29}{63.5} \, \frac{\mathrm{mol}}{\mathrm{g}} \cdot 5 \, \mathrm{cm} \cdot \frac{1 - \frac{1}{2}(0.981031)^2}{1 - (0.981031)^2} \\
&= 44.12 \, \mathrm{MeV}^2 \, .
\end{aligned} \tag{1.16}
$$

Thus the energy resolution of the detector is:

$$
R = \frac{\sqrt{44.12 \, \mathrm{MeV}^2}}{67.35 \, \mathrm{MeV}} = 9.9\% \, . \tag{1.17}
$$