University *of Ljubljana*
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Advanced Particle Detectors and Data Analysis

Notes for Exercises

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1 Interactions of Particles with Photons

1.1 Bethe-Bloch Equation

The Bethe-Bloch equation describes the mean energy loss per distance traveled while traversing through matter. We generally use the Bethe-Bloch equation when we are dealing with **thick absorbers**, such as the ones in calorimeters. Do note that the Bethe-Bloch equation does not accurately describe the energy loss of **electrons** and **positrons** due to their small mass and the fact that they suffer from much larger energy losses due to bremsstrahlung and pair production. For a particle with charge z and velocity $\beta = v/c$, the Bethe-Bloch equation is given as:

$$-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right] , \tag{1.1}$$

where δ is the **density effect correction** and C is the **shell correction**. The rest is as follows:

$$N_a = 6.022 \times 10^{23}\,\mathrm{mol}^{-1} , \quad r_e = 2.818 \times 10^{-15}\,\mathrm{m} , \quad m_e = 9.11 \times 10^{-31}\,\mathrm{kg} , \quad c = 3 \times 10^8\,\mathrm{m/s} ,$$
$$\rho = \text{density of the material} , \quad A = \text{atomic mass of the material} , \quad Z = \text{atomic number of the material} ,$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} , \quad W_{\max} = \text{maximum energy transfer in a single collision} , \quad I = \text{mean excitation energy} .$$

The constant factor in the equation can be written as:

$$\Xi = \boldsymbol{2\pi N_a r_e^2 m_e c^2 = 0.1535\,\mathrm{MeV\ cm^2 mol^{-1}}} , \tag{1.2}$$

where I've chosen to mark this constant factor as Ξ for easier reference in further calculations. We can find the mean excitation energy I from the following experimentally determined formula:

$$I = \begin{cases} Z(12 + \frac{7}{Z}) & \text{for } Z < 13 , \\ Z(9.76 + 58.8 Z^{-1.19}) & \text{for } Z \geq 13 . \end{cases} \tag{1.3}$$

The maximum energy transfer in a single collision $W_{\max}$ can be calculated as:

$$W_{\max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \left({}^{m_e}/_{M} \right) + \left({}^{m_e}/_{M} \right)^2} \approx 2 m_e c^2 \beta^2 \gamma^2 . \tag{1.4}$$

For our purposes we will ignore the density effect correction δ and the shell correction C.

1.1.1 Energy Loss of Charged Kaons

Let us calculate the energy losses for charged kaons K^+ and K^- with a rest mass of 0.493 MeV and momentum of 2.5 GeV in copper which has the following properties:

$$\rho = 8.92\,\mathrm{g/cm^3} ,$$
$$Z = 29 ,$$
$$A = 63.5\,\mathrm{g/mol} .$$

First let us calculate the velocity β and the Lorentz factor γ. We know that

$$\beta = \frac{pc}{E} = \frac{pc}{\sqrt{(pc)^2 + (Mc^2)^2}} , \tag{1.5}$$

where M is the mass of the particle. Thus:

$$\beta = \frac{2.5\,\frac{\mathrm{GeV}}{c}\,c}{\sqrt{\left(2.5\,\frac{\mathrm{GeV}}{c}\,c\right)^2 + \left(0.493\,\frac{\mathrm{MeV}}{c^2}\,c^2\right)^2}} \approx 0.981031\,. \tag{1.6}$$

Remember to take at least 4 significant digits for the velocity β! This is due to the logarithm in the Bethe-Bloch equation. The Lorentz factor γ is then:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \approx 5.159\,. \tag{1.7}$$

Next let us calculate the maximum energy transfer in a single collision $W_{\max}$:

$$W_{\max} = 2m_e c^2\beta^2\gamma^2 = 2\cdot 0.511\,\frac{\mathrm{MeV}}{c^2}\,c^2\cdot(0.981031)^2(5.159)^2 = 26.7\,\mathrm{MeV}\,. \tag{1.8}$$

Last prerequisite is the mean excitation energy I which we can calculate using the formula (1.3) for $Z \geq 13$:

$$I = 29(9.76 + 58.8\cdot 29^{-1.19})\,\mathrm{eV} = 313.9\,\mathrm{eV}\,. \tag{1.9}$$

Now all that is left is to plug in the values into the Bethe-Bloch equation (1.1):

$$\begin{aligned}
-\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle &= \Xi\,\rho\frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{W_{\max}^2}{I^2}\right) - 2\beta^2\right] \qquad (1.10)\\
&= 0.1535\,\frac{\mathrm{MeV\,cm^2}}{\mathrm{mol}}\cdot 8.92\,\frac{\mathrm{g}}{\mathrm{cm^3}}\cdot\frac{29}{63.5}\,\frac{\mathrm{mol}}{\mathrm{g}}\cdot\frac{1}{0.981031^2}\cdot\left[\ln\left(\frac{(26.7\cdot 10^6\,\mathrm{eV})^2}{(313.9\,\mathrm{eV})^2}\right) - 2\cdot(0.981031)^2\right]\\
&= 13.47\,\frac{\mathrm{MeV}}{\mathrm{cm}}\,.
\end{aligned}$$