STAT 5630 Project Milestone 3

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1. Classification

We implemented several non-linear methods on the data, including non-linear Support Vector Machine (SVM) K-Nearest Neighbors (KNN), Decision Tree, and Random Forest. The corresponding training and test error are shown in table 3. For SVM, the tuning parameter is selected by 10-fold cross-validation, as shown in table 6. The kernels and the corresponding parameters to be tuned are shown in table 6 and the parameters chosen are shown in table 7. For KNN, we also used 10-fold cross-validation to find the best "k" (table 5). For Random Forest and Decision Tree, the tuning parameters are recorded in table 8 and table 9. The result suggests non-linear SVM yields the best performance for both data. The comparison with linear method (table 1) is similar to that of regression task.

2. Regression

For regression, we used non-linear SVM, KNN, neural network, and Polynomial regression. The corresponding mean squared error of the training and test set is shown in table 4. For SVM, the tuning process is the same as that of classification task, see table 6 and table 7. For KNN, we also used 10-fold cross-validation to find the best "k" (table 5). For neural network, we selected the best parameters. For Random Forest and Decision Tree, the tuning parameters are recorded in table 8 and table 9. The result suggests that in terms of regression, the regression tree yields the best performance for math data and non-linear SVM for Português data. This is also true when compared with the linear methods(table 2). As the improvement is insignificant, it remains uncertain whether we should choose linear method for better interpretation and generalization.

3. Model Diagnostic & Variable Selection

We begin by checking multicollinarity in our regression model. For Português, the Value Inflation Factor (VIF) output shows that all of the VIFs are below or equal to 3, which means the effect of multicollinarity is little. For math, the VIF output indicates that two variables have a VIF over 3, so we remove these variables to eliminate multicollinarity.

Next we move on to outliers identification. From the Cook's Distance plots (figure 5), we detect observations 1, 157,193, 199, and 266 should be removed from the math data, and observations 1, 604, 500, 524, 550 should be removed from the Português data.

We then proceed to check error assumptions. We conducted the Box-Cox power transformation and based on the results(figure 6), we let $y_{trans} = \sqrt{y}$.

We further conduct a variable selection based on the summary output using regsubset function. For math, the model includes the number of past class failures, school's extra educational support, Mother's and Father's job, family educational support, study time, going out with friends, and free time. For Português, the model includes past class failures, student's school, student's will of higher education, study time, Father's job, school's extra educational support, number of absences, and workday alcohol consumption.

Having finished the above procedures, We refitted the models and calculated the adjusted R-squared values of the model, which are 0.30 and 0.37 for math and Português.

4. Parameter Interpretation

We choose non-linear SVM to compare predictors' influence on the training error, which we defined as the importance in these models, across tasks and datasets. As shown in table 10, table 14, table 11 and table 12. The Venn diagram shown in figure 7 indicate the common important predictors shared by different task and datasets. It is worth noting that 6 predictors are important in regression task for both data.

Appendix

0. Linear Methods Tables

	math		Português	
	training error	test error	training error	test error
baseline	0.768	0.835	0.802	0.738
SVM	0.642	0.633	0.489	0.661
Logistic regression	0.446	0.633	0.462	0.715
LR + Lasso	0.579	0.620	0.493	0.700
LR + Ridge	0.582	0.696	0.489	0.692
LDA	0.487	0.620	0.484	0.692

Table 1: Performance of different methods for classification task

	math		Português	
	training MSE	test MSE	training MSE	test MSE
baseline	0.997	0.966	0.998	0.988
SVM	0.676	0.844	0.629	0.763
OLS	0.886	0.969	0.601	0.800
Lasso	0.678	0.782	0.641	0.774
Ridge	0.671	0.803	0.617	0.772

Table 2: Performance of different methods for regression task

1. Nonlinear Methods Tables

	Math		Português	
	training error	test error	training error	test error
baseline	0.768	0.835	0.802	0.738
Non-linear SVM	0.468	0.608	0.466	0.653
KNN	0.592	0.646	0.561	0.746
Tree(unpruned)	0.437	0.722	0.410	0.677
Tree(pruned)	0.642	0.633	0.578	0.692
Random Forest	0	0.646	0	0.662

Table 3: Performance of different methods for classification task

	Math		Português	
	training MSE	test MSE	training MSE	test MSE
baseline	0.997	0.966	0.998	0.988
Non-linear SVM	0.585	0.827	0.553	0.751
KNN	0.798	0.888	0.701	0.888
Neural Network	0.757	0.889	0.414	0.829
Tree(unpruned)	0.564	0.757	0.448	0.999
Tree(pruned)	0.855	0.767	0.739	0.855
Random Forest	0.160	0.798	0.139	0.763

Table 4: Performance of different methods for regression task

KNN type	tuning parameters
math regression	k=25
por regression	k=19
math classification	k=27
por classification	k=29

Table 5: Tuning parameters for the k (range from 1 to 50)

kernel	formula	tuning parameters
polynomial	$(\gamma x_1^T x_2)^{degree}$	γ, degree(2,3,4), cost
radial	$\exp(-\gamma\lvert x_1 - x_2\rvert^2)$	γ, cost
sigmoid	$\tanh(\gamma x_1^T x_2)$	γ, cost

Table 6: Tuning parameters for non-linear SVM (γ range from 10^{-4} to 10^{-2}, cost range from 0.1 to 5.)

task	parameter chosen
math regression	radial $\gamma = 0.01$, cost $= 0.5$
Português regression	radial $\gamma = 0.006$, cost $= 0.8$
math classification	radial, $\gamma = 0.002$, cost $= 3.4$
Português classification	radial, $\gamma = 0.0025$, cost $= 5$

Table 7: Parameters chosen for non-linear SVM

RF type	tuning parameters
math regression	ntree=950
por regression	ntree=700
math classification	ntree=700
por classification	ntree=850

Table 8: Tuning parameters for the number of trees (range from 50 to 1000, step=50.)

3. Figures

Tree type	tuning parameters
math regression	cp=0.04
por regression	cp=0.052
math classification	cp=0.05
por classification	cp=0.036

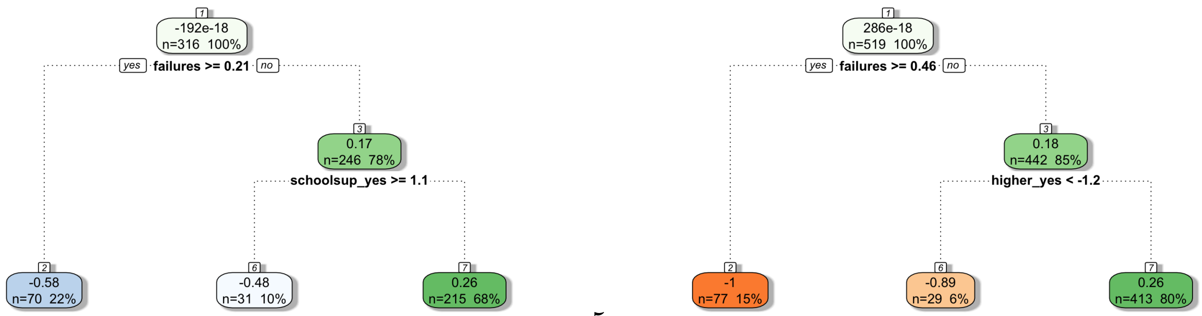
Table 9: Tuning parameters for the cost parameter (range from 50 to 1000, step=50.)

	predictors	importance
32	schoolsup_yes	0.03
8	freetime	0.03
33	famsup_yes	0.03
4	traveltime	0.02
5	studytime	0.02
10	Dalc	0.02
12	health	0.02
16	address_U	0.02
26	Fjob_teacher	0.02
29	reason_reputation	0.02

Table 10: Predictors importance of Non-linear SVM in classification task for math data(top 10)

	predictors	importance
6	failures	0.05
32	schoolsup_yes	0.04
33	famsup_yes	0.02
26	Fjob_teacher	0.02
5	studytime	0.02
37	higher_yes	0.01
15	sex_M	0.01
29	reason_reputation	0.01
36	nursery_yes	0.01
24	Fjob_other	0.01

Table 11: Predictors importance of Non-linear SVM in regression task for math data(top 10)



(a) math-reg-pruned-tree (b) por-reg-pruned-tree

Figure 1: Pruned Trees

	predictors	importance
21	Mjob_services	0.02
14	school_MS	0.02
1	age	0.01
12	health	0.01
37	higher_yes	0.01
10	Dalc	0.01
15	sex_M	0.01
22	Mjob_teacher	0.01
39	romantic_yes	0.01
6	failures	0.01

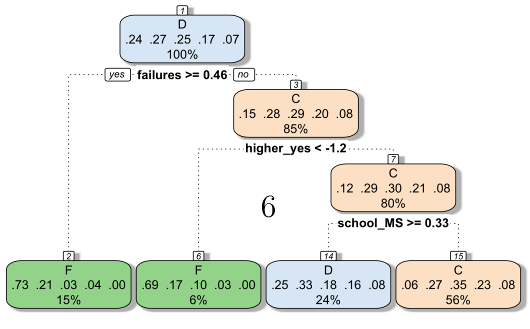
Table 12: Predictors importance of Non-linear SVM in classification for Português data(top 10)

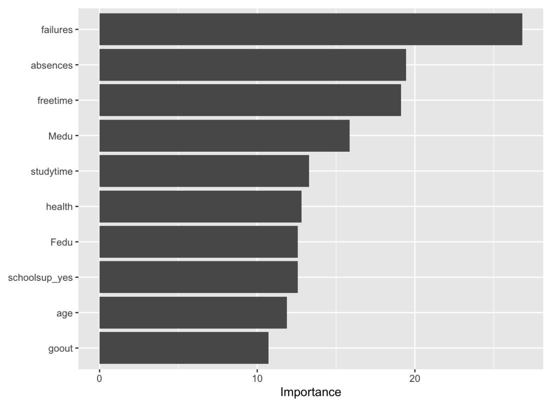
	predictors	importance
6	failures	0.04
14	school_MS	0.03
37	higher_yes	0.03
32	schoolsup_yes	0.01
5	studytime	0.01
13	absences	0.01
26	Fjob_teacher	0.01
31	guardian_other	0.01
38	internet_yes	0.01
15	sex_M	0.01

Table 13: Predictors importance of Non-linear SVM in regression for Português data(top 10)

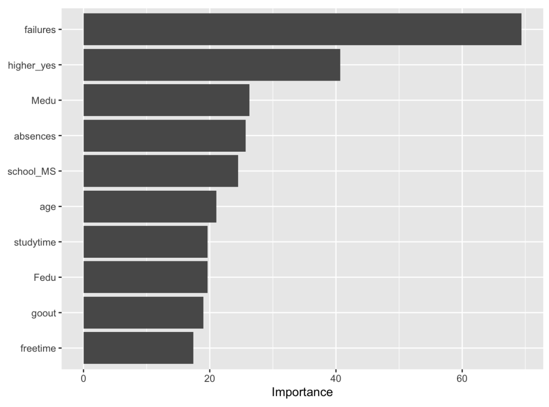
Number of Predictors	Math	Português
1	failures	failures
2	schoolsup_yes	school_MS
3	Mjob_other	higher_yes
4	Fjob_teacher	studytime
5	famsup_yes	Fjob_teacher
6	studytime	schoolsup_yes
7	goout	absences
8	freetime	Dalc

Table 14: best models for each number of variables selected by regsubset()



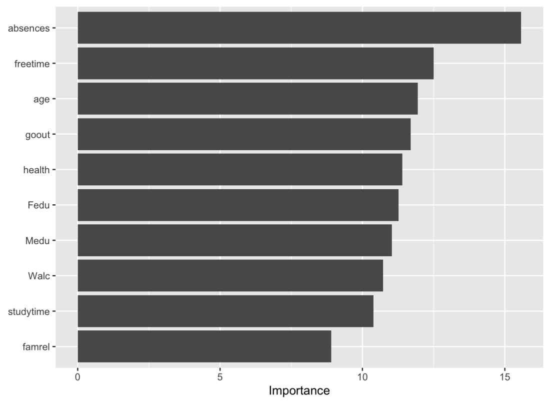


(a) math-reg-vp

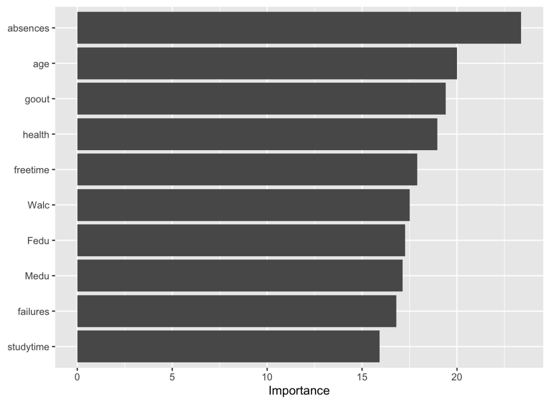


(b) por-reg-vp

Figure 3: Variable Importance Plots for Regression

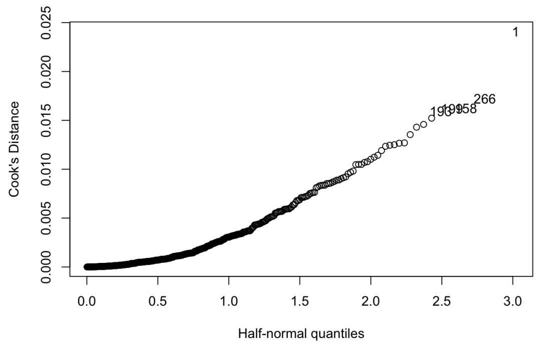


(a) math-class-vp

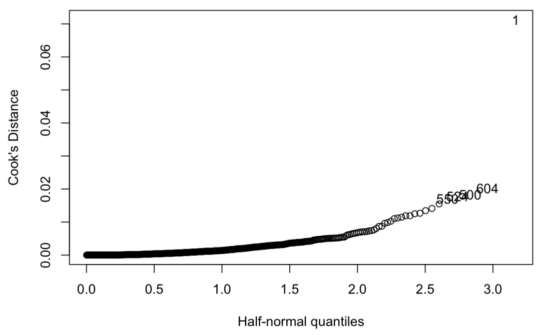


(b) por-class-vp

Figure 4: Variable Importance Plots for Classification

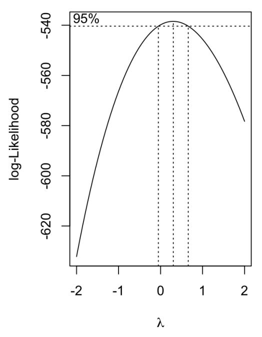


(a) math-outliers

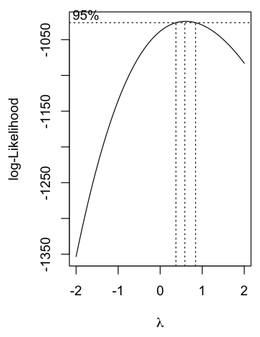


(b) por-outliers

Figure 5: Identification of Outliers using Cook's Distance



(a) math lambda



(b) por lambda

Figure 6: Box-Cox Power Transformation

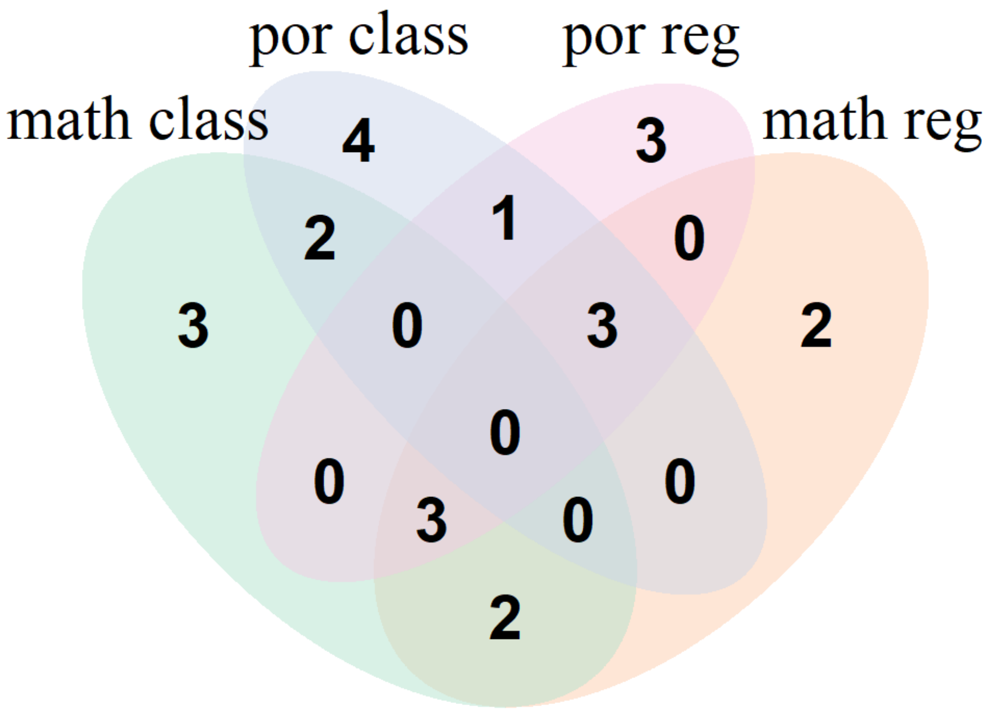


Figure 7: Venn Diagram of important predictors

C. code

```r
## Non-linear SVM ------------------------------
library(e1071)

### math regression -----------------
svm.math.reg = svm.cv.mine(math.train, regression.formula, 10,
    "grade.con")
svm.math.reg
# $kernel [1] 'radial' $gamma [1] 0.01 $cost [1] 0.5

svm.math.reg.fit = do.call(svm, c(list(formula = regression.formula,
    data = math.train), svm.math.reg))
predict.test = predict(svm.math.reg.fit, math.test, decision.values = F)

svm.math.reg.error.test = mean((predict.test - math.test$grade.con)^2)
svm.math.reg.error.train = mean((predict(svm.math.reg.fit) -
    math.train$grade.con)^2)
svm.math.reg.error.test

svm.math.reg.var.imp = numeric(length(predictors))
for (i in 1:length(predictors)) {
    svm.math.reg.fit.inf = do.call(svm, c(list(formula = as.formula(paste("grade.con~",
        paste(predictors[-i], collapse = "+"), sep = "")), data = math.train),
        svm.math.reg))

    svm.math.reg.var.imp[i] = mean((predict(svm.math.reg.fit.inf) -
        math.train$grade.con)^2) - svm.math.reg.error.train
}

svm.math.reg.var.imp = data.frame(predictors = predictors, importance = svm.math.reg.var.imp)
svm.math.reg.var.imp = svm.math.reg.var.imp[order(svm.math.reg.var.imp$importance,
    decreasing = T), ]
xtable(svm.math.reg.var.imp)


### por regression -------------------------
svm.por.reg = svm.cv.mine(por.train, regression.formula, 10,
    "grade.con")
svm.por.reg
# $kernel [1] 'radial' $gamma [1] 0.005959184 $cost [1] 0.8

svm.por.reg.fit = do.call(svm, c(list(formula = regression.formula,
    data = por.train), svm.por.reg))
predict.test = predict(svm.por.reg.fit, por.test, decision.values = F)

svm.por.reg.error.test = mean((predict.test - por.test$grade.con)^2)
svm.por.reg.error.test
svm.por.reg.error.train = mean((predict(svm.por.reg.fit) - por.train$grade.con)^2)

svm.por.reg.var.imp = numeric(length(predictors))
for (i in 1:length(predictors)) {
    svm.por.reg.fit.inf = do.call(svm, c(list(formula = as.formula(paste("grade.con~",
        paste(predictors[-i], collapse = "+"), sep = "")), data = por.train),
        svm.por.reg))

    svm.por.reg.var.imp[i] = mean((predict(svm.por.reg.fit.inf) -
        por.train$grade.con)^2) - svm.por.reg.error.train
}

svm.por.reg.var.imp = data.frame(predictors = predictors, importance = svm.por.reg.var.imp)
svm.por.reg.var.imp = svm.por.reg.var.imp[order(svm.por.reg.var.imp$importance,
    decreasing = T), ]
xtable(svm.por.reg.var.imp[1:10, ])

### por class ------------------------------
svm.por.cat = svm.cv.mine.cat(por.train, classification.formula,
    10, "grade.cat")
svm.por.cat
# $kernel [1] 'radial' $gamma [1] 0.002120408 $cost [1] 3.4

svm.por.cat.fit = do.call(svm, c(list(formula = classification.formula,
    data = por.train), svm.por.cat))
predict.test = predict(svm.por.cat.fit, por.test, decision.values = F)

svm.por.cat.error.test = mean((predict.test != por.test$grade.cat))
svm.por.cat.error.train = mean(predict(svm.por.cat.fit) != por.train$grade.cat)
svm.por.cat.error.test

### math class ----------------
svm.math.cat = svm.cv.mine.cat(math.train, classification.formula,
    10, "grade.cat")
svm.math.cat
# $kernel [1] 'radial' $gamma [1] 0.00252449 $cost [1] 5

svm.math.cat.fit = do.call(svm, c(list(formula = classification.formula,
    data = math.train), svm.math.cat))
predict.test = predict(svm.math.cat.fit, math.test, decision.values = F)

svm.math.cat.error.test = mean((predict.test != math.test$grade.cat))
svm.math.cat.error.train = mean(predict(svm.math.cat.fit) !=
    math.train$grade.cat)
svm.math.cat.error.test

svm.math.cat.var.imp = numeric(length(predictors))
for (i in 1:length(predictors)) {
    svm.math.cat.fit.inf = do.call(svm, c(list(formula = as.formula(paste("grade.cat~",
        paste(predictors[-i], collapse = "+"), sep = "")), data = math.train),
        svm.math.cat))

    svm.math.cat.var.imp[i] = mean((predict(svm.math.cat.fit.inf) !=
        math.train$grade.cat)) - svm.math.cat.error.train
}

svm.math.cat.var.imp = data.frame(predictors = predictors, importance = svm.math.cat.var.imp)
svm.math.cat.var.imp = svm.math.cat.var.imp[order(svm.math.cat.var.imp$importance,
    decreasing = T), ]
xtable(svm.math.cat.var.imp[1:10, ])

svm.por.cat.var.imp = numeric(length(predictors))
for (i in 1:length(predictors)) {
    svm.por.cat.fit.inf = do.call(svm, c(list(formula = as.formula(paste("grade.cat~",
        paste(predictors[-i], collapse = "+"), sep = "")), data = por.train),
        svm.por.cat))

    svm.por.cat.var.imp[i] = mean((predict(svm.por.cat.fit.inf) !=
        por.train$grade.cat)) - svm.por.cat.error.train
}

svm.por.cat.var.imp = data.frame(predictors = predictors, importance = svm.por.cat.var.imp)
svm.por.cat.var.imp = svm.por.cat.var.imp[order(svm.por.cat.var.imp$importance,
    decreasing = T), ]
xtable(svm.por.cat.var.imp[1:10, ])

intersect(svm.por.cat.var.imp[1:10, 1], svm.math.cat.var.imp[1:10,
    1])


## KNN ----------------------------

## Neural network -------------------
library(keras)
library(dplyr)

nn.model.math.reg = keras_model_sequential()


nunits = c(10, 5)
act_fun = c("relu", "elu")

nn.model.math.reg %>%
    layer_dense(units = nunits[1], activation = act_fun[1], input_shape = c(39)) %>%
    layer_dense(units = 1)

nn.model.math.reg %>%
    compile(loss = "mse", optimizer = "rmsprop", metrics = "mse")

nn.model.math.reg.fit <- nn.model.math.reg %>%
    fit(as.matrix(math.train[, predictors]), math.train$grade.con,
        epochs = 20, batch_size = 8, validation_split = 0.2)
mean((nn.model.math.reg %>%
    predict(as.matrix(math.test[, predictors])) - math.test$grade.con)^2)

nn.model.math.reg %>%
```

```r
    evaluate(as.matrix(math.test[, predictors]), math.test$grade.con)


paras_list = expand.grid(c(5, 10, 20), c(2, 4, 5, 10), c("relu",
    "elu", "selu", "sigmoid"), c("relu", "elu", "selu", "sigmoid"),
    c(8, 16, 32), c(10, 20, 40, 60, 80))
# set up a validation set to select parameters
val_d.idx = sample(nrow(math.train), 100, replace = F)
val_d = math.train[val_d.idx, ]
train_d = math.train[-val_d.idx, ]

val_error = numeric(nrow(paras_list))

for (i in 1:nrow(paras_list)) {
    nn.model.math.reg = keras_model_sequential()
    nn.model.math.reg %>%
        layer_dense(units = paras_list[i, 1], activation = paras_list[i,
            3], input_shape = c(39)) %>%
        layer_dense(units = paras_list[i, 2], activation = paras_list[i,
            4]) %>%
        layer_dense(units = 1)

    nn.model.math.reg %>%
        compile(loss = "mse", optimizer = "rmsprop", metrics = "mse")

    nn.model.math.reg.fit <- nn.model.math.reg %>%
        fit(as.matrix(train_d[, predictors]), train_d$grade.con,
            epochs = paras_list[i, 6], batch_size = paras_list[i,
                5], )

    val_error[i] = mean((nn.model.math.reg %>%
        predict(as.matrix(val_d[, predictors])) - val_d$grade.con)^2)
}


paras_list[which.min(val_error), ]
# Var1 Var2 Var3 Var4 Var5 Var6 580 5 4 relu relu 8 20

idx = 179
nn.model.math.reg = keras_model_sequential()
nn.model.math.reg %>%
    layer_dense(units = paras_list[idx, 1], activation = paras_list[idx,
        3], input_shape = c(39)) %>%
    layer_dense(units = paras_list[idx, 2], activation = paras_list[idx,
        4]) %>%
    layer_dense(units = 1)

nn.model.math.reg %>%
    compile(loss = "mse", optimizer = "rmsprop", metrics = "mse")

nn.model.math.reg.fit <- nn.model.math.reg %>%
    fit(as.matrix(math.train[, predictors]), math.train$grade.con,
        epochs = paras_list[idx, 6], batch_size = paras_list[idx,
            5], validation_split = 0.2)

mean((nn.model.math.reg %>%
    predict(as.matrix(math.test[, predictors])) - math.test$grade.con)^2)

nn.model.math.reg = keras_model_sequential()
nn.model.math.reg %>%
    layer_dense(units = 5, activation = "relu", input_shape = c(39)) %>%
    layer_dense(units = 4, activation = "relu") %>%
    layer_dense(units = 1)

nn.model.math.reg %>%
    compile(loss = "mse", optimizer = "rmsprop", metrics = "mse")

nn.model.math.reg.fit <- nn.model.math.reg %>%
    fit(as.matrix(math.train[, predictors]), math.train$grade.con,
        epochs = 20, batch_size = 8, validation_split = 0.2)

mean((nn.model.math.reg %>%
    predict(as.matrix(math.test[, predictors])) - math.test$grade.con)^2)
mean((nn.model.math.reg %>%
    predict(as.matrix(math.train[, predictors])) - math.train$grade.con)^2)

library(keras)
library(dplyr)





nn.model.por.reg = keras_model_sequential()


nunits = c(10, 5)
act_fun = c("relu", "elu")

nn.model.por.reg %>%
    layer_dense(units = nunits[1], activation = act_fun[1], input_shape = c(39)) %>%
    layer_dense(units = 1)

nn.model.por.reg %>%
    compile(loss = "mse", optimizer = "rmsprop", metrics = "mse")

nn.model.por.reg.fit <- nn.model.por.reg %>%
    fit(as.matrix(por.train[, predictors]), por.train$grade.con,
        epochs = 20, batch_size = 8, validation_split = 0.2)
mean((nn.model.por.reg %>%
    predict(as.matrix(por.test[, predictors])) - por.test$grade.con)^2)

nn.model.por.reg %>%
    evaluate(as.matrix(por.test[, predictors]), por.test$grade.con)


paras_list = expand.grid(c(5, 10, 20), c(2, 4, 5, 10), c("relu",
    "elu", "selu", "sigmoid"), c("relu", "elu", "selu", "sigmoid"),
    c(8, 16, 32), c(10, 20, 40, 60, 80))
# set up a validation set to select parameters
set.seed(1)
val_d.idx = sample(nrow(por.train), 100, replace = F)
val_d = por.train[val_d.idx, ]
train_d = por.train[-val_d.idx, ]

val_error = numeric(nrow(paras_list))


options(keras.view_metrics = FALSE)

set.seed(1)
for (i in 1:nrow(paras_list)) {
    nn.model.math.reg = keras_model_sequential()
    nn.model.math.reg %>%
        layer_dense(units = paras_list[i, 1], activation = paras_list[i,
            3], input_shape = c(39)) %>%
        layer_dense(units = paras_list[i, 2], activation = paras_list[i,
            4]) %>%
        layer_dense(units = 1)

    nn.model.math.reg %>%
        compile(loss = "mse", optimizer = "rmsprop", metrics = "mse")

    nn.model.math.reg.fit <- nn.model.math.reg %>%
        fit(as.matrix(train_d[, predictors]), train_d$grade.con,
            epochs = paras_list[i, 6], batch_size = paras_list[i,
                5], )

    val_error[i] = mean((nn.model.math.reg %>%
        predict(as.matrix(val_d[, predictors])) - val_d$grade.con)^2)
}


paras_list[which.min(val_error), ]


paras_list[which.min(val_error), ]
# Var1 Var2 Var3 Var4 Var5 Var6 2100 20 10 selu sigmoid 16
# 60

idx = 2100
nn.model.por.reg = keras_model_sequential()
nn.model.por.reg %>%
    layer_dense(units = paras_list[idx, 1], activation = paras_list[idx,
        3], input_shape = c(39)) %>%
    layer_dense(units = paras_list[idx, 2], activation = paras_list[idx,
        4]) %>%
    layer_dense(units = 1)

nn.model.por.reg %>%
    compile(loss = "mse", optimizer = "rmsprop", metrics = "mse")

nn.model.por.reg.fit <- nn.model.por.reg %>%
    fit(as.matrix(por.train[, predictors]), por.train$grade.con,
        epochs = paras_list[idx, 6], batch_size = paras_list[idx,
            5], validation_split = 0.2)

mean((nn.model.por.reg %>%
    predict(as.matrix(por.test[, predictors])) - por.test$grade.con)^2)
mean((nn.model.por.reg %>%
    predict(as.matrix(por.train[, predictors])) - por.train$grade.con)^2)

## Tree based ---------------------------
```

Tree-based Regression————

```
library(rpart.plot)
library(rattle)
library(rpart)
library(caret)
library(randomForest)
library(gbm)
## Decision Tree for math--------- Unpruned Tree----------
tree.math <- rpart(regression.formula, data = math.train, control = rpart.control(cp = 0))
# predicted result for tree
train.math <- predict(tree.math, newdata = math.train)
tree.pred.math <- predict(tree.math, newdata = math.test)
tree.pred.math
# MSE for unpruned
mean((train.math - math.train$grade.con)^2)
mean((tree.pred.math - math.test$grade.con)^2)
### Pruned Tree-------
set.seed(114514)
tree.tune.math <- train(regression.formula, data = math.train,
    method = "rpart", preProcess = c("center", "scale"), tuneGrid = data.frame(.cp = seq(0,
        0.2, by = 0.004)), trControl = trainControl(method = "repeatedcv",
        repeats = 5, number = 10))
plot(tree.tune.math)
# best cost parameter
best <- tree.tune.math$bestTune
best
tree.math.pruned <- rpart(regression.formula, data = math.train,
    control = rpart.control(cp = best))
fancyRpartPlot(tree.math.pruned, caption = NULL)
# predicted result for pruned tree
train.math <- predict(tree.math.pruned, newdata = math.train)
tree.pruned.math.pred <- predict(tree.math.pruned, newdata = math.test)
tree.pruned.math.pred
# MSE for pruned
mean((train.math - math.train$grade.con)^2)
mean((tree.pruned.math.pred - math.test$grade.con)^2)
```

```
## Decision Tree for Por---------
tree.por <- rpart(regression.formula, data = por.train, control = rpart.control(cp = 0))
# predicted result for tree
train.por <- predict(tree.por, newdata = por.train)
tree.pred.por <- predict(tree.por, newdata = por.test)
tree.pred.por

# MSE for unpruned
mean((train.por - por.train$grade.con)^2)
mean((tree.pred.por - por.test$grade.con)^2)
### Pruned Tree-------
set.seed(114514)
tree.tune.por <- train(regression.formula, data = por.train,
    method = "rpart", preProcess = c("center", "scale"), tuneGrid = data.frame(.cp = seq(0,
        0.2, by = 0.004)), trControl = trainControl(method = "repeatedcv",
        repeats = 5, number = 10))
plot(tree.tune.por)
# best cost parameter
best <- tree.tune.por$bestTune
best
tree.por.pruned <- rpart(regression.formula, data = por.train,
    control = rpart.control(cp = best))
fancyRpartPlot(tree.por.pruned, caption = NULL)
# predicted result for pruned tree
train.por <- predict(tree.por.pruned, newdata = por.train)
tree.pruned.por.pred <- predict(tree.por.pruned, newdata = ,
    por.test)
tree.pruned.por.pred
# MSE
mean((train.por - por.train$grade.con)^2)
mean((tree.pruned.por.pred - por.test$grade.con)^2)
```

Random Forrest regression

```
library(randomForest)
library(caret)
### math-------- choice of m
37/3
# cv on number of trees
test.error <- rep(1, 10)
set.seed(114514)
folds <- createFolds(1:nrow(math.train), k = 10, returnTrain = FALSE)
tree.vec <- seq(50, 1000, by = 50)
tree.error <- rep(1, length(tree.vec))
for (k in 1:length(tree.vec)) {
    for (i in 1:10) {
        set.seed(114514)
        train.rand.for.math <- randomForest(regression.formula,
            data = math.train[-folds[[i]], ], mtry = 12, ntree = k)
        train.pred.rand.for.math <- predict(train.rand.for.math,
            newdata = math.train[folds[[i]], ])
        test.error[i] <- mean((train.pred.rand.for.math - math.train[folds[[i]],
            ]$grade.con)^2)
    }
    tree.error[k] <- mean(test.error)
}
# the choice of number of trees
tree.vec[which.min(tree.error)]
best.rd.math <- tree.vec[which.min(tree.error)]
set.seed(114514)
train.rand.for.math <- randomForest(regression.formula, data = math.train,
    mtry = 14, ntree = best.rd.math)
train.rand.for.math
test.pred.rand.for.math <- predict(train.rand.for.math, newdata = math.test)
train.pred.rand.for.math <- predict(train.rand.for.math, newdata = math.train)
# test result
test.pred.rand.for.math
# misclassifictaion rate
mean((train.pred.rand.for.math - math.train$grade.con)^2)
mean((test.pred.rand.for.math - math.test$grade.con)^2)
library(vip)
vip::vip(train.rand.for.math)


### por--------- choice of m
37/3
# cv on number of trees
test.error <- rep(1, 10)
set.seed(114514)
folds <- createFolds(1:nrow(por.train), k = 10, returnTrain = FALSE)
tree.vec <- seq(50, 1000, by = 50)
tree.error <- rep(1, length(tree.vec))
for (k in 1:length(tree.vec)) {
    for (i in 1:10) {
        set.seed(114514)
        train.rand.for.por <- randomForest(regression.formula,
            data = por.train[-folds[[i]], ], mtry = 12, ntree = k)
        train.pred.rand.for.por <- predict(train.rand.for.por,
            newdata = por.train[folds[[i]], ])
        test.error[i] <- mean((train.pred.rand.for.por - por.train[folds[[i]],
            ]$grade.con)^2)
    }
    tree.error[k] <- mean(test.error)
}
# the choice of number of trees
tree.vec[which.min(tree.error)]
best.rd.por <- tree.vec[which.min(tree.error)]
set.seed(114514)
train.rand.for.por <- randomForest(regression.formula, data = por.train,
    mtry = 14, ntree = best.rd.por)
train.rand.for.por
test.pred.rand.for.por <- predict(train.rand.for.por, newdata = por.test)
train.pred.rand.for.por <- predict(train.rand.for.por, newdata = por.train)
# test result
```

```r
test.pred.rand.for.por
# misclassifictaion rate
mean((train.pred.rand.for.por - por.train$grade.con)^2)
mean((test.pred.rand.for.por - por.test$grade.con)^2)
library(vip)
vip::vip(train.rand.for.por)
```

TreeBased Classification

```r
library(rpart.plot)
library(rattle)
library(rpart)
library(caret)
library(gbm)
## Decision Tree for math--------- Unpruned Tree----------
tree.math <- rpart(classification.formula, data = math.train,
    control = rpart.control(cp = 0))
# predicted result for tree
tree.pred.math <- predict(tree.math, newdata = math.test, type = "class")
tree.pred.math.train <- predict(tree.math, newdata = math.train,
    type = "class")
tree.pred.math
# misclassification for unpruned
1 - mean(tree.pred.math.train == math.train$grade.cat)
1 - mean(tree.pred.math == math.test$grade.cat)
### Pruned Tree-------
set.seed(114514)
tree.tune.math <- train(classification.formula, data = math.train,
    method = "rpart", preProcess = c("center", "scale"), tuneGrid = data.frame(cp = seq(0,
        0.05, by = 0.001)), trControl = trainControl(method = "repeatedcv",
        repeats = 5, number = 10))
plot(tree.tune.math)
# best cost parameter
best <- tree.tune.math$bestTune
best
tree.math.pruned <- rpart(classification.formula, data = math.train,
    control = rpart.control(cp = best))
## We can't plot this because it's only a root--------
## predicted result for pruned tree
tree.pruned.math.pred <- predict(tree.math.pruned, newdata = math.test,
    type = "class")
tree.pruned.math.train <- predict(tree.math.pruned, newdata = math.train,
    type = "class")
tree.pruned.math.pred
# misclassification for pruned
1 - mean(tree.pruned.math.train == math.train$grade.cat)
1 - mean(tree.pruned.math.pred == math.test$grade.cat)



## Decision Tree for Por---------
tree.por <- rpart(classification.formula, data = por.train, control = rpart.control(cp = 0))
# predicted result for tree
```

```r
tree.pred.por <- predict(tree.por, newdata = por.test, type = "class")
tree.pred.por.train <- predict(tree.por, newdata = por.train,
    type = "class")
tree.pred.por
# misclassification rate for unpruned
1 - mean(tree.pred.por.train == por.train$grade.cat)
1 - mean(tree.pred.por == por.test$grade.cat)
### Pruned Tree-------
set.seed(114514)
tree.tune.por <- train(classification.formula, data = por.train,
    method = "rpart", preProcess = c("center", "scale"), tuneGrid = data.frame(cp = seq(0,
        0.2, by = 0.004)), trControl = trainControl(method = "repeatedcv",
        repeats = 5, number = 10))
plot(tree.tune.por)
# best cost parameter
best <- tree.tune.por$bestTune
best
tree.por.pruned <- rpart(classification.formula, data = por.train,
    control = rpart.control(cp = best))
fancyRpartPlot(tree.por.pruned, caption = NULL)
# predicted result for pruned tree
tree.pruned.por.pred <- predict(tree.por.pruned, newdata = por.test,
    type = "class")
tree.pruned.por.train <- predict(tree.por.pruned, newdata = por.train,
    type = "class")
tree.pruned.por.pred
# misclassification rate for pruned
1 - mean(tree.pruned.por.train == por.train$grade.cat)
1 - mean(tree.pruned.por.pred == por.test$grade.cat)
```

Random Forrest Classification

```r
library(randomForest)
### math-------- choice of m
sqrt(37)
# cv on number of trees
test.error <- rep(1, 10)
set.seed(114514)
folds <- createFolds(1:nrow(math.train), k = 10, returnTrain = FALSE)
tree.vec <- seq(50, 1000, by = 50)
tree.error <- rep(1, length(tree.vec))
for (k in 1:length(tree.vec)) {
    for (i in 1:10) {
        set.seed(114514)
        train.rand.for.math <- randomForest(classification.formula,
            data = math.train[-folds[[i]], ], mtry = 6, ntree = k)
        train.pred.rand.for.math <- predict(train.rand.for.math,
            newdata = math.train[folds[[i]], ], type = "class")
        test.error[i] <- 1 - mean(train.pred.rand.for.math ==
            math.train[folds[[i]], ]$grade.cat)
    }
    tree.error[k] <- mean(test.error)
}
```

```r
# the choice of number of trees
tree.vec[which.min(tree.error)]
best.rd.math <- tree.vec[which.min(tree.error)]
set.seed(114514)
train.rand.for.math <- randomForest(classification.formula, data = math.train,
    mtry = 6, ntree = best.rd.math)
train.rand.for.math
test.pred.rand.for.math <- predict(train.rand.for.math, math.test,
    type = "class")
train.pred.rand.for.math <- predict(train.rand.for.math, math.train,
    type = "class")
# test result
test.pred.rand.for.math
# misclassifictaion rate
1 - mean(train.pred.rand.for.math == math.train$grade.cat)
1 - mean(test.pred.rand.for.math == math.test$grade.cat)
library(vip)
vip::vip(train.rand.for.math)
### por--------- por-------- choice of m
sqrt(37)
# cv on number of trees
test.error <- rep(1, 10)
set.seed(114514)
folds <- createFolds(1:nrow(por.train), k = 10, returnTrain = FALSE)
tree.vec <- seq(50, 1000, by = 50)
tree.error <- rep(1, length(tree.vec))
for (k in 1:length(tree.vec)) {
    for (i in 1:10) {
        set.seed(114514)
        train.rand.for.por <- randomForest(classification.formula,
            data = por.train[-folds[[i]], ], mtry = 6, ntree = k)
        train.pred.rand.for.por <- predict(train.rand.for.por,
            newdata = por.train[folds[[i]], ], type = "class")
        test.error[i] <- 1 - mean(train.pred.rand.for.por ==
            por.train[folds[[i]], ]$grade.cat)
    }
    tree.error[k] <- mean(test.error)
}
# the choice of number of trees
tree.vec[which.min(tree.error)]
best.rd.por <- tree.vec[which.min(tree.error)]
set.seed(114514)
train.rand.for.por <- randomForest(classification.formula, data = por.train,
    mtry = 6, ntree = best.rd.por)
train.rand.for.por
test.pred.rand.for.por <- predict(train.rand.for.por, por.test,
    type = "class")
train.pred.rand.for.por <- predict(train.rand.for.por, por.train,
    type = "class")
# test result
test.pred.rand.for.por
# misclassifictaion rate
1 - mean(train.pred.rand.for.por == por.train$grade.cat)
```

```r
1 - mean(test.pred.rand.for.por == por.test$grade.cat)
library(vip)
vip::vip(train.rand.for.por)
```

KNN Regression

```r
library(plotrix)
library(FNN)
library(e1071)
## Math------------------

ctrl <- trainControl(method = "cv", number = 10)

knn.model.math <- train(regression.formula, data = math.train,
    preProcess = c("center", "scale"), method = "knn", trControl = ctrl,
    tuneLength = 20)
plot(knn.model.math)

bestK.math <- knn.model.math$bestTune$k
bestK.math
Knn.for.math <- knn.reg(train = math.train[, -c(14, 15, 16, 43,
    44)], test = math.test[, -c(14, 15, 16, 43, 44)], y = math.train$grade.con,
    k = bestK.math)

# MSE
mean((math.test$grade.con - Knn.for.math$pred)^2)

Knn.for.math <- knn.reg(train = math.train[, -c(14, 15, 16, 43,
    44)], test = math.train[, -c(14, 15, 16, 43, 44)], y = math.train$grade.con,
    k = bestK.math)

# MSE
mean((math.train$grade.con - Knn.for.math$pred)^2)
## Por---------------- For Portuguese

knn.model.por <- train(regression.formula, data = por.train,
    preProcess = c("center", "scale"), method = "knn", trControl = ctrl,
    tuneLength = 20)
plot(knn.model.por)
bestK.por <- knn.model.por$bestTune$k
bestK.por

Knn.for.por <- knn.reg(train = por.train[, -c(14, 15, 16, 43,
    44)], test = por.test[, -c(14, 15, 16, 43, 44)], y = por.train$grade.con,
    k = bestK.por)

# MSE
mean((por.test$grade.con - Knn.for.por$pred)^2)

Knn.for.por <- knn.reg(train = por.train[, -c(14, 15, 16, 43,
    44)], test = por.train[, -c(14, 15, 16, 43, 44)], y = por.train$grade.con,
    k = bestK.por)

# MSE
mean((por.train$grade.con - Knn.for.por$pred)^2)
```

KNN Classification

```r
## math----------
library(class)
set.seed(114514)
ctrl <- trainControl(method = "cv", number = 10)

knn.model.math <- train(classification.formula, data = math.train,
    preProcess = c("center", "scale"), method = "knn", trControl = ctrl,
    tuneLength = 20)
plot(knn.model.math)
bestK.math <- knn.model.math$bestTune$k
bestK.math
math.train$grade.cat <- as.numeric(math.train$grade.cat)
math.test$grade.cat <- as.numeric(math.test$grade.cat)
Knn.for.math <- knn(train = math.train[, -c(14, 15, 16, 43, 44)],
    test = math.test[, -c(14, 15, 16, 43, 44)], cl = math.train$grade.cat,
    k = bestK.math)
# Misclassification Error
1 - mean(math.test$grade.cat == Knn.for.math)
Knn.for.math <- knn(train = math.train[, -c(14, 15, 16, 43, 44)],
    test = math.train[, -c(14, 15, 16, 43, 44)], cl = math.train$grade.cat,
    k = bestK.math)
# Misclassification Error
1 - mean(math.train$grade.cat == Knn.for.math)
## por----------
set.seed(114514)
ctrl <- trainControl(method = "cv", number = 10)

knn.model.por <- train(classification.formula, data = por.train,
    preProcess = c("center", "scale"), method = "knn", trControl = ctrl,
    tuneLength = 20)
plot(knn.model.por)
bestK.por <- knn.model.por$bestTune$k
bestK.por
por.train$grade.cat <- as.numeric(por.train$grade.cat)
por.test$grade.cat <- as.numeric(por.test$grade.cat)
Knn.for.por <- knn(train = por.train[, -c(14, 15, 16, 43, 44)],
    test = por.test[, -c(14, 15, 16, 43, 44)], cl = por.train$grade.cat,
    k = bestK.por)
# Misclassification Error
1 - mean(por.test$grade.cat == Knn.for.por)
Knn.for.por <- knn(train = por.train[, -c(14, 15, 16, 43, 44)],
    test = por.train[, -c(14, 15, 16, 43, 44)], cl = por.train$grade.cat,
    k = bestK.por)
# Misclassification Error
1 - mean(por.train$grade.cat == Knn.for.por)
```

Model Diagnostics

Check Multicollinearity

```r
library(faraway)
## math--------------------
math.ols <- lm(regression.formula, data = math)
# conduct multicollinearity detection
vif(math.ols)
math.mult <- math[, -c(27, 28)]
math.ols.mult <- lm(grade.con ~ . - G1 - G2 - G3 - grade.cat,
    data = math.mult)
summary(math.ols.mult)
# R^2=0.2595 por------------------
por.ols <- lm(regression.formula, data = por)
# conduct multicollinearity detection
vif(por.ols)
por.mult <- por
por.ols.mult <- lm(grade.con ~ . - G1 - G2 - G3 - grade.cat,
    data = por)
summary(por.ols.mult)
# R^2=0.3247
```

Identifying Outlier

```r
library(faraway)
## math--------------------
cook.math <- cooks.distance(math.ols.mult)
halfnorm(cook.math, 5, ylab = "Cook's Distance")
math.mult.out <- math.mult[-c(1, 158, 193, 199, 266), ]
math.ols.mult.out <- lm(grade.con ~ . - G1 - G2 - G3 - grade.cat,
    data = math.mult.out)
summary(math.ols.mult.out)
# R^2=0.2858 por-------------------
cook.por <- cooks.distance(por.ols.mult)
halfnorm(cook.por, 5, ylab = "Cook's Distance")
por.mult.out <- por.mult[-c(1, 604, 500, 524, 550), ]
por.ols.mult.out <- lm(grade.con ~ . - G1 - G2 - G3 - grade.cat,
    data = por.mult.out)
summary(por.ols.mult.out)
# R^2=0.358
```

Check Error Assumption

```r
library(MASS)
## math-------------
rn <- rnorm(300, 0, 1)
par(mfrow = c(1, 2))
qqnorm(rn, pch = 1, ylab = "normally generated data points",
    frame = FALSE)
```

```r
qqnorm(math.ols.mult.out$residuals, pch = 1, ylab = "residual of OLS",
    frame = FALSE)
math.mult.out.error <- math.mult.out
math.mult.out.error$grade.con <- math.mult.out.error$grade.con +
    4
math.ols.mult.out.error <- lm(grade.con ~ . - G1 - G2 - G3 -
    grade.cat, data = math.mult.out.error)
bc <- boxcox(math.ols.mult.out.error, plotit = T)
lambda <- bc$x[which.max(bc$y)]
lambda
math.mult.out.error.sq <- math.mult.out.error
math.mult.out.error.sq$grade.con <- (math.mult.out.error.sq$grade.con)^0.5
math.ols.trans <- lm(grade.con ~ . - grade.cat - G1 - G2 - G3,
    data = math.mult.out.error.sq)
summary(math.ols.trans)
# R^2=0.2954 por-------------
rn <- rnorm(300, 0, 1)
par(mfrow = c(1, 2))
qqnorm(rn, pch = 1, ylab = "normally generated data points",
    frame = FALSE)
qqnorm(por.ols.mult.out$residuals, pch = 1, ylab = "residual of OLS",
    frame = FALSE)
por.mult.out.error <- por.mult.out
por.mult.out.error$grade.con <- por.mult.out.error$grade.con +
    4
por.ols.mult.out.error <- lm(grade.con ~ . - G1 - G2 - G3 - grade.cat,
    data = por.mult.out.error)
bc <- boxcox(por.ols.mult.out.error, plotit = T)
lambda <- bc$x[which.max(bc$y)]
lambda
por.mult.out.error.sq <- por.mult.out.error
por.mult.out.error.sq$grade.con <- (por.mult.out.error.sq$grade.con)^0.5
por.ols.trans <- lm(grade.con ~ . - grade.cat - G1 - G2 - G3,
    data = por.mult.out.error.sq)
summary(por.ols.trans)
# R^2=0.3708
```

Nonlinearity Detection

```r
## math--------
library(car)
crPlots(math.ols.trans)
## por--------
library(car)
crPlots(por.ols.trans)
```

Inference About the Equivalence between two regressions

```r
beta1 <- as.vector(math.ols.trans$coefficients)
beta1.cov <- cov(beta1, beta1)
beta2 <- as.vector(por.ols.trans$coefficients)
beta2.cov <- cov(beta2, beta2)
Z <- (beta2.cov - beta1.cov)/sqrt(beta1.cov + beta2.cov)
```

```r
Z
1 - pnorm(Z)
# Two models are more or less equivalent
```

Variable Selection

```r
## math--------------
library(leaps)
math.reg <- regsubsets(grade.con ~ . - grade.cat - G1 - G2 -
    G3, data = math.mult.out.error.sq)
summary(math.reg)
# variables:
# failures+schoolsup_yes+Mjob_other+Fjob_teacher+famsup_yes+studytime+goout+freetime
math.ols.select <- lm(grade.con ~ failures + schoolsup_yes +
    Mjob_other + Fjob_teacher + famsup_yes + studytime + goout +
    freetime, data = math.mult.out.error.sq)
summary(math.ols.select)
## por--------------
por.reg <- regsubsets(grade.con ~ . - grade.cat - G1 - G2 - G3,
    data = por.mult.out.error.sq)
summary(por.reg)
# variables:
# failures+school_MS+higher_yes+studytime+Fjob_teacher+schoolsup_yes+absences+Dalc
por.ols.select <- lm(grade.con ~ failures + school_MS + higher_yes +
    studytime + Fjob_teacher + schoolsup_yes + absences + Dalc,
    data = por.mult.out.error.sq)
summary(por.ols.select)
```