Chapter 1

Linear Equations in Linear Algebra

1.1 Systems of Linear Equations

Definition 1.1.1. A Linear Equations is the variables $x_1, x_2...x_n$ is an equation that can be written in the form $a_1x_1 + ... + a_nx_n = b$ where $a_1...a_n$ are real coefficient and b is a real number (and known)

Definition 1.1.2. A System of Linear Equations $a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m$ m number of equations, n number unknowns (standard form) (first index row number, second index col number)

Definition 1.1.3. A solution of the system is a list $(s_1, s_2, ..., s_n)$ of numbers that makes each equation a true statement when the values are substituted for $x_1, x_2, ..., x_n$

Definition 1.1.4. Solution Set is the set of all possible solutions

1.1.1 Geometric Interpretations

Ex. Find the Solution set of the system

(a)
$$x_1 - x_2 = 5$$

$$2x_1 + x_2 = 7$$

Definition 1.1.5. A linear system is consistant if it has either one solution or infinitely many solutions

Definition 1.1.6. Matrix of Coefficients
$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{11} & a_{12} & \dots & a_{1n} \end{vmatrix}$$
$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{1n} & \dots & a_{nn} \end{vmatrix}$$

Definition 1.1.7. Augmented Matrix of the System
$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{11} & a_{12} & \dots & a_{1n} & b_1 \end{vmatrix}$$